Logic and Computation I, Autumn 2024

Exercise 01-03
Due Date:

Exercise 1.3.1
Let $f(x_1, \ldots, x_n, y)$ be a primitive recursive function. Prove the following functions are also primitive recursive.

$$F(x_1, \ldots, x_n, z) = \Sigma_{y<z} f(x_1, \ldots, x_n, y),$$
$$G(x_1, \ldots, x_n, z) = \Pi_{y<z} f(x_1, \ldots, x_n, y).$$

Solution:

Exercise 1.3.2
Let $f(x_1, \ldots, x_n, y)$ be a primitive recursive function. Prove the following functions are also primitive recursive.

$$F(x_1, \ldots, x_n, z) = \Sigma_{y<z} f(x_1, \ldots, x_n, y),$$
$$G(x_1, \ldots, x_n, z) = \Pi_{y<z} f(x_1, \ldots, x_n, y).$$

Solution:

Exercise 1.3.3
Prove that if $A(x_1, \ldots, x_n, y)$ is primitive recursive, $\forall y < z\, A(x_1, \ldots, x_n, y)$ and $\exists y < z\, A(x_1, \ldots, x_n, y)$ are also primitive recursive.

Solution:

Exercise 1.3.4

The symbol set Ω is the same as the example above. "Terms" are defined as below

(1) 0, 1 are terms.

(2) if s and t are terms, so is $(s+t)$. e.g., $((1+0)+1)$ is a term, but $(1+0)+1$ is not a term.

Show that the predicate $\mathrm{Term}(x)$ expressing "x is the Gödel number of a term" is primitive recursive.

<u>Solution:</u>