Tsinghua University

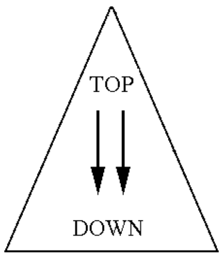


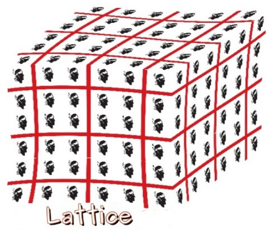
Summary report of history and recent development for our late twenty four year's efforts

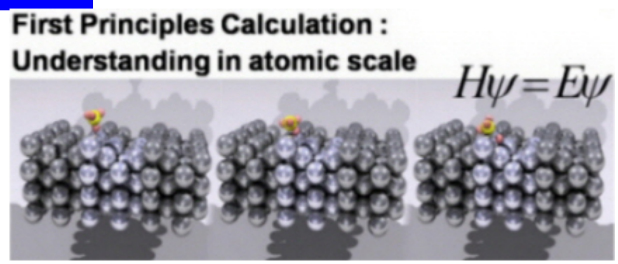
# First Principle Calculation

of

# Low Energy Effective Theory



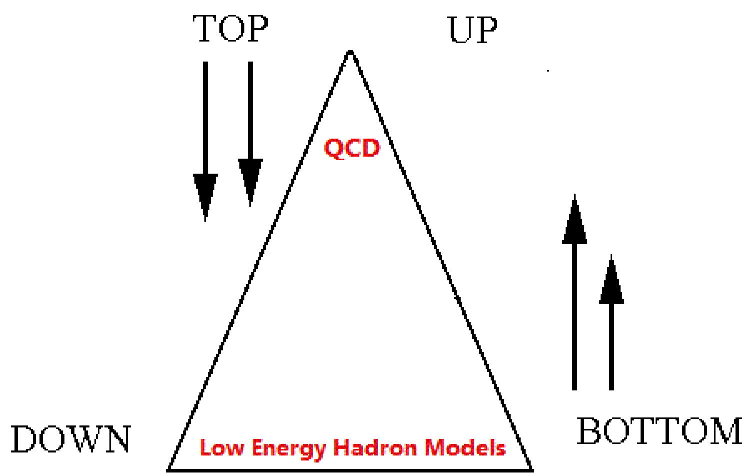




**Qing Wang**

Collaborated with Prof.Kuang and Students

**November 13, 2012**

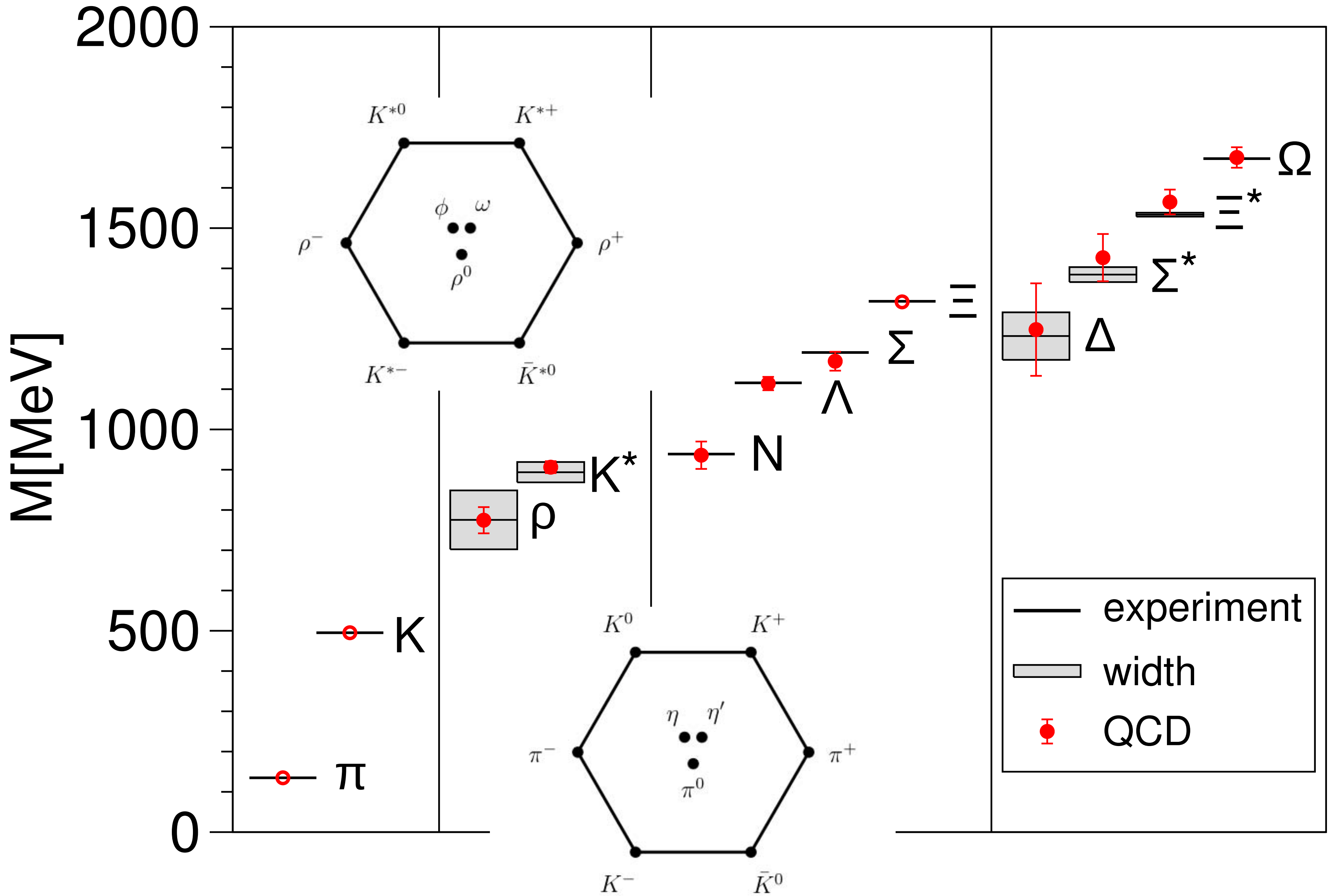
# Low Energy Hadron Models & QCD




# First Principle Derivation and Calculation

**2nd Part: Apply First Principle Calculation to Electroweak Chiral Lagrangian**

# Light Peudoscalar & Vector Mesons and Baryons




BMW; *cf.* Ryan, Zanotti, Edwards

# Different levels of Low Energy Hadron Models

- **Pure Pheno Level:** $\pi, K, \eta, \eta';\ \rho, \omega, \phi, K^*;\ n, p$

  linear,nonlinear$-\sigma$ models; $\chi$EFTs; Hidden Symmetry; Skyrmion Model; Baryon EFT

- **Half Pheno Level:** quark $+\ \pi, K, \eta, \eta';\ \rho, \omega, \phi, K^*;\ n, p$

  various chiral quark models

- **Quark Level:** quark

  various NJL type models; GCM

- **Quark Gluon Level:** quark $+$ gluon

  traditional constituent chiral quark model; String Model, Bag Model, ......

# Relations among QCD & Low Energy Hadron Models

- **Ask for the same physics from QCD and low energy hadron model**

- **Build Relations among QCD & low energy hadron models**

$$\mathbf{Z[J]} \equiv \underbrace{\int \mathcal{D}G\mathcal{D}\Psi\mathcal{D}\overline{\Psi}\mathcal{D}q\mathcal{D}\overline{q}\ e^{i\int d^4x[\mathcal{L}_{\mathrm{QCD}}(q,\Psi,G)+F(q,G,J)]}}_{\text{G,q,}\Psi\text{: gluon、light quark and heavy quark fields}} = \underbrace{\int \mathcal{D}\phi_1\ e^{iS_{\mathrm{eff1}}[\phi_1,J]}}_{\phi_1\text{: various fields of model-1}} = \underbrace{\int \mathcal{D}\phi_2\ e^{iS_{\mathrm{eff2}}[\phi_2,J]}}_{\phi_2\text{: various fields of model-2}}$$

- **Build relations among various hadron models**

- **Only perform derivation for pseudo scalar meson part**

unsatisfactory for vector mesons，not successful for baryons！

# Steps of QCD First Principle Derivation

- Integrate out gluon fields $\Rightarrow$ Nonlocal Multi-quark Theory
  - Introduce bilocal meson field $\Rightarrow$ Bilocal Meson Field Theory
  - Introduce bilocal PS meson field $\Rightarrow$ Pseudoscalar Meson EFT
- Drop out 6、8...quark terms $\rightarrow$ GCM
- Localization approximation to GCM $\rightarrow$ NJL Model

Interest as PhD student $\Updownarrow$ know how to derive from each other

- Introduce local meson field $\Rightarrow$ Chiral Quark Model

# 非微扰真空、手征对称性自发破缺与有效相互作用

（申请清华大学理学博士学位论文）

培养单位：清华大学物理系

专　　业：理论物理

研 究 生：王　青

指导教师：张　礼　教授

　　　　　邝宇平　教授

一九八九年五月

TUTP-89/29

DERIVATION OF LOW ENERGY EFFECTIVE ACTION FOR MESONS FROM QCD *

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2. Department of Physics, Tsinghua University, Beijing 100084, China **

3. Institute of Theoretical Physics, Academia Sinica, Beijing, 100080, China

ABSTRACT

Low energy effective action for mesons is derived from the fundamental theory of QCD in different approaches. The obtained effective action is of the form of that in the chiral σ-model with vector and axial-vector mesons included but with fewer free parameters. Some of its consequences are discussed.

* Talk presented by Q.Wang at the Workshop on Weak Interactions and CP Violation, Aug.1989, Inst. High Energy Phys., Academia Sinica, Beijing, China. This work is supported by the National Natural Science Foundation of China

** Mailing Address

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TUIMP-TH-92/48

# AN ATTEMPT TO CALCULATE THE EFFECTIVE LAGRANGIAN FOR LOW LYING PSEUDOSCALAR MESONS FROM QCD STRONG COUPLING EXPANSION[1]

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and

Institute of Morden Physics, Tsinghua University, Beijing 100084, China[2]

and

Institute of Theoretical Physics, Academia Sinica, Beijing 100080, China

ABSTRACT

An attempt to derive the effective Lagrangian for low lying pseudoscalar mesons is given in QCD strong coupling expansion based on the idea of the effective field theory with a physical cut-off Λ.This theory provides the information about chiral symmetry breaking, and the quark condensates are calculable. The obtained effective Lagrangian contains the exact Wess-Zumino-Witten term and the complete Gasser-Leutwyler chiral Lagrangian with all the coefficients $F_0,B_0,L_1,L_2,\cdots,L_{10},H_1$ and $H_2$ given analytically as functions of the two fundamental parameters Λ and $g_s$ (effective coupling constant in the cut-off QCD theory). Λ and $g_s$ are then determined by taking the data of $m_\pi$ and $m_k$ as inputs. Up to order-$1/g_s^2$ contributions, the calculated $m_\eta,F_\pi,F_k,F_\eta$, quark condensates, pion- pion scattering lengths,and pseudoscalar-meson form factors are all in reasonable agreement with experiments.

[1] Work supported by the National Natural Science Foundation of China.

[2] Mailing address

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**Derivation with approximations ?**

# From QCD to NJL type Model present view point

$$\int \mathcal{D}q\mathcal{D}\bar{q}\int \mathcal{D}\Psi\mathcal{D}\bar{\Psi}\mathcal{D}G_\mu\, \Delta_F(G_\mu)\exp\left\{i\int d^4x\,[\bar{q}(i\partial\!\!\!/ + J)q - g\bar{q}\frac{\lambda^\alpha}{2}\gamma^\mu q G^\alpha_\mu]\right\}$$

$$\times\exp\left\{i\int d^4x\Big[-\frac{1}{4}G^\alpha_{\mu\nu}G^{\alpha\mu\nu} - \frac{1}{2\xi}[F^i(G_\mu)]^2 + \bar{\Psi}(i\partial\!\!\!/ - M_h - gG\!\!\!/^\alpha\frac{\lambda^\alpha}{2})\Psi\Big]\right\}$$

$$\overset{\text{Abelian Approx}}{=\!=\!=\!=\!=}\int\mathcal{D}q\mathcal{D}\bar{q}\exp\left\{i\int d^4x\Big[\bar{q}(i\partial\!\!\!/+J)q+\int d^4y\frac{(-i)^2g^2}{2!}G^{\alpha\beta}_{\mu\nu}(x,y)\bar{q}(x)\frac{\lambda^\alpha}{2}\gamma^\mu q(x)\bar{q}(y)\frac{\lambda^\beta}{2}\gamma^\nu q(y)\Big]\right\}$$

$$\overset{G^{\alpha\beta}_{\mu\nu}(x,y)\approx\delta(x-y)g_{\mu\nu}\delta^{\alpha\beta}/\tilde{\Lambda}^2}{=\!=\!=\!=\!=\!=\!=\!=\!=\!=\!=\!=}\int\mathcal{D}q\mathcal{D}\bar{q}\exp\left\{i\int d^4x\Big[\bar{q}(i\partial\!\!\!/+J)q - \frac{g^2}{2\tilde{\Lambda}^2}\underbrace{\bar{q}(x)\frac{\lambda^\alpha}{2}\gamma^\mu q(x)\bar{q}(x)\frac{\lambda^\alpha}{2}\gamma_\mu q(x)}_{I}\Big]\right\}$$

$$I\overset{\text{Fierz}}{=\!=\!=\!=}-\frac{1}{2N_f}(\bar{q}q)^2-\frac{1}{2N_f}(\bar{q}i\gamma_5q)^2+(\frac{1}{4N_f}-\frac{1}{2N_c})\bar{q}\gamma^\mu q\bar{q}\gamma_\mu q+\frac{1}{4N_f}\bar{q}\gamma^\mu\gamma_5q\bar{q}\gamma_\mu\gamma_5q$$
$$-\frac{1}{2}\bar{q}\lambda^a q\bar{q}\lambda^a q-\frac{1}{2}\bar{q}i\gamma_5\lambda^a q\bar{q}i\gamma_5\lambda^a q+\frac{1}{4}\bar{q}\gamma^\mu\lambda^a q\bar{q}\gamma_\mu\lambda^a q+\frac{1}{4}\bar{q}\gamma^\mu\gamma_5\lambda^a q\bar{q}\gamma_\mu\gamma_5\lambda^a q$$

$$G^{\alpha\beta}_{\mu\nu}(x,y)\overset{\text{tree,Feynman gauge}}{=\!=\!=\!=\!=\!=\!=\!=\!=\!=}g_{\mu\nu}\delta^{\alpha\beta}\int\frac{d^4p}{(2\pi)^4}e^{-ip\cdot(x-y)}\frac{-1}{p^2}=\frac{1}{\tilde{\Lambda}^2}g_{\mu\nu}\delta^{\alpha\beta}\delta(x-y)\qquad\left\{\begin{array}{l}\frac{-1}{p^2-\tilde{\Lambda}^2}\\ \frac{-1}{p^2}\underbrace{[1-e^{\frac{p^2}{\tilde{\Lambda}^2}}]}_{\text{high Mom}}\end{array}\right.$$

**Earlier time:** Due to divergence from quark loop，add in mass counter term $\frac{1}{2}\tilde{\Lambda}^2$ to the gluon Lagrangian

PHYSICAL REVIEW D, VOLUME 61, 054011

# Derivation of the effective chiral Lagrangian for pseudoscalar mesons from QCD

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(Received 1 March 1999; published 9 February 2000)

We formally derive the chiral Lagrangian for low lying pseudoscalar mesons from the first principles of QCD considering the contributions from the normal part of the theory without taking an approximation. The derivation is based on the standard generating functional of QCD in the path integral formalism. The gluon-field integration is formally carried out by expressing the result in terms of the physical Green's functions of the gluon. To integrate over the quark field, we introduce a bilocal auxiliary field $\Phi(x,y)$ representing the mesons. We then develop a consistent way of extracting the local pseudoscalar degree of freedom $U(x)$ in $\Phi(x,y)$ and integrating out the rest degrees of freedom such that the complete pseudoscalar degree of freedom resides in $U(x)$. With certain techniques, we work out the explicit $U(x)$ dependence of the effective action up to the $p^4$ terms in the momentum expansion, which leads to the desired chiral Lagrangian in which all the coefficients contributed from the normal part of the theory are expressed in terms of certain quark Green's functions in QCD. Together with the exsisting QCD formulas for the anomaly contributions, the present results lead to the complete effective chiral Lagrangian for pseudoscalar mesons. The final result can be regarded as the fundamental QCD definition of the coefficients in the chiral Lagrangian. The relation between the present QCD definition of the $p^2$-order coefficient $F_0^2$ and the well-known appoximate result given by Pagels and Stokar is discussed.

**Vectors:** X.L.Wang and Q.Wang, Commun. Theor. Phys. 34 ,519(2000);
**Nonet Pseudo Scalars:** X.L.Wang，Z.M.Wang and Q.Wang， Commun. Theor. Phys. 34 ,683(2000);
**General Technicolor:** Z.M.Wang and Q.Wang, Commun. Theor. Phys. 36 ,417(2001)

# Why we take the QCD first principle derivation?

- **Understanding hadron interations in a more fundamental and unified way**
- **Test QCD**
- **Independent of symmetry constraints, easily generalized to other theories**
-  **Compute parameters of hadron models**
- **Technique to Investigate QCD non-perturbative effects in terms of Exp data**

# Low Energy Constants

As a phenomenological Lagrangian, CL has abundance of low energy constants LECs

For Pseudo scalar mesons

- $p^2$ **LECs:** $F$, $B$ **2 flavour**; $F_0$, $B_0$ **3 flavour**
- $p^4$ **LECs:** $l_1, \cdots, l_7$, $h_1, h_2, h_3$ **2 flavour**; $L_1, \cdots, L_{10}$, $H_1, H_2$ **3 flavour**
- $p^6$ **LECs (normal):** $c_1, \cdots, c_{52}$, $+4$ **2 flavour**; $C_1, \cdots, C_{90}$, $+4$ **3 flavour**
- $p^6$ **LECs (anomalous):** $c_1, \cdots, c_5$, $+8$ **2 flavour**; $C_1, \cdots, C_{23}$ **3 flavour**

Number of LECs increase rapidly for high order terms !

- **Up to $p^6$, lack of enough exp data to fix all LECs**
- **To high orders, CL lost prediction power !**

**Calculation of LECs will improve prediction power !**

# Anomaly Approach

**Anomaly approach: Quark flavor anomaly can induce Chiral Lagrangian !**

$$S_{\rm eff}\Big|_{\text{anomaly approach}} = -i\ln\frac{\int\mathcal{D}\psi_\Omega\mathcal{D}\overline{\psi}_\Omega\ e^{i\int d^4x\overline{\psi}(i\partial\!\!\!/+J)\psi}}{\int\mathcal{D}\psi\mathcal{D}\overline{\psi}\ e^{i\int d^4x\overline{\psi}(i\partial\!\!\!/+J)\psi}} = i\ln\frac{\int\mathcal{D}\psi\mathcal{D}\overline{\psi}\ e^{i\int d^4x\overline{\psi}(i\partial\!\!\!/+J_\Omega)\psi}}{\int\mathcal{D}\psi\mathcal{D}\overline{\psi}\ e^{i\int d^4x\overline{\psi}(i\partial\!\!\!/+J)\psi}}$$

$$= -i\ln\mathrm{Det}[i\partial\!\!\!/ + J_\Omega] + i\ln\mathrm{Det}[i\partial\!\!\!/ + J]$$

Lead WZW anomaly terms and **nonzero CL LECs:** $8L_1=4L_2=-2L_3\underbrace{=24L_7=-8L_8}_{\text{wrong signs}}=L_9=-2L_{10}=\frac{N_c}{48\pi^2}$

But:

- **Difficult to obtain SCSB** $F_0^2\propto 0,\ \Lambda^2$
- **Non trivial LECs** CL$\neq 0$ **in absence of color interaction** $\alpha_s=0$
- **Some $p^4$ order coefficients have wrong signs**
- **No or divergent higher order terms** J.F.Donoghue,D.Wyler;J.Bijnens, A.Bramon,F.Cornet;R.Akhoury,A.Alfakih; H.W.Fearing,S.Scherer;J.Bijnens,L.Girlanda,P.Talavera; T.Ebertshauser,H.W.Fearing,S.Scherer

♣ $\boxed{\text{Re } S_{\rm eff}\Big|_{\text{anomaly approach}}=0}$

PHYSICAL REVIEW D **66**, 014019 (2002)

# Calculation of the chiral Lagrangian coefficients from the underlying theory of QCD: A simple approach

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We calculate the coefficients in the chiral Lagrangian approximately from QCD based on a previous study of deriving the chiral Lagrangian from the first principles of QCD in which the chiral Lagrangian coefficients are defined in terms of certain Green's functions in QCD. We first show that, in the large-$N_c$ limit, the anomaly part contributions to the coefficients are exactly cancelled by certain terms in the normal part contributions, and the final results of the coefficients only concern the remaining normal part contributions depending on QCD interactions. We then do the calculation in a simple approach with the approximations of taking the large-$N_c$ limit, the leading order in dynamical perturbation theory, and the improved ladder approximation; thereby the relevant Green's functions are expressed in terms of the quark self-energy $\Sigma(p^2)$. By solving the Schwinger-Dyson equation for $\Sigma(p^2)$, we obtain the approximate QCD predicted coefficients and quark condensate which are consistent with the experimental values.

DOI: 10.1103/PhysRevD.66.014019 PACS number(s): 12.39.Fe, 11.30.Rd, 12.38.Aw, 12.38.Lg

**Fermion determinant:** Q.Lu, H.Yang, Q.Wang, Commun. Theor. Phys. 38, 185(2002)

**Phenomenological gauge invariant,nonlocal, dynamical quark model:** H.Yang, Q.Wang, Q.Lu, Phys.Lett. B532, 240 (2002)

**Calculation formalism: Y.H.An, H.Yang, Q.Wang, Eur. Phys. J. C29, 65(2003)**

**The LECs are in units of** $10^{-3}\mathrm{GeV}^{-2}$

| $i$ | $C_i$ | $j$ | $c_j$ | $i$ | $C_i$ | $j$ | $c_j$ | $i$ | $C_i$ | $j$ | $c_j$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | $3.79^{+0.10}_{-0.17}$ | 1 | $3.58^{+0.09}_{-0.15}$ | 31 | $-0.63^{+0.05}_{-0.09}$ | 17 | $-1.10^{+0.12}_{-0.19}$ | 61 | $2.88^{-0.22}_{+0.26}$ | 34 | $2.84^{-0.22}_{+0.26}$ |
| 2 | $0.00^{+.00}_{-0.00}$ | | | 32 | $0.18^{-0.03}_{+0.04}$ | 18 | $0.43^{-0.07}_{+0.08}$ | 62 | $0.00^{+0.00}_{-0.00}$ | | |
| 3 | $-0.05^{+0.01}_{-0.01}$ | 2 | $-0.03^{+0.01}_{-0.01}$ | 33 | $0.09^{-0.00}_{+0.03}$ | 19 | $0.41^{-0.06}_{+0.10}$ | 63 | $2.99^{-0.24}_{+0.30}$ | | |
| 4 | $3.10^{+0.09}_{-0.15}$ | 3 | $2.89^{+0.08}_{-0.13}$ | 34 | $1.59^{-0.10}_{+0.16}$ | 20 | $1.56^{-0.10}_{+0.17}$ | 64 | $0.00^{+0.00}_{-0.00}$ | | |
| 5 | $-1.01^{+0.08}_{-0.11}$ | 4 | $1.21^{-0.07}_{+0.06}$ | 35 | $0.17^{-0.12}_{+0.17}$ | 21 | $0.29^{-0.18}_{+0.24}$ | 65 | $-2.43^{+0.15}_{-0.16}$ | 35 | $3.39^{-0.32}_{+0.41}$ |
| 6 | $0.00^{+0.00}_{-0.00}$ | | | 36 | $0.00^{+0.00}_{-0.00}$ | | | 66 | $1.71^{+0.07}_{-0.12}$ | 36 | $1.57^{+0.06}_{-0.10}$ |
| 7 | $0.00^{+0.00}_{-0.00}$ | | | 37 | $-0.56^{+0.09}_{-0.11}$ | | | 67 | $0.00^{+0.00}_{-0.00}$ | | |
| 8 | $2.31^{-0.16}_{+0.18}$ | | | 38 | $0.41^{-0.08}_{+0.07}$ | 22 | $-1.32^{+0.18}_{-0.25}$ | 68 | $0.00^{+0.00}_{-0.00}$ | | |
| 9 | $0.00^{+0.00}_{-0.00}$ | | | 39 | $0.00^{+0.00}_{-0.00}$ | 23 | $0.86^{-0.12}_{+0.15}$ | 69 | $-0.86^{-0.04}_{+0.06}$ | 38 | $-0.68^{-0.03}_{+0.05}$ |
| 10 | $-1.05^{+0.08}_{-0.09}$ | 5 | $-0.98^{+0.07}_{-0.09}$ | 40 | $-6.35^{-0.18}_{+0.32}$ | 24 | $-4.84^{-0.14}_{+0.25}$ | 70 | $1.73^{-0.08}_{+0.07}$ | 39 | $1.81^{-0.08}_{+0.07}$ |
| 11 | $0.00^{+0.00}_{-0.00}$ | | | 41 | $0.00^{+0.00}_{-0.00}$ | | | 71 | $0.00^{+0.00}_{-0.00}$ | | |
| 12 | $-0.34^{+0.02}_{-0.01}$ | 6 | $-0.33^{+0.01}_{-0.01}$ | 42 | $0.60^{+0.00}_{-0.00}$ | | | 72 | $-3.30^{+0.05}_{-0.00}$ | 40 | $-3.17^{+0.05}_{-0.02}$ |
| 13 | $0.00^{+0.00}_{-0.00}$ | | | 43 | $0.00^{+0.00}_{-0.00}$ | | | 73 | $0.50^{+0.43}_{-0.56}$ | 41 | $0.30^{+0.42}_{-0.54}$ |
| 14 | $-0.83^{+0.12}_{-0.19}$ | 7 | $-1.72^{+0.25}_{-0.35}$ | 44 | $6.32^{+0.20}_{-0.36}$ | 25 | $6.03^{+0.19}_{-0.33}$ | 74 | $-5.07^{-0.16}_{+0.27}$ | 42 | $-4.74^{-0.14}_{+0.24}$ |
| 15 | $0.00^{+0.00}_{-0.00}$ | 8 | $0.86^{-0.12}_{+0.15}$ | 45 | $0.00^{+0.00}_{-0.00}$ | | | 75 | $0.00^{+0.00}_{+0.00}$ | | |

**Serial Numbers: J. Bijnens, G. Colangelo, G. Ecker, JHEP02(1999)020**

**S. Z. Jiang, Y Zhang, C. Li and Q. Wang, PRD81, 014001(2010)**

**The LECs are in units of $10^{-3}\mathrm{GeV}^{-2}$**

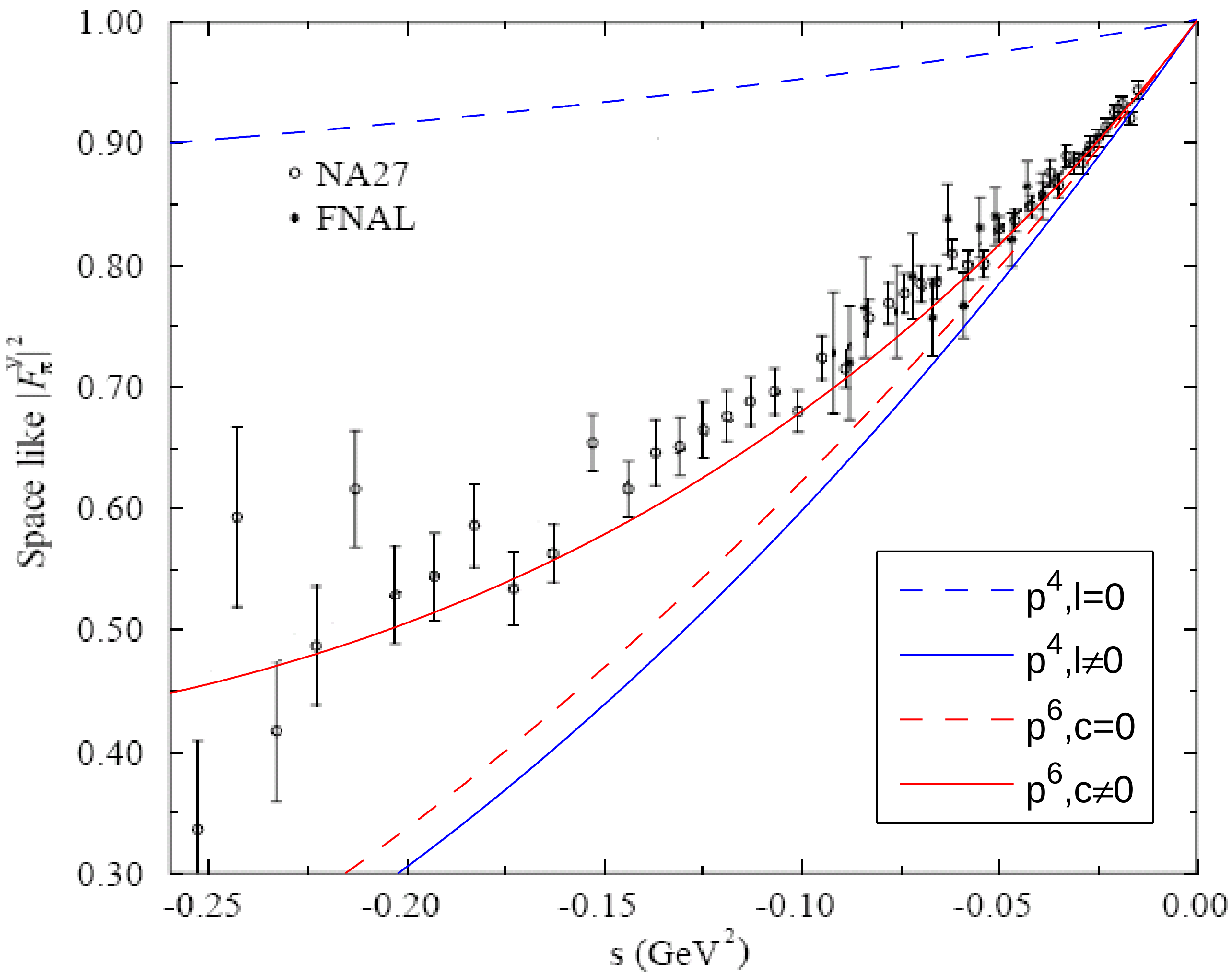
| $i$ | $C_i$ | $j$ | $c_j$ | $i$ | $C_i$ | $j$ | $c_j$ | $i$ | $C_i$ | $j$ | $c_j$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 16 | $0.00^{+0.00}_{-0.00}$ | | | 46 | $-0.60^{-0.02}_{+0.04}$ | 26 | $-1.14^{-0.05}_{+0.07}$ | 76 | $-1.44^{-0.23}_{+0.31}$ | 43 | $-1.29^{-0.23}_{+0.30}$ |
| 17 | $0.01^{-0.01}_{-0.01}$ | 9 | $-0.84^{+0.12}_{-0.17}$ | 47 | $0.08^{+0.01}_{-0.00}$ | | | 77 | $0.00^{+0.00}_{-0.00}$ | | |
| 18 | $-0.56^{+0.09}_{-0.11}$ | | | 48 | $3.41^{+0.06}_{-0.10}$ | | | 78 | $17.51^{+1.02}_{-1.59}$ | 44 | $16.16^{+0.94}_{-1.45}$ |
| 19 | $-0.48^{+0.09}_{-0.13}$ | 10 | $-0.37^{+0.07}_{-0.10}$ | 49 | $0.00^{+0.00}_{-0.00}$ | | | 79 | $-0.56^{-0.30}_{+0.40}$ | 45 | $0.26^{-0.26}_{+0.34}$ |
| 20 | $0.18^{-0.03}_{+0.04}$ | 11 | $0.00^{+0.00}_{-0.00}$ | 50 | $8.71^{+0.78}_{-1.12}$ | 27 | $13.57^{+1.41}_{-2.00}$ | 80 | $0.87^{-0.04}_{+0.03}$ | 46 | $0.85^{-0.04}_{+0.02}$ |
| 21 | $-0.06^{+0.01}_{-0.01}$ | | | 51 | $-11.49^{+0.18}_{-0.09}$ | 28 | $0.93^{+0.98}_{-1.25}$ | 81 | $0.00^{+0.00}_{-0.00}$ | | |
| 22 | $0.27^{+0.19}_{-0.25}$ | 12 | $0.15^{+0.18}_{-0.24}$ | 52 | $-5.04^{-0.67}_{+0.93}$ | | | 82 | $-7.13^{-0.32}_{+0.51}$ | 47 | $-6.73^{-0.29}_{+0.47}$ |
| 23 | $0.00^{+0.00}_{-0.00}$ | | | 53 | $-11.99^{-0.87}_{+1.33}$ | 29 | $-11.01^{-0.81}_{+1.23}$ | 83 | $0.07^{+0.20}_{-0.27}$ | 48 | $-0.22^{+0.18}_{-0.25}$ |
| 24 | $1.62^{+0.04}_{-0.07}$ | | | 54 | $0.00^{+0.00}_{-0.00}$ | | | 84 | $0.00^{+0.00}_{-0.00}$ | | |
| 25 | $-5.98^{-0.49}_{+0.72}$ | 13 | $-5.39^{-0.45}_{+0.66}$ | 55 | $16.79^{+0.96}_{-1.49}$ | 30 | $15.72^{+0.89}_{-1.38}$ | 85 | $-0.82^{+0.03}_{-0.02}$ | 49 | $-0.78^{+0.03}_{-0.01}$ |
| 26 | $3.35^{+0.29}_{-0.47}$ | 14 | $4.17^{+0.30}_{-0.49}$ | 56 | $19.34^{+0.52}_{-0.98}$ | 31 | $17.57^{+0.42}_{-0.82}$ | 86 | $0.00^{+0.00}_{-0.00}$ | | |
| 27 | $-1.54^{+0.15}_{-0.18}$ | 15 | $-2.71^{+0.21}_{-0.25}$ | 57 | $7.92^{+1.34}_{-1.85}$ | 32 | $7.18^{+1.28}_{-1.76}$ | 87 | $7.57^{+0.37}_{-0.60}$ | 50 | $7.18^{+0.34}_{-0.55}$ |
| 28 | $0.30^{+0.01}_{-0.01}$ | | | 58 | $0.00^{+0.00}_{-0.00}$ | | | 88 | $-5.47^{-0.73}_{+1.03}$ | 51 | $-4.85^{-0.69}_{+0.97}$ |
| 29 | $-3.08^{-0.26}_{+0.32}$ | 16 | $-2.22^{-0.22}_{+0.27}$ | 59 | $-22.49^{-1.21}_{+1.89}$ | 33 | $-21.19^{-1.12}_{+1.76}$ | 89 | $34.74^{+1.61}_{-2.62}$ | 52 | $32.19^{+1.46}_{-2.37}$ |
| 30 | $0.60^{+0.02}_{-0.03}$ | | | 60 | $0.00^{+0.00}_{-0.00}$ | | | 90 | $2.44^{-0.38}_{+0.46}$ | 53 | $2.51^{-0.37}_{+0.46}$ |

**Serial Numbers: J. Bijnens, G. Colangelo, G. Ecker, JHEP02(1999)020**

**S. Z. Jiang, Y Zhang, C. Li and Q. Wang, PRD81, 014001(2010)**

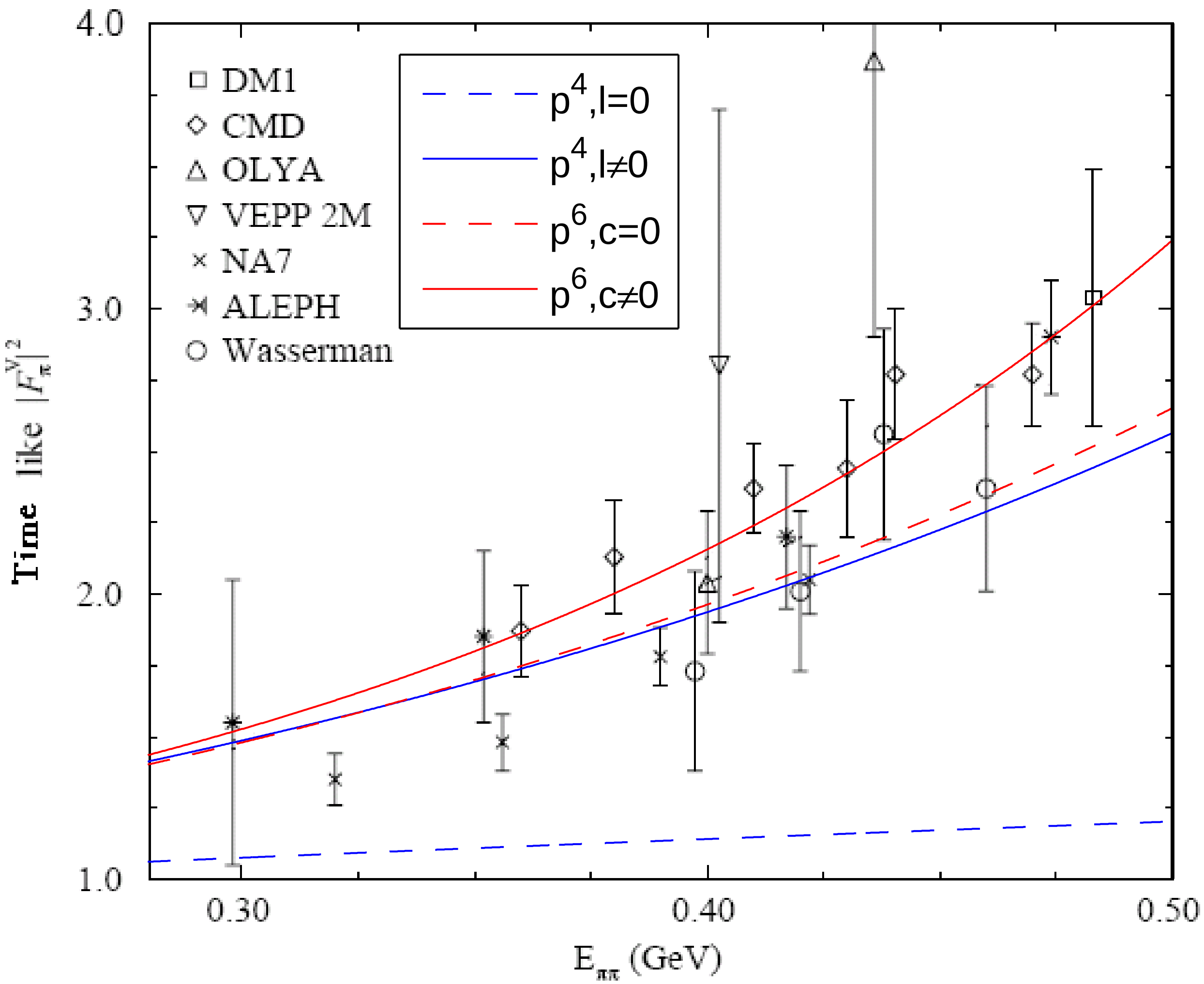
# Vector Form Factor (space-like)

$$\langle\pi^i(p_2)|\frac{1}{2}(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d)|\pi^j(p_1)\rangle = i\epsilon^{i3j}(p_{1\mu}+p_{2\mu})F_V[(p_2-p_1)^2]$$



# Vector Form Factor (time-like)

$$\langle \pi^i(p_2)|\frac{1}{2}(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d)|\pi^j(p_1)\rangle = i\epsilon^{i3j}(p_{1\mu}+p_{2\mu})F_V[(p_2-p_1)^2]$$



# Calculating Anomalous Part LECs

$$S_{\rm eff}[U,J] \overset{\text{Euclidean Space}}{=\!=\!=\!=\!=\!=} \mathrm{Tr}\ln[i\partial\!\!\!/ + J_{\Omega(t)} + \Sigma(-\nabla_t^2)] = \int_0^1 dt\ \frac{d}{dt}\mathrm{Tr}\ln[i\partial\!\!\!/ + J_{\Omega(t)} + \Sigma(-\nabla_t^2)]\Big|_{\Sigma\ \text{dep}}$$

$$= \int_0^1 dt\ \mathrm{Tr}\Big[[\frac{\partial J_{\Omega(t)}}{\partial t} + \frac{\partial\Sigma(-\nabla_t^2)}{\partial t}][i\partial\!\!\!/ + J_{\Omega(t)} + \Sigma(-\nabla_t^2)]^{-1}\Big]_{\Sigma\ \text{dependent}}$$

$$= \int_0^1 dt\ \mathrm{Tr}\Big[\Big([\frac{\partial U_t}{\partial t}U_t^\dagger\gamma_5, \partial\!\!\!/ + J_{\Omega(t)}]_+ + \frac{\partial\Sigma(-\nabla_t^2)}{\partial t}\Big)[i\partial\!\!\!/ + J_{\Omega(t)} + \Sigma(-\nabla_t^2)]^{-1}\Big]_{\Sigma\ \text{dependent}}$$

$$J_{\Omega(t)}(x) = -i v\!\!\!/_{\Omega(t)}(x) - i a\!\!\!/_{\Omega(t)}(x)\gamma_5 - s_{\Omega(t)}(x) + i p_{\Omega(t)}(x)\gamma_5$$

$$S_{\rm eff}[U,J]\Big|_{\mathbf{a},p^4} = \frac{1}{32\pi^2}\epsilon_{\mu\nu\alpha\beta}\int d^4x \int_0^1 dt\ \mathrm{tr}_f\Big[\frac{\partial U_t}{\partial t}U_t^\dagger\ \Big(V_t^{\mu\nu}V_t^{\alpha\beta} + \frac{2i}{3}\{a_t^\mu a_t^\nu, V_t^{\alpha\beta}\}$$
$$+\frac{4}{3}d_t^\mu a_t^\nu d_t^\alpha a_t^\beta + \frac{8i}{3}a_t^\mu V_t^{\nu\alpha}a_t^\beta + \frac{4}{3}a_t^\mu a_t^\nu a_t^\alpha a_t^\beta\Big)\Big]$$

**Y. L. Ma and Q. Wang, PLB560, 188(2003)**

## Finish the integration of fifth dimension

**G. C. Rossi, M. Testa and K. Yoshida, PLB134, 78(1984); N. K. Pak and P. Rossi, NPB250, 279(1985)**

# Anomalous $p^6$ terms – three flavors

**unit:** $10^{-3}\mathrm{GeV}^{-2}$

| $n$ | $C_n^W$ ours | arXiv:0302064[hep-ph](I) | (II) | (III) | (IV) | (V) | EPJC55,273(2008) |
|---|---|---|---|---|---|---|---|
| 1 | $4.97^{+0.55}_{-0.79}$ | | | | | | PRL89,061803(2002 |
| 2 | $-1.43^{+0.10}_{-0.12}$ | $-0.32 \pm 10.4$ | | $0.78 \pm 12.7$ | $4.96 \pm 9.70$ | $-0.074 \pm 13.3$ | JHEP05,052,273(200 |
| 3 | $\equiv 0$ | | | | | | |
| 4 | $-0.96^{+0.22}_{-0.29}$ | $0.28 \pm 9.19$ | | $0.67 \pm 10.9$ | $6.32 \pm 6.09$ | $-0.55 \pm 9.05$ | PRD79,076005(2009 |
| 5 | $3.26^{+0.34}_{-0.49}$ | $28.50 \pm 28.83$ | | $9.38 \pm 152.2$ | $33.05 \pm 28.66$ | $34.51 \pm 41.13$ | |
| 6 | $0.91^{+0.03}_{-0.04}$ | | | | | | |
| 7 | $1.68^{-0.24}_{+0.31}$ | $0.013 \pm 1.17$ | | | $0.51 \pm 0.06$ | | $0.1 \pm 1.2$ |
| | | $20.3 \pm 18.7$ | | | | | $0.1^*$ |
| 8 | $0.41^{+0.01}_{-0.02}$ | $0.76 \pm 0.18$ | | | | | $0.58 \pm 0.20$ |
| | | | | | | | $0.5^*$ |
| 9 | $1.15^{-0.03}_{+0.03}$ | | | | | | |
| 10 | $-0.18^{-0.01}_{+0.01}$ | | | | | | |
| 11 | $-1.15^{+0.08}_{-0.10}$ | $-6.37 \pm 4.54$ | | | $-0.00143 \pm 0.03$ | | $0.68 \pm 0.21$ |
| 12 | $-5.13^{-0.15}_{+0.25}$ | | | | | | |
| 13 | $-6.37^{-0.18}_{+0.31}$ | $-74.09 \pm 55.89$ | $-20.00$ | $-8.44 \pm 69.9$ | $14.15 \pm 15.22$ | $-7.46 \pm 19.62$ | |

**Serial Numbers: J. Bijnens, L. Girlanda, P. Talavera, Eur. Phys. J. C23,539(2002)**

**S. Z. Jiang and Q. Wang, PRD81, 094037(2010)**

# Anomalous $p^6$ terms – three flavors

**unit:** $10^{-3}\mathrm{GeV}^{-2}$

| $n$ | $C_n^W$ **ours** | **arXiv:0302064[hep-ph](I)** | **(II)** | **(III)** | **(IV)** | **(V)** | **EPJC55,273(2008)** |
|---|---|---|---|---|---|---|---|
| 14 | $-2.00^{-0.06}_{+0.10}$ | $29.99 \pm 11.14$ | $-6.01$ | $0.72 \pm 15.3$ | $10.23 \pm 7.56$ | $-0.58 \pm 9.77$ | |
| 15 | $4.17^{+0.12}_{-0.20}$ | $-25.30 \pm 23.93$ | $2.00$ | $-3.10 \pm 28.6$ | $19.70 \pm 7.49$ | $8.89 \pm 9.72$ | |
| 16 | $3.58^{+0.10}_{-0.17}$ | | | | | | |
| 17 | $1.98^{+0.06}_{-0.10}$ | | | | | | |
| 18 | $\equiv 0$ | | | | | | |
| 19 | $0.29^{+0.01}_{-0.01}$ | | | | | | |
| 20 | $1.83^{+0.05}_{-0.09}$ | | | | | | |
| 21 | $2.48^{+0.07}_{-0.12}$ | | | | | | |
| 22 | $5.01^{+0.14}_{-0.24}$ | $6.52 \pm 0.78$ | $8.01$ | | $3.94 \pm 0.43$ | | $5.4 \pm 0.8$ |
| | | $5.07 \pm 0.71$ | | | $3.94 \pm 0.43$ | | |
| 23 | $2.74^{+0.08}_{-0.13}$ | | | | | | |

**Serial Numbers: J. Bijnens, L. Girlanda, P. Talavera, Eur. Phys. J. C23,539(2002)**

**S. Z. Jiang and Q. Wang, PRD81, 094037(2010)**

Tsinghua University

TUHEP-TH-12176

# Computation of the $p^6$ order low-energy constants with tensor sources

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We present the results of computing the $p^4$ and $p^6$ order low-energy constants of the chiral Lagrangian with tensor sources for both two and three flavors pseudoscalar mesons. This is a generalization of our previous work on calculating the $p^4$ and $p^6$ order coefficients of the chiral Lagrangian without tensor sources in terms of the quark self-energy $\Sigma(p^2)$. We find that some $p^6$ order operators with tenor sources used in the literature are related to each other with the help of some epsilon relations. There leaves 100 independent terms for $n$-flavor, 94 terms for 3-flavor, and 67 terms for 2-flavor cases. We also find that the odd-intrinsic-parity chiral Lagrangian with tensor sources can not exist.

PACS numbers: 12.39.Fe, 11.30.Rd, 12.38.Aw, 12.38.Lg

arXiv:1203.0712v1 [hep-ph] 4 Mar 2012

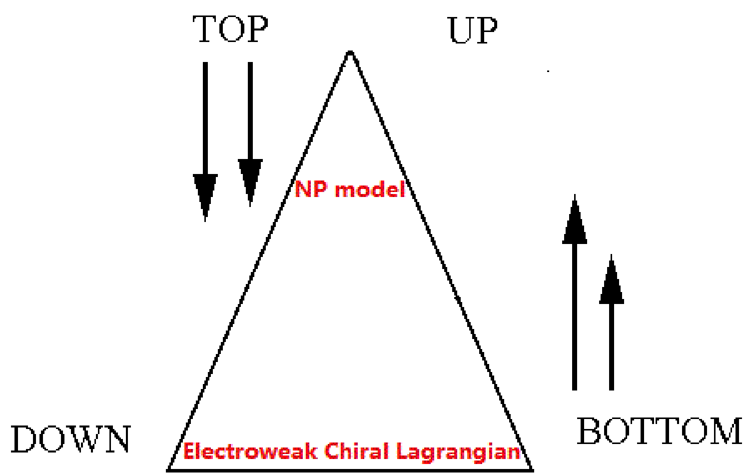
## I. INTRODUCTION

In the low-energy region, conventional perturbation theory is ineffective for the strong interaction. If we focus on the pseudoscalar mesons $(\pi, K, \eta)$, chiral perturbation theory provides us an effective way to deal with the system. It can be applied not only to the strong interaction, but also to the weak and electromagnetic interactions. It was first introduced by Weinberg [1]. The idea was to expand the meson part Lagrangian in terms of powers of external momenta. Then, Gasser and Leutwyler [2, 3] extended it to the $p^4$ order, and built up the path integral formalism which enables us to compute the various Green's functions of the light-quark scalar, pseudoscalar, vector and axial
vector curr ... later. The form of
the normal ... pon the anomalous
(or odd pa ... the antisymmetric
tensor curr ... his may be partly
due to the ... scussed in Ref.[10],
researches ... currents. Further,
antisymme ... e more exotic $1^{+-}$
mesons. T ... nteractions. More
importantl ... egrees of freedoms
and ones u ... plies that $\Gamma$ can be
expanded in terms of the 16 $\gamma$-matrices, and one is used to name the currents according to their $\gamma$-matrices structures.
Due to this incompleteness of the $\gamma$-matrices structures, just scalar, pseudo scalar, vector and axial vector currents

**Further improvements :**

- **Increase the precision of the calculations**
- **Try to go beyond low energy expansion**

# Apply First Principle Calculation to




# Electroweak Chiral Lagrangian

# Existing TC2 Models

- **C: Topcolor assisted tecnicolor** C.T.Hill, Phys.Lett.B345(1995)483 formulation
  H.H.Zhang, S.Z.Jiang, J.Y.Lang, Q.Wang, Phys.Rev.D77(2008)055003

- **C: Natural Topcolor-assisted tecnicolor** K.Lane,E.Eichten, Phys.Lett.B352(1995)382 ETC
  J.Y.Lang, S.Z.Jiang, Q.Wang, Phys.Rev.D79,(2009)015002

- **F: New strong interactions at the Tevatron?**
  R.S.Chivukula, A.G.Cohen, E.H.Simmons, Phys.Lett.B380(1996)92

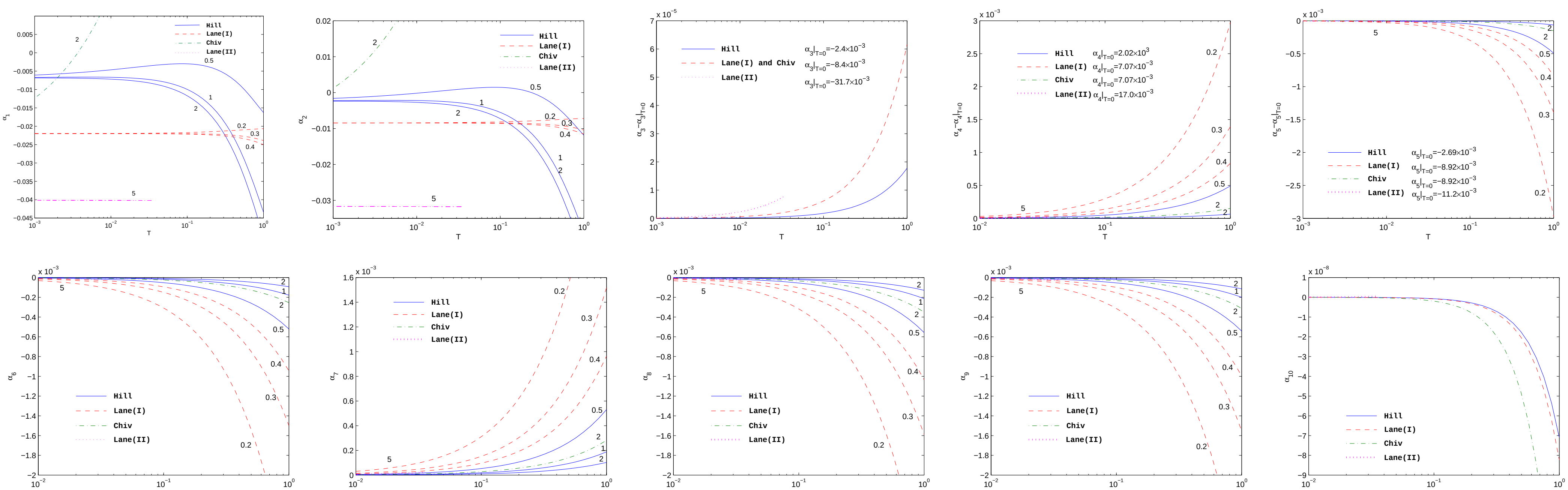
- **C: Symmetry breaking and generational mixing in top-color-assisted technicolor**
  K.Lane, Phys.Rev.D54(1996)2204 Walking effects F.J.Ge, S.Z.Jiang, Q.Wang, Phys.Rev.D84(2011)015009

- **F: A heavy top quark from flavor-universal colorons**
  M.B.Popovic, E.H.Simmons, Phys.Rev.D58(1998)095007

- **F: A new model of topcolor-assisted technicolor**
  K.Lane, Phys.Lett.B433(1998)96

- **H: Hypercharge-universal topcolor** F.Braam, M.Flossdorf, R.S.Chivukula, S.D.Chiara, E.H.Simmons, Phys.Rev.D77,(2008)055005 Hypercharge effects J.Y.Lang,S.Z.Jiang,Q.Wang, Phys.Lett.B673(2009)63

| Property or LEC | Schematic TC2[1] | Natural TC2 | Hypercharge Universal | Walking TC2 |
|---|---|---|---|---|
| Upper bound of $M_{Z'}$ | $\surd$ | $\surd$ | $\surd$ | $\times$ |
| Negative $S$ | $M_{Z'}<0.44$**TeV** or $T>0.17$ | $\times$ | $T\geq 10^{-1}$ | choose hypercharges |
| Typical $S=-16\pi\alpha_1$ | $\sim 0.3$ | $\sim 0.8$ | $\sim 1$ | $\sim 2$ |
| $\alpha_2$ | $-10^{-3}$ | $-10^{-3}$ | $-10^{-3}$ | $-10^{-2}$ |
| $\alpha_3$ | $-10^{-3}$ | $3\times$ result of [1] | $-10^{-3}$ | $-10^{-2}$ |
| $\alpha_4$ | $10^{-3}$ | $3\times$ result of [1] | $10^{-3}$ | $10^{-2}$ |
| $\alpha_5$ | $-10^{-3}$ | $3\times$ result of [1] | $-10^{-3}$ | $-10^{-2}$ |
| $\alpha_6$ | $\sim -10^{-4}$ | $\sim -10^{-3}$ | $\sim -10^{-4}$ | $\sim -10^{-5}$ |
| $\alpha_7$ | $\sim 10^{-4}$ | $\sim 10^{-3}$ | $\sim 10^{-4}$ | $\sim 10^{-5}$ |
| $\alpha_8=-\frac{U}{16\pi}$ | $\sim -10^{-4}$ | $3\times$ result of [1] | $\sim -10^{-4}$ | $\sim -10^{-5}$ |
| $\alpha_9$ | $\sim -10^{-4}$ | $3\times$ result of [1] | $\sim -10^{-4}$ | $\sim -10^{-5}$ |
| $\alpha_{10}$ | $\sim -10^{-8}$ | $\sim -10^{-8}$ | $\sim -10^{-7}$ | $\sim 10^{-10}$ |



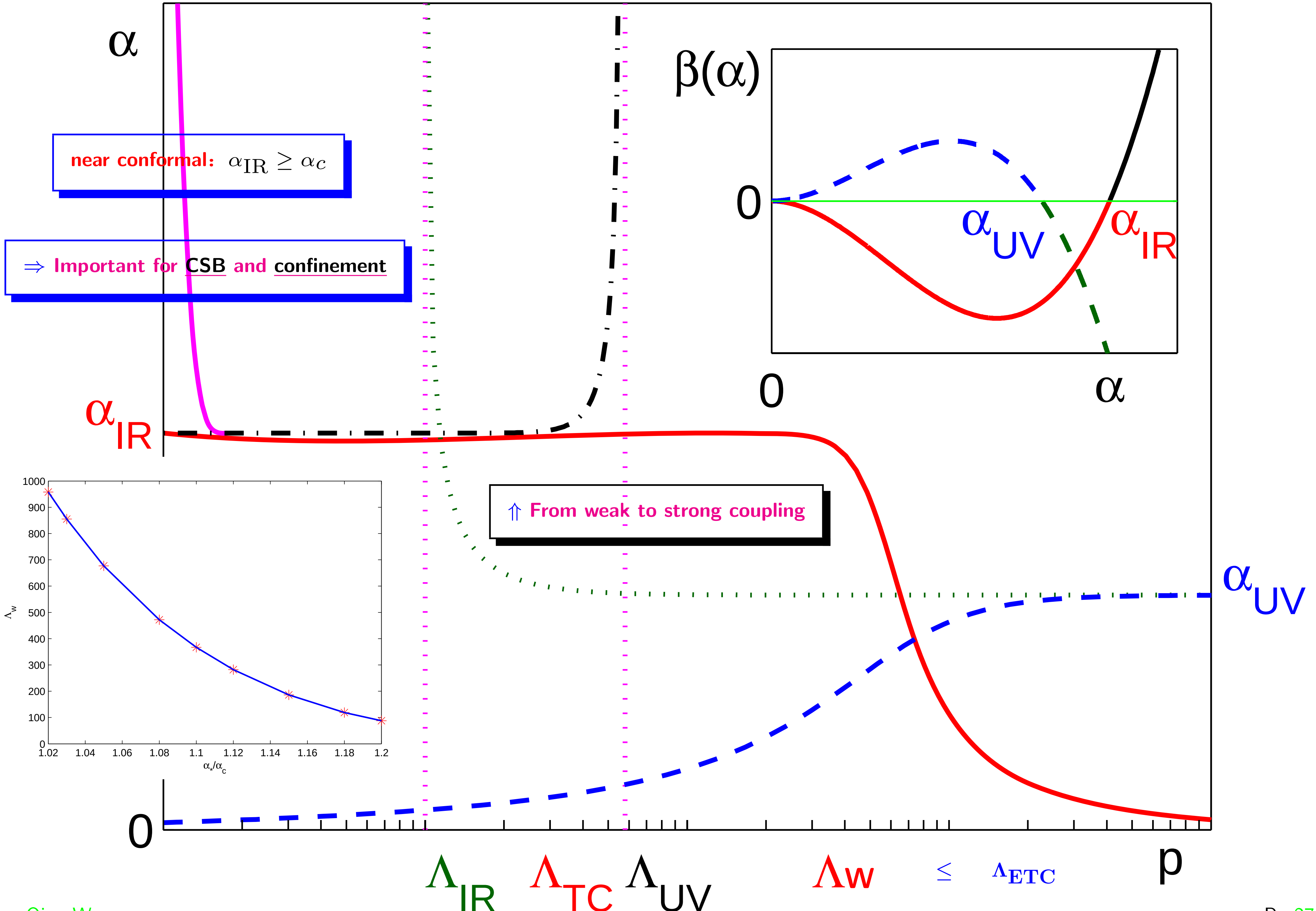
# TC2 contribution to EWCL coefficients

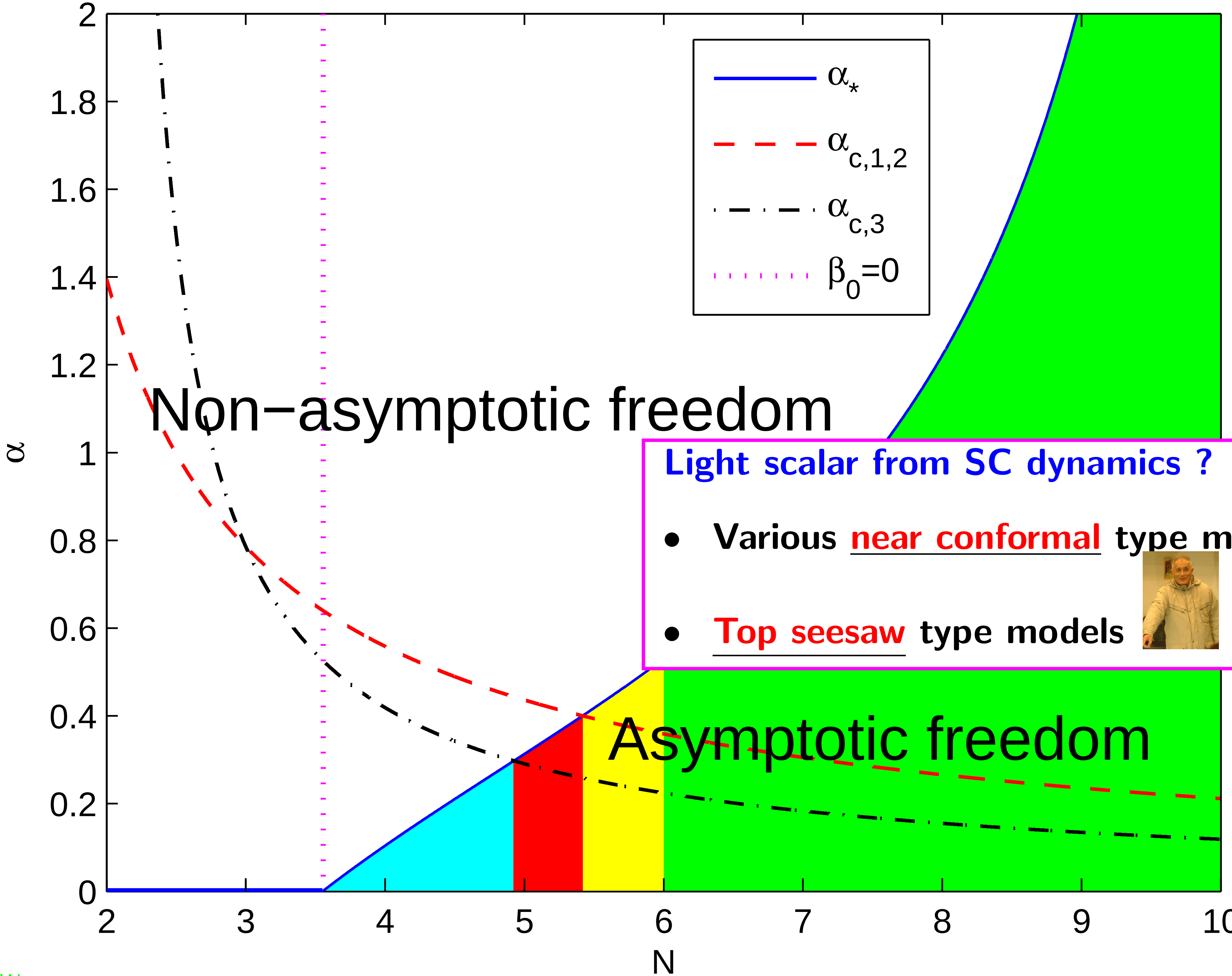
- Pure TC contributions one or three doublet TC model result
- $Z'$ contributions
- Ordinary quarks and leptons contributions not investigated
- Colorons make no contributions at the leading order
- ETC effects are small
- WTC effects  different understandings
- $\alpha T$ is positive and bounded above: $\frac{1}{25}$ Hill $\frac{9}{40}$ Lane $\frac{9}{34}$ Chivukula

near conformal: $\alpha_{\rm IR} \geq \alpha_c$

$\Rightarrow$ Important for CSB and confinement

$\Uparrow$ From weak to strong coupling






**Light scalar from SC dynamics ?**

- **Various near conformal type models**
- **Top seesaw type models**

# Summary

- First principle derivation of QCD is achieved without approximation
- With approximations, we have calculated LECs up to $p^6$
- The technique is applied to electroweak chiral Lagrangian
- Improvement on the calculation precision is under consideration
- Prof. Kuang plays key role in guiding the direction of research

# Thanks !