Threshold Resummation for Single Top Production in Soft Collinear Effective Theory

Hua Xing Zhu
In Collaboration with Prof.Chong Sheng Li, Jian Wang, Jia Jun Zhang

work in progress

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Outline

1. Short review of threshold resummation.
2. Factorization in Soft-Collinear effective field theory (SCET).
3. $\mathcal{O}(\alpha_s)$ corrections in SCET.
4. Renormalization group evolution and resummation in SCET.
5. Conclusion.

Short Review of Threshold Resummation

Resummation=organization of large logarithms in perturbative expansions.
For observables at hadronic colliders:

$$\begin{aligned}\langle \hat{O} \rangle &= 1 + \alpha_s(L^2 + L + 1) + \alpha_s^2(L^4 + L^3 + L^2 + L + 1) + \cdots \\ &= \exp\left(\underbrace{\underbrace{L g_1(\alpha_s L)}_{LL} + g_2(\alpha_s L)}_{NLL} + \alpha_s g_3(\alpha_s L) \right) \underbrace{C(\alpha_s)}_{\texttt{matching coefficient}} \\ &\quad \texttt{+suppressed terms}\end{aligned}$$

We benefits from resummation:

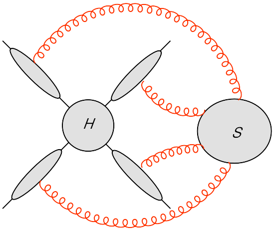
- better theoretical accuracy

Resummation of threshold double logs $L^2 \sim \log^2\left(1 - \frac{Q^2}{s}\right)$:

- G. Sterman, 1987
- Catani, L. Trentadue, 1989,1991

Basic Idea: The general structure of any such observable falls into a form that can be represented schematically as

$$\frac{d\sigma_{a+b\to jets\{i\}}(Q)}{dQ} = f_{a'/a} \otimes f_{b'/b} \otimes H_{IK}^{a'+b'\to\{d_i\}} \otimes S_{IK}^{a'+b'\to\{d_i\}} \otimes \prod_{jets\{i\}} J_{d_i}.$$



- Factorize naturally in axial gauge

Resummation in SCET–Motivation

Drawbacks in conventional formalism:

1. Landau pole ambiguity–unavoidable; not necessarily imply a renormalon ambiguity!
2. The separation of the hard, jet and soft scales is not transparent.

Advantages of using SCET to do resummation:

1. Landau pole ambiguity can be avoided in SCET formalism.
2. SCET gives resummation a better field theorectical interpretation.
3. SCET formalism exhibits a better numerical perturbative convengence.

Resummation in SCET

- Effective field treatment possible since invention of SCET. Bauer, Fleming, Pirjol, Rothstein, Stewart ('02)
- P_T resummation in SCET: Yang Gao, Chong Sheng Li, Jian Jun Liu ('05); Idilbi, Xiang-Dong Ji ('05)
- Threshold resummation in SCET: Idilbi, Xiang-Dong Ji, Jian-Ping Ma, Feng Yuan ('05); Li Lin Yang, Chong Sheng Li, Yang Gao, Jian Jun Liu ('05)
- Resummation in momentum space: Becher, Neubert, Pecjak, G. Xu ('06,'07)
- Resum large π^2 terms from analytic continuation: Ahrens, Becher, Neubert, Li Lin Yang ('08)

Basic of SCET

- SCET is an effective theory containing only soft and collinear D.O.F.
- SCET is reproducing the long distance behavior of QCD, short distance physics can be encoded in Wilson coefficients.
- the momentum scaling of field in SCET:

$$\begin{array}{ll} \text{collinear} & (n\cdot p, \bar{n}\cdot p, p_\perp) \sim (\lambda^2, 1, \lambda) \\ \text{soft} & (n\cdot p, \bar{n}\cdot p, p_\perp) \sim (\lambda^2, \lambda^2, \lambda^2) \end{array}$$

- No interaction between collinear field of different direction
- soft interaction decoupled into soft Wilson line

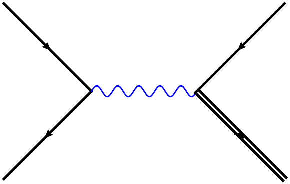
Single Top production

In this talk, I will talk about the resummation of large logs in single top production via SCET. To be more specific, I will concentrate on s-channel case for the purpose of illustration.

Single top production is important:

- Already observed at Tevatron
- Unique direct probe to V_{tb}
- sensitive to new physics: W', FCNC or modified top quark interaction
- NLO QCD corrections: M. C. Smith, Willenbrock; S. H. Zhu; Harris, Laenen, Phaf, Sullivan, Weinzierl; Campbell, R. K. Ellis, Tramontano; Q.-H. Cao, C.-P. Yuan
- NNNLO soft and Virtual corrections: Kidonakis ('07)

Single Top production–con'd



- Define $s_4 = s + t + u - m_t^2$, s, t, u are Mandelstam variables
- At absolute threshold, $s_4 = 0$, s_4 measure the distance from threshold

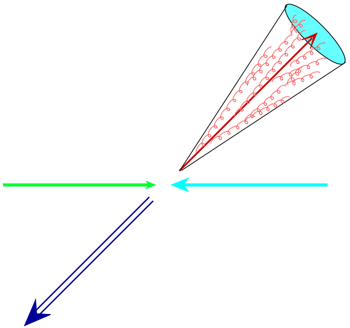
- The total Xsec is a convolution of s_4

$$\sigma \sim \int_0^{s_4^{\max}} ds_4 \frac{d\sigma}{dtdu}$$

the partonic differential Xsec $\frac{d\sigma}{dtdu}$ contains threshold logarithms of the form $[\log^i \frac{s_4}{m_t^2}/s_4]_+$, where the conventional plus distribution is defined as

$$\int_0^1 f(x)g(x)_+ \equiv \int_0^1 (f(x) - f(1))g(x)$$

Single Top production–con'd

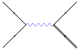
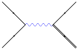


- Initial states: n_1-collinear fields $\xi_{n_1}, \xi_{\bar{n}_1}$, final state b-quark: n_2-collinear field ξ_{n_2}
- top quark: heavy quark field h_v as in HQET. (Li Lin Yang, Chong Sheng Li, Yang Gao, Jian Jun Liu '05)

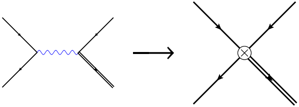
There are four typical scales in this process:

1. hard scale: $\mu_h \sim \sqrt{s}$
2. jet scale: typically the invariant mass of the jet $\mu_j^2 \sim p_{\text{jet}}^2$
3. soft scale: energy of the soft gluon μ_s
4. factorization scale: μ_F

In the first step, we write down the Xsec in operator formalism

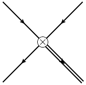
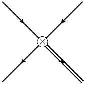
$$\frac{d^2\sigma}{dtdu} = \sum_X \langle N| \Big($$  $$\Big) |X\rangle \delta(s_4 - \hat{s}_4) \langle X| \Big($$  $$\Big)^\dagger |N\rangle$$

Then we match the operator in the full theory onto SCET



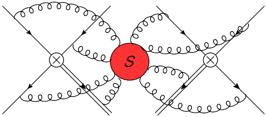
$$O_{\rm full} = \overline{\psi}_u \Gamma_\mu \psi_d \overline{\psi}_t \Gamma^\mu \psi_b \longrightarrow O_{\rm SCET} = \overline{\chi}_{\bar{n}_1,\omega_1} \Gamma_\mu \chi_{n_1,\omega_1} \bar{h}_v \Gamma^\mu \chi_{n_2,\omega_2}$$

At this stage, the underlying hard interaction is encoded into a Wilson coefficient: hard function.

$$\frac{d^2\sigma}{dtdu} = \mathcal{H} \otimes \sum_X \langle N| \Big($$  $$\Big)\Big($$  $$\Big)^\dagger |N\rangle$$

Next we deal with the soft gluon interaction. In general soft gluon can attach on all colored legs

$$\frac{d^2\sigma}{dtdu} = \mathcal{H} \otimes \langle N|$$



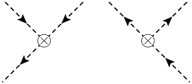
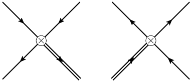
$$|N\rangle$$

The advantage of SCET is that one can decouple the soft interaction by simple field redefinition

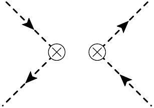
$$\chi_n = Y_n \chi_n^{(0)} \qquad A_n = Y_n A_n^{(0)} Y_n^\dagger$$

where Y_n is a soft Wilson line describes eikonal interaction between collinear field with soft field

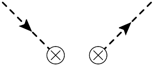
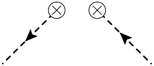
$$Y_n = \mathrm{P}\exp\left[i g_s \int_0^\infty ds\, n\cdot A_n(ns) \right]$$

$$\frac{d^2\sigma}{dtdu} = \mathcal{H} \otimes \langle N|$$

$$|N\rangle \otimes \underbrace{\langle 0|\ \cdots\ |0\rangle}_{\mathcal{S}}$$


Because collinear fields of different direction decouple in SCET, one can factorizes and integrate out the final state quark jet, which usually has a large jet invariant mass

$$\frac{d^2\sigma}{dtdu} = \mathcal{H} \otimes \mathcal{S} \otimes \langle N|$$

$$|N\rangle \otimes \underbrace{\sum \langle 0|\ \cdots\ |0\rangle}_{\mathcal{J}}$$


Finally we match the initial state matrix onto PDFs

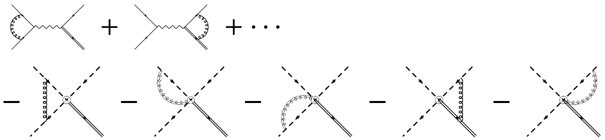
$$\frac{d^2\sigma}{dtdu} = \mathcal{H} \otimes \mathcal{S} \otimes \mathcal{J} \otimes \underbrace{\langle N_a|\ \cdots\ |N_a\rangle}_{f_a} \otimes \underbrace{\langle N_b|\ \cdots\ |N_b\rangle}_{f_b}$$



$$= \mathcal{H} \otimes \mathcal{S} \otimes \mathcal{J} \otimes f_a \otimes f_b$$

$\mathcal{O}(\alpha_s)$ Corrections in SCET

Now we derive the $\mathcal{O}(\alpha_s)$ corrections of the matching coefficients. In on-shell dimensional regularization, virtual corrections in SCET vanishes since no explicit mass scale appears in the loop integrals. This greatly simplify the matching calculation.

$$\frac{1}{2}\mathcal{H} =$$



+counterterms

From the counter terms one can derive the anomalous dimension

$$\gamma_H = \frac{\alpha_s}{2\pi} \times \left[\Gamma^{(0)}_{\text{cusp}} \log \frac{s}{\mu^2} - 6C_F + \Gamma^{(0)}_{\text{cusp}} \log \frac{s - m_t^2}{\mu m_t} - 5C_F \right] \quad (1)$$

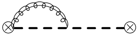
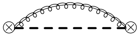
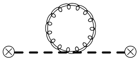
This result is in agreement with the general results of Becher & Neubert '09.

Next we come to the jet function. The SCET jet function is just the imaginary part of the propagator of a collinear quark. It's a universal quantity in SCET. (Manohar '03; Bauer, Manohar '04) It is defined by

$$\mathcal{J}(p^2,\mu) = \frac{1}{\pi(\bar{n}\cdot p)} \mathrm{Im}\left[i \int d^4x\, e^{-ipx} \langle 0 | \mathrm{T}\left\{ \bar{\chi}_n(x) \frac{\bar{n}\!\!\!/}{2} \chi_n(0) \right\} |0\rangle \right]$$

At NLO, we have

$$\mathcal{J} \sim \mathrm{Disc}\Big\{$$

 $+$  $+$ 

$$+ \cdots \Big\}$$

$$\sim \frac{\alpha_s}{\pi} C_F \times \left(-\frac{3}{4} \frac{1}{[s_4]_+} + \left[\frac{\log \frac{s_4}{m_t^2}}{s_4} \right]_+ + \texttt{non leading terms} \right)$$

Its anomalous dimension is

$$\gamma_J = \frac{\alpha_s}{2\pi} \times \left(3C_F - \frac{4C_F}{[s_4/m_t^2]_+} \right)$$

The soft function $\mathcal{S}$ is

$$\mathcal{S} = \frac{1}{N_c^2} \sum_{X_s} \langle 0|O_s|X_s\rangle\langle X_s|O_s^\dagger|0\rangle$$

with $O_s = Y_{n_1}^\dagger Y_{\bar{n}_1} Y_v^\dagger Y_{n_2}$. In principle, the soft function is non-perturbative, because the soft scale can be smaller than Λ_{QCD}. However, it will be calculable if we restrict $\mu_s > \Lambda_{\text{QCD}}$. Even if $\mathcal{S}$ is non-perturbative, the anomalous dimension can still dictates its evolution.

At NLO it is given by

$$\mathcal{S} \sim \frac{\alpha_s}{\pi} C_F \left(4 \left[\frac{\log \frac{s_4}{m_t^2}}{s_4} \right]_+ - 2 \left[\frac{\log \frac{s_4}{m_t^2}}{s_4} \right]_+ - \frac{1}{[s_4]_+} + 2 \frac{\log \frac{s}{\mu^2}}{[s_4]}_+ + 2 \frac{\log \frac{s-m_t^2}{\mu m_t}}{[s_4]}_+ \right)$$

and its anomalous dimension

$$\gamma_S = \frac{\alpha_s}{2\pi} \left[2C_F + \frac{4C_F}{[s_4/m_t^2]_+} + \Gamma_{\text{cusp}}^{(0)} \left(\log \frac{\mu m_t}{s - m_t^2} + \log \frac{\mu^2}{s} \right) \right]$$

Finally we are left with the PDFs. Although it can not be calculated by perturbative theory, its anomalous dimension is well and is given by the well-known Altarelli-Parisi kernel $\gamma_{\rm AP}$.

Combining all these corrections and neglecting the suppressed terms, we have

$$\left.\frac{d\sigma}{dtdu}^{(1)}\right|_{\rm leading} = \frac{\alpha_s}{\pi} C_F \left(3\left[\frac{\log\frac{s_4}{m_t^2}}{s_4}\right]_+ - \frac{7}{4}\frac{1}{[s_4]_+} + \frac{\log\frac{s}{\mu^2}}{[s_4]}_+ + \frac{\log\frac{s-m_t^2}{\mu m_t}}{[s_4]}_+ \right)$$

This is precisely the leading singular terms obtained by Kidonakis '07, up to some kinemetical factors.

Also we found that

$$2\gamma_{\rm AP} + \gamma_H + \gamma_J + \gamma_S = 0$$

This relation is expected because the cross section is a physical observable and should be indenpendent of the arbitrary 't Hooft scale μ.

Renormalization Group Evolution in SCET

With the anomalous dimensions of hard, jet and soft functions at hand, large logarithms can be resummed by renormalization group evolution. This is similar to, `e.g.`, B-decay, where logs of the form $\alpha_s(M_W) \log \frac{M_W}{\mu}$ are resummed via RG equation.

$\mathcal{H}(\mu_h)$	$U(\mu_h, \mu_j)$	$\mathcal{J}(\mu_j)$	$U(\mu_j, \mu_s)$	$\mathcal{S}(\mu_s)$	$U(\mu_s, \mu_F)$	$PDFs(\mu_F)$
match	$\rightarrow$ run	match	$\rightarrow$ run	match	$\rightarrow$ run	match

- U is the evolution kernel
- can have analytic solution by the method of Laplace transformation. Becher, Neubert ('06)

The final results take the form

$$\frac{d\sigma}{dtdu} = \mathcal{H}(\mu_h, \mu_F) \otimes \mathcal{J}(\mu_j, \mu_F) \otimes \mathcal{S}(\mu_s, \mu_F) \otimes f_a(\mu_F) \otimes f_b(\mu_F),$$

Where each function has been evolved from its intrinsic scale to the same scale μ_F. For example, the jet function now is (Becher, Neubert '06)

$$\mathcal{J}(s_4, \mu_j, \mu_F) = \exp\left[-4S(\mu_j, \mu_F) + 2a_{\gamma_J}(\mu_j, \mu_F)\right]\tilde{j}(\partial_\eta, \mu_j)\left[\frac{1}{s_4}\left(\frac{s_4}{\mu_j}\right)^\eta\right]_* \frac{e^{-\gamma_E \eta}}{\Gamma(\eta)},$$

and similarly for other functions. It's remarkable that we have fully analytical control over the solution of RG equation, in contrast to the tranditonal formalism in Mellin moment space:

- The scale seperation become transparent now.
- We can explicitly adjust the soft scale now, in contrast to the Mellin moment space formalism, where μ_s is implicitly chosen as Q/N.
- Also Landau pole ambiguity is avoided now, if we restrict $\mu_s > \Lambda_{\rm QCD}$.

- general two loop soft anomalous dimensions are known. Ferroglia, Neubert, Pecjak, Li Lin Yang ('09)
- two loop jet anomalous dimension is known. Becher, Neubert ('06)
- We also have three loop DGLAP kernels. Moch, Vermaseren, Vogt ('04)

Can perform resummation to Next-to-Next-to-Leading-Logarithms.

Conclusion so far

- We have derived the factorized expression for single top production in SCET, suitable for threshold resummation.
- The cross section factorized into a hard function, a jet function, a soft function and the PDFs, each satisfies a RG equation.
- We solved the evolution kernel analytically, which leads to resummation in momentum space.
- Numerical results is in preparation.