

Logic and Computation I, Autumn 2022

Homework No.3

Due Date: November 14, 11:59 pm (Beijing)

Name:

Problem 1

Let $\sigma(x, y)$ be a computable function. Prove that there exists a computable function k such that

$$\{k(y)\}^n(x_1, \dots, x_n) \sim \{\sigma(k(y), y)\}^n(x_1, \dots, x_n).$$

Hint: Consider a computable function $h(x)$ such that $\{\{x\}(x)\} \sim \{h(x)\}$ and then $\sigma(h(x), y)$ is expressed as $\{S(y)\}(x)$ by the parameter theorem.

Solution:

Problem 2 Show that if a CE set is infinite, it contains computable infinite subsets.

Solution:

Problem 3 Show the following two sets are noncomputable CE set.

$$K_0 = \{(x, e) : \{e\}(x) \downarrow\},$$

$$K_1 = \{e : \text{dom}(\{e\}) \neq \emptyset\}.$$

Solution: