

Logic and Computation I, Autumn 2022

Homework No.10

Happy holidays

Name:

Problem 1

Prove $\mathbb{Q} \vdash 0 + 1 = 1$. (See Slide p.11 of Lecture03-06)

Solution:

Problem 2

In a Σ_1 complete theory T , show that 1-consistency of T is equivalent to the following: for any Σ_0 formula $\varphi(x)$, if $\varphi(\bar{n})$ is provable in T for all n , then $\exists x \neg \varphi(x)$ is not provable in T .

Solution:

Problem 3

Let A, B be two disjoint CE sets. Assume a theory T is Σ_1 -complete. Show that there exists a Σ_1 formula $\psi(x)$ such that

$$n \in A \Rightarrow T \vdash \psi(\bar{n}), \quad n \in B \Rightarrow T \vdash \neg\psi(\bar{n}).$$

From this, deduce that $\{\ulcorner \sigma \urcorner : T \vdash \sigma\}$ and $\{\ulcorner \sigma \urcorner : T \vdash \neg\sigma\}$ are computably inseparable. (See Lecture01-06, Slide p.25.) In particular, $\{\ulcorner \sigma \urcorner : T \vdash \sigma\}$ is not computable.

Solution: