Experimental Tests of Quantum Mechanics

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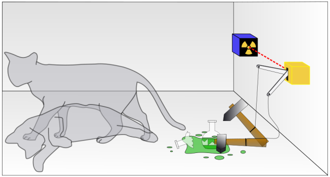
Outline

- *Basics*
- *Bell's Inequalities*
- *Early Experiments*
- *Parametric Down-Conversion*
- *Advanced Topics*
- *Summary*



➜ heated (philosophical) discussions

- relation between QM and the understanding of nature
  - ➜ mathematical structure: (mostly) accepted
  - ➜ core issue: interpretation of QM
    - ✘ causality and chance
    - ✘ relation to classical physics
- persona remarks
  - ➜ classical picture: causality in space and time
  - ➜ QM: (perhaps) theorie regarding information
    - ✘ information is intrinsically quantized
    - ✘ relevant for sufficiently small physical systems
    - ✘ QM was developed when atomic scales became accessible
  - ➜ general observation
    - ✘ information given: classical behavior
    - ✘ information missing: chance

❖ in the following:

Try to get a better understanding from comparing theory and experiment

➜ some Nobelprize awarded results. . .

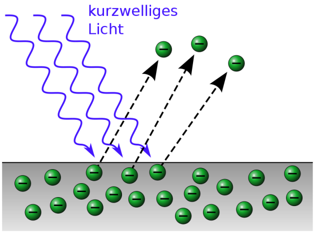
Max Planck

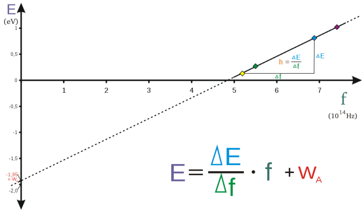
- discovery of the quantization of action $h \neq 0$
  - energy of light-waves is continuous
  - interaction with matter is quantized

$$E = h\,\nu$$

(Nobelprize 1918)

experiment: photo-effect





(explanation by Einstein, Nobelprize 1921)

➜ Albert Einstein

- mass and energy are equivalent

$$E = m\,c^2$$

➜ De Broglie

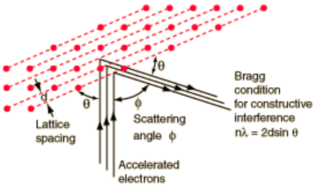
- matter has wave-like properties, too
- particles cannot be perfectly localized

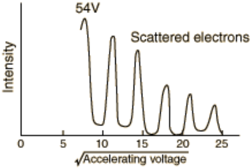
$$E = h\,\nu = m\,c^2 = (mc)\,c = p\,c$$

$$\nu = \frac{c}{\lambda}$$

Resultat ➜ $\lambda = \frac{h}{p}$

(Nobelprize 1929)





❖ experiment: Davisson-Germer experiment

- diffraction in electron-scattering off crystals
- same phenomenology as X-ray scattering

(Nobelprize 1937)

Φ Formal description

Physics	$\Longleftrightarrow$	Mathematics
state of a system		**normalized wavefunction** $\mid \psi \rangle$
observable S		**hermitian operator** S
Measurement		**Eigenvalue und Eigenfunction**

➜ discussion:

- $\mid \psi \rangle$ is element of a linear vector space: wavefunctions can be linearly superposed and it exsist an inner product. The normalization is $\langle \psi \mid \psi \rangle$=1.
- On the linear space of the wavefunctions S is a matrix with real eigenvalues λ_k and an ortho-normalsystem of eigenvectors $\mid \phi_k \rangle$, i.e. $\langle \phi_k \mid \phi_l \rangle = \delta_{kl}$.
- A measurement always yields an eigenvalue λ_k of the respective operator. After the measurement the wavefunction is the eigenvector $\mid \phi_k \rangle$ (“collapse of the wavefunction”, or “decoherence”).

➜ the statistical interpretation of Quantum Mechanics

wanted: distribution of the measured values for a given state

$$| \psi \rangle = \sum_k a_k | \phi_k \rangle$$

with (in general) complex-valued coefficients a_k.

A priori a measurement can return any eigenvalue λ_k, i.e. the question is what are the relative frequencies p_k (probabilities). Exploit that the wavefunction the p_k are normalized:

$$1 = \sum_k p_k = \langle \psi | \psi \rangle = \sum_{k,l} a_k a_l^* \langle \phi_k | \phi_l \rangle = \sum_{k,l} a_k a_l^* \delta_{kl} = \sum_k |a_k|^2$$

and thus $p_k = |a_k|^2$ (Nobelprize 1954)

- in general the result of a measurement cannot be predicted, however . . .
- relative frequencies are fixed by the wavefunction
- only a system in the eigenstate ϕ_k deterministically yields the eigenvalue λ_k
- general predictability would contradict with relativity

Φ Linear algebra...

➜ determination of the expansion coefficients a_k

$$\langle\, \phi_k \mid \psi \,\rangle = \langle\, \phi_k \mid \left(\sum_i a_i \mid \phi_i \,\rangle \right) = \sum_i a_i \langle\, \phi_k \mid \phi_i \,\rangle = \sum_i a_i \delta_{ki} = a_k$$

➜ expectation values

$$\langle S \rangle \equiv \sum_k p_k \lambda_k = \langle\, \psi \mid S \mid \psi \,\rangle$$

❖ proof:

$$\langle\, \psi \mid S \mid \psi \,\rangle = \sum_{kl} a_k\, a_l^* \langle\, \phi_l \mid S \mid \phi_k \,\rangle = \sum_{kl} a_k\, a_l^* \lambda_k \langle\, \phi_l \mid \phi_k \,\rangle$$

$$= \sum_{kl} a_k\, a_l^* \lambda_k \delta_{kl} = \sum_k |a_k|^2 \lambda_k = \sum_k p_k \lambda_k$$

in the following: 2-state systems with $\lambda_{1,2} = \pm 1$ ➜

Spin-1/2 particles (i)

➜ operators for spin-components in x, y, z

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

➜ Eigenstates for σ_z and σ_x

$$|\uparrow\rangle_z = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \ \lambda = +1 \quad \text{und} \quad |\downarrow\rangle_z = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \ \lambda = -1$$

$$|\uparrow\rangle_x = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \ \lambda = +1 \quad \text{und} \quad |\downarrow\rangle_x = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \ \lambda = -1$$

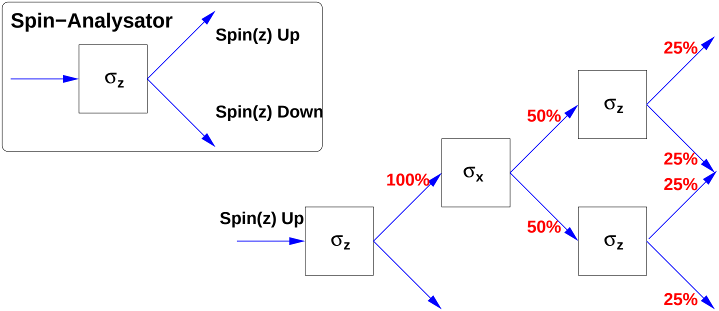
➜ transformation between the two bases

$$|\uparrow\rangle_x = \frac{1}{\sqrt{2}} \left(|\uparrow\rangle_z + |\downarrow\rangle_z \right) \quad \text{und} \quad |\downarrow\rangle_x = \frac{1}{\sqrt{2}} \left(|\uparrow\rangle_z - |\downarrow\rangle_z \right)$$

$$|\uparrow\rangle_z = \frac{1}{\sqrt{2}} \left(|\uparrow\rangle_x - |\downarrow\rangle_x \right) \quad \text{und} \quad |\downarrow\rangle_z = \frac{1}{\sqrt{2}} \left(|\uparrow\rangle_x + |\downarrow\rangle_x \right)$$

➜ example: consequence for Stern-Gerlach type experiments

- initial stat: $|\uparrow\rangle_z$
- start with measurement of the z-component of the spins
- then measure the x-component
- then measure the z-component



After a measurement all information about earlier states has been erased!

Φ *Two-particle systems*

➜ construction of “classical” product states

z.B. $|\,\psi\,\rangle = |\uparrow\rangle_1 \otimes |\uparrow\rangle_2 \equiv |\uparrow\uparrow\rangle$ oder $|\,\psi\,\rangle = |\uparrow\rangle_1 \otimes |\downarrow\rangle_2 \equiv |\uparrow\downarrow\rangle$

- direct product of single particle states
- use a basis (here and below) == eigenstates of σ_z

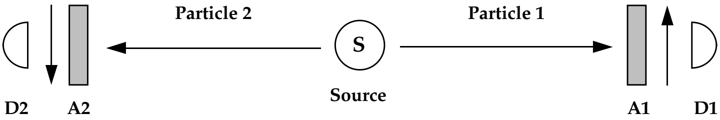
➜ new: “entagled states”

z.B. $|\,\psi\,\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$ (spin-singlet)

- possible because of the superposition principle in QM
- no classical interpretation - both particles are simultaneously “up” and “down”
- interesting phenomenology when measuring both spins . . .

Φ Quantum mechanical prediction (i)

➜ spin-correlation for the spin-singlet state



- xz-direction of spin measurement: particle-1: α, particle-2: β
- operators for those observables (e.g. α)

$$\sigma_\alpha = \cos\alpha \cdot \sigma_z - \sin\alpha \cdot \sigma_x$$

- effects of base-operators

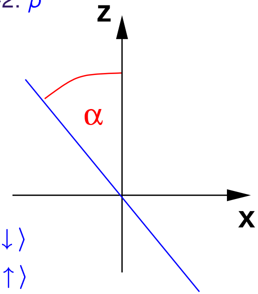
$$\sigma_z \,|\uparrow\rangle = |\uparrow\rangle \quad \text{und} \quad \sigma_z \,|\downarrow\rangle = -|\downarrow\rangle$$

$$\sigma_x \,|\uparrow\rangle = |\downarrow\rangle \quad \text{und} \quad \sigma_x \,|\downarrow\rangle = |\uparrow\rangle$$

- effects of the operators for the actual observables

$$\sigma_\alpha \,|\uparrow\rangle = \cos\alpha\,|\uparrow\rangle - \sin\alpha\,|\downarrow\rangle \equiv c_\alpha\,|\uparrow\rangle - s_\alpha\,|\downarrow\rangle$$

$$\sigma_\alpha \,|\downarrow\rangle = -\cos\alpha\,|\downarrow\rangle - \sin\alpha\,|\uparrow\rangle \equiv -c_\alpha\,|\downarrow\rangle - s_\alpha\,|\uparrow\rangle$$



then calculate. . .

Quantum mechanical prediction (ii)

- expectation values of individual measurements

$$\begin{aligned}
\langle\sigma_\alpha\rangle &= \frac{1}{2}\left[\langle\uparrow\downarrow| - \langle\downarrow\uparrow|\right](\sigma_\alpha)\left[|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right] \\
&= \frac{1}{2}\left[\langle\uparrow\downarrow| - \langle\downarrow\uparrow|\right]\left[\sigma_\alpha|\uparrow\downarrow\rangle - \sigma_\alpha|\downarrow\uparrow\rangle\right] \\
&= \frac{1}{2}\left[\langle\uparrow\downarrow| - \langle\downarrow\uparrow|\right]\left[(c_\alpha|\uparrow\downarrow\rangle - s_\alpha|\downarrow\downarrow\rangle) - (-c_\alpha|\downarrow\uparrow\rangle - s_\alpha|\uparrow\uparrow\rangle)\right] \\
&= \frac{1}{2}\left[\langle\uparrow\downarrow| - \langle\downarrow\uparrow|\right]\left[c_\alpha(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle) - s_\alpha(|\downarrow\downarrow\rangle - |\uparrow\uparrow\rangle)\right] \\
&= \frac{1}{2}c_\alpha\left[\langle\uparrow\downarrow|\uparrow\downarrow\rangle - \langle\downarrow\uparrow|\downarrow\uparrow\rangle\right] = 0
\end{aligned}$$

- note:
  - σ_α only acts on the first particle
  - σ_β would only act on the other particle
  - formally everything can be expressed by 4×4 matrices
  - inner products of orthogonal states are zero
  - single measurements are random with equal probability for $\uparrow_\alpha$ und $\downarrow_\alpha$

Φ Quantum mechanical prediction (iii)

❖ expectation value of the product (correlation)

$$\langle \sigma_\alpha \sigma_\beta \rangle = \frac{1}{2} \left[\langle \uparrow\downarrow | - \langle \downarrow\uparrow | \right] (\sigma_\alpha \sigma_\beta) \left[| \uparrow\downarrow \rangle - | \downarrow\uparrow \rangle \right]$$

$$= \frac{1}{2} \left[\langle \uparrow\downarrow | - \langle \downarrow\uparrow | \right] \left[| \, (c_\alpha \uparrow - s_\alpha \downarrow)(-c_\beta \downarrow - s_\beta \uparrow) \rangle - | \, (-c_\alpha \downarrow - s_\alpha \uparrow)(c_\beta \uparrow - s_\beta \downarrow) \rangle \right]$$

$$= \frac{1}{2} \left[\langle \uparrow\downarrow | - \langle \downarrow\uparrow | \right] \left[| \uparrow\downarrow \rangle (-c_\alpha c_\beta - s_\alpha s_\beta) + | \downarrow\uparrow \rangle (s_\alpha s_\beta + c_\alpha c_\beta) \right.$$

$$\left. + | \uparrow\uparrow \rangle (-c_\alpha s_\beta - s_\alpha c_\beta) + | \downarrow\downarrow \rangle (s_\alpha c_\beta - c_\alpha s_\beta) \right]$$

$$= \frac{1}{2} (-c_\alpha c_\beta - s_\alpha s_\beta) \langle \uparrow\downarrow | \uparrow\downarrow \rangle - \frac{1}{2} (s_\alpha s_\beta + c_\alpha c_\beta) \langle \downarrow\uparrow | \downarrow\uparrow \rangle$$

$$= -(c_\alpha c_\beta + c_\alpha c_\beta) = -\cos(\alpha - \beta) \equiv -\cos(\phi)$$

➜ the correlation is only a function of the opening angle $\phi = \alpha - \beta$

spin-1/2 particles $\langle \sigma_\alpha \sigma_\beta \rangle = -\cos\phi$

photons (spin-1) $\langle \sigma_\alpha \sigma_\beta \rangle = -\cos 2\phi$

(180deg between orthogonal spin-1/2 states, 90deg between orthogonal photon polarisations)

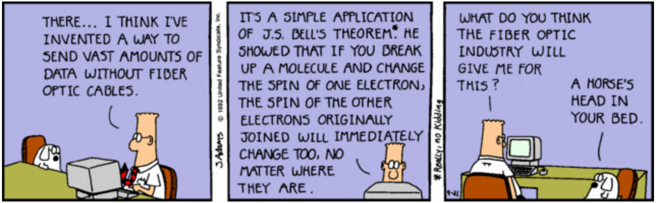
➜ Interpretation

➜ $\langle \sigma_\alpha \sigma_\beta \rangle$ is only a function of ϕ

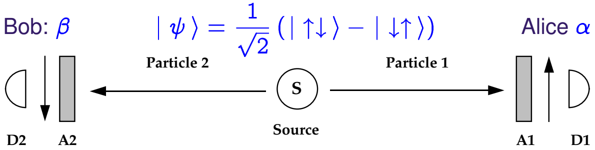
- single measurements are perfectly random
- equal probability to measure “Spin-up” or “Spin-down”
- perfect anti-correlation of both measurements refer to the same direction
- independent of space and time, i.e.
  - ➜ independent of the time ordering of the measurement
  - ➜ independent of the spatial separation

➜ obvious(?) questions:

- Is there “spooky action at a distance” which causes perfect synchronisation?
- can one use this to transmit information with $v = \infty$?



Communication with $v = \infty$?



- analyzer setting $\alpha \parallel \beta$: perfect anti-correlation
- analyser setting $\alpha \perp \beta$: uncorrelated measurements

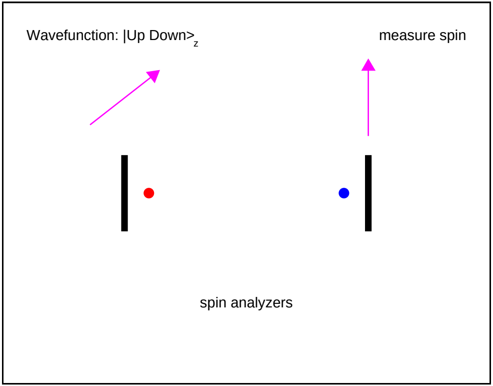
➜ Alice knows β and sends one bit to Bob by causing an excess of -1

- case 1: Alice can influence her result
  - ➜ set $\alpha \parallel \beta$ and cause an excess of $+1$ at her side
  - ➜ Bob observes the same excess of -1
- case 2: Alice kann predict her result
  - ➜ prediction $+1$: set $\alpha \parallel \beta$ and Bob always sees -1
  - ➜ prediction -1: set $\alpha \perp \beta$ and Bob measures equal numbers of ± 1

❖ insight:

If a quantum mechanical measurement is truly random, i.e. neither predictable, nor controllable then communication with $v > c$ is impossible.

➜ measurement of spin correlations for spin-1/2 particles



- for ideal detektors and different wavefunctions consider . . .
  - ➜ measurements of spin correlations
  - ➜ coincidence measurements