Physics at a Linear Collider

— Basic Knowledge and Techniques

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Univ. of Wisconsin - Madison

ILC Workshop/Summer School,
TsingHua University, Beijing, China
(July 15 – 20, 2005)
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I. Basics for $e^+e^-$ Physics: SM expectations
II. Beyond the SM: SUSY and CP Violation
III. Other Operational Modes in a Linear Collider
IV. Techniques and Tools
V. High Energy Physics: Where We Are
I. Basics for $e^+e^-$ Physics: SM expectations

About an $e^+e^-$ Collider

The collisions between $e^-$ and $e^+$ have major advantages:

- The system of an electron and a positron has zero charge, zero lepton number etc., so that it is suitable to **create new particles** after $e^+e^-$ annihilation.

- With symmetric beams between the electrons and positrons, the laboratory frame is the same as the c.m. frame, so that the **total c.m. energy** is fully exploited to reach the highest possible physics threshold.

- With well-understood beam properties, the **scattering kinematics** is well-constrained.

- **Backgrounds low** and well-undercontrol.
• It is possible to achieve high degrees of beam polarizations, so that chiral couplings and other asymmetries can be effectively explored.

**Disadvantages**

• Large synchrotron radiation due to acceleration, 

\[ \Delta E \sim \frac{1}{R} \left( \frac{E}{m_e} \right)^4. \]

Thus, a multi-hundred GeV \( e^+e^- \) collider will have to be made a linear accelerator.

• This becomes a major challenge for achieving a high luminosity when a storage ring is not utilized; beamsstrahlung severe.
Our magnifying glass: Detector complex

hadronic calorimeter
E-CAL
tracking
(vertex detector
beam
pipe
muon chambers

photons
\( e^\pm \)
muons
\( \pi^\pm, p, n \)

Innermost Layer...

...Outermost Layer
**Something a theorist should know:**
— What do the SM particles look like in a detector?

Although gauge interactions present remarkable universality among the SM fermions, we theorists should always remember the significant differences how the SM particles actually look like in a detector...

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<td>$\to q\bar{q}'$</td>
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<td>2 jets</td>
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<tr>
<td>$Z^0 \to \ell^+ \ell^-$</td>
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<td>2 jets</td>
<td>$\times$</td>
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How to search for new particles?

Leptons (e, μ)

Photons

H → γγ

H → ZZ → ℓ⁺ℓ⁻τ⁺τ⁻
H → WW → ℓ⁺νℓ⁻ν

Taus

Jets

Z' → ℓ⁺ℓ⁻
W' → ℓν

H → ZZ → 4 leptons

H → ZZ → ℓνjj
H → WW → ℓνjj

Missing E_T

b-Jet-tag

unpredicted discovery

χ⁰, χ⁺ → n leptons + x
χ⁻ → n jets + E_T

H → ZZ → γγ
H → γγ

H → ZZ → 4 leptons

H → ZZ → ℓνjj
H → ZZ → ℓνjj

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H → ZZ → ℓνjj
— How to search for new particles?

Homework I-1: For $Z'$ particle that couples to SM fermions and $W^+W^-$. List the experimental final states.
We know that at the LHC ...
Huge reach for physics goals:

Event rate very high, but background suppression $10^{-6} - 10^{-10}$!
Baisc Standard Model Processes at an $e^+e^-$ Collider

— more details to come ...
The Mandelstam variables are defined for \( a + b \rightarrow 1 + 2 \):

\[
\begin{align*}
    s &= (p_a + p_b)^2 = (p_1 + p_2)^2 = E_{cm}^2, \\
    t &= (p_a - p_1)^2 = (p_b - p_2)^2 = m_a^2 + m_1^2 - 2(E_aE_1 - p_ap_1\cos\theta_{a1}), \\
    u &= (p_a - p_2)^2 = (p_b - p_1)^2 = m_a^2 + m_2^2 - 2(E_aE_2 - p_ap_2\cos\theta_{a2}).
\end{align*}
\]

Assume that \( m_a = m_1 \) and \( m_b = m_2 \), (Homework I-2: work out the kinematics.)

\[
\begin{align*}
    t &= -2p_{cm}^2(1 - \cos\theta_{a1}^*), \\
    u &= -2p_{cm}^2(1 + \cos\theta_{a1}^*) + \frac{(m_1^2 - m_2^2)^2}{s},
\end{align*}
\]

where \( p_{cm} = \lambda^{1/2}(s, m_1^2, m_2^2)/2\sqrt{s} \) is the momentum magnitude in the c.m. frame.

\( s \)- and \( t \)-channel singularities characteristic.
• $e^+e^- \rightarrow e^+e^-$

The Bhabha scattering: The leading order QED process; It presents $s,t$-channel “singularities” (see last page).

(Think 1: which diagrams? are they physical? observable?)
\[ e^+ e^- \rightarrow f \bar{f} \ (\mu \mu, \tau \tau, \ b\bar{b}, \ t\bar{t}, \ ... ) \]

The simplest reaction would be the QED process \( e^+ e^- \rightarrow \gamma^* \rightarrow \mu^+ \mu^- \):

\[
\sigma(e^+ e^- \rightarrow \gamma^* \rightarrow \mu^+ \mu^-) \equiv \sigma_{pt} = \frac{4\pi\alpha^2}{3s} \approx \frac{100 \text{ fb}}{(E_{cm}/1 \text{ TeV})^2}.
\]
• \( e^+e^- \rightarrow f \bar{f} \) (\( \mu \mu, \tau \tau, b \bar{b}, t \bar{t}, \ldots \))

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\]

\( \sigma_{pt} \) has become standard units.

Z contribution becomes important at \( \sqrt{s} \sim M_Z \).

Diagrams by MadGraph

Very important to produce new particle states (\( J = 1 \) channel).
• $e^+e^- \rightarrow W^+W^-, \ ZZ, \ \gamma\gamma$

Important to test non-Abelian gauge field self interactions. Important to produce new states (via $J = 1$ channel).
\( e^+e^- \to W^+W^-Z, \ W^+W^-\gamma \ldots \)

Important to check 4-point couplings;
First time for \( H \to WW, ZZ \) to contribute.
Leading Higgs production near threshold.

Diagrams by MadGraph

\( e^+ e^- \rightarrow Zh \)
\( e^+ e^- \rightarrow Zh \)

Leading Higgs production near threshold.

Diagrams by MadGraph

\[ \begin{array}{c}
\text{graph 1}
\end{array} \]

\( e^+ e^- \rightarrow W^* W^* h \rightarrow \bar{\nu} \nu h \)

Leading Higgs production for heavier \( H \).

Diagrams by MadGraph

\[ \begin{array}{c}
\text{graph 2}
\end{array} \]
Simple Formalism

For the production of two-particle $a,b$, the differential cross section is given by

$$\frac{d\sigma(e^+e^- \rightarrow ab)}{d\cos \theta} = \frac{\beta}{32\pi s} \sum |M|^2$$

where
- $\beta = \lambda^{1/2}(1, m^2_a/s, m^2_b/s)$, is the speed factor for the out-going particles in the c.m. frame, and $p_{cm} = \beta \sqrt{s}/2$,
- $\sum |M|^2$ the squared matrix element, summed and averaged over quantum numbers (like color and spins etc.)
- unpolarized beams so that the azimuthal angle trivially integrated out,
The $Z$ resonance prominent: Breit-Wigner resonance

$$\frac{1}{(s - M_V^2)^2 + \Gamma_V^2 M_V^2}$$
Resonant production:

For an arbitrary resonance $V$,

If the energy spread $\delta \sqrt{s} \ll \Gamma_V$, the line-shape mapped out:

$$\sigma(e^+e^- \rightarrow V \rightarrow X) = \frac{4\pi(2j + 1)\Gamma(V \rightarrow e^+e^-)\Gamma(V \rightarrow X)}{(s - M_V^2)^2 + \Gamma_V^2 M_V^2} \frac{s}{M_V^2},$$
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$$

If $\delta \sqrt{s} \gg \Gamma_V$, the narrow-width approximation:

$$
\sigma(e^+e^- \rightarrow V \rightarrow X) = \frac{1}{(s - M_V^2)^2 + \Gamma_V^2 M_V^2} \frac{\pi}{M_V \Gamma_V} \delta(s = M_V^2),
$$

$$
\frac{4\pi^2(2j + 1)\Gamma(V \rightarrow e^+e^-)BF(V \rightarrow X)}{M_V^3} \frac{dL(\hat{s} = M_V^2)}{d\tau}
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If $\delta \sqrt{s} \gg \Gamma_V$, the narrow-width approximation:

$$\frac{1}{(s - M_V^2)^2 + \Gamma_V^2 M_V^2} \to \frac{\pi}{M_V \Gamma_V} \delta(s = M_V^2),$$

$$\sigma(e^+e^- \to V \to X) = \frac{4\pi^2(2j+1)\Gamma(V \to e^+e^-)BF(V \to X)}{M_V^3} \frac{dL(\hat{s} = M_V^2)}{d\tau}.$$

Homework I-3: sketch the derivation of these two formulas.

For a more general relation between $\delta \sqrt{s} - \Gamma_V$, need more careful convolution $dL/d\tau$. 
Away from the resonance and for finite-angle scattering:

\[ \sigma \sim \frac{1}{s} \quad \text{or} \quad \sigma \sim \frac{1}{M^2_V} \ln^2 \frac{s}{M^2_V}. \]
Fermion production:

Common processes: \( e^- e^+ \rightarrow f \bar{f} \).

For most of the situations, the scattering matrix element can be casted into a \( V \pm A \) chiral structure of the form (sometimes with the help of Fierz transformations)

\[
M = \frac{e^2}{s} Q_{\alpha\beta} \left[ \bar{v}_{e^+}(p_2)\gamma^\mu P_\alpha u_{e^-}(p_1) \right] \left[ \bar{\psi}_f(q_1)\gamma_\mu P_\beta \psi'_f(q_2) \right],
\]

where \( P_\pm = (1 \mp \gamma_5)/2 \) are the \( L, R \) chirality projection operators, and \( Q_{\alpha\beta} \) are the bilinear couplings governed by the underlying physics of the interactions with the intermediate propagating fields.

With this structure, the scattering matrix element squared:

\[
\sum |M|^2 = \frac{e^4}{s^2} \left[ (|Q_{LL}|^2 + |Q_{RR}|^2) u_i u_j + (|Q_{LL}|^2 + |Q_{RL}|^2) t_i t_j \right. \\
\left. + 2 \text{Re}(Q^*_{LL}Q_{LR} + Q^*_{RR}Q_{RL}) m_f \bar{m}_f \right],
\]

where \( t_i = t - m_i^2 = (p_1 - q_1)^2 - m_i^2 \) and \( u_i = u - m_i^2 = (p_1 - q_2)^2 - m_i^2 \).

Homework I-4: Verify this formula.
Typical size of the cross sections:

- The $Z$ resonance prominent (or other $M_V$),
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- At the ILC $\sqrt{s} = 500$ GeV,

  $$\sigma(e^+e^- \rightarrow e^+e^-) \sim 100\sigma_{pt} \sim 40 \text{ pb}. $$

  (angular cut dependent.)
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  \[ \sigma_{pt} \sim \sigma(ZZ) \sim \sigma(tt) \sim 400 \text{ fb;} \]

  \[ \sigma(u,d,s) \sim 9\sigma_{pt} \sim 3.6 \text{ pb;} \]

  \[ \sigma(WW) \sim 20\sigma_{pt} \sim 8 \text{ pb.} \]
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$$\sigma(WW) \sim 20\sigma_{pt} \sim 8 \text{ pb}.$$ 

and

$$\sigma(ZH) \sim \sigma(WW \rightarrow H) \sim \sigma_{pt}/4 \sim 100 \text{ fb};$$
$$\sigma(WWZ) \sim 0.1\sigma_{pt} \sim 40 \text{ fb}.$$
Gauge boson radiation:

A qualitatively different process is initiated from gauge boson radiation, typically off fermions:

\[ f f' \rightarrow a, \gamma^* e^- \rightarrow e^+ e^- \]

The simplest case is the photon radiation off an electron, like:

\[ e^+ e^- \rightarrow e^+, \gamma^* e^- \rightarrow e^+ e^- \]

The dominant features are due to the result of a \( t \)-channel singularity, induced by the collinear photon splitting:

\[ \sigma(e^- a \rightarrow e^- X) \approx \int dx \ P_{\gamma/e}(x) \ \sigma(\gamma a \rightarrow X) \]

The so called the effective photon approximation.
For an electron of energy $E$, the probability of finding a collinear photon of energy $xE$ is given by

$$P_{\gamma/e}(x) = \frac{\alpha}{2\pi} \frac{1 + (1 - x)^2}{x} \ln \frac{E^2}{m_e^2},$$

known as the Weizsäcker-Williams spectrum.

(Think 2: How to derive this splitting function?)

We see that:

- $m_e$ enters the log to regularize the collinear singularity;
- $1/x$ leads to the infrared behavior of the photon;
- This picture of the photon probability distribution is also valid for other photon spectrum:

**Based on the back-scattering laser technique, it has been proposed to produce much harder photon spectrum, to construct a “photon collider”...**
(massive) Gauge boson radiation:

A similar picture may be envisioned for the electroweak massive gauge bosons, \( V = W^\pm, Z \).

Consider a fermion \( f \) of energy \( E \), the probability of finding a (nearly) collinear gauge boson \( V \) of energy \( xE \) and transverse momentum \( p_T \) (with respect to \( \vec{p}_f \)) is approximated by

\[
P_{V/f}^T(x, p_T^2) = \frac{g_V^2 + g_A^2}{8\pi^2} \frac{1 + (1 - x)^2}{x} \frac{p_T^2}{(p_T^2 + (1 - x)M_V^2)^2},
\]

\[
P_{V/f}^L(x, p_T^2) = \frac{g_V^2 + g_A^2}{4\pi^2} \frac{1 - x}{x} \frac{(1 - x)M_V^2}{(p_T^2 + (1 - x)M_V^2)^2}.
\]

Although the collinear scattering would not be a good approximation until reaching very high energies \( \sqrt{s} \gg M_V \), it is instructive to consider the qualitative features.

Think 3: Compare the qualitative features of the \( p_T \) spectra for \( P^T, P^L \) distributions.
Beam polarization:

One of the merits for an $e^+e^-$ linear collider is the possible high polarization for both beams. Consider first the longitudinal polarization along the beam line direction. Denote the average $e^\pm$ beam polarization by $P_{\pm}$, with $P_{\pm} = -1$ purely left-handed and $+1$ purely right-handed.

The polarized squared matrix element, based on the helicity amplitudes $M_{\sigma_e\sigma_{e'}}$:

$$\sum |M|^2 = \frac{1}{4} [(1 - P_L^\pm)(1 - P_L^\mp)|M_{--}|^2 + (1 - P_L^\pm)(1 + P_L^\mp)|M_{-+}|^2$$

$$+ (1 + P_L^\pm(1 - P_L^\mp)|M_{+-}|^2 + (1 + P_L^\pm(1 + P_L^\mp)|M_{++}|^2].$$

Since the electroweak interactions of the SM and beyond are chiral: Certain helicity amplitudes can be suppressed or enhanced by properly choosing the beam polarizations: e.g., $W^\pm$ exchange ...
Furthermore, it is possible to produce transversely polarized beams with the help of a spin-rotator. If the beams present average polarizations with respect to a specific direction perpendicular to the beam line direction, $-1 < P^T_\pm < 1$, then there will be one additional term in the limit $m_e \to 0$,

$$\frac{1}{4} 2 P^- P^+ \text{Re}(M_- + M^*_+).$$

The transverse polarization is particularly important when the interactions produce an asymmetry in azimuthal angle, such as the effect of CP violation.