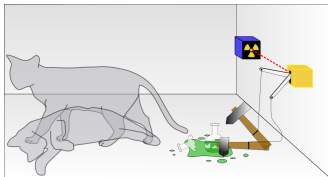


Experimental Tests of Quantum Mechanics

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Outline

- *Basics*
- *Bell's Inequalities*
- *Early Experiments*
- *Parametric Down-Conversion*
- *Advanced Topics*
- *Summary*





→ heated (philosophical) discussions

- relation between QM and the understanding of nature
 - mathematical structure: (mostly) accepted
 - core issue: interpretation of QM
 - ✗ causality and chance
 - ✗ relation to classical physics
- persona remarks
 - classical picture: causality in space and time
 - QM: (perhaps) theorie regarding information
 - ✗ information is intrinsically quantized
 - ✗ relevant for sufficiently small physical systems
 - ✗ QM was developed when atomic scales became accessible
 - general observation
 - ✗ information given: classical behavior
 - ✗ information missing: chance

❖ in the following:

Try to get a better understanding from comparing theory and experiment

→ some Nobelprize awarded results. . .



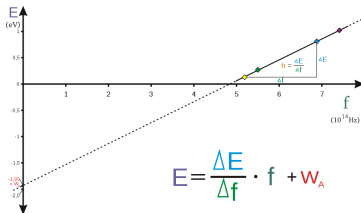
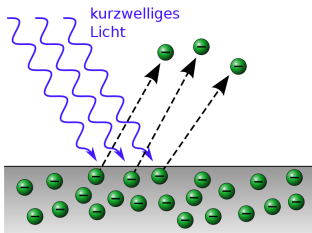
→ Max Planck

- discovery of the quantization of action $h \neq 0$
 - energy of light-waves is continuous
 - interaction with matter is quantized

$$E = h \nu$$

(Nobelprize 1918)

❖ experiment: photo-effect



(explanation by Einstein, Nobelprize 1921)



→ Albert Einstein

- mass and energy are equivalent

$$E = m c^2$$

→ De Broglie

- matter has wave-like properties, too
- particles cannot be perfectly localized

$$E = h \nu = m c^2 = (m c) c = p c$$

$$\nu = \frac{c}{\lambda}$$

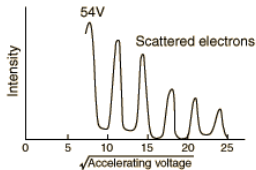
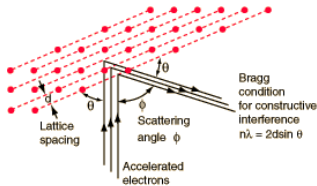
$$\text{Resultat} \rightarrow \lambda = \frac{h}{p}$$

(Nobelprize 1929)

❖ experiment: Davisson-Germer experiment

- diffraction in electron-scattering off crystals
- same phenomenology as X-ray scattering

(Nobelprize 1937)





Physics



Mathematics

state of a system

normalized wavefunction $|\psi\rangle$

observable S

hermitian operator S

Measurement

Eigenvalue und Eigenfunktion

→ discussion:

- $|\psi\rangle$ is element of a linear vector space: wavefunctions can be linearly superposed and it exist an inner product. The normalization is $\langle \psi | \psi \rangle = 1$.
- On the linear space of the wavefunctions S is a matrix with real eigenvalues λ_k and an ortho-normalsystem of eigenvectors $|\phi_k\rangle$, i.e. $\langle \phi_k | \phi_l \rangle = \delta_{kl}$.
- A measurement always yields an eigenvalue λ_k of the respective operator. After the measurement the wavefunction is the eigenvector $|\phi_k\rangle$ (“collapse of the wavefunction”, or “decoherence”).



→ the statistical interpretation of Quantum Mechanics

wanted: distribution of the measured values for a given state

$$|\psi\rangle = \sum_k a_k |\phi_k\rangle$$

with (in general) complex-valued coefficients a_k .

A priori a measurement can return any eigenvalue λ_k , i.e. the question is what are the relative frequencies p_k (probabilities). Exploit that the wavefunction the p_k are normalized:

$$1 = \sum_k p_k = \langle \psi | \psi \rangle = \sum_{k,l} a_k a_l^* \langle \phi_k | \phi_l \rangle = \sum_{k,l} a_k a_l^* \delta_{kl} = \sum_k |a_k|^2$$

and thus $p_k = |a_k|^2$ (Nobelprize 1954)

- in general the result of a measurement cannot be predicted, however . . .
- relative frequencies are fixed by the wavefunction
- only a system in the eigenstate ϕ_k deterministically yields the eigenvalue λ_k
- general predictability would contradict with relativity



→ determination of the expansion coefficients a_k

$$\langle \phi_k | \psi \rangle = \langle \phi_k | \left(\sum_i a_i | \phi_i \rangle \right) = \sum_i a_i \langle \phi_k | \phi_i \rangle = \sum_i a_i \delta_{ki} = a_k$$

→ expectation values

$$\langle S \rangle \equiv \sum_k p_k \lambda_k = \langle \psi | S | \psi \rangle$$

❖ proof:

$$\begin{aligned} \langle \psi | S | \psi \rangle &= \sum_{kl} a_k a_l^* \langle \phi_l | S | \phi_k \rangle = \sum_{kl} a_k a_l^* \lambda_k \langle \phi_l | \phi_k \rangle \\ &= \sum_{kl} a_k a_l^* \lambda_k \delta_{kl} = \sum_k |a_k|^2 \lambda_k = \sum_k p_k \lambda_k \end{aligned}$$

in the following: 2-state systems with $\lambda_{1,2} = \pm 1$ →



→ operators for spin-components in x, y, z

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

→ Eigenstates for σ_z and σ_x

$$\begin{aligned} |\uparrow\rangle_z &= \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \lambda = +1 & \text{und} & \quad |\downarrow\rangle_z &= \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \lambda = -1 \\ |\uparrow\rangle_x &= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \lambda = +1 & \text{und} & \quad |\downarrow\rangle_x &= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \lambda = -1 \end{aligned}$$

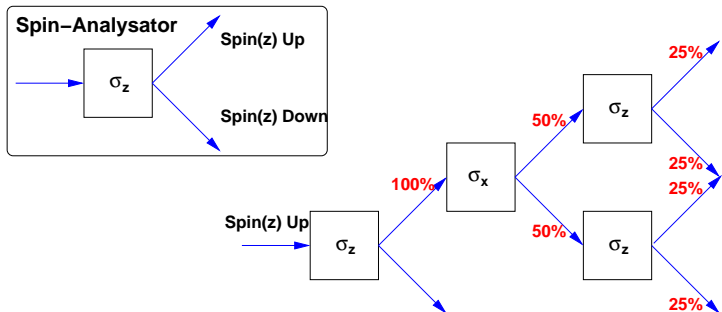
→ transformation between the two bases

$$\begin{aligned} |\uparrow\rangle_x &= \frac{1}{\sqrt{2}} (|\uparrow\rangle_z + |\downarrow\rangle_z) & \text{und} & \quad |\downarrow\rangle_x &= \frac{1}{\sqrt{2}} (|\uparrow\rangle_z - |\downarrow\rangle_z) \\ |\uparrow\rangle_z &= \frac{1}{\sqrt{2}} (|\uparrow\rangle_x + |\downarrow\rangle_x) & \text{und} & \quad |\downarrow\rangle_z &= \frac{1}{\sqrt{2}} (|\uparrow\rangle_x - |\downarrow\rangle_x) \end{aligned}$$



→ example: consequence for Stern-Gerlach type experiments

- initial stat: $|\uparrow\rangle_z$
- start with measurement of the z -component of the spins
- then measure the x -component
- then measure the z -component



After a measurement all information about earlier states has been erased!



→ construction of “classical” product states

z.B. $|\psi\rangle = |\uparrow\rangle_1 \otimes |\uparrow\rangle_2 \equiv |\uparrow\uparrow\rangle$ oder $|\psi\rangle = |\uparrow\rangle_1 \otimes |\downarrow\rangle_2 \equiv |\uparrow\downarrow\rangle$

- direct product of single particle states
- use a basis (here and below) == eigenstates of σ_z

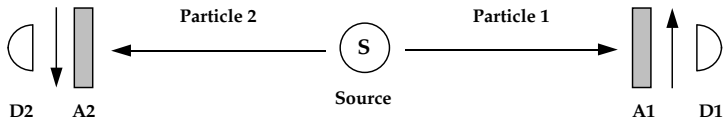
→ new: “entangled states”

z.B. $|\psi\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$ (spin-singlet)

- possible because of the superposition principle in QM
- no classical interpretation - both particles are simultaneously “up” and “down”
- interesting phenomenology when measuring both spins ...



→ spin-correlation for the spin-singlet state



- xz -direction of spin measurement: particle-1: α , particle-2: β
- operators for those observables (e.g. α)

$$\sigma_\alpha = \cos \alpha \cdot \sigma_z - \sin \alpha \cdot \sigma_x$$

- effects of base-operators

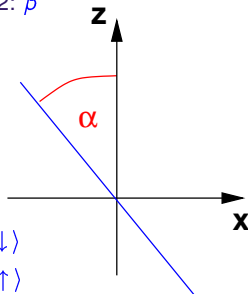
$$\sigma_z |\uparrow\rangle = |\uparrow\rangle \quad \text{und} \quad \sigma_z |\downarrow\rangle = -|\downarrow\rangle$$

$$\sigma_x |\uparrow\rangle = |\downarrow\rangle \quad \text{und} \quad \sigma_x |\downarrow\rangle = |\uparrow\rangle$$

- effects of the operators for the actual observables

$$\sigma_\alpha |\uparrow\rangle = \cos \alpha |\uparrow\rangle - \sin \alpha |\downarrow\rangle \equiv c_\alpha |\uparrow\rangle - s_\alpha |\downarrow\rangle$$

$$\sigma_\alpha |\downarrow\rangle = -\cos \alpha |\downarrow\rangle - \sin \alpha |\uparrow\rangle \equiv -c_\alpha |\downarrow\rangle - s_\alpha |\uparrow\rangle$$



then calculate...



❖ expectation values of individual measurements

$$\begin{aligned}\langle \sigma_\alpha \rangle &= \frac{1}{2} [\langle \uparrow\downarrow | - \langle \downarrow\uparrow |] (\sigma_\alpha) [| \uparrow\downarrow \rangle - | \downarrow\uparrow \rangle] \\ &= \frac{1}{2} [\langle \uparrow\downarrow | - \langle \downarrow\uparrow |] [\sigma_\alpha | \uparrow\downarrow \rangle - \sigma_\alpha | \downarrow\uparrow \rangle] \\ &= \frac{1}{2} [\langle \uparrow\downarrow | - \langle \downarrow\uparrow |] [(c_\alpha | \uparrow\downarrow \rangle - s_\alpha | \downarrow\downarrow \rangle) - (-c_\alpha | \downarrow\uparrow \rangle - s_\alpha | \uparrow\uparrow \rangle)] \\ &= \frac{1}{2} [\langle \uparrow\downarrow | - \langle \downarrow\uparrow |] [c_\alpha (| \uparrow\downarrow \rangle + | \downarrow\uparrow \rangle) - s_\alpha (| \downarrow\downarrow \rangle - | \uparrow\uparrow \rangle)] \\ &= \frac{1}{2} c_\alpha [\langle \uparrow\downarrow | \uparrow\downarrow \rangle - \langle \downarrow\uparrow | \downarrow\uparrow \rangle] = 0\end{aligned}$$

❖ note:

- σ_α only acts on the first particle
- σ_β would only act on the other particle
- formally everything can be expressed by 4×4 matrices
- inner products of orthogonal states are zero
- single measurements are random with equal probability for \uparrow_α und \downarrow_α



❖ expectation value of the product (correlation)

$$\begin{aligned}\langle \sigma_\alpha \sigma_\beta \rangle &= \frac{1}{2} [\langle \uparrow\downarrow | - \langle \downarrow\uparrow |] (\sigma_\alpha \sigma_\beta) [| \uparrow\downarrow \rangle - | \downarrow\uparrow \rangle] \\ &= \frac{1}{2} [\langle \uparrow\downarrow | - \langle \downarrow\uparrow |] [| (c_\alpha \uparrow - s_\alpha \downarrow)(-c_\beta \downarrow - s_\beta \uparrow) \rangle - | (-c_\alpha \downarrow - s_\alpha \uparrow)(c_\beta \uparrow - s_\beta \downarrow) \rangle] \\ &= \frac{1}{2} [\langle \uparrow\downarrow | - \langle \downarrow\uparrow |] [| \uparrow\downarrow \rangle (-c_\alpha c_\beta - s_\alpha s_\beta) + | \downarrow\uparrow \rangle (s_\alpha s_\beta + c_\alpha c_\beta) \\ &\quad + | \uparrow\uparrow \rangle (-c_\alpha s_\beta - s_\alpha c_\beta) + | \downarrow\downarrow \rangle (s_\alpha c_\beta - c_\alpha s_\beta)] \\ &= \frac{1}{2} (-c_\alpha c_\beta - s_\alpha s_\beta) \langle \uparrow\downarrow | \uparrow\downarrow \rangle - \frac{1}{2} (s_\alpha s_\beta + c_\alpha c_\beta) \langle \downarrow\uparrow | \downarrow\uparrow \rangle \\ &= -(c_\alpha c_\beta + c_\alpha c_\beta) = -\cos(\alpha - \beta) \equiv -\cos(\phi)\end{aligned}$$

→ the correlation is only a function of the opening angle $\phi = \alpha - \beta$

spin-1/2 particles $\langle \sigma_\alpha \sigma_\beta \rangle = -\cos \phi$

photons (spin-1) $\langle \sigma_\alpha \sigma_\beta \rangle = -\cos 2\phi$

(180deg between orthogonal spin-1/2 states, 90deg between orthogonal photon polarisations)

→ Interpretation

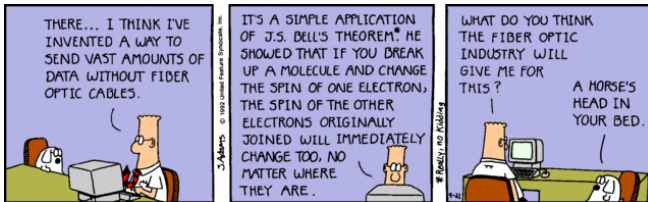


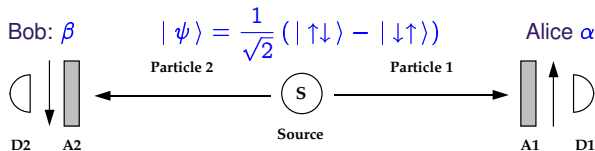
→ $\langle \sigma_\alpha \sigma_\beta \rangle$ is only a function of ϕ

- single measurements are perfectly random
- equal probability to measure “Spin-up” or “Spin-down”
- perfect anti-correlation of both measurements refer to the same direction
- independent of space and time, i.e.
 - independent of the time ordering of the measurement
 - independent of the spatial separation

→ obvious(?) questions:

- Is there “spooky action at a distance” which causes perfect synchronisation?
- can one use this to transmit information with $v = \infty$?





- analyzer setting $\alpha \parallel \beta$: perfect anti-correlation
- analyser setting $\alpha \perp \beta$: uncorrelated measurements

→ Alice knows β and sends one bit to Bob by causing an excess of -1

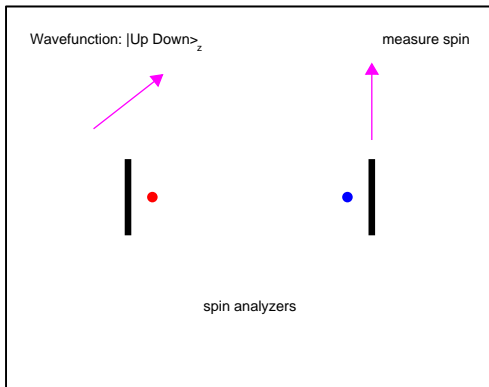
- case 1: Alice can influence her result
 - set $\alpha \parallel \beta$ and cause an excess of $+1$ at her side
 - Bob observes the same excess of -1
- case 2: Alice kann predict her result
 - prediction $+1$: set $\alpha \parallel \beta$ and Bob always sees -1
 - prediction -1 : set $\alpha \perp \beta$ and Bob measures equal numbers of ± 1

❖ insight:

If a quantum mechanical measurement is truly random, i.e. neither predictable, nor controllable then communication with $v > c$ is impossible.



→ measurement of spin correlations for spin-1/2 particles



- for **ideal detectors** and different wavefunctions consider . . .
 - measurements of spin correlations
 - coincidence measurements



→ The EPR-paradox (Einstein, Podolski, Rosen, 1935)

❖ quantum mechanics versus physical reality

■ definition: Element of reality

There exists a certain prediction ($p = 1$) for an observable, which can be obtained without perturbing the system.

■ definition: Complete theorie

Each element of reality is represented in the theory.

■ discussion:

→ plausible concepts

→ inconsistent with quantum mechanics

→ EPR: consider the wavefunction of a two-body decay $M \rightarrow m_1 m_2$, which is an eigenstate of both $(x_1 - x_2)$ and $(p_1 + p_2)$

$$(x_1 - x_2) | \psi \rangle = a | \psi \rangle$$

$$(p_1 + p_2) | \psi \rangle = P | \psi \rangle$$

→ allowed by quantum mechanics, since:

$$[x_1 - x_2, p_1 + p_2] = [x_1, p_1] - [x_2, p_1] + [x_1, p_2] - [x_2, p_2] = i\hbar - 0 + 0 - i\hbar = 0$$



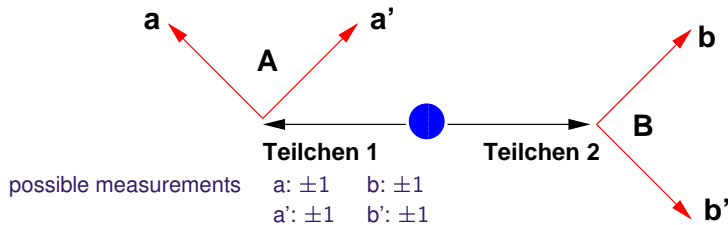
→ measurement and interpretation

- measure x_1
 - predict $x_2 = a + x_1$
 - always found when measured
- measure p_2
 - predict $p_1 = P - p_2$
 - always found when measured
- **result:** A measurement of x_1 and p_2 not only determines those, but also x_2 and p_1 . For each of the 4 variables one has a sure prediction.
 - x_1 AND p_1 as well as x_2 AND p_2 are elements of reality.
- quantum mechanics:
 - x_1 exclusive OR p_1 and x_2 /exclusive OR p_2 are elements of reality.
- solution to the contradiction
 - Option 1:
Quantum mechanics is incomplete. There are additional hidden parameters which also explain the statistical properties and action at a distance.
 - Option 2:
Einstein's concept of reality is not realized by nature.



→ consider two-particle systems and 2×2 analyser settings

J.S. Bell: *Physics* 1 (1964) 195, *On the Einstein Podolski Rosen paradox*



❖ local-deterministic assumption:

During the decay some hidden variables determine which measurements the two particles produce for given analyser settings. Each possible outcome has a fixed probability to be realized. For example:

$$\rho(a = +1, a' = -1, b = -1, b' = +1) :$$

probability that particle-1 for analyzer setting $a, (a')$ yields the measurement $+1, (-1)$, and particle-2 for analyzer setting $b, (b')$ the measurement Messwert $-1, (+1)$.



→ configuration space and possible measurements

k	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
a	-	-	-	-	-	-	-	-	+	+	+	+	+	+	+	+
b	-	-	-	-	+	+	+	+	-	-	-	-	+	+	+	+
a'	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+	+
b'	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-	+

For the configurations k one has:

$$\rho_k \geq 0 \quad \forall k \quad \text{and} \quad \sum_k \rho_k = 1$$

❖ measurements:

$E(x, y) = \langle x \cdot y \rangle$ expectation value of the product of the measurements x and y

$F(x, y)$ probability for $x = +1 \wedge y = +1$

The expectation values $E(x, y)$ are theoretically nice, the $F(x, y)$ are experimentally easier to determine coincidence probabilities, where $+1$ means that a particles passes the analyzer and is recorded in a detector.



→ linear combination of expectation values

$$T = E(a, b) - E(a, b') + E(a', b) + E(a', b') \equiv E_1 - E_2 + E_3 + E_4$$

❖ straightforward calculation:

k	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
a	-	-	-	-	-	-	-	-	+	+	+	+	+	+	+	+
b	-	-	-	-	+	+	+	+	-	-	-	-	+	+	+	+
a'	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+	+
b'	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-	+

$$E_1 : \rho_0 + \rho_1 + \rho_2 + \rho_3 - \rho_4 - \rho_5 - \rho_6 - \rho_7 - \rho_8 - \rho_9 - \rho_{10} - \rho_{11} + \rho_{12} + \rho_{13} + \rho_{14} + \rho_{15}$$

$$E_2 : \rho_0 - \rho_1 + \rho_2 - \rho_3 + \rho_4 - \rho_5 + \rho_6 - \rho_7 - \rho_8 + \rho_9 - \rho_{10} + \rho_{11} - \rho_{12} + \rho_{13} - \rho_{14} + \rho_{15}$$

$$E_3 : \rho_0 + \rho_1 - \rho_2 - \rho_3 - \rho_4 - \rho_5 + \rho_6 + \rho_7 + \rho_8 + \rho_9 - \rho_{10} - \rho_{11} - \rho_{12} - \rho_{13} + \rho_{14} + \rho_{15}$$

$$E_4 : \rho_0 - \rho_1 - \rho_2 + \rho_3 + \rho_4 - \rho_5 - \rho_6 + \rho_7 + \rho_8 - \rho_9 - \rho_{10} + \rho_{11} + \rho_{12} - \rho_{13} - \rho_{14} + \rho_{15}$$

collecting all terms:

$$\begin{aligned} T &= 2 \cdot (\rho_0 + \rho_1 + \rho_3 + \rho_7 + \rho_8 + \rho_{12} + \rho_{14} + \rho_{15}) \\ &\quad - 2 \cdot (\rho_2 + \rho_4 + \rho_5 + \rho_6 + \rho_9 + \rho_{10} + \rho_{11} + \rho_{13}) \end{aligned}$$

and using $\sum \rho_k \leq 1$ one finds:

$$-2 \leq T \leq 2$$



→ linear combination of coincidence probabilities

$$S = F(a, b) - F(a, b') + F(a', b) + F(a', b') - F(a') - F(b)$$

❖ straightforward calculation:

k	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
a	-	-	-	-	-	-	-	-	+	+	+	+	+	+	+	+
b	-	-	-	-	+	+	+	+	-	-	-	-	+	+	+	+
a'	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+	+
b'	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-	+

$$F(a, b) = \rho_{12} + \rho_{13} + \rho_{14} + \rho_{15}$$

$$F(a, b') = \rho_9 + \rho_{11} + \rho_{13} + \rho_{15}$$

$$F(a', b) = \rho_6 + \rho_7 + \rho_{14} + \rho_{15}$$

$$F(a', b') = \rho_3 + \rho_7 + \rho_{11} + \rho_{15}$$

$$F(a') = \rho_2 + \rho_3 + \rho_6 + \rho_7 + \rho_{10} + \rho_{11} + \rho_{14} + \rho_{15}$$

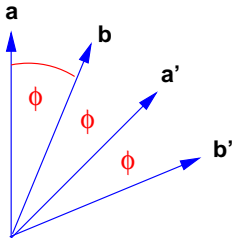
$$F(b) = \rho_4 + \rho_5 + \rho_6 + \rho_7 + \rho_{12} + \rho_{13} + \rho_{14} + \rho_{15}$$

collecting all terms:

$$S = -\rho_2 - \rho_3 - \rho_4 - \rho_5 - \rho_6 - \rho_9 - \rho_{11} - \rho_{13} \quad \text{and thus} \quad S \leq 0$$



→ consider symmetric configurations



❖ predictions

Spin-1/2 particles: $E(x, y) = -\cos \phi_{xy}$

Photons: $E(x, y) = -\cos 2\phi_{xy}$

experimental: $E(x, y) \rightarrow V \cdot E(x, y)$

attenuation of the correlation by Visibility $V < 1$

→ predictions for T

$$\begin{aligned} T(\text{Spin } 1/2) &= E(a, b) - E(a, b') + E(a', b) + E(a', b') \\ &= V \cdot (-\cos(\phi) + \cos(3\phi) - \cos(\phi) - \cos(\phi)) \\ &= V \cdot (\cos(3\phi) - 3\cos(\phi)) \end{aligned}$$

$$T(\text{Spin } 1) = V \cdot (\cos(6\phi) - 3\cos(2\phi))$$

→ result: QM: $|T| \leq V \cdot 2\sqrt{2}$ vs Bell: $|T| < 2$



→ predictions for S

relation between coincidence probability $F(x, y)$ and $E(x, y)$:

$$E(x, y) = F^{++}(x, y) + F^{--}(x, y) - F^{+-}(x, y) - F^{-+}(x, y)$$

with F^{ab} the probability to observe simultaneously a at analyzer setting x for particle 1, and b at analyzer setting y for particle 2. Using

$$F = F^{++} = F^{--} \quad , \quad F^{+-} = F^{-+} \quad , \quad F^{++} + F^{+-} = \frac{1}{2}$$

one finds

$$E(x, y) = 2 \cdot F(x, y) - 2 \cdot \left(\frac{1}{2} - F(x, y) \right) \quad \text{or} \quad F(x, y) = \frac{1}{4}(1 + E(x, y))$$

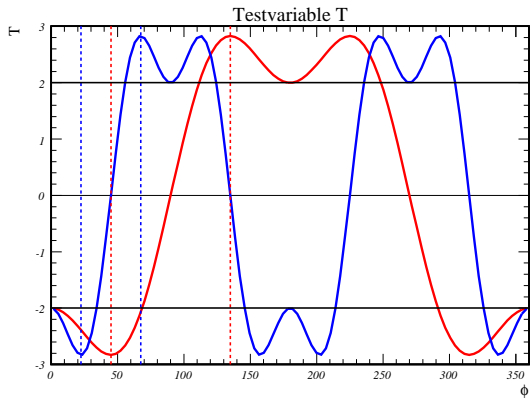
and with $F(a') = F(b) = 1/2$ finally

$$\begin{aligned} S &= F(a, b) - F(a, b') + F(a', b) + F(a', b') - F(a') - F(b) \\ &= -\frac{1}{2} + \frac{1}{4} (E(a, b) - E(a, b') + E(a', b) + E(a', b')) = -\frac{1}{2} + \frac{T}{4} \end{aligned}$$

→ result: QM: $S < (\sqrt{2} - 1)/2 \approx 0.207$ vs Bell: $S < 0$



→ the test variables S and T are equivalent:



❖ most sensitive settings

Photons $\phi = 22.5^\circ, 67.5^\circ$

spin 1/2 $\phi = 45^\circ, 135^\circ$

→ blue: QM-prediction for photons

→ red: QM-prediction for Spin-1/2 particles



→ quantum mechanics permits violations of Bell's inequalities

❖ consequences of experimental confirmation

- ❑ local-realistic theories are falsified
 - there are no hidden variables
 - nature is non-local
- ❑ the experiments did not really test quantum mechanics

❖ Loopholes

- ❑ static experimental setup affects hidden parameters
 - decide on analyser setting only after emission of the particles
- ❑ analyser are not spacial separated (i.e. within light cone)
 - use large distanced
 - measure in moving reference frames (each observer sees his particle first)
- ❑ das "Detection Loophole"
 - only a small fraction of all particles is recorded
 - this fraction is not a fair sampling of the total

→ (increasingly better) experiments. . .