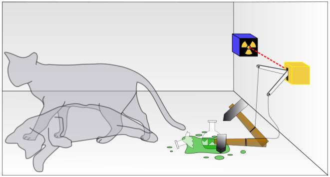
Experimental Tests of Quantum Mechanics

Michael Schmelling – MPI for Nuclear Physics
e-Mail: Michael.Schmelling@mpi-hd.mpg.de

Outline

- *Basics*
- *Bell's Inequalities*
- *Early Experiments*
- *Parametric Down-Conversion*
- *Advanced Topics*
- *Summary*



Φ 1. BASICS

➜ heated (philosophical) discussions

- relation between QM and the understanding of nature
  - ➜ mathematical structure: (mostly) accepted
  - ➜ core issue: interpretation of QM
    - ✘ causality and chance
    - ✘ relation to classical physics
- persona remarks
  - ➜ classical picture: causality in space and time
  - ➜ QM: (perhaps) theorie regarding information
    - ✘ information is intrinsically quantized
    - ✘ relevant for sufficiently small physical systems
    - ✘ QM was developed when atomic scales became accessible
  - ➜ general observation
    - ✘ information given: classical behavior
    - ✘ information missing: chance

❖ in the following:

Try to get a better understanding from comparing theory and experiment

➜ some Nobelprize awarded results. . .

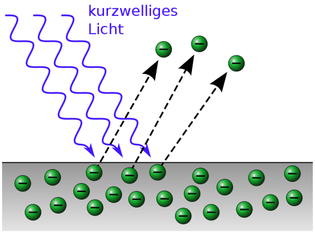
➜ Max Planck

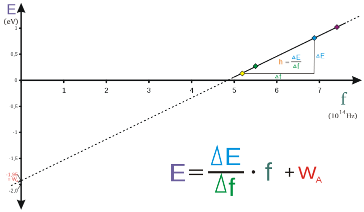
- discovery of the quantization of action $h \neq 0$
  - ➜ energy of light-waves is continuous
  - ➜ interaction with matter is quantized

$$E = h\,\nu$$

(Nobelprize 1918)

❖ experiment: photo-effect





(explanation by Einstein, Nobelprize 1921)

Φ *Historical milestones (ii)*

➜ Albert Einstein

- mass and energy are equivalent

$$E = m\,c^2$$

➜ De Broglie

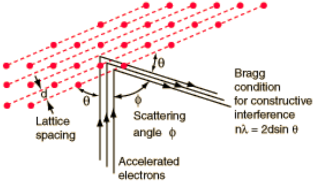
- matter has wave-like properties, too
- particles cannot be perfectly localized

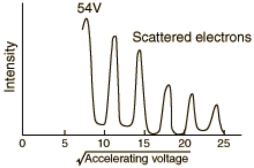
$$E = h\,\nu = m\,c^2 = (mc)\,c = p\,c$$

$$\nu = \frac{c}{\lambda}$$

Resultat ➜ $$\lambda = \frac{h}{p}$$

(Nobelprize 1929)





❖ experiment: Davisson-Germer experiment

- diffraction in electron-scattering off crystals
- same phenomenology as X-ray scattering

(Nobelprize 1937)

Φ Formal description

Physics	$\Longleftrightarrow$	Mathematics
state of a system		**normalized wavefunction** $\mid \psi \rangle$
observable S		**hermitian operator** S
Measurement		**Eigenvalue und Eigenfunction**

➜ discussion:

- $\mid \psi \rangle$ is element of a linear vector space: wavefunctions can be linearly superposed and it exsist an inner product. The normalization is $\langle \psi \mid \psi \rangle$=1.
- On the linear space of the wavefunctions S is a matrix with real eigenvalues λ_k and an ortho-normalsystem of eigenvectors $\mid \phi_k \rangle$, i.e. $\langle \phi_k \mid \phi_l \rangle = \delta_{kl}$.
- A measurement always yields an eigenvalue λ_k of the respective operator. After the measurement the wavefunction is the eigenvector $\mid \phi_k \rangle$ ("collapse of the wavefunction", or "decoherence").

➜ the statistical interpretation of Quantum Mechanics

wanted: distribution of the measured values for a given state

$$| \psi \rangle = \sum_k a_k | \phi_k \rangle$$

with (in general) complex-valued coefficients a_k.

A priori a measurement can return any eigenvalue λ_k, i.e. the question is what are the relative frequencies p_k (probabilities). Exploit that the wavefunction the p_k are normalized:

$$1 = \sum_k p_k = \langle \psi | \psi \rangle = \sum_{k,l} a_k a_l^* \langle \phi_k | \phi_l \rangle = \sum_{k,l} a_k a_l^* \delta_{kl} = \sum_k |a_k|^2$$

and thus $\quad p_k = |a_k|^2$ (Nobelprize 1954)

- in general the result of a measurement cannot be predicted, however . . .
- relative frequencies are fixed by the wavefunction
- only a system in the eigenstate ϕ_k deterministically yields the eigenvalue λ_k
- general predictability would contradict with relativity

➜ determination of the expansion coefficients a_k

$$\langle \phi_k \mid \psi \rangle = \langle \phi_k \mid \left(\sum_i a_i \mid \phi_i \rangle \right) = \sum_i a_i \langle \phi_k \mid \phi_i \rangle = \sum_i a_i \delta_{ki} = a_k$$

➜ expectation values

$$\langle S \rangle \equiv \sum_k p_k \lambda_k = \langle \psi \mid S \mid \psi \rangle$$

❖ proof:

$$\begin{aligned}\langle \psi \mid S \mid \psi \rangle &= \sum_{kl} a_k a_l^* \langle \phi_l \mid S \mid \phi_k \rangle = \sum_{kl} a_k a_l^* \lambda_k \langle \phi_l \mid \phi_k \rangle \\ &= \sum_{kl} a_k a_l^* \lambda_k \delta_{kl} = \sum_k |a_k|^2 \lambda_k = \sum_k p_k \lambda_k\end{aligned}$$

in the following: 2-state systems with $\lambda_{1,2} = \pm 1$ ➜

Spin-1/2 particles (i)

➜ operators for spin-components in x, y, z

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

➜ Eigenstates for σ_z and σ_x

$$|\uparrow\rangle_z = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \; \lambda = +1 \quad \text{und} \quad |\downarrow\rangle_z = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \; \lambda = -1$$

$$|\uparrow\rangle_x = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \; \lambda = +1 \quad \text{und} \quad |\downarrow\rangle_x = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \; \lambda = -1$$

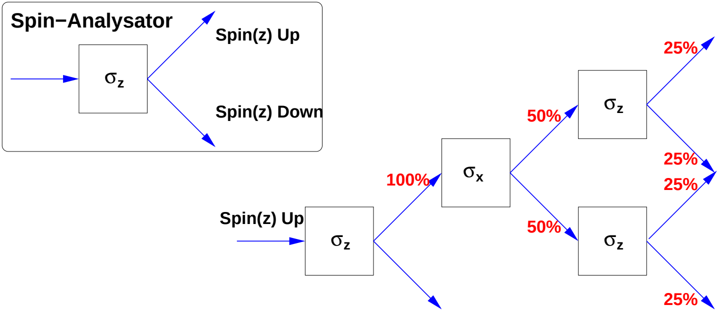
➜ transformation between the two bases

$$|\uparrow\rangle_x = \frac{1}{\sqrt{2}} \left(|\uparrow\rangle_z + |\downarrow\rangle_z \right) \quad \text{und} \quad |\downarrow\rangle_x = \frac{1}{\sqrt{2}} \left(|\uparrow\rangle_z - |\downarrow\rangle_z \right)$$

$$|\uparrow\rangle_z = \frac{1}{\sqrt{2}} \left(|\uparrow\rangle_x - |\downarrow\rangle_x \right) \quad \text{und} \quad |\downarrow\rangle_z = \frac{1}{\sqrt{2}} \left(|\uparrow\rangle_x + |\downarrow\rangle_x \right)$$

➜ example: consequence for Stern-Gerlach type experiments

- initial stat: $|\uparrow\rangle_z$
- start with measurement of the z-component of the spins
- then measure the x-component
- then measure the z-component



After a measurement all information about earlier states has been erased!

Φ *Two-particle systems*

➜ construction of “classical” product states

z.B. $|\,\psi\,\rangle = |\uparrow\rangle_1 \otimes |\uparrow\rangle_2 \equiv |\uparrow\uparrow\rangle$ oder $|\,\psi\,\rangle = |\uparrow\rangle_1 \otimes |\downarrow\rangle_2 \equiv |\uparrow\downarrow\rangle$

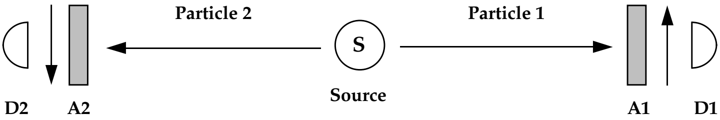
- direct product of single particle states
- use a basis (here and below) == eigenstates of σ_z

➜ new: “entagled states”

z.B. $|\,\psi\,\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$ (spin-singlet)

- possible because of the superposition principle in QM
- no classical interpretation - both particles are simultaneously “up” and “down”
- interesting phenomenology when measuring both spins . . .

➜ spin-correlation for the spin-singlet state



- xz-direction of spin measurement: particle-1: α, particle-2: β
- operators for those observables (e.g. α)

$$\sigma_\alpha = \cos\alpha \cdot \sigma_z - \sin\alpha \cdot \sigma_x$$

- effects of base-operators

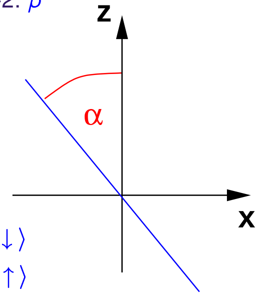
$$\sigma_z \,|\uparrow\rangle = |\uparrow\rangle \quad \text{und} \quad \sigma_z \,|\downarrow\rangle = -|\downarrow\rangle$$

$$\sigma_x \,|\uparrow\rangle = |\downarrow\rangle \quad \text{und} \quad \sigma_x \,|\downarrow\rangle = |\uparrow\rangle$$

- effects of the operators for the actual observables

$$\sigma_\alpha \,|\uparrow\rangle = \cos\alpha\,|\uparrow\rangle - \sin\alpha\,|\downarrow\rangle \equiv c_\alpha\,|\uparrow\rangle - s_\alpha\,|\downarrow\rangle$$

$$\sigma_\alpha \,|\downarrow\rangle = -\cos\alpha\,|\downarrow\rangle - \sin\alpha\,|\uparrow\rangle \equiv -c_\alpha\,|\downarrow\rangle - s_\alpha\,|\uparrow\rangle$$



then calculate. . .

❖ expectation values of individual measurements

$$\begin{aligned}\langle\sigma_\alpha\rangle &= \frac{1}{2}\left[\langle\uparrow\downarrow| - \langle\downarrow\uparrow|\right](\sigma_\alpha)\left[|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right]\\ &= \frac{1}{2}\left[\langle\uparrow\downarrow| - \langle\downarrow\uparrow|\right]\left[\sigma_\alpha|\uparrow\downarrow\rangle - \sigma_\alpha|\downarrow\uparrow\rangle\right]\\ &= \frac{1}{2}\left[\langle\uparrow\downarrow| - \langle\downarrow\uparrow|\right]\left[(c_\alpha|\uparrow\downarrow\rangle - s_\alpha|\downarrow\downarrow\rangle) - (-c_\alpha|\downarrow\uparrow\rangle - s_\alpha|\uparrow\uparrow\rangle)\right]\\ &= \frac{1}{2}\left[\langle\uparrow\downarrow| - \langle\downarrow\uparrow|\right]\left[c_\alpha(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle) - s_\alpha(|\downarrow\downarrow\rangle - |\uparrow\uparrow\rangle)\right]\\ &= \frac{1}{2}c_\alpha\left[\langle\uparrow\downarrow|\uparrow\downarrow\rangle - \langle\downarrow\uparrow|\downarrow\uparrow\rangle\right] = 0\end{aligned}$$

❖ note:

- σ_α only acts on the first particle
- σ_β would only act on the other particle
- formally everything can be expressed by 4×4 matrices
- inner products of orthogonal states are zero
- single measurements are random with equal probability for $\uparrow_\alpha$ und $\downarrow_\alpha$

Quantum mechanical prediction (iii)

❖ expectation value of the product (correlation)

$$\langle\sigma_\alpha\sigma_\beta\rangle = \frac{1}{2}\left[\langle\uparrow\downarrow| - \langle\downarrow\uparrow|\right](\sigma_\alpha\sigma_\beta)\left[|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right]$$

$$= \frac{1}{2}\left[\langle\uparrow\downarrow| - \langle\downarrow\uparrow|\right]\left[|(c_\alpha\uparrow - s_\alpha\downarrow)(-c_\beta\downarrow - s_\beta\uparrow)\rangle - |(-c_\alpha\downarrow - s_\alpha\uparrow)(c_\beta\uparrow - s_\beta\downarrow)\rangle\right]$$

$$= \frac{1}{2}\left[\langle\uparrow\downarrow| - \langle\downarrow\uparrow|\right]\left[|\uparrow\downarrow\rangle(-c_\alpha c_\beta - s_\alpha s_\beta) + |\downarrow\uparrow\rangle(s_\alpha s_\beta + c_\alpha c_\beta)\right.$$

$$\left. + |\uparrow\uparrow\rangle(-c_\alpha s_\beta - s_\alpha c_\beta) + |\downarrow\downarrow\rangle(s_\alpha c_\beta - c_\alpha s_\beta)\right]$$

$$= \frac{1}{2}(-c_\alpha c_\beta - s_\alpha s_\beta)\langle\uparrow\downarrow|\uparrow\downarrow\rangle - \frac{1}{2}(s_\alpha s_\beta + c_\alpha c_\beta)\langle\downarrow\uparrow|\downarrow\uparrow\rangle$$

$$= -(c_\alpha c_\beta + c_\alpha c_\beta) = -\cos(\alpha-\beta) \equiv -\cos(\phi)$$

➜ the correlation is only a function of the opening angle $\phi = \alpha - \beta$

spin-1/2 particles	$\langle\sigma_\alpha\sigma_\beta\rangle = -\cos\phi$
photons (spin-1)	$\langle\sigma_\alpha\sigma_\beta\rangle = -\cos 2\phi$

(180deg between orthogonal spin-1/2 states, 90deg between orthogonal photon polarisations)

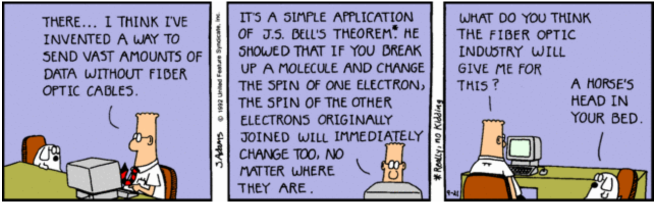
➜ Interpretation

➜ $\langle\sigma_\alpha\sigma_\beta\rangle$ is only a function of ϕ

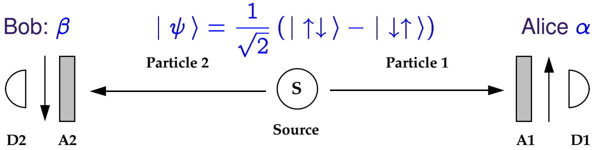
- single measurements are perfectly random
- equal probability to measure “Spin-up” or “Spin-down”
- perfect anti-correlation of both measurements refer to the same direction
- independent of space and time, i.e.
  - ➜ independent of the time ordering of the measurement
  - ➜ independent of the spatial separation

➜ obvious(?) questions:

- Is there “spooky action at a distance” which causes perfect synchronisation?
- can one use this to transmit information with $v = \infty$?



Communication with $v = \infty$?



- analyzer setting $\alpha \parallel \beta$: perfect anti-correlation
- analyser setting $\alpha \perp \beta$: uncorrelated measurements

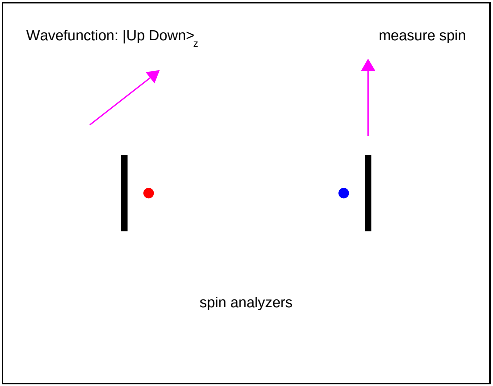
➜ Alice knows β and sends one bit to Bob by causing an excess of -1

- case 1: Alice can influence her result
  - ➜ set $\alpha \parallel \beta$ and cause an excess of $+1$ at her side
  - ➜ Bob observes the same excess of -1
- case 2: Alice kann predict her result
  - ➜ prediction $+1$: set $\alpha \parallel \beta$ and Bob always sees -1
  - ➜ prediction -1: set $\alpha \perp \beta$ and Bob measures equal numbers of ± 1

❖ insight:

If a quantum mechanical measurement is truly random, i.e. neither predictable, nor controllable then communication with $v > c$ is impossible.

➜ measurement of spin correlations for spin-1/2 particles



- for ideal detektors and different wavefunctions consider . . .
  - ➜ measurements of spin correlations
  - ➜ coincidence measurements

➜ The EPR-paradox (Einstein, Podolski, Rosen, 1935)

❖ quantum mechanics versus physical reality

- definition: Element of reality
  There exists a certain prediction ($p = 1$) for an observable, which can be obtained without perturbing the system.
- definition: Complete theorie
  Each element of reality is represented in the theory.
- discussion:
  - ➜ plausible concepts
  - ➜ inconsistent with quantum mechanics

➜ EPR: consider the wavefunction of a two-body decay $M \rightarrow m_1 m_2$, which is an eigenstate of both $(x_1 - x_2)$ and $(p_1 + p_2)$

$$(x_1 - x_2) \mid \psi \rangle = a \mid \psi \rangle$$
$$(p_1 + p_2) \mid \psi \rangle = P \mid \psi \rangle$$

➜ allowed by quantum mechanics, since:

$$[x_1 - x_2, p_1 + p_2] = [x_1, p_1] - [x_2, p_1] + [x_1, p_2] - [x_2, p_2] = i\hbar - 0 + 0 - i\hbar = 0$$

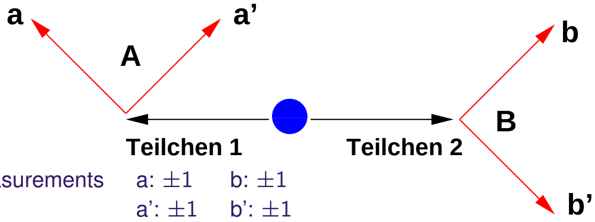
➜ measurement and interpretation

- measure x_1
  - ➜ predict $x_2 = a + x_1$
  - ➜ always found when measured
- measure p_2
  - ➜ predict $p_1 = P - p_2$
  - ➜ always found when measured
- result: A measurement of x_1 and p_2 not only determines those, but also x_2 and p_1. For each of the 4 variables one has a sure prediction.
  - ➜ x_1 AND p_1 as well as x_2 AND p_2 are elements of reality.
- quantum mechanics:
  - ➜ x_1 exclusive OR p_1 and x_2 /exclusive OR p_2 are elements of reality.
- solution to the contradiction
  - ➜ Option 1:
    Quantum mechanics is incomplete. There are additional hidden parameters which also explain the statistical properties and action at a distance.
  - ➜ Option 2:
    Einstein's concept of reality is not realized by nature.

Φ *Bell´s inequalities (i)*

➜ consider two-particle systems and 2×2 analyser settings

J.S. Bell: Physics 1 (1964) 195, *On the Einstein Podolski Rosen paradox*



possible measurements a: ± 1 b: ± 1
a': ± 1 b': ± 1

❖ local-deterministic assumption:

During the decay some hidden variables determine which measurements the two particles produce for given analyser settings. Each possible outcome has a fixed probability to be realized. For example:

$$\rho(a = +1, a' = -1, b = -1, b' = +1):$$

probability that particle-1 for analyzer setting a, (a') yields the measurement $+1$, (-1), and particle-2 for analyzer setting b, (b') the measurement Messwert -1, $(+1)$.

Φ *Bell´s inequalities (ii)*

➜ configuration space and possible measurements

k	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
a	−	−	−	−	−	−	−	−	+	+	+	+	+	+	+	+
b	−	−	−	−	+	+	+	+	−	−	−	−	+	+	+	+
a′	−	−	+	+	−	−	+	+	−	−	+	+	−	−	+	+
b′	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+

For the configurations k one has:

$$\rho_k \geq 0 \;\forall\; k \qquad \text{and} \qquad \sum_k \rho_k = 1$$

❖ measurements:

$E(x,y) = \langle x \cdot y \rangle$ expectation value of the product of the measurements x und y

$F(x,y)$ probability for $x = +1 \wedge y = +1$

The expectation values $E(x,y)$ are theoretically nice, the $F(x,y)$ are experimentally easier to determine coincidence probabilities, where $+1$ means that a particles passes the analyzer and is recorded in a detector.

➜ linear combination of expectation values

$$T = E(a,b) - E(a,b') + E(a',b) + E(a',b') \equiv E_1 - E_2 + E_3 + E_4$$

❖ straightforward calculation:

k	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
a	−	−	−	−	−	−	−	−	+	+	+	+	+	+	+	+
b	−	−	−	−	+	+	+	+	−	−	−	−	+	+	+	+
a′	−	−	+	+	−	−	+	+	−	−	+	+	−	−	+	+
b′	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+

$E_1 : \rho_0 + \rho_1 + \rho_2 + \rho_3 - \rho_4 - \rho_5 - \rho_6 - \rho_7 - \rho_8 - \rho_9 - \rho_{10} - \rho_{11} + \rho_{12} + \rho_{13} + \rho_{14} + \rho_{15}$

$E_2 : \rho_0 - \rho_1 + \rho_2 - \rho_3 + \rho_4 - \rho_5 + \rho_6 - \rho_7 - \rho_8 + \rho_9 - \rho_{10} + \rho_{11} - \rho_{12} + \rho_{13} - \rho_{14} + \rho_{15}$

$E_3 : \rho_0 + \rho_1 - \rho_2 - \rho_3 - \rho_4 - \rho_5 + \rho_6 + \rho_7 + \rho_8 + \rho_9 - \rho_{10} - \rho_{11} - \rho_{12} - \rho_{13} + \rho_{14} + \rho_{15}$

$E_4 : \rho_0 - \rho_1 - \rho_2 + \rho_3 + \rho_4 - \rho_5 - \rho_6 + \rho_7 + \rho_8 - \rho_9 - \rho_{10} + \rho_{11} + \rho_{12} - \rho_{13} - \rho_{14} + \rho_{15}$

collecting all terms:

$$T = 2 \cdot (\rho_0 + \rho_1 + \rho_3 + \rho_7 + \rho_8 + \rho_{12} + \rho_{14} + \rho_{15})$$
$$- 2 \cdot (\rho_2 + \rho_4 + \rho_5 + \rho_6 + \rho_9 + \rho_{10} + \rho_{11} + \rho_{13})$$

and using $\sum \rho_k \leq 1$ one finds:

$$-2 \leq T \leq 2$$

➜ linear combination of coincidence probabilities

$$S = F(a,b) - F(a,b') + F(a',b) + F(a',b') - F(a') - F(b)$$

❖ straightforward calculation:

k	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
a	−	−	−	−	−	−	−	−	+	+	+	+	+	+	+	+
b	−	−	−	−	+	+	+	+	−	−	−	−	+	+	+	+
a′	−	−	+	+	−	−	+	+	−	−	+	+	−	−	+	+
b′	−	+	−	+	−	+	−	+	−	+	−	+	−	+	−	+

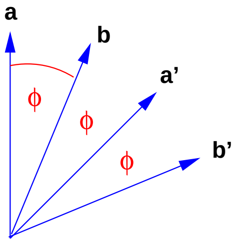
$$F(a,b) = \rho_{12} + \rho_{13} + \rho_{14} + \rho_{15}$$
$$F(a,b') = \rho_9 + \rho_{11} + \rho_{13} + \rho_{15}$$
$$F(a',b) = \rho_6 + \rho_7 + \rho_{14} + \rho_{15}$$
$$F(a',b') = \rho_3 + \rho_7 + \rho_{11} + \rho_{15}$$
$$F(a') = \rho_2 + \rho_3 + \rho_6 + \rho_7 + \rho_{10} + \rho_{11} + \rho_{14} + \rho_{15}$$
$$F(b) = \rho_4 + \rho_5 + \rho_6 + \rho_7 + \rho_{12} + \rho_{13} + \rho_{14} + \rho_{15}$$

collecting all terms:

$$S = -\rho_2 - \rho_3 - \rho_4 - \rho_5 - \rho_6 - \rho_9 - \rho_{11} - \rho_{13} \quad \text{and thus} \quad S \leq 0$$

Φ Predictions of quantum mechanics (i)

➜ consider symmetric configurations



❖ predictions

Spin-1/2 particles: $E(x,y) = -\cos\phi_{xy}$

Photons: $E(x,y) = -\cos 2\phi_{xy}$

experimental: $E(x,y) \to V \cdot E(x,y)$

attenuation of the correlation by Visibility $V < 1$

➜ predictions for T

$$\begin{aligned} T(\text{Spin } 1/2) &= E(a,b) - E(a,b') + E(a',b) + E(a',b') \\ &= V \cdot (-\cos(\phi) + \cos(3\phi) - \cos(\phi) - \cos(\phi)) \\ &= V \cdot (\cos(3\phi) - 3\cos(\phi)) \end{aligned}$$

$$T(\text{Spin } 1) = V \cdot (\cos(6\phi) - 3\cos(2\phi))$$

➜ result: QM: $|T| \leq V \cdot 2\sqrt{2}$ vs Bell: $|T| < 2$

Predictions of quantum mechanics (ii)

➜ predictions for S

relation between coincidence probability $F(x,y)$ and $E(x,y)$:

$$E(x,y) = F^{++}(x,y) + F^{--}(x,y) - F^{+-}(x,y) - F^{-+}(x,y)$$

with F^{ab} the probability to observe simultaneously a at analyzer setting x for particle 1, and b at analyzer setting y for particle 2. Using

$$F = F^{++} = F^{--} \quad , \quad F^{+-} = F^{-+} \quad , \quad F^{++} + F^{+-} = \frac{1}{2}$$

one finds

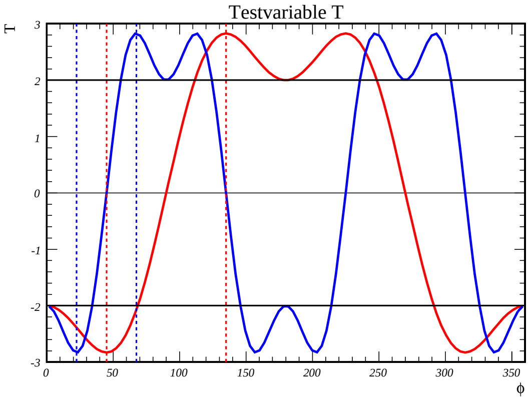
$$E(x,y) = 2 \cdot F(x,y) - 2 \cdot \left(\frac{1}{2} - F(x,y)\right) \quad \text{or} \quad F(x,y) = \frac{1}{4}(1 - E(x,y))$$

and with $F(a') = F(b) = 1/2$ finally

$$\begin{aligned} S &= F(a,b) - F(a,b') + F(a',b) + F(a',b') - F(a') - F(b) \\ &= -\frac{1}{2} + \frac{1}{4}\left(E(a,b) - E(a,b') + E(a',b) + E(a',b')\right) = -\frac{1}{2} + \frac{T}{4} \end{aligned}$$

➜ result: QM: $S < (\sqrt{2} - 1)/2 \approx 0.207$ vs Bell: $S < 0$

➜ the test variables S and T are equivalent:



❖ most sensitive settings

Photons $\phi = 22.5^\circ, 67.5^\circ$

spin 1/2 $\phi = 45^\circ, 135^\circ$

➜ blue: QM-prediction for photons

➜ red: QM-prediction for Spin-1/2 particles

Φ Loopholes

➜ quantum mechanics permits violations of Bell's inequalities

❖ consequences of experimental confirmation

- local-realistic theories are falsified
  - ➜ there are no hidden variables
  - ➜ nature is non-local
- the experiments did not really test quantum mechanics

❖ Loopholes

- static experimental setup affects hidden parameters
  - ➜ decide on analyser setting only after emission of the particles
- analyser are not spacial separated (i.e. within light cone)
  - ➜ use large distanced
  - ➜ measure in moving reference frames (each observer sees his particle first)
- das "Detection Loophole"
  - ➜ only a small fraction of all particles is recorded
  - ➜ this fraction is not a fair sampling of the total

➜ (increasingly better) experiments. . .

➜ early particle physics experiments

❖ proton-proton scattering

M. Lamehi-Rachti and W. Mittig: Phys. Rev. D14 (1976) 2543
Quantum mechanics and hidden variables: A test of Bell's inequality by the measurement of the spin correlation in low-energy proton-proton scattering

❖ Positronium-decay $e^+e^- \rightarrow \gamma\gamma$

M. Bruno, M d'Agostino and C. Maroni: Il Nuovo Cimento (1977) 143
Measurement of linear polarization of positronium annihilation photons

➜ experiments with atomic transitions

A. Aspect, J. Dalibard and G. Roger: Phys. Rev. Lett. 47 (1981) 460
Experimental test of realistic local theories via Bell's theorem

A. Aspect, J. Dalibard and G. Roger: Phys. Rev. Lett. 49 (1982) 91
Experimental realization of Einstein-Podolski-Rosen Gedankenexperiment: a new violation of Bell's inequalities

A. Aspect, J. Dalibard and G. Roger: Phys. Rev. Lett. 49 (1982) 1804
Experimental test of Bell's inequalities using time-varying analyzers

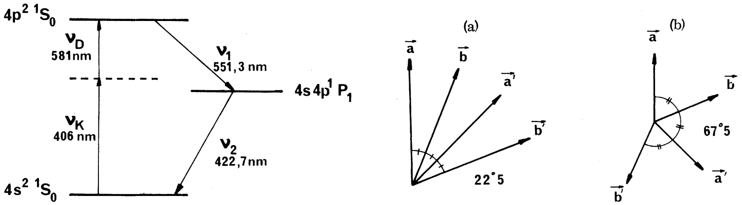
➜ some detail. . .

➜ Calcium $0 \to 1 \to 0$-cascade

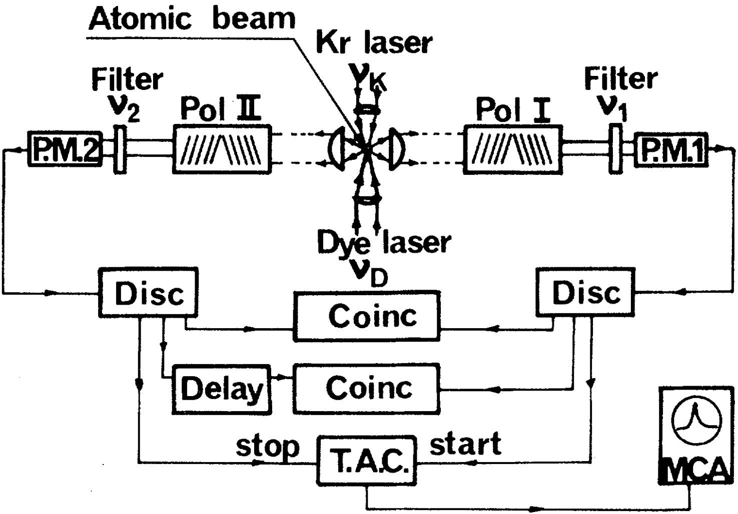
- 2-photon absorption with 2 pump-lasers
- 2-photon decay into spin-Singlet state

$$\frac{1}{\sqrt{2}}(|\,HV\,\rangle - |\,VH\,\rangle)$$

- lifetime of the intermediate state: $\tau = 5\,\mathrm{ns}$
- measure polarisation-dependent coincidence rates ➜ test-variable S



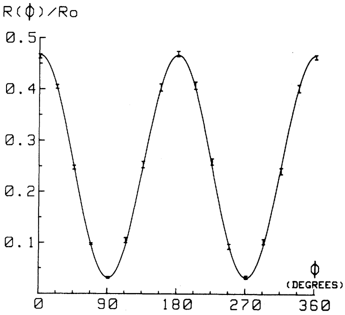
Φ Experimental setup



Φ Experimental details

- large aperture of the optical system
- efficient polarizers
  - ➜ `PolI` : $\varepsilon_{||} = 0.971$ $\varepsilon_{\perp} = 0.029$
  - ➜ `PolII`: $\varepsilon_{||} = 0.968$ $\varepsilon_{\perp} = 0.028$
- narrow-band optical filter to suppress stray (laser) light
- measurement of chance-coincidences at 100 ns delay
- check that signal follows lifetime of the intermediate state
- time window for coincidences 19 ns
- rates
  - ➜ `P.M.1:` 40 kHz, `P.M.2:` 120 kHz, dark rate ca. 200 Hz
  - ➜ coincidence rate 240 Hz, random coincidences 90 Hz
  - ➜ 100 s per measurement
- measure coincidence probabilities $F(\phi) = R(\phi)/R_0$
  - ➜ R_0: coincidence rate without filters
  - ➜ $R(\phi)$: coinciden rate as a function of the angle between the filters
- Check: maximize the distance between the filters to $D = 6.5\,\mathrm{m}$
  - ➜ ca. 4 coherence lengths of the wavepacket associated with the lifetime of the intermediate state

➜ coincidence rate as a funktion of the angle between the filters



❖ Visiblity V limits Modulation

$$R(\phi)/R_0 = \frac{1}{4}(1 + V\ \cos(2\phi))$$

➜ $V \approx 0.88$

consequence:

$$S_{\text{max}} = -\frac{1}{2} + \frac{1}{4} T_{\text{max}}$$
$$= \frac{1}{2}(V \cdot \sqrt{2} - 1)$$

- quantum mechanical expectation: $S = 0.118 \pm 0.005$
- measurement: $S = 0.126 \pm 0.014$ violation of Bell's inequality with $9\,\sigma$
- better sensitivity when assuming rotational invariance
  - ➜ Bell's inequality: $\delta = |F(22.5^\circ) - F(67.5^\circ| - 1/4 \leq 0$
  - ➜ measurement: $\delta = (5.72 \pm 0.43) \cdot 10^{-2}$ larger than zero with $13\,\sigma$

Φ Discussion

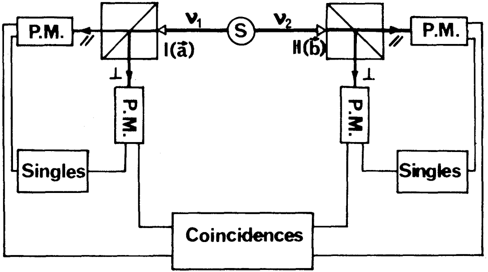
- measurements favor quantum mechanics
  - ➜ statistically very significant results
- point of criticism:
  - ➜ only a tiny fraction of all photon pairs is recorded
  - ➜ only sensitive to one polarisation direction per measurement
  - ➜ setting of the filters is static
- improved measurements
  - ➜ simultaneous measurement of all combinations of polarisations
  - ➜ determination of analyser settings only after photon emission

➜

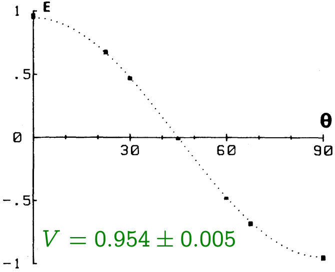
Simultaneous measurement of all polarisations

➜ direct determination of the expectation values $E(\phi)$

experimental setup



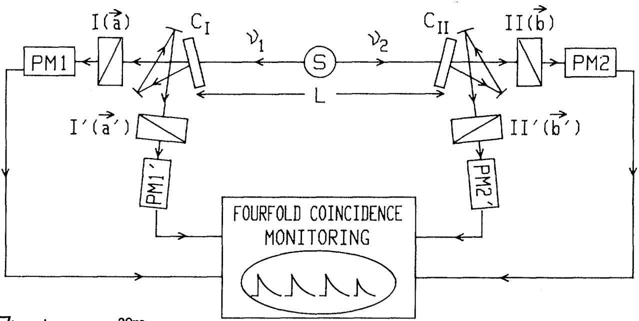
results

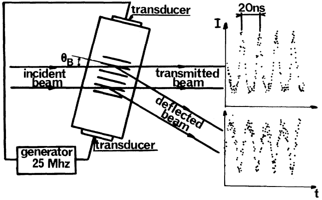


- simultaneous measurement of coincidence rates $F^{++}, F^{--}, F^{+-}, F^{-+}$
- expectation values: $E(\phi) = F^{++}(\phi) + F^{--}(\phi) - F^{+-}(\phi) - F^{-+}(\phi)$
- result: $T = 2.697 \pm 0.015$
- $46\,\sigma$ violation of Bell's inequality $|T| \leq 2$

Φ Dynamic selektion of polarisation

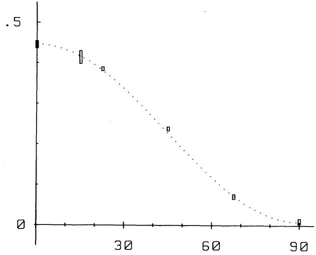
➜ fast switching between different optical paths





- periodic switching of optical paths by standing waves
- measurements of coincidence rates
  - ➜ $L/c = 40\,\mathrm{ns}$
  - ➜ definition of observable after 2γ-emission

➜ study the test-variable S



❖ 5 σ violation of $S \leq 0$

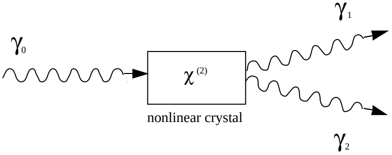
$$S = 0.101 \pm 0.0020$$

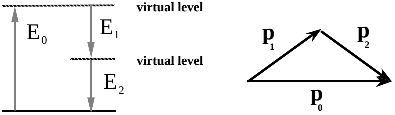
consistent with quantum mechanics

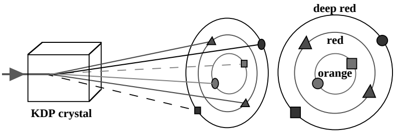
➜ remarks:

- extremely difficult experiment
- periodische switching of optical path in principle predictable
- Loopholes existed in all tests using atomic transitions
- progress through new techniques . . .

➜ generate photon pairs by “Parametric Down Conversion”







Kalium-Di-Hydrogen-Phosphate

- initial state: UV-Laser
  - ➜ large coherence length
  - ➜ polarisation ⊥ drawing plane
- final state: visible light
  - ➜ polarisation in drawing plane
  - ➜ short coherence length
- energy- and momentum conservation

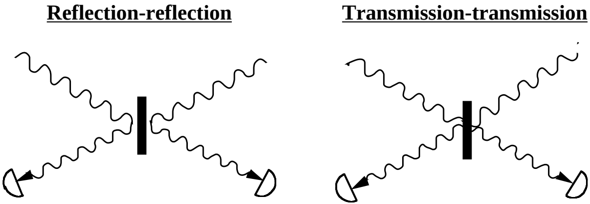
$$\omega_0 = \omega_1 + \omega_2$$

$$\vec{p}_0 = \vec{p}_1 + \vec{p}_2$$

➜ QM-Tests:

P.G. Kwiat, A.E. Steinberg and R.Y. Chiao: Phys. Rev. A 45 (1992) 7729. *Observation of a “quantum eraser”: a revival of coherence in a two-photon interference experiment*

→ consider coincidences at a beam splitter:



amplitude $= \frac{i}{\sqrt{2}} \cdot \frac{i}{\sqrt{2}} = -\frac{1}{2}$ **amplitude** $= \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = +\frac{1}{2}$

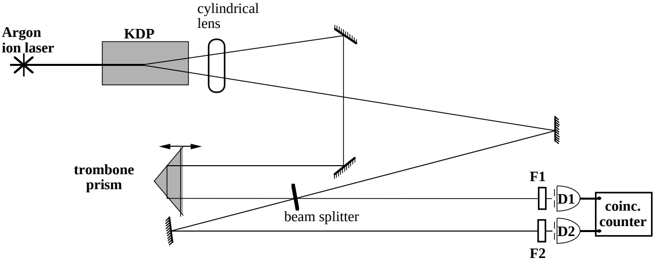
- superposition of two configurations leading to coincidences
- for simultaneous arrival (within coherence length) of the two photons:

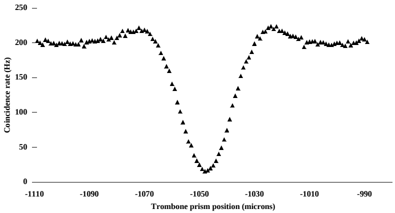
$$P_{coinc} = |A_{RR} + A_{TT}|^2 = |-1/2 + 1/2|^2 = 0$$

→ no coincidences

- exploit for experiments to destroy and recover QM coherence

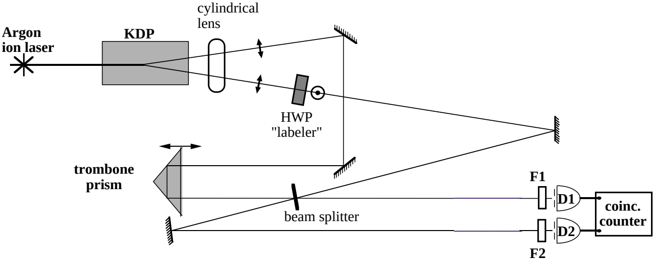
➜ demonstration of quantum-mechanical interference

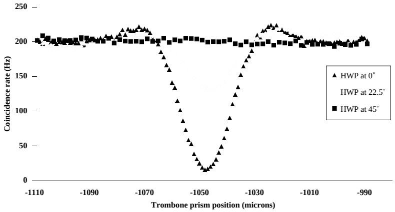




- narrow bandwidth filters to select photons that can interfere
- destructive interference at the beam splitter
  - ➜ length of photons ca. 40 μm
  - ➜ time resolution O(fs)

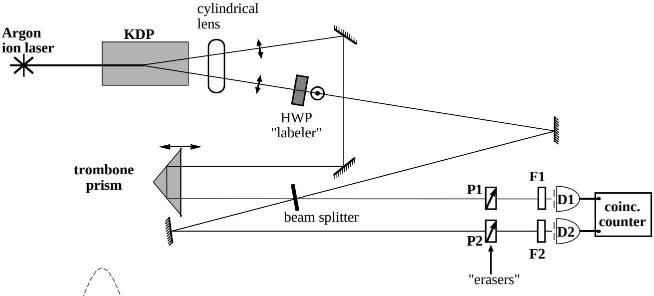
➜ labeling by polarisation

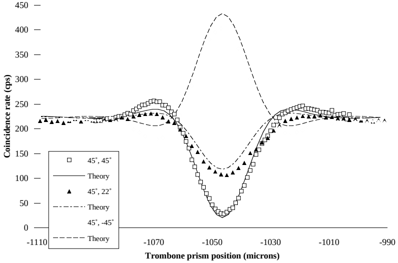




- destructive interference disappears for orthogonal polarisation states
- interference destroyed even though the path is not observed. It's sufficient that it is in principle observable.

➜ a posteriori erasure of polarisations labeling



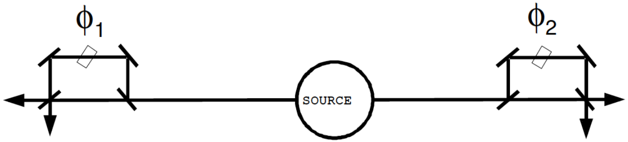


- restauration of interference after the beam splitter "delayed choice"
- a posteriori phase-shift even permits interference-minimum ➜ -maximum

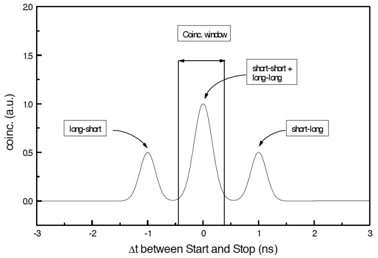
➜ correlations between interferometers

J.D. Franson: Phys. Rev. Lett. 62 (1989) 2205, *Bell inequality for position and time*

❖ principle:

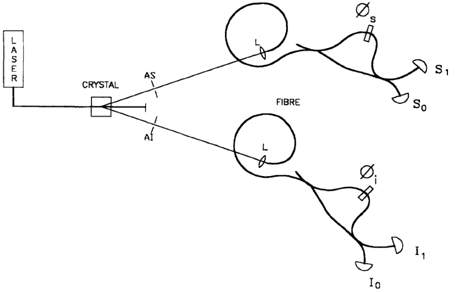


- no direct 2-photon interference
  - ➜ consider coincidence measurements
- two final states on each side
  - ➜ coincidence measurements a la Bell
- "long-long" and "short-short" indistinguishable
  - ➜ QM-interference
  - ➜ rates by $\phi_{1,2}$ adjustable



➜ experimental setup

P.R. Tapster, J.G. Rarity and P.C.M Owens: Phys. Rev. Lett. 73 (1994) 1923
Violation of Bell's inequality over 4 km optical fiber



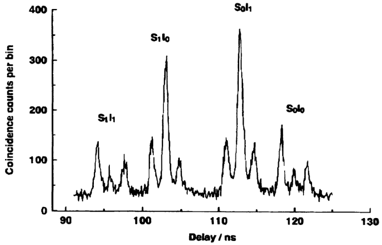
- photon pairs from Parametric-Down-Conversion
- coupling into optical fibres
- phase shift by air gap in one of the fibre interferometers
- measure coincidence rates of "signal" and "idler"

➜ determine correlation function

$$C(\phi) = F(0,0) + F(1,1) - F(1,0) - F(0,1) = V \cdot \cos\phi$$

➜ modulation with $V > 1/\sqrt{2}$ violates Bell's inequality

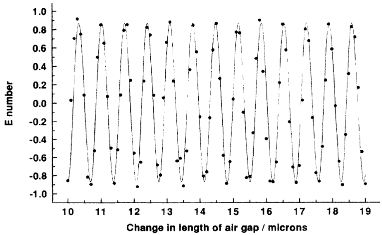
➜ results



- central peaks of the 4 triplets:

$$\Psi \sim |\, LL \,\rangle + |\, SS \,\rangle$$

- study QM-entanglement



- modulation with $V = 0.864 \pm 0.012$
- Bell's inequality violated with $13\,\sigma$

Φ Down-Conversion with entanglement (i)

➜ new intense source of entangles photons

P.G. Kwiat et al.: Phys. Rev. Lett. 75 (1995) 4337

New High-Intensity Source of Polarization-Entangled Photon Pairs

❖ so far:

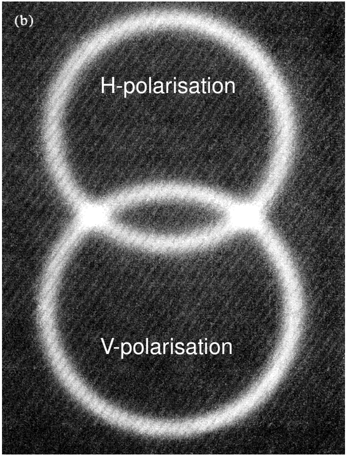
- Down-Conversion creates product states, i.e. pairs of independent photons
- entanglement by external means, e.g. beam splitter, interferometers etc.

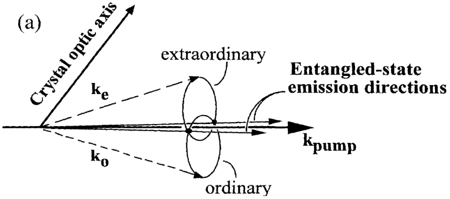
❖ new approach:

- Down-Conversion in birefringent crystal
  - ➜ BBO = Beta-Barium-Borat
- direct generation of polarisation-entangles states
  - ➜ similar to states from atomic transitions
  - ➜ “down-conversion with type-II phase matching”

Down-Conversion with entanglement (ii)

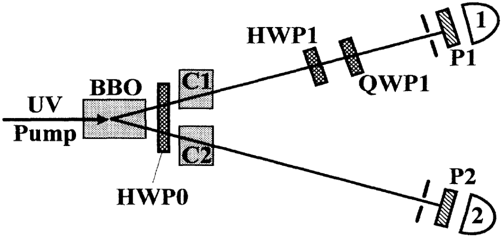
➜ characteristics of the generation of photon pairs





- polarisation defined by direction in the birefringent crystal
- entanglement in the overlap of the Parametric-Down-Conversion cones
- wavefunvtion: $(|\, HV \,\rangle + e^{i\alpha}\,|\, VH \,\rangle)/\sqrt{2}$
  - ➜ phase α from birefringence
  - ➜ adjustable to 0 or π by external birefringent phase shifter

➜ experimental setup



- ➜ HWP: $\lambda/2$-plate: $H \leftrightarrow V$
- ➜ QWP: $\lambda/4$-plate: $H \rightarrow -H$
- ➜ C: birefringent crystal
- ➜ P: polarisation filter

- HWP0, C1, und C2:
  - ➜ compensation of time delays by polarisation dependence of c
- P1 und P2:
  - ➜ polarisers for correlation measurements at all Bell-states
- HWP1 and QWP1: acting on the 2nd photon
  - ➜ QWP1: $|\,HV\,\rangle + |\,VH\,\rangle \quad \rightarrow \quad |\,HV\,\rangle - |\,VH\,\rangle$
  - ➜ HWP1: $|\,HV\,\rangle + |\,VH\,\rangle \quad \rightarrow \quad |\,HH\,\rangle + |\,VV\,\rangle$
  - ➜ HWP1 und QWP1: $|\,HV\,\rangle + |\,VH\,\rangle \quad \rightarrow \quad |\,HH\,\rangle - |\,VV\,\rangle$

Down-Conversion with entanglement (iv)

➜ tests of Bell's inequalities at all 4 Bell-states

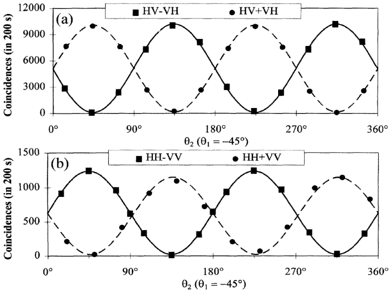
- maximal entangled base for two-particle states

$$|\,\Psi^{\pm}\,\rangle = (|\,HV\,\rangle \pm |\,VH\,\rangle)/\sqrt{2} \quad \text{and} \quad |\,\phi^{\pm}\,\rangle = (|\,HH\,\rangle \pm |\,VV\,\rangle)/\sqrt{2}$$

- measure coincidence rates as a function of the angle between polarisers
- extract the test-variable S; Bell's inequality: $|S| \leq 2$
  - ➜ modulation $V = 97.8 \pm 1.0\%$
  - ➜ 100σ-violation in 5 min

❖ results:

EPR-Bell state	$C(\theta_1, \theta_2)$	S [a]
$\|\psi^+\rangle$	$\sin^2(\theta_1 + \theta_2)$	-2.6489 ± 0.0064
$\|\psi^-\rangle$	$\sin^2(\theta_1 - \theta_2)$	-2.6900 ± 0.0066
$\|\phi^+\rangle$	$\cos^2(\theta_1 - \theta_2)$	2.557 ± 0.014
$\|\phi^-\rangle$	$\cos^2(\theta_1 + \theta_2)$	2.529 ± 0.013



Φ Quantum teleportation (i)

C.H. Bennet et al.: Phys. Rev. Lett. 70 (1993) 1895

Teleporting an unknown quantum state via dual classical and Einstein-Podolski-Rosen channels

D. Bouwmeester et al.: Nature 390 (1997) 575

Experimental quantum teleportation

D. Boschi, et al.: Phys. Rev. Lett. 80 (1998) 1121

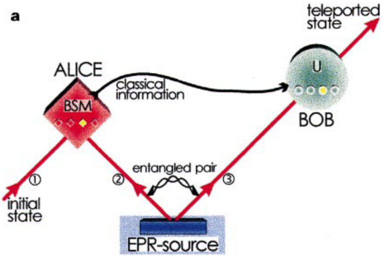
Experimental Realization of Teleporting an Unknown Pure Quantum State via Dual Classical and Einstein-Podolsky-Rosen Channels

➜ quantum teleportation

- restauration of a quantum state "without" transport of matter
- must satisfy the Quanten No-Cloning-Theorem
  - ➜ no knowledge of the initial state required
  - ➜ is destroyed and the reconstructed at a remote place without explicit knowledge
- cannot be done with speed $v > c$

➜ principle

- problem: wavefunction in general cannot be determined
  - ➜ only possible for an ensemble of identically prepared states
  - ➜ basis of the “No-Cloning Theorems”
- solution: exploit entanglement
  - ➜ entangle the unknown state ϕ_1 with partner of an EPR-pair
  - ➜ project this state onto a known entangled state
  - ➜ this fixes the wavefunction of the second partner
    - ✘ it is not known, but
    - ✘ can be transformed to $|\,\phi_1\,\rangle$
- note:
  - ➜ ϕ_1 is never explicitly determined
  - ➜ can itself be a part of an EPR pair
    - ✘ “entanglement swapping”
    - ✘ relevant for quantum computers



➜ formal description: Bell-basis for entangles states

$$|\,\Psi^{\pm}\,\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle\right) \quad \text{and} \quad |\,\Phi^{\pm}\,\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\uparrow\rangle \pm |\downarrow\downarrow\rangle\right)$$

or

$$|\uparrow\downarrow\rangle = \frac{1}{\sqrt{2}}\Big(|\,\Psi^{+}\,\rangle + |\,\Psi^{-}\,\rangle\Big) \qquad |\downarrow\uparrow\rangle = \frac{1}{\sqrt{2}}\Big(|\,\Psi^{+}\,\rangle - |\,\Psi^{-}\,\rangle\Big)$$

$$|\uparrow\uparrow\rangle = \frac{1}{\sqrt{2}}\Big(|\,\Phi^{+}\,\rangle + |\,\Phi^{-}\,\rangle\Big) \qquad |\downarrow\downarrow\rangle = \frac{1}{\sqrt{2}}\Big(|\,\Phi^{+}\,\rangle - |\,\Phi^{-}\,\rangle\Big)$$

With

$$|\,\phi_1\,\rangle = a\,|\uparrow\rangle + b\,|\downarrow\rangle \quad \text{where} \quad |a|^2 + |b|^2 = 1$$

and the wavefunction ϕ_{23} of an entangled EPR-pair

$$|\,\phi_{23}\,\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

the total wavefunction becomes

$$|\,\phi_{123}\,\rangle = \frac{a}{\sqrt{2}}\left(|\uparrow_1\uparrow_2\downarrow_3\rangle - |\uparrow_1\downarrow_2\uparrow_3\rangle\right) + \frac{b}{\sqrt{2}}\left(|\downarrow_1\uparrow_2\downarrow_3\rangle - |\downarrow_1\downarrow_2\uparrow_3\rangle\right)$$

and re-writing in the Bell-Basis for particles 1 and 2 yields: ➜

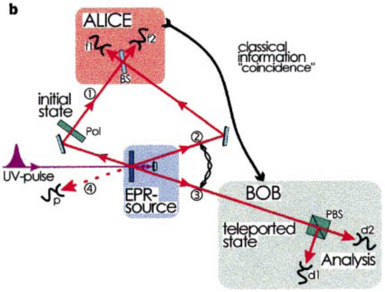
Quantum teleportation (iv)

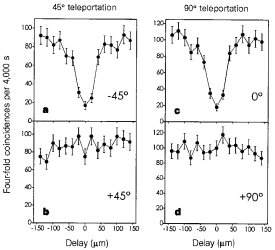
$$
\begin{aligned}
|\,\phi_{123}\,\rangle = \frac{a}{2}\Big(|\,(\Phi^+_{12}+\Phi^-_{12})\downarrow_3\,\rangle - |\,(\Psi^+_{12}+\Psi^-_{12})\uparrow_3\,\rangle\Big) \\
+\frac{b}{2}\Big(|\,(\Psi^+_{12}-\Psi^-_{12})\downarrow_3\,\rangle - |\,(\Phi^+_{12}-\Phi^-_{12})\uparrow_3\,\rangle\Big) \\
=\frac{1}{2}|\,\Phi^+_{12}\,\rangle(-b|\uparrow_3\rangle + a|\downarrow_3\rangle) + \frac{1}{2}|\,\Phi^-_{12}\,\rangle(b|\uparrow_3\rangle + a|\downarrow_3\rangle) \\
+\frac{1}{2}|\,\Psi^+_{12}\,\rangle(-a|\uparrow_3\rangle + b|\downarrow_3\rangle) + \frac{1}{2}|\,\Psi^-_{12}\,\rangle(-a|\uparrow_3\rangle - b|\downarrow_3\rangle)
\end{aligned}
$$

- Fazit:
  - projection of $|\,\phi_{12}\,\rangle$ on a Bell state yields one of 4 possible results
  - all results are equally probable, independent of $|\,\phi_1\,\rangle$
    - ➜ 2 bits classical information
    - ➜ no knowledge of coefficients a and b
    - ➜ initial state $|\,\phi_1\,\rangle$ is destroyed
  - by entanglement $|\,\phi_3\,\rangle$ is fixed
  - Bell-state of $|\,\phi_{12}\,\rangle$ contains information about transformation $|\,\phi_3\,\rangle \rightarrow |\,\phi_1\,\rangle$
  - classical channel with $v \leq c$ needed to use the information
  - 2 bit classical information als “left-over” (?!)

➜ experimental realisation

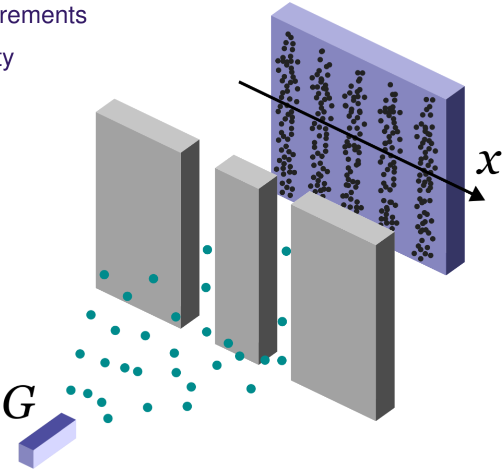
- 2 EPR-Paare from TypeII-Down-Conversion
  - ➜ photon 1 for teleportation
  - ➜ photonen 2 und 3 for entanglement
  - ➜ photon 4 as trigger (optional)
- preparation of photon 1 by polarizers
- entanglement of photon 1 and Photon 2 by beam splitter
  - ➜ coincidence selects the only antisymmetric Bell-state $|\Psi^-\rangle$
  - ➜ photons 1 and 3 in the same state (modula a global phase -1)
  - ➜ scan the entanglement by variation of the optical path of photon 2
- signature for teleportation:
  - ➜ suppression of coincidences with wrong polarisation for photon 3





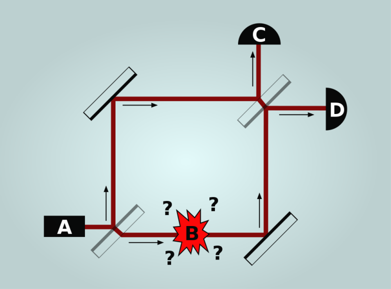
➜ other interesting aspects of quantum physics . . .

- interaction-free measurements
- test of Einstein's locality
- quantum cryptography
- decoherence
- . . .



➜ registration of an object without direct interaction

- motivation: object will be destroyed by any interaction
- quantum mechanics: in principle possible
- consider photon in an interferometer



❖ probability of a signal in C&D

- equal length in both arms
- case 1: no object “B” in the path
  - ✘ $p(C) \propto |i^3 + i \ |^2 = 0$
  - ✘ $p(D) \propto |i^2 + i^2|^2 = 4$
  - ➜ only detector D gets a signal
- case 2: object “B” blocks the lower branch
  - ➜ in 50% of the cases B will be destroyed
  - ➜ in 25% of the D sees a photon ➜ no information
  - ➜ in 25% of the cases C sees a photon ➜ object B registered without interaction