Congratulation to Prof. Yu-Ping Kuang, the old times friend

W. Wetzel , Universität Heidelberg

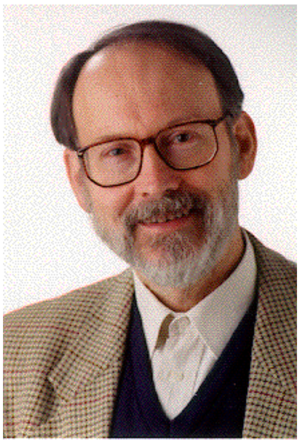
November 13, 2012

1 YPK

Figure 1: Ithaca 1980, Niagara Falls

Figure 2: Ithaca 1981 Cayuga Lake

Figure 3: Ithaca 1981, Taughannock State Park



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Testing the anomalous color-electric dipole moment of the c quark from $\psi' \to J/\psi + \pi^+ + \pi^-$ at the Beijing Spectrometer

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If the c quark has an anomalous color-electric dipole moment (CEDM), it may serve as a new source of CP violation. The strength of such a CP violation depends on the size of the CEDM, d_c'. We propose two effective ways of testing it from the large sample of $\psi' \to J/\psi + \pi^+ + \pi^-$ at the Beijing Spectrometer, and the obtained result, $|d_c'| < 3 \times 10^{-14}$ e cm (95% confidence level), gives the first experimentally determined upper bound on the CEDM of the c quark.

UV-Divergence Structure of φ^4 Theory in the Broken Phase

Part I : Continuum Form

Part II : Lattice Form

2 Lagrangian

$v \ := \ <\varphi>$
$H \ := \ \varphi \ - \ v$
self coupling: $\lambda/4!$
Symmetry: $H + v \to -(H + v)$

$$L = \frac{1}{2}(\partial H)^2 \ - \ \frac{1}{2}\, m^2 \, H^2 \ - \ \frac{m\sqrt{3}\sqrt{\lambda}}{3!}\, H^3 \ - \ \frac{\lambda}{4!}\, H^4$$
$$+ \ \delta^v \, \frac{m^3\sqrt{3}}{2\sqrt{\lambda}}\, H \ + \ \delta^v \, \frac{m^2}{4}\, H^2$$
$$m^2 := \frac{\lambda}{3}\, v^2 \qquad\qquad [\, -2\,\mu_2 \ = \ m'^{\,2} \ = \ m^2\,(1 - \delta^v)\,]$$

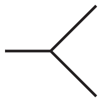
δ^v : dimensionless, (Lagrange multiplier) $\leftrightarrow$ $<H> = 0$

(1) 

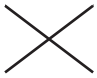
$$i \, \frac{\sqrt{3}}{2} \, m^3 \, \frac{1}{\sqrt{\lambda}} \, \delta^v$$

(2) 

$$i \, \frac{1}{2} \, m^2 \, \delta^v$$

(3) 

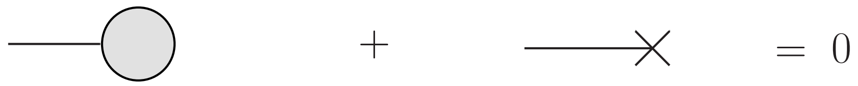
$$-i \, \sqrt{3} \, m \, \sqrt{\lambda}$$

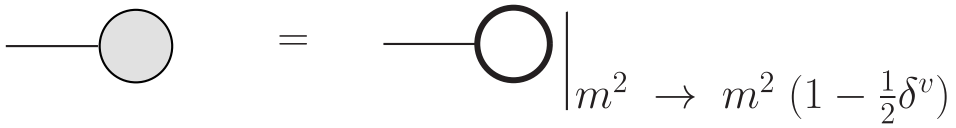
(4) 

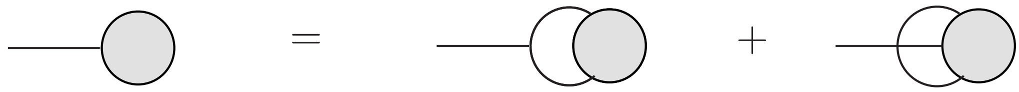
$$-i \, \lambda$$

$$(\mathbf{1}) \; / \; (\mathbf{2}) \;\; = \;\; (\mathbf{3}) \; / \; (\mathbf{4})$$

3 Recursion for δ^v

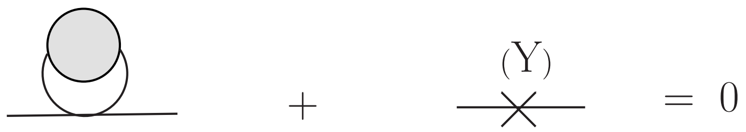






$$\delta^v \quad = \quad \delta^v(\mathrm{Y}) \quad + \quad \delta^v(\Psi)$$

Diagrams with 'ears' are cancelled by $\delta^v(\mathrm{Y})$ insertions :



And more . . .

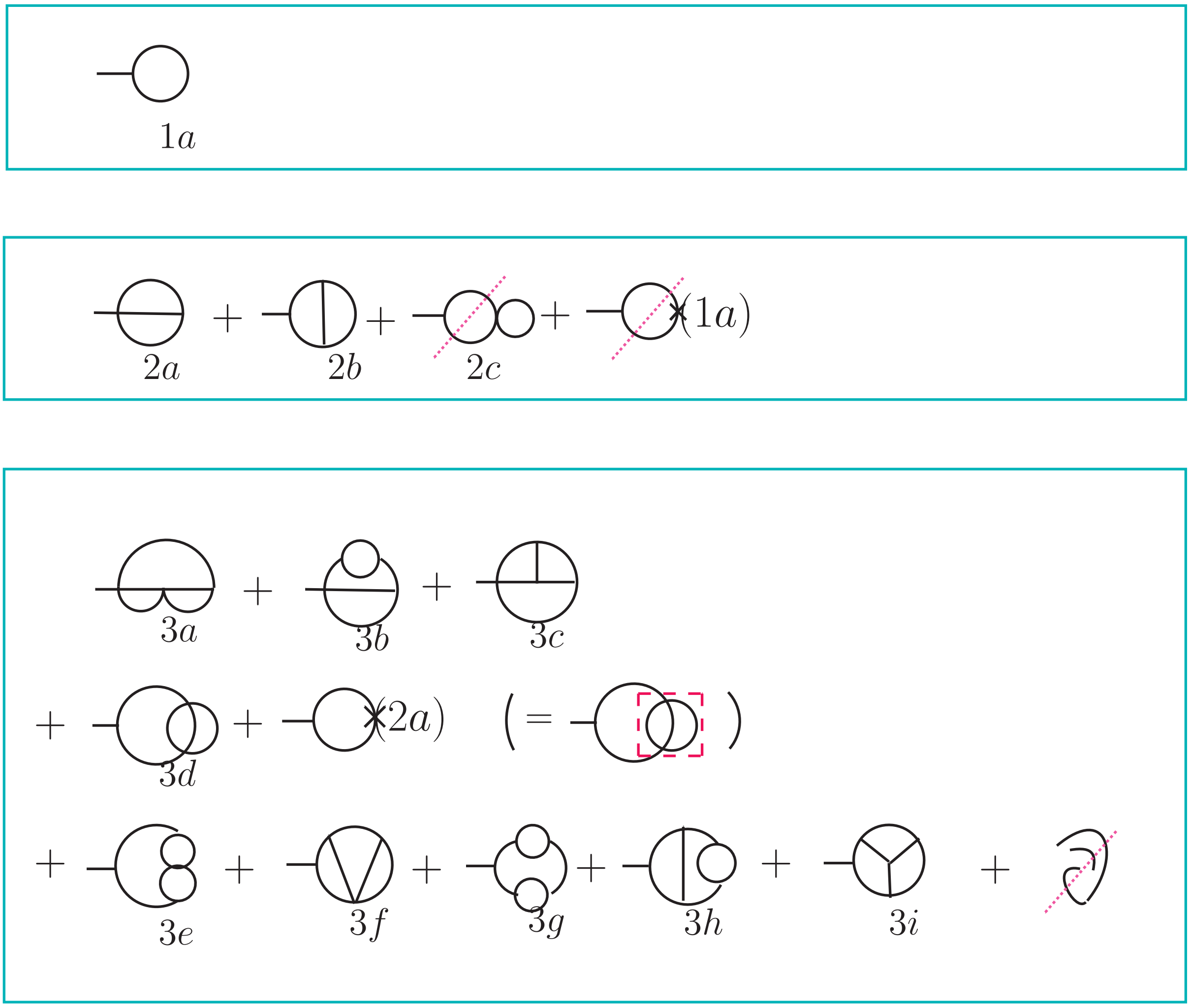


Figure 4: Diagrams for δ^v

4 Poles of δ^v in dim. Regularisation

$$\tilde{\lambda} := \frac{\lambda}{(m^2)^\epsilon} \qquad , \qquad \epsilon = 2 - d/2$$

$$\delta^v = \delta^v(1) + \delta^v(2) + \dots = \tilde{\lambda}\, \delta^v_1(\epsilon) + \tilde{\lambda}^2\, \delta^v_2(\epsilon) + \dots$$

Loop-integrals ($l = 1, 2$):

$$\tilde{F}_1(n; d) = \int_k \frac{1}{(k^2 - 1)^n} = i\, \frac{(1)^n}{(4\pi)^{d/2}} \frac{\Gamma(n - d/2)}{\Gamma(n)}$$

$$\tilde{F}_2(n_1, n_2, n_3; d) = \int_{k_1, k_2} \frac{1}{(k_1^2 - 1)^{n_1}\, (k_2^2 - 1)^{n_2}\, ((k_1 + k_2)^2 - 1)^{n_3}}$$

IBP : Intergration By Parts identities

Results for $\delta^v_l(\epsilon)$, $l \leq 3$:

Simple pole at $d = 4 - 2/l$ (overall quadr. divergence)

Poles up to order l at $d = 4$ (log. (sub)-divergence)

Expectation : pattern valid to all orders

5 No quad. Divergence in Self-energy

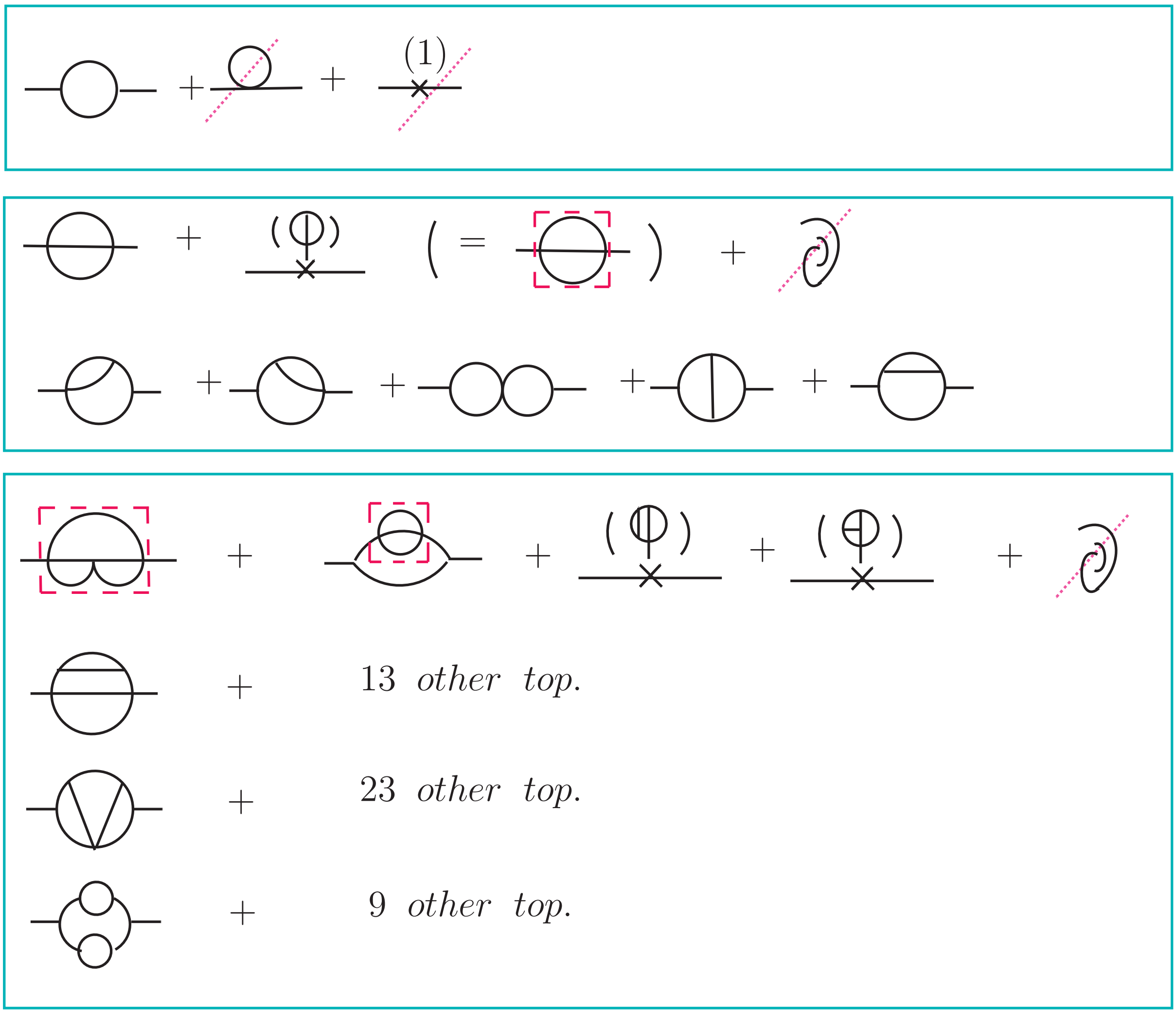
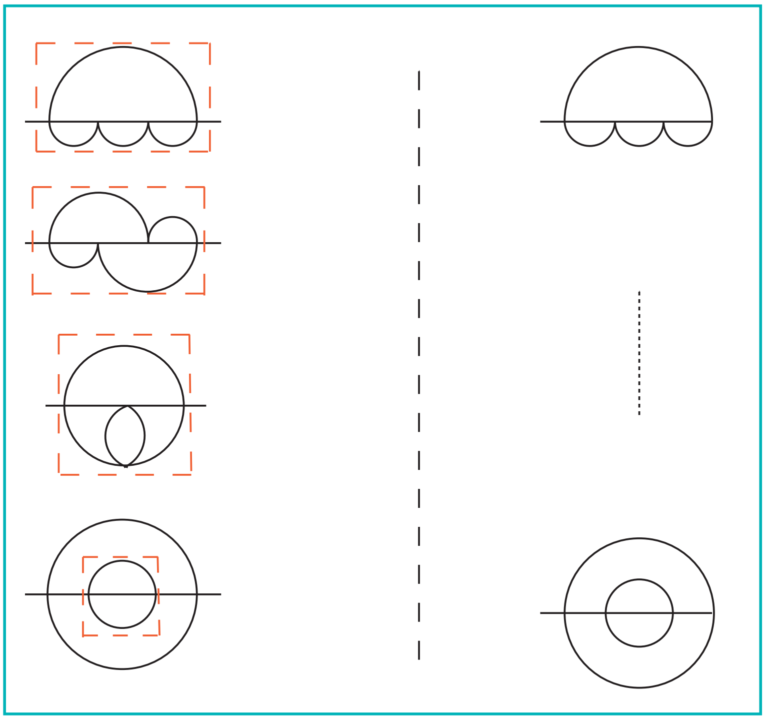


Figure 5: self-energy to 3 loops

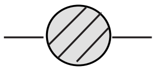
4 loop self-energy

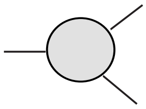
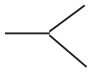
- 627 qgraf top. + tower of insertions
- 275 qgraf 'ear' top. are cancelled
- overall quadratic div. incl. 1 quadr. subdiv

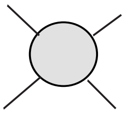


- diagrams with 2- and 3- loop quadr. subdiv.

6 Renormalisation

 $= -i\, m^2\, \tilde{\Sigma} \quad , \quad \tilde{\Sigma}|_{p=0} = \sigma \quad ; \quad -m^2 \frac{d}{dp^2}\tilde{\Sigma}|_{p=0} = \sigma'$

 $=$  $\cdot\, (\, 1 + \tilde{\Gamma}^3) \quad , \quad \tilde{\Gamma}^3|_{(p=0)} = \gamma^3$

 $=$  $\cdot\, (\, 1 + \tilde{\Gamma}^4) \quad , \quad \tilde{\Gamma}^4|_{(p=0)} = \gamma^4$

$Z = 1 + \delta^Z \;\; , \;\; m^2/m_R^2 = 1 + \delta^m \;\; , \; \lambda/\lambda_n = 1 + \delta^n$

$$\begin{aligned} 1 &= (1+\delta^Z)\cdot(1+\sigma') \\ 1 &= (1+\delta^Z)\cdot(1+\delta^m)\cdot(1+\sigma) \\ 1 &= (1+\delta^Z)^3\cdot(1+\delta^m)\cdot(1+\delta^3)\cdot(1+\gamma^3) \\ 1 &= (1+\delta^Z)^2\cdot(1+\delta^4)\cdot(1+\gamma^4) \end{aligned}$$

$$v_R^2 \;=\; \frac{3\cdot m_R^2}{\lambda_3}\cdot\left(\frac{1+\gamma^3}{1+\sigma}\right)^2 \;=\; \frac{3\cdot m_R^2}{\lambda_4}\cdot\left(\frac{1+\gamma^4}{1+\sigma}\right)$$

7 2-loop Results

All quantities (σ_l , . . . , δ_l^4) expressed by
$l = 1 :\quad \tilde{F}_1(2) \quad , \quad l = 2 :\quad \tilde{F}_1(2)^2 \,,\; \tilde{F}_2(2,2,1)$
multiplied by polynomials in ϵ , e.g.

$$\delta_2^4 \;=\; -1/(48\,(1-\epsilon)(2-\epsilon))$$

$$\cdot \;\; \{ \;\epsilon\,(80 + 334\epsilon - 2769\epsilon^2 + 6915\epsilon^3 - 4752\epsilon^4 + 972\epsilon^5\;)\;\tilde{F}_1(2)^2$$

$$- \;\; 4\,(112 - 926\epsilon + 1975\epsilon^2 - 2718\epsilon^3 + 972\epsilon^4)\;\tilde{F}_2(2,2,1)\;\;\}$$

$\Rightarrow$ Relations between renormalized quantities:

$$v_R^2 = \frac{3\,m_R^2}{\lambda_3}(\;1 + 3\,\frac{\lambda_3}{(4\pi)^2} - \frac{21}{4}\,(\frac{\lambda_3}{(4\pi)^2})^2 + \ldots\;)$$

$$= \;\frac{3\,m_R^2}{\lambda_4}(\;1 + \frac{9}{2}\,\frac{\lambda_4}{(4\pi)^2} - 9\,(\frac{\lambda_4}{(4\pi)^2})^2 + \ldots\;)$$

$$\lambda_3 = \lambda_4\;(\;1 - \frac{3}{2}\,\frac{\lambda_4}{(4\pi)^2} + 6\,(\frac{\lambda_4}{(4\pi)^2})^2 + \ldots\;)$$

$$\lambda_4 = \lambda_3\;(\;1 + \frac{3}{2}\,\frac{\lambda_3}{(4\pi)^2} - \frac{3}{2}\,(\frac{\lambda_3}{(4\pi)^2})^2 + \ldots\;)$$

8 MS β-functions

$$\lambda = g_{MS}^2 \,(\, 1 + \tilde{g}_{MS}^2\, a_1(\epsilon) + (\tilde{g}_{MS}^2)^2\, a_2(\epsilon) + \ldots\,)$$

$$m^2 = m_{MS}^2 \,(\, 1 + \tilde{g}_{MS}^2\, b_1(\epsilon) + (\tilde{g}_{MS}^2)^2\, b_2(\epsilon) + \ldots\,)$$

$$\tilde{g}_{MS}^2 = g_{MS}^2/(\mu^2)^\epsilon$$

$$\mu^2 \frac{dg_{MS}^2}{d\mu^2} = \beta_g = \sum_{l=1}^{\infty} l\,(g_{MS}^2)^{(l+1)}\, a_{l,1}$$

$$\frac{\mu^2}{m_{MS}^2}\frac{dm_{MS}^2}{d\mu^2} = \beta_m = \sum_{l=1}^{\infty} l\,(g_{MS}^2)^{l}\, b_{l,1}$$

$$\beta_g = \frac{3}{2}\,(\frac{g_{MS}^2}{(4\pi)^2})^2 - \frac{17}{6}\,(\frac{g_{MS}^2}{(4\pi)^2})^3 \qquad \beta_m = \frac{3}{2}\,\frac{g_{MS}^2}{(4\pi)^2} - \frac{35}{12}\,(\frac{g_{MS}^2}{(4\pi)^2})^2$$

$$m^2 \to m'^{\,2} : \quad \frac{3}{2} \to \frac{1}{2} \quad , \quad \frac{35}{12} \to \frac{5}{12}$$

$$Z_{MS} = ?$$

$$v^2 = \frac{3m^2}{\lambda} = \frac{3m_{MS}^2}{g_{MS}^2}\,\frac{1 + \tilde{g}_{MS}^2\, b_1 + \ldots}{1 + \tilde{g}_{MS}^2\, a_1 + \ldots}$$

$$v_{MS}^2 = Z_{MS}^{-1}\, v^2 \quad \Rightarrow Z_{MS} = \frac{1 + \tilde{g}_{MS}^2\, b_1 + \ldots}{1 + \tilde{g}_{MS}^2\, a_1 + \ldots}$$

9 2-loop Effective Potential

Closely related to zero momentum scheme, yields $\sigma \; \ldots \; \gamma^4$ in more compact form.

Euclidean formulation:

$$V = \frac{1}{2}\,\mu_2\,\varphi^2 + \frac{\lambda}{4!}\,\varphi^4 + \frac{1}{2}\int_p ln[1] \;-\; \frac{1}{12}\lambda^2\varphi^2\int_{p_3=p_1+p_2}\frac{1}{[1]\,[2]\,[3]} \;+\; \frac{1}{8}\lambda\,(\int_p\frac{1}{[1]}\;)^2$$

$$[i] = M_2 + {p_i}^2 \quad , \quad M_2 = \mu_2 + \frac{1}{2}\,\lambda\,\varphi^2 \;=\; M^2 - \frac{1}{2}\,m^2\,\delta^v$$

$$M^2 \;=\; m^2\,\tilde{M}^2 \;=\; m^2\,(\,1 \;+\; \frac{3}{2}\,\lambda\,(\frac{\varphi^2}{v^2} - 1)) = m^2\,(1+y)$$

$$V \;=\; const \;-\; \frac{1}{4}\,m^2\,\delta^v\,(\varphi^2 \;-\; v^2\,) \;+\; \frac{\lambda}{4!}\,(\varphi^2 \;-\; v^2\,)^2$$
$$+\,\frac{1}{2}\int ln[1] \;-\; \frac{1}{12}\,\lambda^2\,\tilde{F}_2(1,1,1)\,\varphi^2(M_2)^{1-2\epsilon} \;+\; \frac{1}{8}\,\lambda\,(\tilde{F}_1(1))^2\,(M_2)^{2-2\epsilon}$$

2-loop terms are easy , $\int ln[1]$ needs 2 subtractions

$$\frac{\partial^{\,n}}{\partial\varphi}V|_{\varphi=v} \quad\longleftrightarrow\quad \delta^v \;,\; \sigma \;,\; \gamma^3 \;,\; \gamma^4$$

3 Issues :

- $\tilde{V}(\tilde{\lambda}, y)\,|_0^y$ regular for $d < 4$? , $(\tilde{V} = V/(m^2)^{2-\epsilon})$

  √

- $0 \;=\; \mu^2\,\frac{d}{d\mu^2}\,V(g^2_{MS}, m^2_{MS}, \mu^2, y)$

  √

- $V \;=\; \frac{1}{4}\,(m^2_{MS})^2\,\{\frac{1}{g^2_{MS}}\,V_{-1} \;+\; V_0 \;+\; g^2_{MS}\,V_1\}$, $V_{(\cdot)}(y, \eta_{MS})$

  √

10 φ^4 in dimensionless Form

$$\hat{\varphi} = \frac{\varphi}{v} \quad , \quad \hat{H} = \frac{H}{v} \quad , \quad \hat{x} = x\, v^{\frac{1}{1-\epsilon}} \quad , \quad \hat{\lambda} = \lambda / v^{\frac{\epsilon}{1-\epsilon}}$$

$$S = \int L(H(x))dx = \int \hat{L}(\hat{H}(\hat{x}))d\hat{x} \quad , \quad \hat{L}(\hat{H}(\hat{x})) = \frac{1}{2}(\partial \hat{H})^2 - \hat{V}(\hat{H})$$

$$\hat{V} = \frac{1}{6}\,\hat{\lambda}\,\{ -\delta^v \hat{H} - \frac{1}{2}\,\delta^v \hat{H}^2 + \hat{H}^2 + \hat{H}^3 + \frac{1}{4}\,\hat{H}^4 \}$$

$$= \frac{1}{6}\,\hat{\lambda}\,\{ \frac{1}{2}\,(\frac{1}{2} + \delta^v) - \frac{1}{2}(1 + \delta^v)\,\hat{\varphi}^2 + \frac{1}{4}\hat{\varphi}^4 \}$$

δ^v determined by : $\quad <\hat{H}>= 0 \quad$ or $\quad <\hat{\varphi}>= 1$

$\hat{\lambda}$ can be transfered to kinetic term :

$$\hat{x} = (\frac{6}{\hat{\lambda}})^{1/d}\,\hat{x}' \quad , \quad \hat{\lambda}' = (\frac{\hat{\lambda}}{6})^{1-2/d}$$

$$\frac{1}{2}(\partial \hat{H})^2 = \frac{1}{2\hat{\lambda}'}(\partial' \hat{H}')^2 \quad , \quad \hat{V}'(\hat{H}'(\hat{x}')) = \frac{6}{\hat{\lambda}}\,\hat{V}(\hat{H}(\hat{x}))$$

Properties:

- No wavefunction renormalization
- Mass-term, and interactions all proportional to a common parameter.
  $\Rightarrow$ cannot adjust to a situation where pairwise ratios of
  $(1+\sigma)$, $(1+\gamma^3)$, $(1+\gamma^4)$ diverge

11 Lattice Version of $\hat{L}$ (d=4)

Standard discretization in terms of a dimensionless lattice constant:

$$\hat{x} \to \hat{a}\, n\ ,\ n_\mu \,\epsilon\, \mathbb{Z}$$

$$\hat{\partial}^2_\mu \to \frac{1}{\hat{a}^2}(\ \delta_{n',n+\mu} + \delta_{n',n-\mu} - 2\delta_{n',n}\ ) \quad , \quad \int d\hat{x} \to \hat{a}^4 \sum_n$$

$$\phi := \hat{a}\ \hat{\varphi} \quad , \quad h := \hat{a}\ \hat{H} \quad ,$$

$$\hat{L} \to \sum_n \{ \sum_\mu \chi_n\, \chi_{n+\mu} \ - \ 4\, \chi_n^2 - \frac{\lambda}{6}\, U(\chi_n)\} \ , \ (\ \chi \ = \ \phi\ ,\ h)$$

$$U(\phi)\ =\ U_0\ -\ \frac{1}{2}\hat{a}^2\,(1+\delta^v)\,\phi^2\ +\ \frac{1}{4}\,\phi^4$$

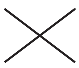
$$U(h)\ =\ -\,\hat{a}^3\,\delta^v\,h\ -\ \frac{1}{2}\hat{a}^2\,\delta^v\,h^2\ +\ \hat{a}^2\,h^2\ +\ \hat{a}\,h^3\ +\ \frac{1}{4}\,h^4$$

$\hat{k}^2\ +\ \hat{m}^2, \quad \hat{m}^2\ =\ \frac{1}{3}\,\lambda\,\hat{a}^2 \quad , \quad \hat{k_\mu}\ =\ 2\ sin(\frac{k_\mu}{2})$

(1)  $\frac{1}{6}\,\lambda\,\hat{a}^3\,\delta^v$

(2)  $\frac{1}{6}\,\lambda\,\hat{a}^2\,\delta^v$

(3)  $-\lambda\,\hat{a}$

(4)  $-\lambda$

(1) / **(2)** = **(3)** / **(4)**

Small $\hat{a}$ expansion of lattice perturbation theory:
(K.Symanzik, H.Kawai et al, T.Reisz,...)

$$\delta^v \; = \; \hat{a}^{-2}\delta^v_{,-1} \; + \; \delta^v_{,0} \; + \; \hat{a}^2 \, \delta^v_{,1} \; + \; \dots \; , \; \delta^v_{,m} = \sum_{l \geq n \geq 0} \delta^v_{,m,n} \, ln(\hat{a}^2)^n$$

$\Rightarrow \; \delta^v_{,-1} \; = \; \lim_{\hat{a} \to 0} \; \hat{a}^2 \, \delta^v$ exists even beyond pert. theory.

$\delta^v_{,0}$: continuum physics (scaling behaviour)

$\delta^v_{,n>0}$: lattice artefacts

Connection of $\delta^v_{,-1}$ with critical line in $(\kappa \, , \, \lambda_{MC})$ plane:

$$(\, \hat{L} \; = \; \sum_n \left\{ \, \kappa \sum_\mu \varphi_n \, \varphi_{n+\mu} \; - \; \varphi_n^2 \; - \; \lambda_{MC} \, (\varphi_n^2 \; - \; 1)^2) \right\})$$

$$\kappa \; = \; \frac{2(1 - 2\lambda_{MC})}{8 \; - \; \hat{a}^2 \delta^v \; \lambda/6} \qquad , \qquad \lambda_{MC} \; = \; \frac{\lambda}{4!} \, \kappa^2$$

$$\downarrow \hat{a} \to 0 \; , \; \lambda \; fixed \qquad (< \hat{H} > = \; \hat{a})$$

$$\kappa_{crit}(\lambda_{MC}) \; = \; \frac{2(1 - 2\lambda_{MC})}{8 \; - \; \delta^v_{,-1} \; \lambda/6}$$

Implicit equation:

$$\left\{ \, \frac{\lambda}{48} \, , \; \delta^v_{,-1} \, \right\} \; = \; \left\{ \, \frac{\lambda_{MC}}{2\kappa_{crit}^2} \, , \; \frac{12}{\lambda} \, \frac{4 \, \kappa_{crit} \; - \; 1 \; + \; 2\lambda_{MC}}{\kappa_{crit}} \, \right\}$$

12 Lüscher-Weisz data for κ_{crit} combined with Mean-Field

κ-series for symmetric phase:

Lüscher-Weisz derive series for susceptibility up to order 15, then estimate its convergence radius ($\Rightarrow \kappa_{crit}^{LW}$) .

Mean-field for broken phase:

$$(\kappa_{crit}^{MF})^{-1} \;=\; 8\, W_2(\lambda_{MC})$$

$$W_2(\lambda_{MC}) \;=\; \frac{\int_\phi \phi^2 \; d\mu(\phi)}{\int_\phi \; d\mu(\phi)} \quad , \quad d\mu(\phi) \;=\; exp\big(-\phi^2 - \lambda_{MC}\,(\phi^2-1)^2\big)$$

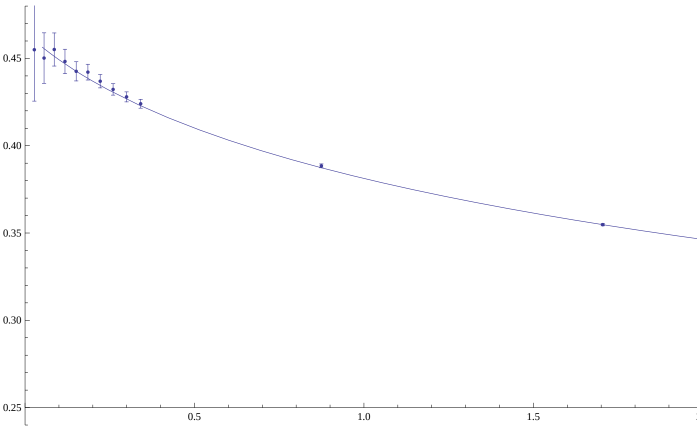
Chain $(X = LW\ ,\ MF)$:

$$\lambda_{MC}$$
$$\downarrow$$
$$\kappa^X_{crit}$$
$$\downarrow$$
$$\lambda \ = \ 4!\ \lambda_{MC}/(\kappa^X_{crit})^2$$
$$\downarrow$$
$$\delta^v_{X,-1} \ = \ 48/\lambda \cdot (4\ \kappa^X_{crit} - 1 + 2\ \lambda_{MC})/(4\ \kappa^X_{crit})$$

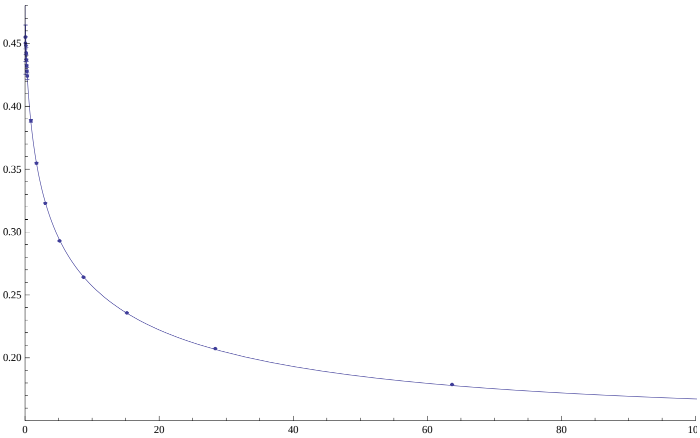
Rescaling to fit LW data :

$$\delta^v_{MFs,-1}(\lambda, s) \ := \ 4(\ \delta^v_{,-1}(0) \ - \ \kappa^{Ising}_{crit}) \cdot (\ \delta^v_{MF,-1}(\lambda/s) \ - \ 3/8\ ) \ + \ \delta^v_{,-1}(0)$$

Best fit : $s \ = \ .65 \quad , \quad \chi^2/20 \approx 2$



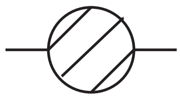
0.45
0.40
0.35
0.30
0.25
0.5
1.0
1.5

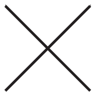


0.45
0.40
0.35
0.30
0.25
0.20
0
20
40
60
80

13 The Theory on the Critical Line

 $\hat{k}^2$

 $\frac{1}{6}\,\lambda\,\delta^v_{,-1}$  $\Big|_{k\,=\,0} = 0$

 $-\lambda$

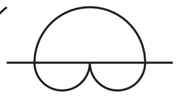
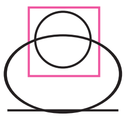
1-loop:

$$\delta^v_{(1),-1} = 3\int\frac{1}{\hat{k}^2} = 0.465.. \approx \frac{1}{2\pi}$$

2-loop:

$$\delta^v_{(2),-1} = -\lambda\int\frac{1}{(1)(2)(1+2)} \approx -\lambda\,(\frac{1}{2\pi})^3$$

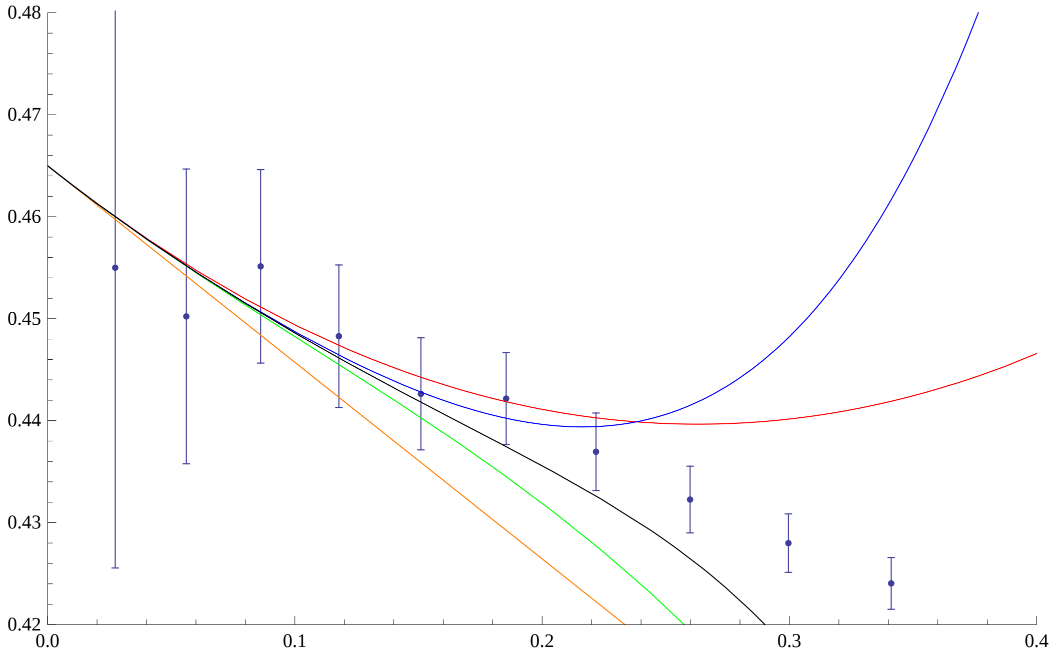
3-loop:

$\delta^v_{(3),-1} = -\frac{6}{\lambda}\Bigg($  $+$  $\Bigg)$

determined up to 6 loops

Subtractions regulate IR-divergencies of lattice integrals (Brillouin zone type)

- magnitude of unsubtracted diagrams: $(\frac{1}{2\pi})^{\#lines}$
- magnitude of subtracted diagrams one order smaller


0.48
0.47
0.46
0.45
0.44
0.43
0.42
0.0
0.1
0.2
0.3
0.4

14 $\delta^{v}_{,-1}$ Borel summable ?

Yes , if Mean-field derived form is suggestive.

15 Scaling behaviour of lattice model

Continuum physics requires the control of $O(\hat{a}^2)$ terms in $\hat{L}$ e.g. in :
$U(\phi) \;=\; U_0 \;-\; \frac{1}{2}\hat{a}^2\,(1+\delta^v)\,\phi^2 \;+\; \frac{1}{4}\,\phi^4$
relative to the leading $\delta^v_{,-1}$ term.

Strategy ? : Make use of $<\phi>=\hat{a}$

1. Choose λ
2. Choose ξ in $\hat{a}^2\,(1+\delta^v) \;=\; \delta^v_{,-1}+\xi$ with appropriate sign and size
3. Determine $<\phi>$ by MonteCarlo simulation (the easiest quantity).
4. The gained $\hat{a}$ yields $\hat{m}^2 \;:=\; \frac{\lambda}{3}\,\hat{a}^2$
5. examine $\hat{a}^2\,\delta^v \;=\; \delta^v_{,-1} \;+\; \xi \;-\; \hat{a}^2$
6. back to item 1.

16 Of Interest

- Polymer expansion (loophole for nontriviality Aizenman,Fröhlich)
- Yukawa
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17 Thanks to the Organizers