Logic and Foundations I, Autumn 2023

Homework No.12
Due:
Name:

Problem 1
(1) T is called ω-consistent if for any formula $\varphi(x)$, if $T \vdash \varphi(\bar{n})$ for all $n \in \mathbb{N}$ then $T \nvdash \exists x \neg \varphi(x)$.

Show that a Σ_1-complete theory T is 1-consistent iff it is ω-consistent with respect to the Σ_0 formulas $\varphi(x)$.

(2) T is called Σ_n-consistent if any Σ_n theorem of T is true. Similarly for Π_n-consistency.

Show that if a Σ_1-complete theory T is ω-consistent, then it is Π_3-consistent, but not necessarily Σ_3-consistent.

Solution: