Logic and Foundations I, Autumn 2023

Homework No.6
Due: 2023.11.07
Name:

Problem 1 (difficult)

Let Σ be a theory in a language $\mathcal{L}$ including an n-ary relation symbol R and some others. Then, R is said to be **explicitly definable** in Σ, if there exists a formula $\varphi(x_0, \ldots, x_{n-1})$ in $\mathcal{L} - \{R\}$ such that

$$\Sigma \vdash \forall x_0, \ldots, x_{n-1}(R(x_0, \ldots, x_{n-1}) \leftrightarrow \varphi(x_0, \ldots, x_{n-1})).$$

Now, we construct Σ' from Σ by replacing all occurrences of R by a new symbol R'. Then, R is said to be **implicitly definable** in Σ, if the following hold

$$\Sigma \cup \Sigma' \vdash \forall x_0, \ldots, x_{n-1}(R(x_0, \ldots, x_{n-1}) \leftrightarrow R'(x_0, \ldots, x_{n-1})).$$

Show that R is explicitly definable in Σ iff R is implicitly definable in Σ.

Solution: