Logic and Computation I, Autumn 2024

Exercise 01-05
Due Date:

Exercise 1.5.1 Let $\sigma(x, y)$ be a computable function. Prove the existence of computable function g such that

$$\{g(y)\}^n(x_1, \cdots, x_n) \sim \{\sigma(g(y), y)\}^n(x_1, \cdots, x_n).$$

Hint: Consider a computable function $h(x)$ such that $\{\{x\}(x)\} \sim \{h(x)\}$ and then $\sigma(h(x), y)$ is expressed as $\{S(y)\}(x)$ by the parameter theorem.

Solution:

Exercise 1.5.2 Show that the Ackermann function is not primitive recursive.
Hint: For any primitive recursive function $g(x, y)$ there exists a c such that

$$g(x, y) < f(c, \max\{x, y\}).$$

Solution:

Exercise 1.5.3 (Challenging)

Show that the graph of the Ackermann function is a primitive recursive set.

Solution:

Exercise 1.5.4 (Homework for everybody)

Show that any infinite CE set contains an infinite computable subset.

Solution: