

Logic and Foundations I, Autumn 2023

Homework No.13

Due: Jan. 03, 2024

Name:

Problem 1

(1) Show that there is a consistent theory T that proves its own contradiction $\neg\text{Con}(T)$.

(2) Let $\text{Bew}_T^\#(x) \equiv (\text{Bew}_T(x) \wedge x \neq \overline{\overline{0 = 1}})$. For any true proposition σ ,

$$\text{Bew}_T^\#(\overline{\overline{\sigma}}) \leftrightarrow \text{Bew}_T(\overline{\overline{\sigma}})$$

and

$$T \vdash \neg\text{Bew}_T^\#(\overline{\overline{0 = 1}}).$$

Does it contradict with the second incompleteness theorem?

Solution: