Logic and Foundations I, Autumn 2023

Homework No.9
Due: 2023.11.28
Name:

Problem 1

Use ultraproducts to show that any field $\mathcal{F}$ has algebraic closure $\overline{\mathcal{F}}$.

(Hint. Let $\mathcal{F}_P$ be the decomposition field of polynomial P, and for each $Q \in \mathcal{F}[X]$,

$$J_Q = \{P \in \mathcal{F}[X] : Q \text{ is decomposed into a product of linear expressions on } \mathcal{F}_P\}.$$

Then, let the ultrafilter containing $\{J_Q : Q \in \mathcal{F}[X]\}$ be $\mathcal{U}$, and consider the ultraproduct $\prod \mathcal{F}_P / \mathcal{U}$.)

Solution: