Testing anomalous couplings at High Energy Colliders

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OUTLINE

- Testing anomalous Higgs couplings via vector boson scattering at LHC
- Testing anomalous gauge couplings of the Higgs boson at high energy photon colliders
- Testing fermion anomalous coupling at future ILC

Testing anomalous Higgs couplings via vector boson scattering on LHC

EWSBM is not known yet. In EW theory, all particle masses come from the VEV . Probing EWSBM concerns the understanding of the original of all particle masses.

• SM Higgs sector suffers from triviality and unnaturalness.
It is currently accepted that there should be new physics beyond the SM above certain high energy scale **Λ**.

• New physics:

SUSY **TC** **top seesaw** **extra dim.** **little Higgs$\cdots$**

$\searrow$ $\downarrow$ $\swarrow$ $\swarrow$

nature=?

no-lose probe of *EWSBM* and *new physics* is expected!

No-lose Probe of New Physics

Effective theory is a way of No-lose Probe of New Physics :

Effective Lagrangian provide a model-independent description and the anomalous couplings reflect the effect of the new physics.

Finding sensitive processes to measure the coefficients in general EL is needed to obtain effective Lagrangian reflecting the nature.

Testing effective anomalous gauge couplings of Higgs boson can discriminate the EWSB sector of the new physics model from SM

ANOMALOUS HVV COUPLINGS

We take linear realization as example.

SM: only dim-4 ΦVV couplings,

New physics: can contain extra dim-6 ΦVV couplings.

M. C. Gonzalea-Garcia [Int. J. Mod. Phys. A**14**, 3121 (1999)]:

$$\mathcal{L}_{\text{eff}}^{(6)} = \sum_n \frac{\mathrm{f_n}}{\Lambda^2} \mathcal{O}_n(\Phi, W^a, B)$$

$$\mathcal{O}_{\Phi,1} = (D_\mu \Phi)^\dagger \Phi^\dagger \Phi (D^\mu \Phi), \qquad \mathcal{O}_{BW} = \Phi^\dagger \hat{B}_{\mu\nu} \hat{W}^{\mu\nu} \Phi,$$

$$\mathcal{O}_{DW} = \mathrm{Tr}\left(\left[D_\mu, \hat{W}_{\nu\rho}\right]\left[D^\mu, \hat{W}^{\nu\rho}\right]\right), \qquad \mathcal{O}_{DB} = -\frac{g'^2}{2}(\partial_\mu B_{\nu\rho})(\partial^\mu B^{\nu\rho}),$$

$$\mathcal{O}_{\Phi,2} = \frac{1}{2}\partial^\mu\left(\Phi^\dagger\Phi\right)\partial_\mu\left(\Phi^\dagger\Phi\right), \qquad \mathcal{O}_{\Phi,3} = \frac{1}{3}\left(\Phi^\dagger\Phi\right)^3,$$

$$\mathcal{O}_{WWW} = \mathrm{Tr}[\hat{W}_{\mu\nu}\hat{W}^{\nu\rho}\hat{W}_\rho^\mu], \qquad \mathcal{O}_{WW} = \Phi^\dagger \hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\Phi,$$

$$\mathcal{O}_{BB} = \Phi^\dagger \hat{B}_{\mu\nu}\hat{B}^{\mu\nu}\Phi, \qquad \mathcal{O}_W = (D_\mu\Phi)^\dagger \hat{W}^{\mu\nu}(D_\nu\Phi),$$

$$\mathcal{O}_B = (D_\mu\Phi)^\dagger \hat{B}^{\mu\nu}(D_\nu\Phi), \qquad \hat{B}_{\mu\nu} = i\frac{g'}{2}B_{\mu\nu}, \quad \hat{W}_{\mu\nu} = i\frac{g}{2}\sigma^a W_{\mu\nu}^a.$$

Anomalous HVV Couplings

relations between anomalous Higgs coupling constants gn's and linear realized effective coupling constants fn's

$$
\begin{aligned}
\mathcal{L}_{\text{eff}}^{HVV} = \;& \mathbf{g}_{\mathbf{H}\gamma\gamma} H A_{\mu\nu} A^{\mu\nu} + \mathbf{g}^{(1)}_{\mathbf{HZ}\gamma} A_{\mu\nu} Z^{\mu} \partial^{\nu} H + \mathbf{g}^{(2)}_{\mathbf{HZ}\gamma} H A_{\mu\nu} Z^{\mu\nu} \\
& + \mathbf{g}^{(1)}_{\mathbf{HZZ}} Z_{\mu\nu} Z^{\mu} \partial^{\nu} H + \mathbf{g}^{(2)}_{\mathbf{HZZ}} H Z_{\mu\nu} Z^{\mu\nu} \\
& + \mathbf{g}^{(1)}_{\mathbf{HWW}} (W^{+}_{\mu\nu} W^{\mu}_{-} \partial^{\nu} H + h.c.) + \mathbf{g}^{(2)}_{\mathbf{HWW}} H W^{+}_{\mu\nu} W^{\mu\nu}_{-},
\end{aligned}
$$

$$
\mathbf{g}_{\mathbf{H}\gamma\gamma} = -\left(\frac{gM_W}{\Lambda^2}\right)\frac{s^2(\mathbf{f}_{\mathbf{BB}} + \mathbf{f}_{\mathbf{WW}})}{2}, \quad \mathbf{g}^{(1)}_{\mathbf{HZ}\gamma} = \left(\frac{gM_W}{\Lambda^2}\right)\frac{s(\mathbf{f}_{\mathbf{W}} - \mathbf{f}_{\mathbf{B}})}{2c},
$$

$$
\mathbf{g}^{(2)}_{\mathbf{HZ}\gamma} = \left(\frac{gM_W}{\Lambda^2}\right)\frac{s[s^2\mathbf{f}_{\mathbf{BB}} - c^2\mathbf{f}_{\mathbf{WW}}]}{c}, \quad \mathbf{g}^{(1)}_{\mathbf{HZZ}} = \left(\frac{gM_W}{\Lambda^2}\right)\frac{c^2\mathbf{f}_{\mathbf{W}} + s^2\mathbf{f}_{\mathbf{B}}}{2c^2},
$$

$$
\mathbf{g}^{(2)}_{\mathbf{HZZ}} = -\left(\frac{gM_W}{\Lambda^2}\right)\frac{s^4\mathbf{f}_{\mathbf{BB}} + c^4\mathbf{f}_{\mathbf{WW}}}{2c^2}, \quad \mathbf{g}^{(1)}_{\mathbf{HWW}} = \left(\frac{gM_W}{\Lambda^2}\right)\frac{\mathbf{f}_{\mathbf{W}}}{2},
$$

$$
\mathbf{g}^{(2)}_{\mathbf{HWW}} = -\left(\frac{gM_W}{\Lambda^2}\right)\mathbf{f}_{\mathbf{WW}}, \qquad s \equiv \sin\theta_W, \; c \equiv \cos\theta_W.
$$

Testing HVV Anomalous Couplings

Now, a light Higgs candidate is found, is it a Higgs in SM or new physics?

−testing the HVV couplings ($V = W,\ Z,\ \gamma$).

$$\mathcal{L}_{eff}(H,V) = \sum_{V=W,Z,\gamma} \sum_{i=1}^{2} \mathbf{g}^{(\mathbf{i})}_{\mathbf{HVV}}\ \mathcal{O}^{(i)}_{HVV}(H,V,\varphi).$$

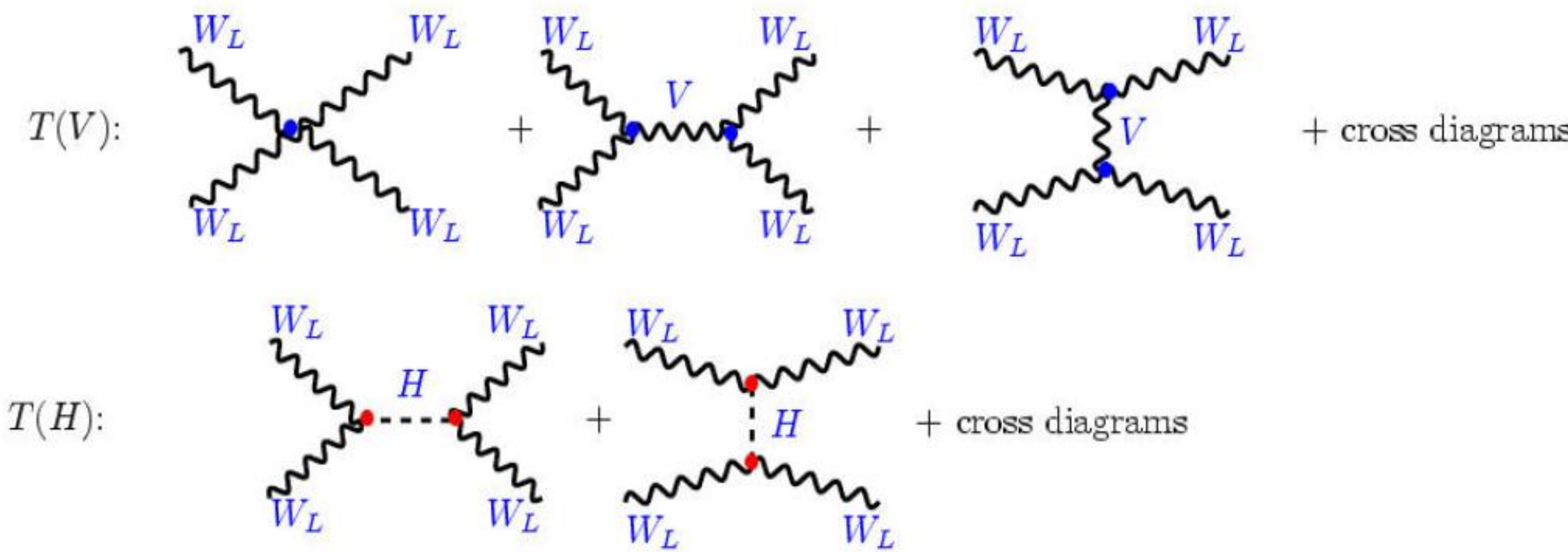
$\mathbf{W^+W^+}$ scattering is the most sensitive test of $\mathbf{g}^{(\mathbf{i})}_{\mathbf{HVV}}$ at the LHC.

- Phys. Lett. **B554** (2003) 64;
- Phys. Rev. D **67** (2003) 114024;
- CERN Yellow Report "The Higgs Working Group: Summary Report", p.44 hep-ph/0406152.

Testing HVV via WW Scatterings

Why sensitive?

Scattering amplitude contains two parts: (a) T(V), (b) T(H).



Why sensitive?

Consider $$W_L W_L \to W_L W_L.$$

- In SM, HVV coupling constant is g. At high energies,

$$T(V) \sim E^2, \qquad T(H) \sim E^2$$

$$\Downarrow \ \text{standard} \quad HVV$$

$$T(V) + T(H) \sim E^0$$

guaranteeing unitarity of the S-matrix.

- With anomalous $g^{(i)}_{HVV}$ due to new physics, $E < \Lambda$,

$$T(V) \sim E^2, \qquad T(H) \sim E^2$$

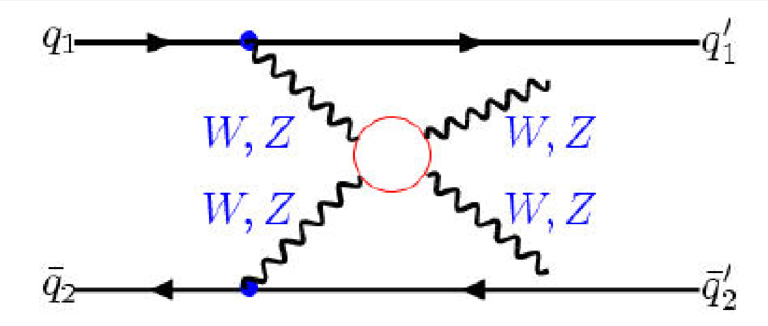
$$\Downarrow \ \text{anomalous} \quad HVV$$

$$T(V) + T(H) \sim E^2$$

Cross section is sensitive to anomalous $g^{(i)}_{HVV}$.

WW Scatterings on LHC

- **Singal**: $W, Z \to leptons$ (gold-plated mode)



- **Backgrounds** [J. Bagger *et al.*, PRD **49** (1994) 1246]:

  (a), QCD bkgd:

  (b), top quark bkgd:

  (c), EW bkgd:

Numerical results

We calcuated the full tree level cross sections of

$$pp \to Z_L Z_L jj \to l^+ l^- l^+ l^- jj, l^+ l^- \nu\bar{\nu} jj,$$
$$pp \to W_L^+ W_L^- jj \to l^+ \nu l^- \bar{\nu} jj,$$
$$pp \to W_L^+ W_L^+ jj \to l^+ \nu l^+ \nu jj,$$
$$pp \to W_L^- W_L^- jj \to l^- \bar{\nu} l^- \bar{\nu} jj,$$
$$pp \to Z_L W_L^+ jj \to l^+ l^- l^+ \nu jj,$$
$$pp \to Z_L W_L^- jj \to l^+ l^- l^- \bar{\nu} jj.$$

Our result shows: $\mathbf{W^+W^+ \to W^+W^+}$ is the most sensitive channel.

Numerical results

Obtained 1σ detectable limit (sensitivity):

$$f_W/\Lambda^2 > 0.85\ \mathrm{TeV}^{-2}, \qquad f_W/\Lambda^2 < -1.0\ \mathrm{TeV}^{-2},$$
$$\text{or} \quad f_{WW}/\Lambda^2 > 1.6\ \mathrm{TeV}^{-2} \qquad f_{WW}/\Lambda^2 < -1.6\ \mathrm{TeV}^{-2}.$$

Correspondingly (in units of TeV^{-1}),

$$1\sigma: \quad |g^{(1)}_{HWW}| > 0.026, \qquad |g^{(1)}_{HZZ}| > 0.026,$$
$$|g^{(2)}_{HWW}| > 0.083, \qquad |g^{(2)}_{HZZ}| > 0.032, \qquad |g^{(2)}_{HZ\gamma}| > 0.018,$$

More sensitive than the existing LHC 1σ limit:

$$1\sigma: \quad |g^{(2)}_{HWW}| \geq 0.1\ \mathrm{TeV}^{-1} \quad (pp \to HX,\ H \to \gamma\gamma,\ \tau^+\tau^-).$$

Numerical results

Obtained 2σ detectable limit (sensitivity):

$$f_W/\Lambda^2 > 1.2\ \mathrm{TeV}^{-2}, \qquad f_W/\Lambda^2 < -1.4\ \mathrm{TeV}^{-2},$$

$$\text{or} \quad f_{WW}/\Lambda^2 > 2.2\ \mathrm{TeV}^{-2} \qquad f_{WW}/\Lambda^2 < -2.2\ \mathrm{TeV}^{-2}.$$

Correspondingly (in units of TeV^{-1}),

$$2\sigma: \ |g^{(1)}_{HWW}| > 0.036, \qquad |g^{(1)}_{HZZ}| > 0.036,$$

$$|g^{(2)}_{HWW}| > 0.11, \qquad |g^{(2)}_{HZZ}| > 0.044, \qquad |g^{(2)}_{HZ\gamma}| > 0.024,$$

Close to the sensitivity of the existing LC 2σ limit:

$|g^{(i)}_{HZZ}|,\ |g^{(i)}_{HZ\gamma}| \geq 10^{-3} - 10^{-2}\ \mathrm{TeV}^{-1}$.

Semi-leptonic mode in WW scatterings

the most sensitive test at the LHC is via the pure leptonic mode in W+W+ scattering, However, the required integrated luminosity 300 fb^{-1}.

$$pp \rightarrow W^+W^+j_1^fj_2^f \rightarrow l^+\nu_l l^+\nu_l j_1^f j_2^f$$

we study the possibility of taking the Semi-leptonic mode which can have a larger cross section.

$$pp \rightarrow W^+W^\pm j_1^fj_2^f \rightarrow l^+\nu_l j_1 j_2 j_1^f j_2^f$$

Semileptonic Mode

Semileptonic mode which can have a larger cross section. Since it is not possible to distinguish W+ → jj and W− → jj experimentally, we have to study the scatterings

$$pp \;\rightarrow\; W^+W^\pm j_1^f j_2^f \;\rightarrow\; l^+\nu_l j_1 j_2 j_1^f j_2^f$$

Now the final state contains four jets, so that the parton level study is not sufficient for finding out the suitable kinematic cuts to suppress the large backgrounds.

We worked at the hadron level, calculating the full tree level contributions to the signal and backgrounds using the helicity amplitude methods and the package PYTHIA with its default fragmentation model.

Semileptonic Mode

The required integrated luminosity for reaching the 3σ sensitivity can be reduced to $100\ fb^{-1}$, reduced by a factor of 3.

If the anomalous couplings in nature are actually not small, about 1σ, the experiment can start the test for an integrated luminosity of $50\ fb^{-1}$.

Testing anomalous gauge couplings of the Higgs boson at high energy photon colliders

At the LHC, the most sensitive constraints on f_W/Λ^2 and f_{WW}/Λ^2 will be from the measurement of the gauge-boson scattering $W^+W^+ \to W^+W^+$.

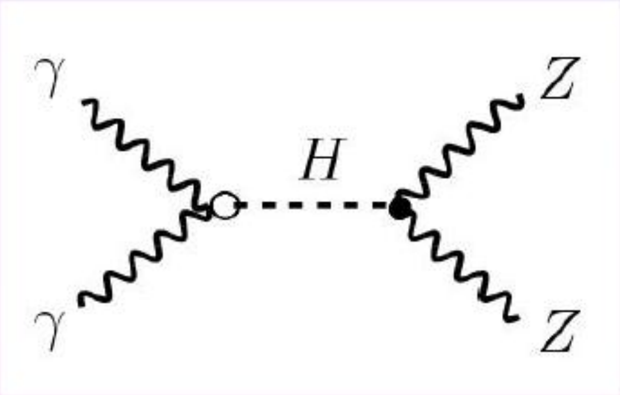
Those processes are insensitive to f_B/Λ^2 and f_{BB}/Λ^2

At e^+e^- linear colliders on the other hand, the anomalous couplings $g^{(1)}_{HZZ}$ and $g^{(2)}_{HZZ}$ can be constrained at the 2σ sensitivity to (10^{-3} –10^{-2}) TeV^{-1} from the Higgs-strahlung process $e^+e^- \to Z^* \to ZH$

At photon colliders, the sensitivities to probe those couplings can be improved.

$\gamma\gamma \to ZZ$

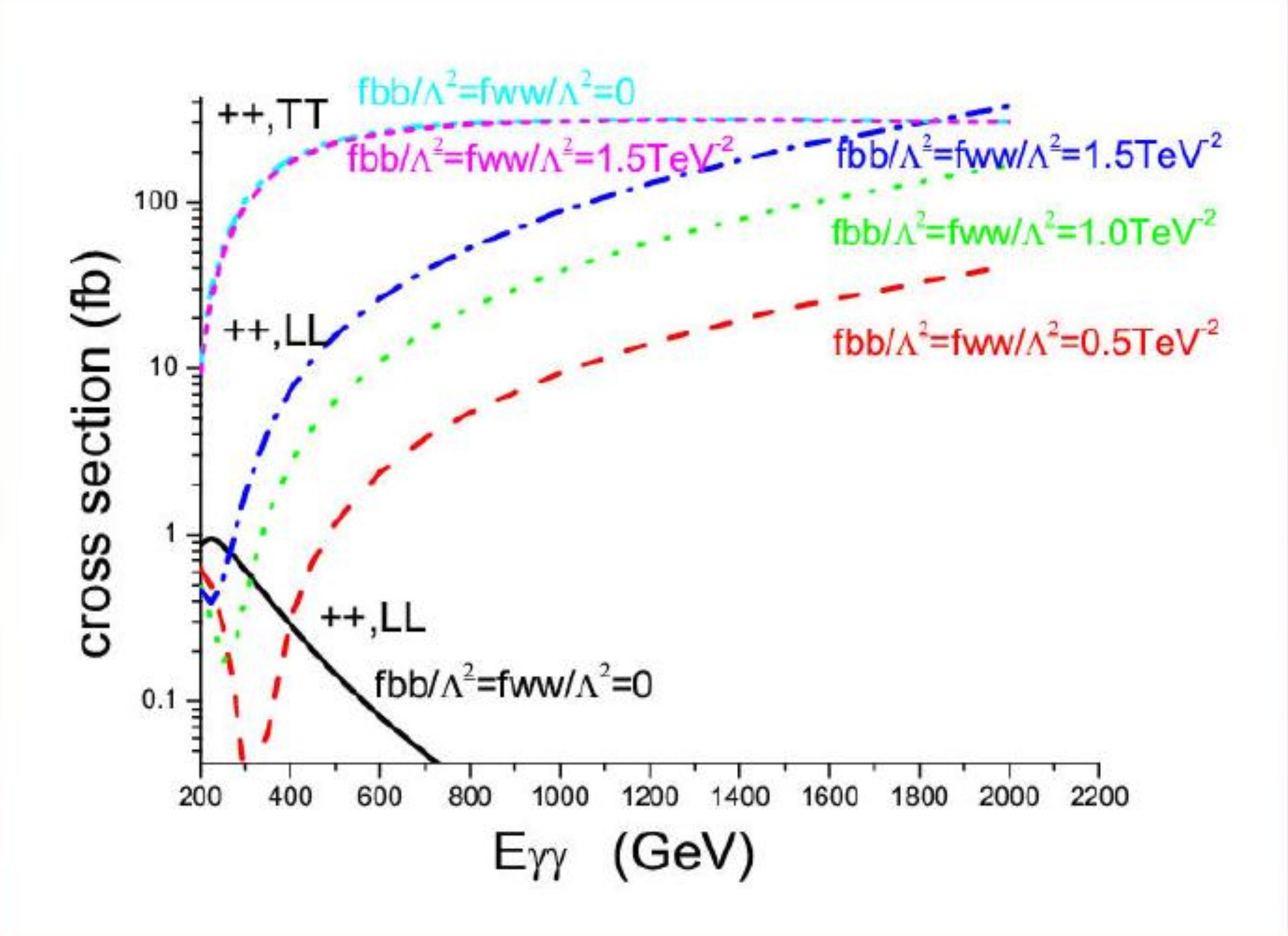
With the anomalous $H\gamma\gamma$ ($g_{H\gamma\gamma} \propto f_{BB} + f_{WW}$) coupling, the signal process $\gamma\gamma \to ZZ$ can have a tree level contribution



vertex $\circ$ contains only the anomalous $H\gamma\gamma$ vertex $\bullet$ contains the SM and anomalous HZZ interactions .

the contribution of the anomalous HZZ interactions in $\bullet$ to the cross section is only a few percent, thus the process $\gamma\gamma \to ZZ$ mainly tests the anomalous couplings $g_{H\gamma\gamma}$

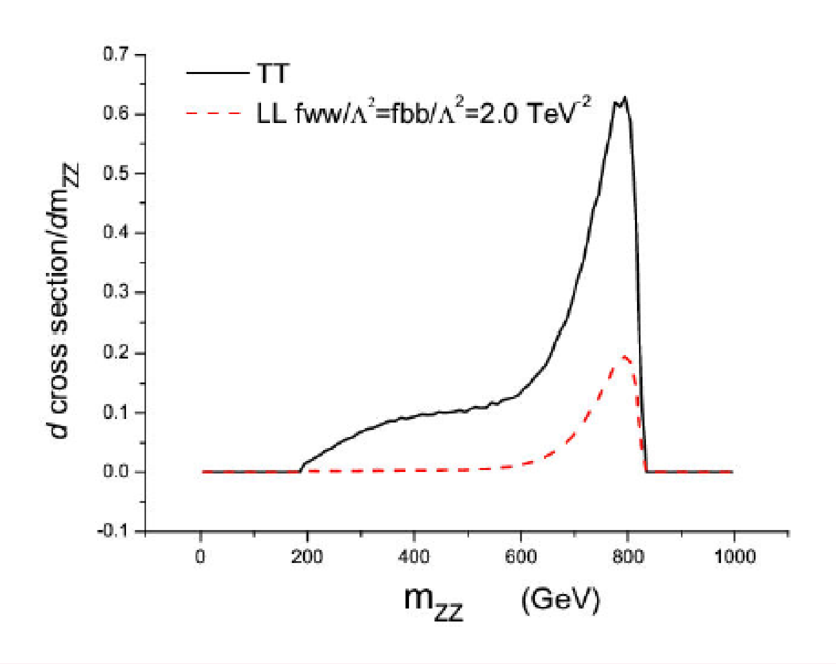
signal and background



The signal is in $\gamma_+\gamma_+ \rightarrow Z_L Z_L$ channel ,and raise with energy. The main background is $\gamma\gamma(\gamma_+\gamma_+, \gamma_+\gamma_-) \rightarrow Z_T Z_T$ loop contributions

Z pair center-of-mass energy distribution

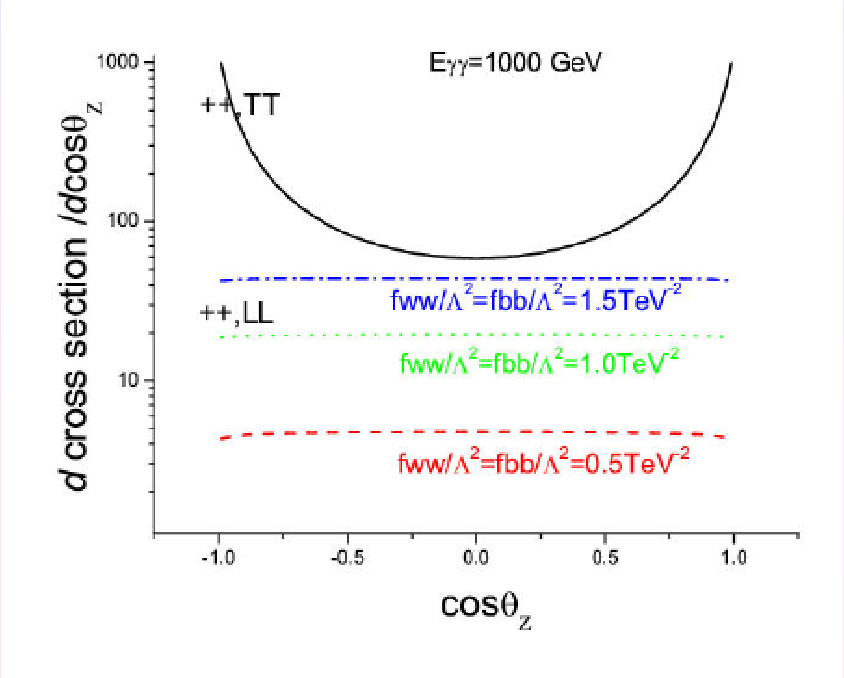
Z pair center-of-mass energy distribution of signal (LL) and background(TT) on $2\lambda_e P_c = -1$ polarized $\sqrt{s_{ee}} = 1$ TeV photon collider

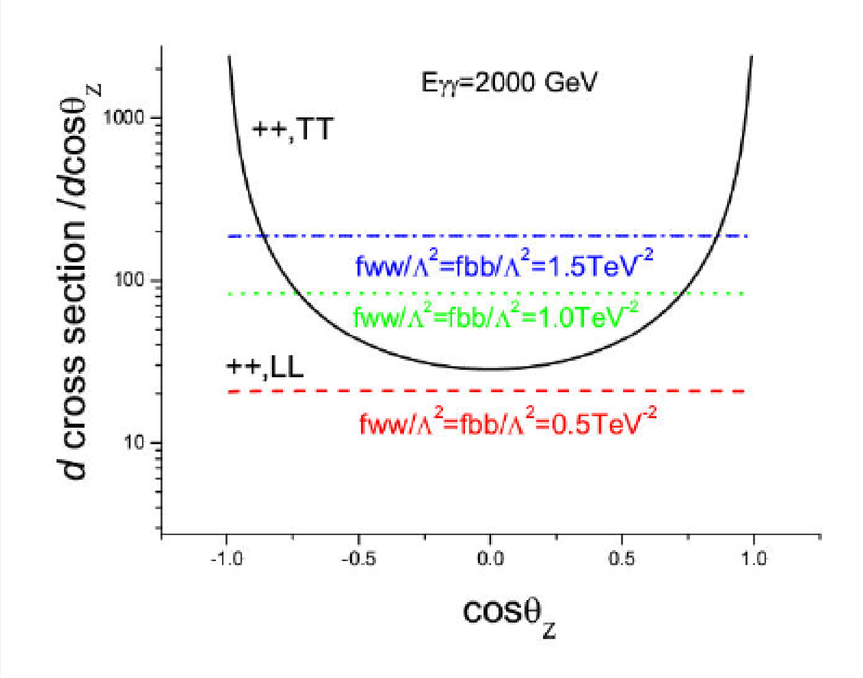


$$M_{ZZ} > 0.65\sqrt{s_{ee}}.$$

θ_Z angle distribution

Final Z bosons outgoing angle distribution





$$-0.5 < \cos\theta_z < 0.5$$

Jets energy difference

the energy difference between the two leptons (jets) is

$$\Delta E_{ll(jj)} \equiv |q_1^0 - q_2^0| = |\mathbf{P}_Z| \cos\theta'.$$

θ' distribution for Z_T decay $\propto (1 \pm \cos\theta')^2$, while
θ' distribution for Z_L decay $\propto \sin^2\theta'$.
So the leptons (jets) from Z_T decay are mainly in the region near $\theta' = 0 \text{ or } \pi$,
while the leptons (jets) from Z_L decay are mainly in the region near $\theta' = \pi/2$.

If we make a cut on the upper limit of $\Delta E_{ll(jj)}$ which picks up the region near $\theta' = \pi/2$, the Z_T decay mode will be suppressed. So we propose our third cut requiring

$$\Delta E_{ll(jj)} < \frac{1}{2}|\mathbf{P}_Z|.$$

The efficiency of cuts

$\sigma(LL, f_{WW}/\Lambda^2 = f_{BB}/\Lambda^2 = 2\ \mathrm{TeV}^{-2})$ as signal on $\sqrt{s_{ee}} = 1$ TeV,
$\sigma(LL, f_{WW}/\Lambda^2 = f_{BB}/\Lambda^2 = 0.67\ \mathrm{TeV}^{-2})$ as signal on $\sqrt{s_{ee}} = 3$ TeV and
$\sigma(TT, f_{WW}/\Lambda^2 = f_{BB}/\Lambda^2 = 0\ \mathrm{TeV}^{-2})$ as background

	without cut	M_{ZZ} cut	$+\ \theta_Z$ cut	$+\ \Delta E_{jj(ll)}$ cut	
$\sqrt{s_{ee}} = 1$ TeV					
σ_{LL} (fb)	22	20	9.2	4.9	
σ_{TT} (fb)	115	72	16	3.5	
efficiency	–	$\frac{90\%}{63\%} \approx 1.4$	$\frac{47\%}{22\%} \approx 2.1$	$\frac{54\%}{22\%} \approx 2.5$	9
$\sqrt{s_{ee}} = 3$ TeV					
σ_{LL} (fb)	27	23	9.9	5.6	
σ_{TT} (fb)	190	85	6.5	1.1	
efficiency	–	$\frac{85\%}{45\%} \approx 1.9$	$\frac{43\%}{7.6\%} \approx 5.7$	$\frac{57\%}{17\%} \approx 3.4$	30

CONCLUSION

The process $\gamma\gamma \to ZZ$ at polarized photon colliders based on e^+e^- linear colliders of 500 GeV, 1 TeV, and 3 TeV is sensitive to test the $H\gamma\gamma$ anomalous coupling . With an integrated luminosity of 1000 fb^{-1}, the 2σ testing sensitivities are

$$\sqrt{s_{ee}} = 500 \text{ GeV} \quad :$$
$$-0.021 \text{ TeV}^{-1} < g_{H\gamma\gamma} < 0.0078 \text{ TeV}^{-1}.$$

$$\sqrt{s_{ee}} = 1 \text{ TeV} \quad :$$
$$-0.0068 \text{ TeV}^{-1} < g_{H\gamma\gamma} < 0.0048 \text{ TeV}^{-1}.$$

$$\sqrt{s_{ee}} = 3 \text{ TeV} \quad :$$
$$-0.0015 \text{ TeV}^{-1} < g_{H\gamma\gamma} < 0.0015 \text{ TeV}^{-1}.$$

Testing fermion anomalous coupling at future ILC

- The flavor physics about leptons, neutrino mass and flavor mixing has become a hot topic.
- Many theoretical models introduced the heavy neutrinos or the fourth generation leptons.
- the effects of those extra massive particles can be reflected in the anomalous couplings of leptons and gauge bosons .
- Detecting these leptonic anomalous gauge couplings on colliders can also test and verify these electroweak new physical models.

The Leptonic Anomalous Gauge Couplings In Effective Lagrangian

$$\mathcal{O}_7^{VF} = i\overline{L}\gamma_\mu W^{\mu\nu}\overleftrightarrow{D}_\nu L\,,$$

$$\mathcal{O}_{11}^{VF} = i\overline{L}\gamma_\mu B^{\mu\nu}\overleftrightarrow{D}_\nu L\,,$$

$$\mathcal{O}_{13}^{VF} = i\overline{E}\gamma_\mu B^{\mu\nu}\overleftrightarrow{D}_\nu E\,,$$

$$\mathcal{O}_{24}^{VF} = \overline{L}\gamma_\mu (D_\nu W^{\mu\nu}) L\,,$$

$$\mathcal{O}_{26}^{VF} = \overline{L}\gamma_\mu \partial_\nu B^{\mu\nu} L\,,$$

$$\mathcal{O}_{27}^{VF} = \overline{E}\gamma_\mu \partial_\nu B^{\mu\nu} E\,,$$

the operator $\mathcal{O}_7$ and $\mathcal{O}_{24}$ can affect all the vertices

Therefore, the process $e^+e^- \to W^+W^-$

is much more sensitive to operators $\mathcal{O}_7$ and $\mathcal{O}_{24}$ than others.

$e^+e^- \to W^+W^-$ at the future e^+e^- inear collider (ILC)

In the standard model e+e- ->W+W—process， the E^2 terms in the amplitude cancel each other between different Feynman diagrams

the constraints from LEP2 are very weak

$$-2.6\ \mathrm{TeV}^{-2} < f_7/\Lambda^2 < 2.6\ \mathrm{TeV}^{-2},$$

$$-9.8\ \mathrm{TeV}^{-2} < f_{24}/\Lambda^2 < 1.8\ \mathrm{TeV}^{-2},$$

$$-11\ \mathrm{TeV}^{-2} < f_{26}/\Lambda^2 < 33\ \mathrm{TeV}^{-2},$$

$$-13\ \mathrm{TeV}^{-2} < f_{27}/\Lambda^2 < 30\ \mathrm{TeV}^{-2}.$$

the detection sensitivities of the anomalous coupling at 500 GeV ILC

$$-0.18\ \mathrm{TeV}^{-2} < \ f_7/\Lambda^2 \ < 0.18\ \mathrm{TeV}^{-2},$$

$$-0.045\ \mathrm{TeV}^{-2} < \ f_{24}/\Lambda^2 \ < 0.15\ \mathrm{TeV}^{-2},$$

$$-2.5\ \mathrm{TeV}^{-2} < \ f_{26}/\Lambda^2 \ < 8.8\ \mathrm{TeV}^{-2},$$

$$-3.3\ \mathrm{TeV}^{-2} < \ f_{27}/\Lambda^2 \ < 6.2\ \mathrm{TeV}^{-2}.$$

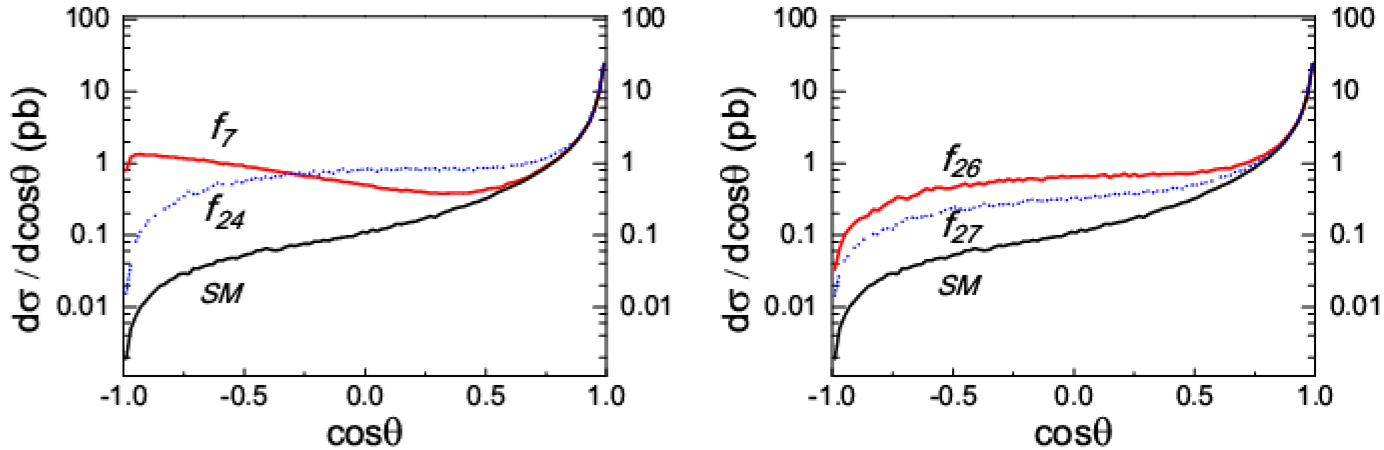
the detection sensitivities of the anomalous coupling at 1 TeV ILC

$$-2.0\times 10^{-2}\ \mathrm{TeV}^{-2} < \ f_7/\Lambda^2 \ < 2.0\times 10^{-2}\ \mathrm{TeV}^{-2},$$

$$-2.5\times 10^{-3}\ \mathrm{TeV}^{-2} < \ f_{24}/\Lambda^2 \ < 8.0\times 10^{-3}\ \mathrm{TeV}^{-2},$$

$$-0.6\ \mathrm{TeV}^{-2} < \ f_{26}/\Lambda^2 \ < 2.3\ \mathrm{TeV}^{-2},$$

$$-1.0\ \mathrm{TeV}^{-2} < \ f_{27}/\Lambda^2 \ < 1.6\ \mathrm{TeV}^{-2}.$$



$\cos\theta$ cut	$f_7/\Lambda^2(10^{-3}\mathrm{TeV}^{-2})$	$f_{24}/\Lambda^2(10^{-4}\mathrm{TeV}^{-2})$	$f_{26}/\Lambda^2(10^{-1}\mathrm{TeV}^{-2})$	$f_{27}/\Lambda^2(10^{-1}\mathrm{TeV}^{-2})$
none	$-20 \sim 20$	$-25 \sim 80$	$-6 \sim 23$	$-9 \sim 15$
$\cos\theta < 0.75$	$-9.5 \sim 9.5$	$-6.7 \sim 10$	$-1.9 \sim 2.3$	$-3.3 \sim 10$
$\cos\theta < 0.5$	$-6.5 \sim 6.5$	$-4.2 \sim 5.0$	$-1.1 \sim 1.4$	$-2.0 \sim 9.2$
$\cos\theta < 0$	$-4.6 \sim 4.6$	$-2.8 \sim 2.9$	$-0.75 \sim 0.78$	$-1.4 \sim 2.5$

After *W* angle cut, the ILC detection sensitivities are increased by 3-4 orders of magnitude than the LEP2.

THANKS

The Polarization Scheme

Right handed polarized initial electrons are useful to detect the anomalous coupling between lepton and *Z* boson

the polarized $\sqrt{s_{ee}} = 500$ GeV ILC can provide a better detection sensitivity

$$-3.5 \times 10^{-2}\ \mathrm{TeV}^{-2} < f_{27}/\Lambda^2 < 3.5 \times 10^{-2}\ \mathrm{TeV}^{-2},$$

And for $\sqrt{s_{ee}} = 1$ TeV polarized ILC, the detection sensitivity is:

$$-9 \times 10^{-3}\ \mathrm{TeV}^{-2} < f_{27}/\Lambda^2 < 9 \times 10^{-3}\ \mathrm{TeV}^{-2},$$