

Logic and Computation I, Autumn 2022

Homework No.7

Due Date: December 12, 11:59 pm (Beijing)

Name:

Problem 1

Prove $\text{DSPACE}(n) \neq \text{P}$ and $\text{DSPACE}(n) \neq \text{NP}$.

Solution:

Problem 2

In the structure \mathbb{N} of natural numbers in the language $\mathcal{L}_{\text{OR}} = \{0, 1, +, \cdot, <\}$, express the following statements by a first-order formula.

- (1) There are infinitely many prime numbers.
- (2) Every even number greater than 2 can be written as the sum of two primes.

Solution:

Problem 3

- (1) In the structure $(\mathbb{R}, <, f)$ of real numbers, construct a formula expressing “the function $f(x)$ is continuous at $x = a$ ”.
(Note: Sum-product operations cannot be used).
- (2) In the structure $(\mathbb{R}, <, f)$, show that there is no formula that expresses “ $f(x)$ is differentiable with respect to $x = a$ ” (A. Padoa’s method).

Solution: