Logic and Computation I, Autumn 2022

Homework No.2
Due Date: November 7, 11:59 pm (Beijing)
Name:

Problem 1
Prove $x \cdot y$, x^y, $x!$, $\max\{x, y\}$, $\min\{x, y\}$ are primitive recursive functions.

Solution:

Problem 2
Let $f(x_1, \ldots, x_n, y)$ be a primitive recursive function. Prove the following functions are also primitive recursive.

$$F(x_1, \ldots, x_n, z) = \Sigma_{y<z} f(x_1, \ldots, x_n, y),$$
$$G(x_1, \ldots, x_n, z) = \Pi_{y<z} f(x_1, \ldots, x_n, y).$$

Solution:

Problem 3 Prove that if $A(x_1, \ldots, x_n, y)$ is primitive recursive, $\forall y < z \; A(x_1, \ldots, x_n, y)$ and $\exists y < z \; A(x_1, \ldots, x_n, y)$ are also primitive recursive.

<u>Solution:</u>

Problem 4 The symbol set $\Omega = \{0, 1, +, (,)\}$. "Terms" are defined as below

(1) 0, 1 are terms.

(2) if s and t are terms, so is $(s + t)$. e.g., $((1 + 0) + 1)$ is a term, but $(1 + 0) + 1$ is not a term.

Show that the predicate $\mathrm{Term}(x)$ expressing "x is the Gödel number of a term" is primitive recursive.

<u>Solution:</u>