Logic and Computation II, Spring 2023

Bonus Homework No.1

Name:

Problem 1

By SωS, we denote the monadic second-order theory of $\mathcal{T}_\omega = (\mathbb{N}^*, \{S_i(x)\}_{i\in\mathbb{N}}, \subset, \preccurlyeq)$, where $S_i(w) = w\hat{\ }i$ ($i \in \mathbb{N}$), $\subset$ is the prefix and $\preccurlyeq$ is the lexicographic order.

Now let $f : \mathbb{N}^* \to \{0,1\}^*$ be

$$f(n_0 n_1 \ldots n_{k-1}) = 0^{n_0} 1 0^{n_1} 1 \cdots 1 0^{n_{k-1}} 1, \quad \text{and } f(\epsilon) = \epsilon.$$

Letting D be the range of f, we have $\mathcal{D} = (D, \{S_i^D(x)\}_{i\in\mathbb{N}}, \subset^D, \preccurlyeq^D) \cong \mathcal{T}_\omega$.

Then show that $\mathcal{D}$ is S2S-definable (Note: $\subset$ and $\preccurlyeq$ cannot be defined in $(\mathbb{N}^*, \{S_i(x)\}_{i\in\mathbb{N}})$). From this, derive that SωS is decidable.

<u>Solution:</u>