Neutrino Physics

陆锦标 Kam-Biu Luk

Tsinghua University
and
University of California, Berkeley
and
Lawrence Berkeley National Laboratory

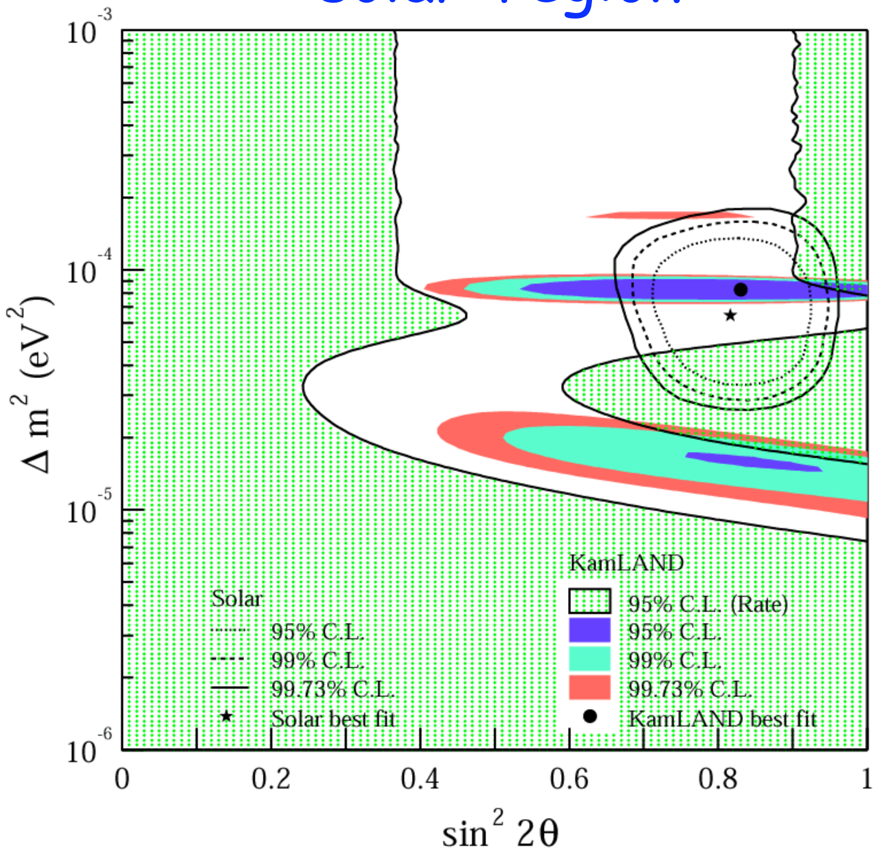
Lecture 10, 15 June, 2007

Outline

- Three-flavour description of neutrino oscillation
- Searching for mixing angle θ_{13}

What We Have Learned So Far

'solar' region

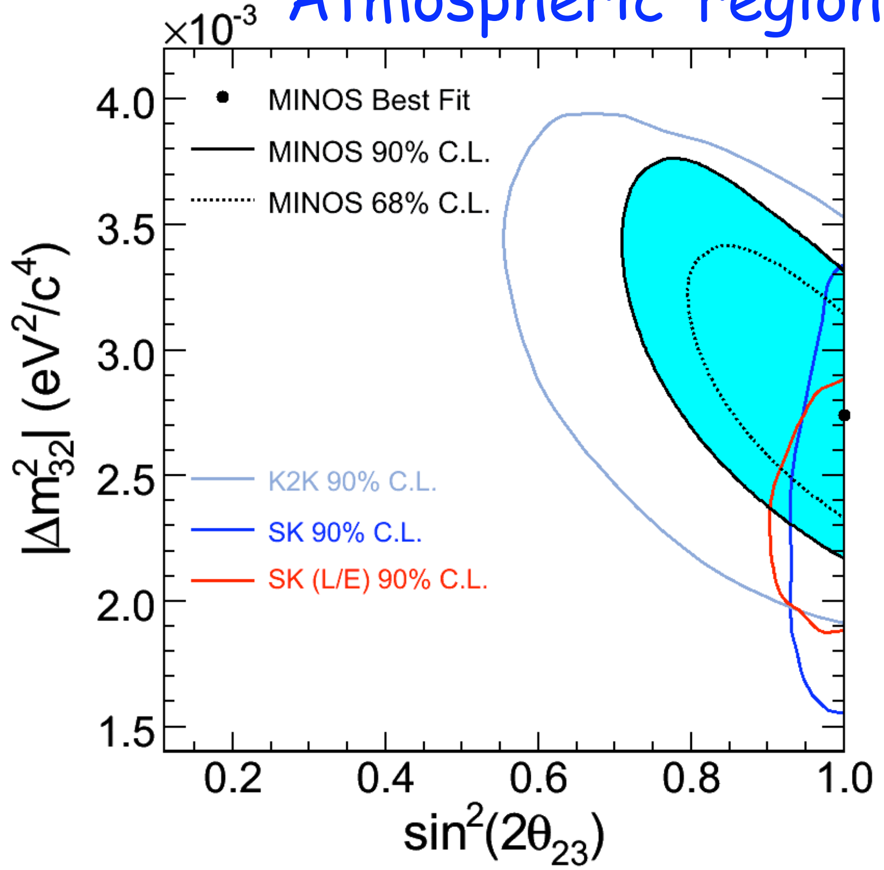


$$\Delta m^2_{12} = 8.2^{+0.6}_{-0.5} \times 10^{-5}\, eV^2$$

$$\tan^2\theta_{12} = 0.40^{+0.09}_{-0.07}$$

$\nu_e \rightarrow \nu_\mu + \nu_\tau$

'Atmospheric' region



$$|\Delta m^2_{32}| = 2.74^{+0.44}_{-0.26}\ (\text{stat} + \text{syst}) \times 10^{-3}\, \text{eV}^2$$

$$\sin^2 2\theta_{23} = 1.00_{-0.13}\ (\text{stat} + \text{syst})$$

$\nu_\mu \rightarrow \nu_\tau$

Neutrino Mixing Of Three Generations

- For three generations, the unitary mixing matrix is 3 x 3:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

- U is called Pontecorvo-Maki-Nakagawa-Sakata matrix.
- the matrix elements are in general complex numbers
- the PMNS matrix depends on four independent variables:

Three mixing angles: θ_{12}, θ_{23}, and θ_{13}

One CP-violating phase: δ

More On The Mixing Matrix

- In terms of the mixing angles and the phase:

$$\begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta_{13} & 0 & \sin\theta_{13} \\ 0 & 1 & 0 \\ -\sin\theta_{13} e^{i\delta} & 0 & \cos\theta_{13} e^{i\delta} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix}$$

- The PMNS matrix can be explicitly written as:

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta} & c_{23} c_{13} \end{pmatrix}$$

c = cos s = sin

- Mixing angle θ_{12} is associated with the 'solar' neutrino oscillation, and θ_{13} is tied to the 'atmospheric' neutrino oscillation.

Mass Hierarchy

- For the three mass eigenstates with masses m_1, m_2, and m_3:

$$m_1^2 - m_1^2 + m_2^2 - m_2^2 + m_3^2 - m_3^2 = 0$$

Hence, only two independent mass-squared differences:

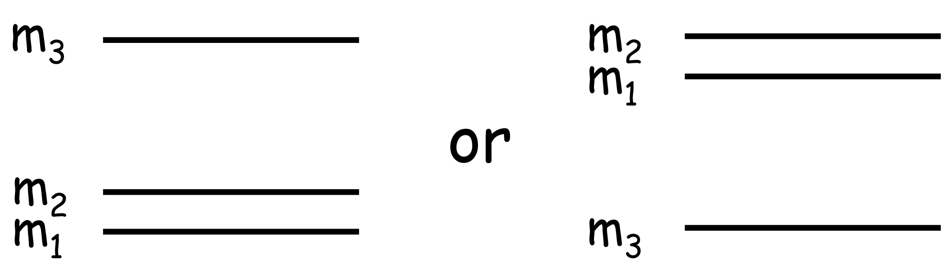
$$\Delta m_{21}^2 \quad \text{and} \quad \Delta m_{32}^2$$

such that $\Delta m_{31}^2 = m_3^2 - m_1^2 = \Delta m_{32}^2 + \Delta m_{21}^2$

- Again, Δm_{21}^2 and Δm_{32}^2 are related to the 'solar' and 'atmospheric' neutrino oscillations.
- The three mass eigenstates can be arranged as:

Convention:

$m_2 > m_1$



Normal hierarchy

$\Delta m_{32}^2 > 0$

Inverted hierarchy

$\Delta m_{32}^2 < 0$

Neutrino Oscillation With Three Flavours

- Define the weak eigenstates as $|\nu_w\rangle$, with w = e, μ, and τ,. and mass eigenstates as $|\nu_i\rangle$, with i = 1, 2, and 3

- In the rest frame of ν_i, the total energy of the neutrino is just the rest mass.

- The time evolution of $|\nu_i\rangle$ is just given by:

$$|\nu_i(\tau_i)\rangle = e^{-iE_i'\tau_i}|\nu_i(0)\rangle = e^{-im_i\tau_i}|\nu_i(0)\rangle$$

- In the laboratory frame,

$$|\nu_i(t)\rangle = e^{-i(E_i t \ - p_i L)}|\nu_i(0)\rangle$$

and the weak eigenstate is

$$|\nu_w(t)\rangle = \sum_{i=1}^{3} U_{wi}^{*}|\nu_i(t)\rangle = \sum_{i=1}^{3} U_{wi}^{*} e^{-i(E_i t - p_i L)}|\nu_i(0)\rangle$$

Neutrino Oscillation With Three Flavours (cont.)

- Assume all the states have the same momentum p.
- If all masses « p, the neutrinos are moving at c=1. Then,

  $t = L, \quad E_i t - p_i L \approx (p + m_i^2/2p)L - pL = m_i^2 L/2p = m_i^2 L/2E,$

  In this case,

$$|\nu_w(L)\rangle = \sum_{i=1}^{3} U^*_{wi} e^{-i\frac{m_i^2 L}{2E}} |\nu_i(0)\rangle$$

- On the other hand,

$$|\nu_i(0)\rangle = \sum_{w'=1}^{3} U_{w'i} |\nu_{w'}(0)\rangle$$

- In other words, the neutrino state at L is related to the initial state by,

$$|\nu_w(L)\rangle = \sum_{w'=1}^{3} \sum_{i=1}^{3} U^*_{wi} e^{-i\frac{m_i^2 L}{2E}} U_{w'i} |\nu_{w'}(0)\rangle$$

Probability Of Oscillation

- The probability of transforming from one flavour to another is:

$$P(\nu_w \to \nu_{w'}) = \left|\langle \nu_{w'} | \nu_w \rangle\right|^2 = \delta_{ww'} - 4\sum_{i>j}^{3} \mathrm{Re}\left(U^*_{wi} U_{w'i} U_{wj} U^*_{w'j}\right) \sin^2\left(\frac{\Delta m^2_{ij} L}{4E}\right)$$

$$+ 2\sum_{i>j}^{3} \mathrm{Im}\left(U^*_{wi} U_{w'i} U_{wj} U^*_{w'j}\right) \sin^2\left(\frac{\Delta m^2_{ij} L}{2E}\right)$$

- As an example for the survival probability:

$$P(\nu_e \to \nu_e) \approx 1 - \sin^2 2\theta_{13} \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right) - \cos^4\theta_{13} \sin^2 2\theta_{12} \sin^2\left(\frac{\Delta m^2_{21} L}{4E}\right)$$

which can be used to analyze the solar and reactor neutrino experiments.

Probability Of Oscillation (cont.)

- An example of the appearance probability is:

$P(\nu_\mu \rightarrow \nu_e) = P_1 + P_2 + P_3 + P_4$

$$P_1 = \sin^2\theta_{23}\sin^2 2\theta_{13}\left(\frac{\Delta_{13}}{B_\pm}\right)^2 \sin^2\frac{B_\pm L}{2}$$

$$P_2 = \cos^2\theta_{23}\sin^2 2\theta_{12}\left(\frac{\Delta_{12}}{A}\right)^2 \sin^2\frac{AL}{2}$$

$$P_3 = J\cos\delta\left(\frac{\Delta_{12}}{A}\right)\left(\frac{\Delta_{13}}{B_\pm}\right)\cos\frac{\Delta_{13}L}{2}\sin\frac{AL}{2}\sin\frac{B_\pm L}{2}$$

$$P_4 = \mp J\sin\delta\left(\frac{\Delta_{12}}{A}\right)\left(\frac{\Delta_{13}}{B_\pm}\right)\sin\frac{\Delta_{13}L}{2}\sin\frac{AL}{2}\sin\frac{B_\pm L}{2}$$

where

$$\Delta_{ij} = \frac{\Delta m^2_{ij}}{2E_\nu} \qquad B_\pm = |A \pm \Delta_{13}|$$

Matter effect $A = \sqrt{2}G_F n_e$

$$J = \cos\theta_{13}\sin 2\theta_{12}\sin 2\theta_{13}\sin 2\theta_{23}$$

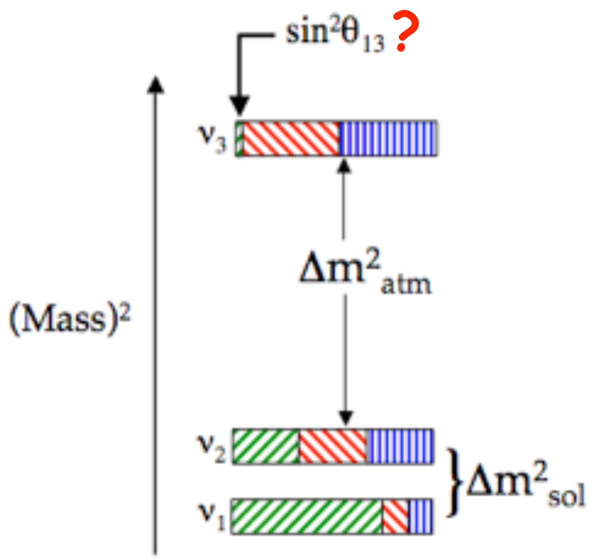
and $\pm$ is for ν (+) and $\bar{\nu}$(-).

What Remain To Be Determined ?

θ_{13},   sign of $\Delta m_{32}{}^2$,   and δ

Importance of θ_{13}

- It is one of the key parameters in determining the leptonic mixing matrix.
- What is ν_e fraction of ν_3?

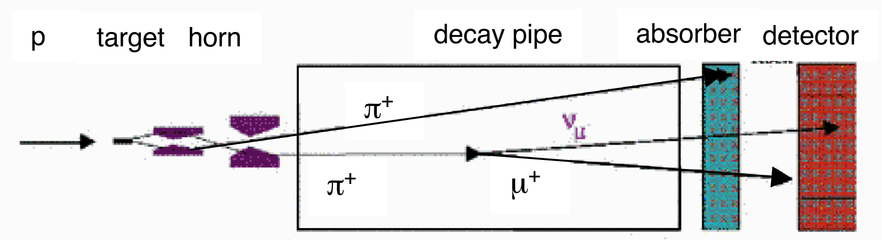


- U_{e3} is the gateway to CP violation in neutrino sector:

$$P(\nu_\mu \to \nu_e) - P(\bar{\nu}_\mu \to \bar{\nu}_e) \propto \sin 2\theta_{12} \sin 2\theta_{23} \cos^2\theta_{13} \sin 2\theta_{13} \sin\delta$$

Some Methods For Determining θ_{13}

Method 1: Accelerator Experiments



$$P_{\mu e} \approx \sin^2 2\theta_{13} \sin^2 2\theta_{23} \sin^2\left(\frac{\Delta m_{31}{}^2 L}{4E_\nu}\right) + \ldots$$

- $\nu_\mu \rightarrow \nu_e$ appearance experiment
- need other mixing parameters to extract θ_{13}
- baseline *O(100-1000 km)*, matter effects present
- expensive

Method 2: Reactor Experiments

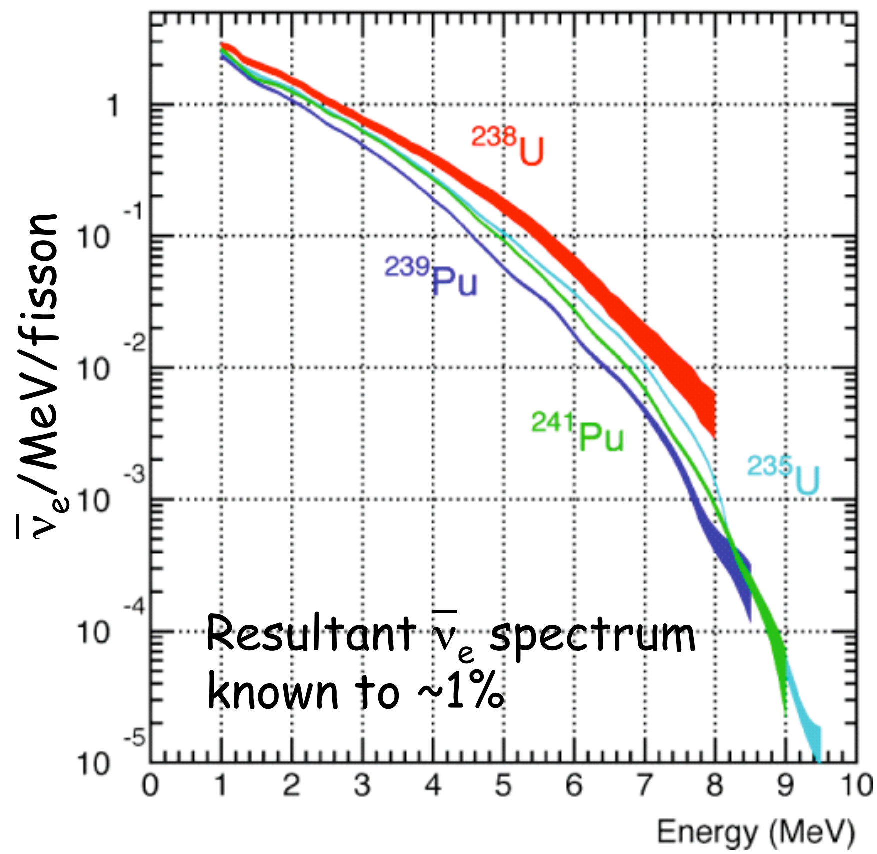
$$P_{ee} \approx 1 - \sin^2 2\theta_{13} \sin^2\left(\frac{\Delta m_{31}{}^2 L}{4E_\nu}\right) - \cos^4\theta_{13} \sin^2 2\theta_{12} \sin^2\left(\frac{\Delta m_{21}{}^2 L}{4E_\nu}\right)$$

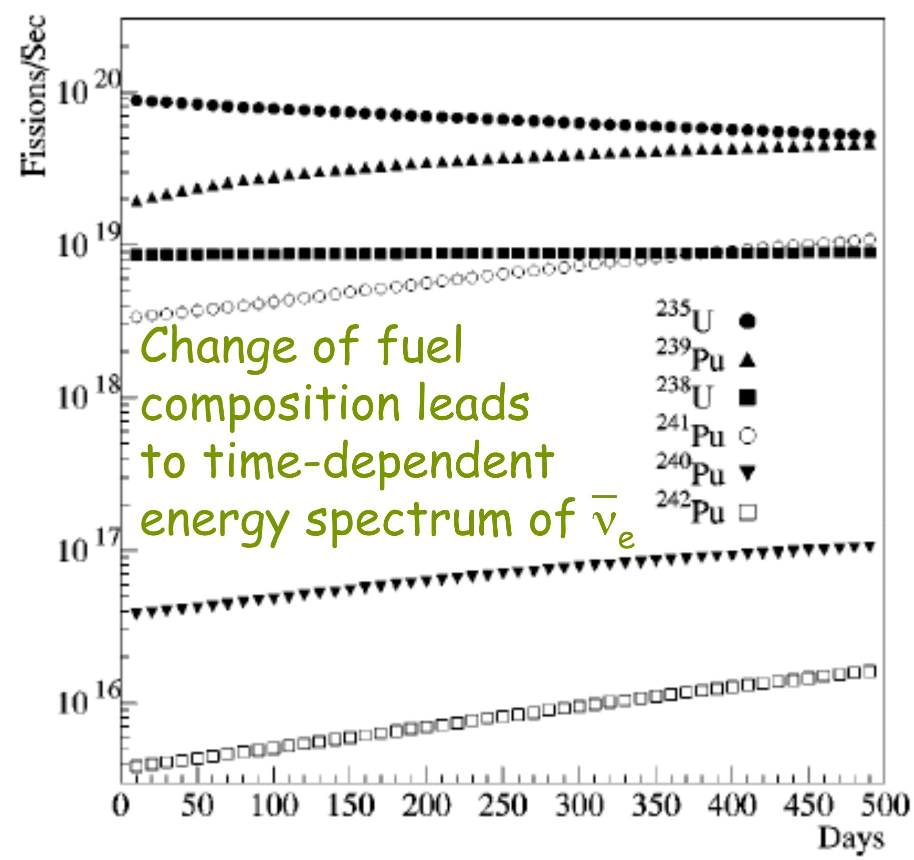
- $\bar{\nu}_e \rightarrow X$ disappearance experiment
- baseline *O(1 km)*, no matter effect, no ambiguity
- relatively cheap

Reactor $\bar{\nu}_e$

- Fission processes in nuclear reactors produce huge number of low-energy $\bar{\nu}_e$:

3 GW_{th} generates 6×10^{20} $\bar{\nu}_e$ per sec





Detecting $\bar{\nu}$ With Liquid Scintillator

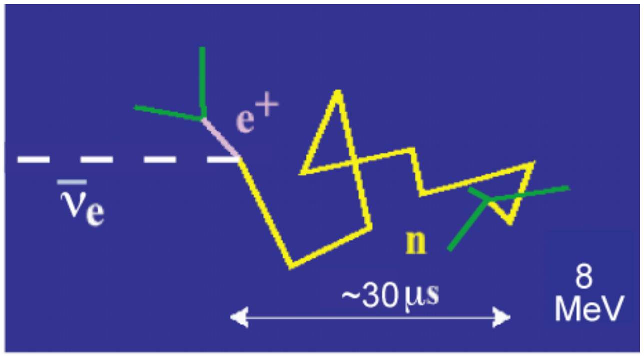
- Use the inverse β-decay reaction in 0.1% Gd-doped liquid scintillator:

$$\bar{\nu}_e + p \rightarrow e^+ + n \quad \text{(prompt)}$$

0.3b → $+ p \rightarrow D + \gamma(2.2\ \text{MeV})$ (delayed)

50,000b → $+ Gd \rightarrow Gd^*$

→ $Gd + \gamma\text{'s}(8\ \text{MeV})$ (delayed)

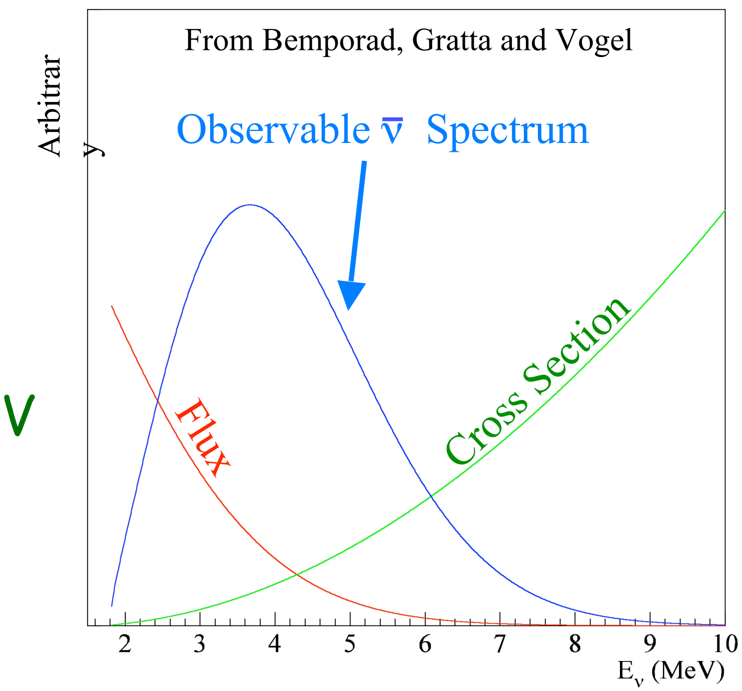


- Time- and energy-tagged signal is a good tool to suppress background events.

- Energy of $\bar{\nu}_e$ is given by:

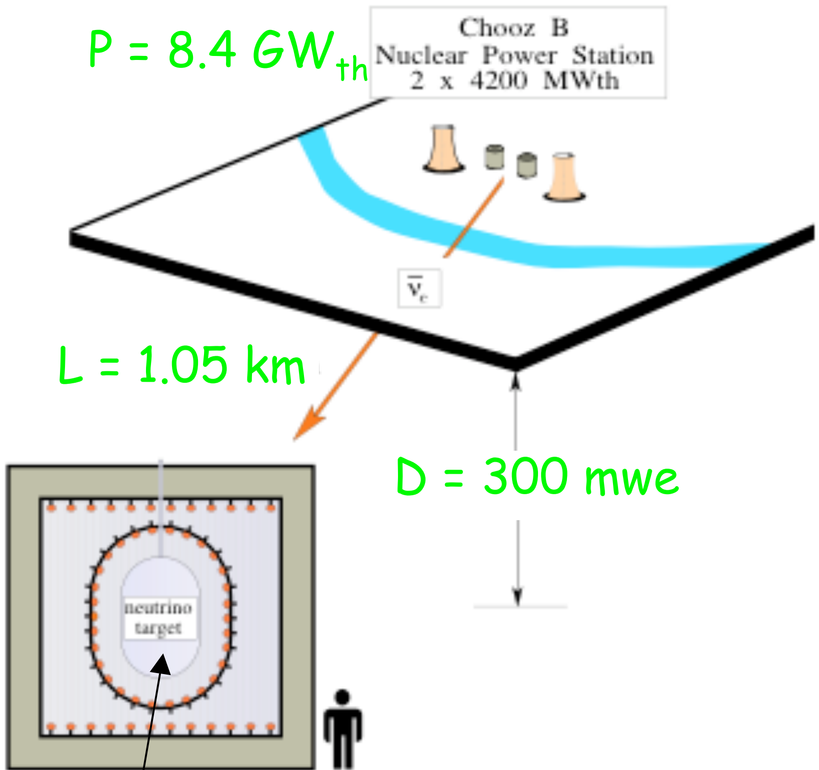
$$E_{\bar{\nu}} \approx T_{e+} + T_n + (m_n - m_p) + m_{e+} \approx T_{e+} + 1.8\ \text{MeV}$$

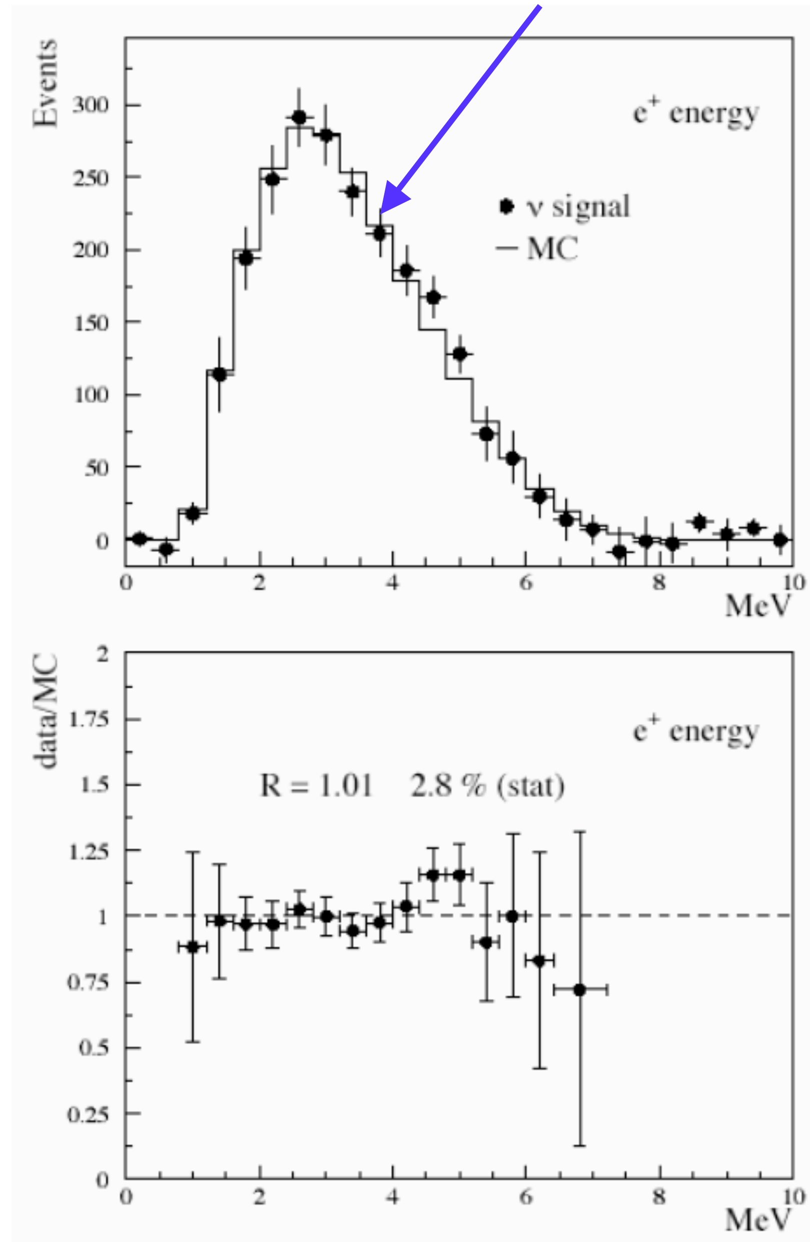
10-40 keV



Chooz: Finding θ_{13}

~3000 $\bar{\nu}_e$ candidates
(excluded 10% bkg)
in 335 days





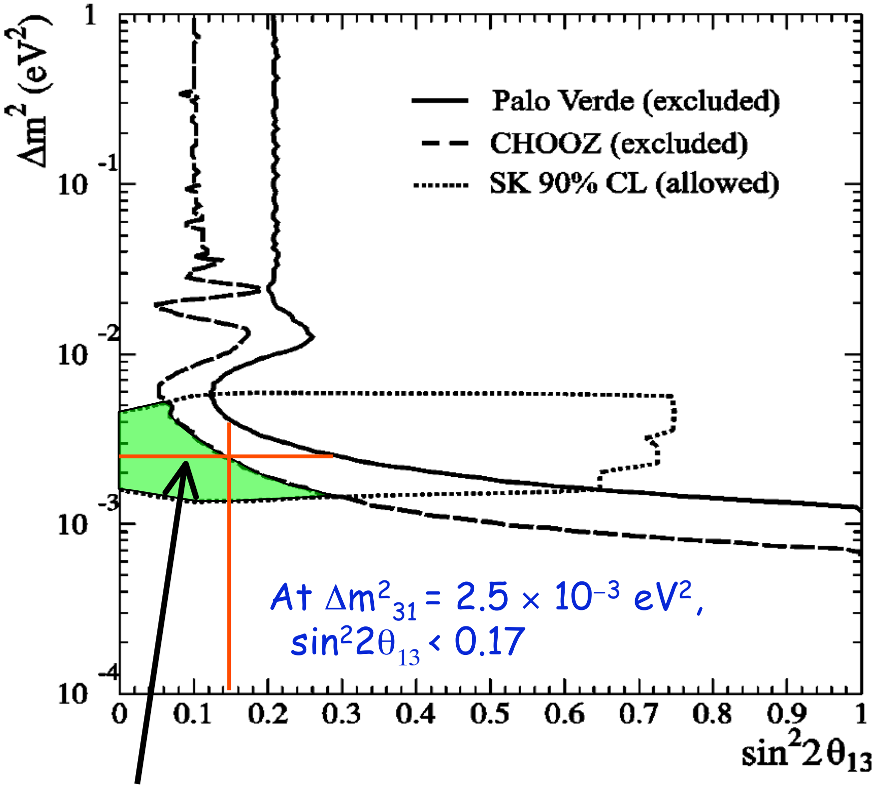
5-ton 0.1% Gd-loaded liquid scintillator
to detect $\bar{\nu}_e + p \rightarrow e^+ + n$

Rate:

~5 evts/day/ton (full power)
including 0.2-0.4 bkg/day/ton

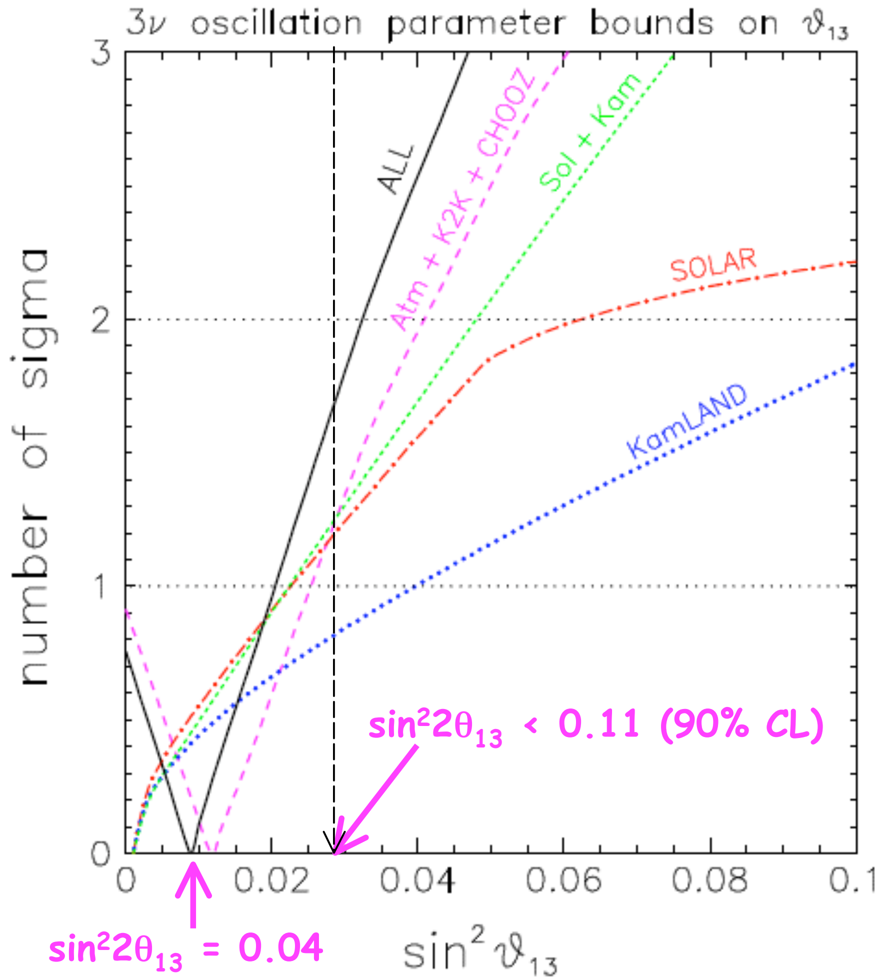
Current Knowledge of θ_{13}

Direct search



allowed region

Global fit



$\sin^2 2\theta_{13} < 0.11$ (90% CL)

$\sin^2 2\theta_{13} = 0.04$

Best fit value of $\Delta m^2_{32} = 2.4 \times 10^{-3}$ eV^2

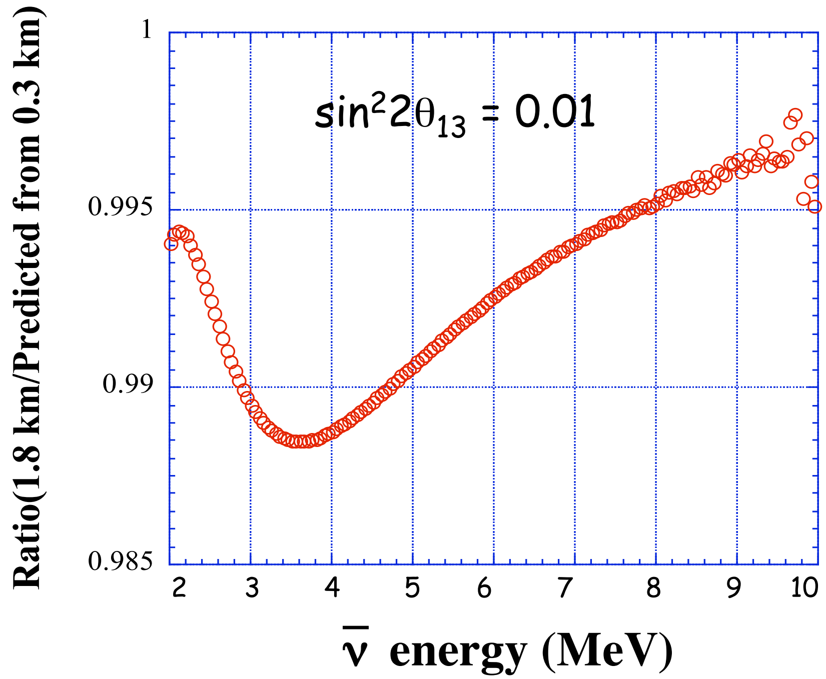
Fogli etal., hep-ph/0506083

Proposed Reactor Neutrino Experiments

Daya Bay: Goal And Approach

- Determine $\sin^2 2\theta_{13}$ with a sensitivity of ≤ 0.01

  by measuring deficit in $\bar{\nu}_e$ rate and spectral distortion.



- If $\sin^2 2\theta_{13} > 0.01$, use conventional neutrino beams from accelerator to look for CP violation;
  If $\sin^2 2\theta_{13} < 0.01$, need to come up with new experimental methods and accelerator technology to explore CP violation

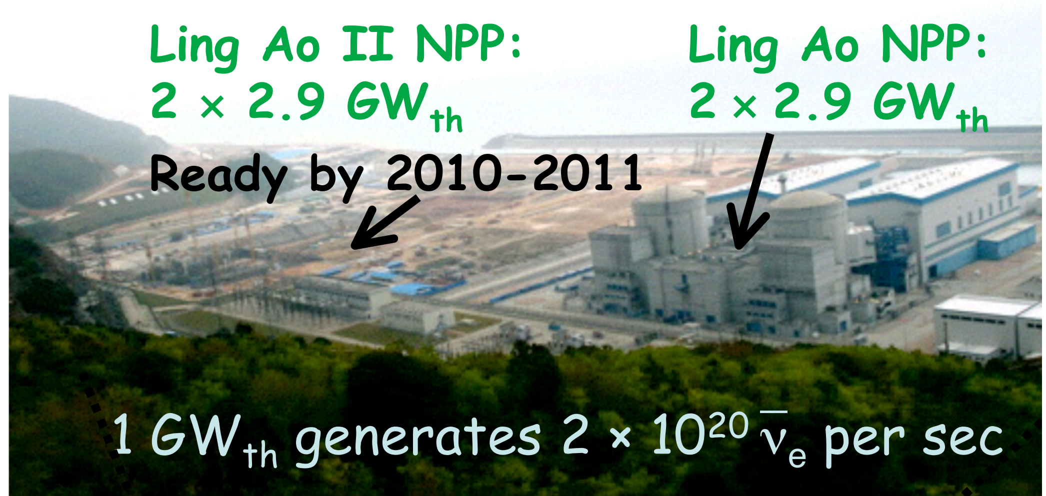
How To Reach A Precision of 0.01 in Daya Bay?

- **Increase statistics:**
  - Use more powerful nuclear reactors
  - Utilize larger target mass, hence larger detectors
- **Suppress background:**
  - Go deeper underground to gain overburden for reducing cosmogenic background
  - Use active shield around the target
- **Reduce systematic uncertainties:**
  - Reactor-related:
    - Optimize baseline for best sensitivity and smaller residual reactor-related errors
    - Near and far detectors to minimize reactor-related errors
  - Detector-related:
    - Use "Identical" pairs of detectors to do ***relative*** measurement
    - Comprehensive program in calibration/monitoring of detectors
    - Interchange near and far detectors (optional)


55 km
HONG KONG
SHENZHEN
LEGEND
Railway
Main road
Secondary road
Built-up area / Cultivation
Airport / Major port
Power station
Peak (in metres)
Contours
(vertical interval 200 metres with supplementary contour at 100 metres)
Sea depth in metres
Boundary of Special Administrative Region
Series AR/18/IRS
Edition 7 2002
Survey and Mapping Office, Lands Department
© Copyright reserved -- reproduction by permission only

The Daya Bay Nuclear Power Complex

- 12th most powerful in the world (11.6 GW_{th})
- Fifth most powerful by 2011 (17.4 GW_{th})
- Adjacent to mountain, easy to construct tunnels to reach underground labs with sufficient overburden to suppress cosmic rays



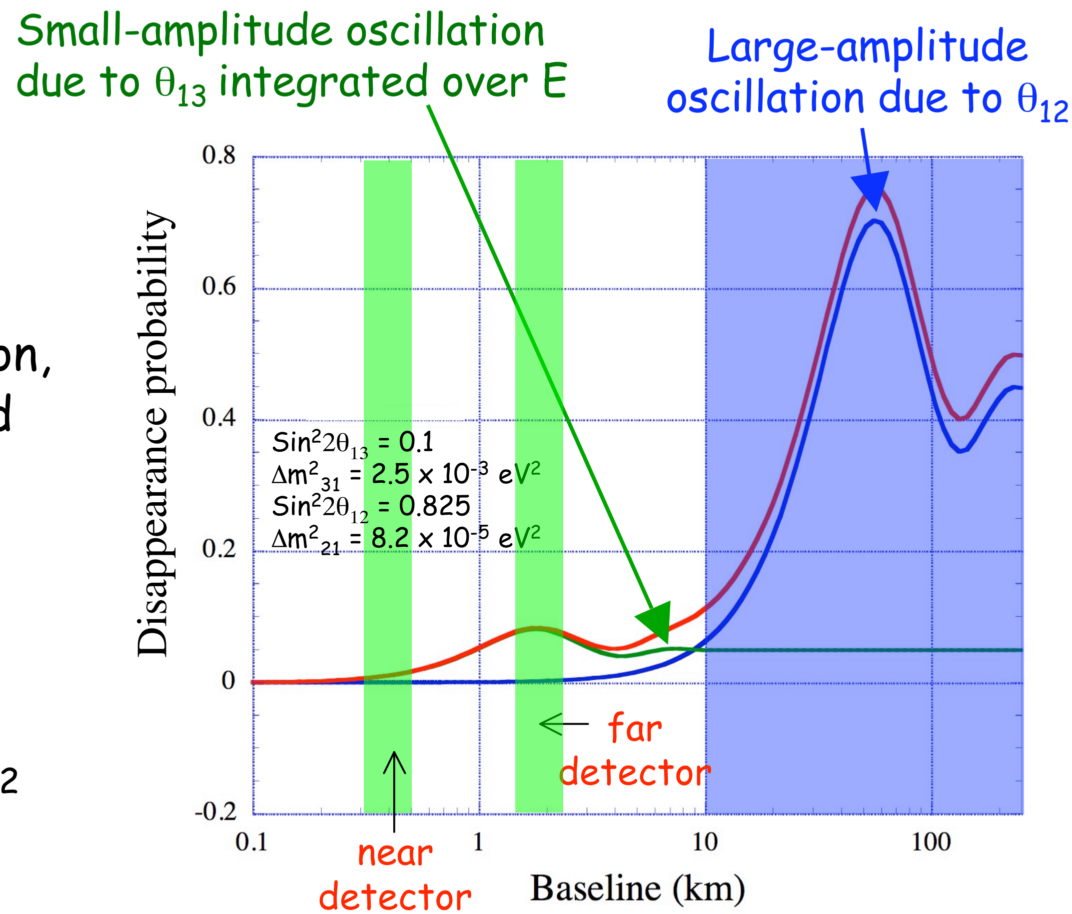
Where To Place The Detectors ?

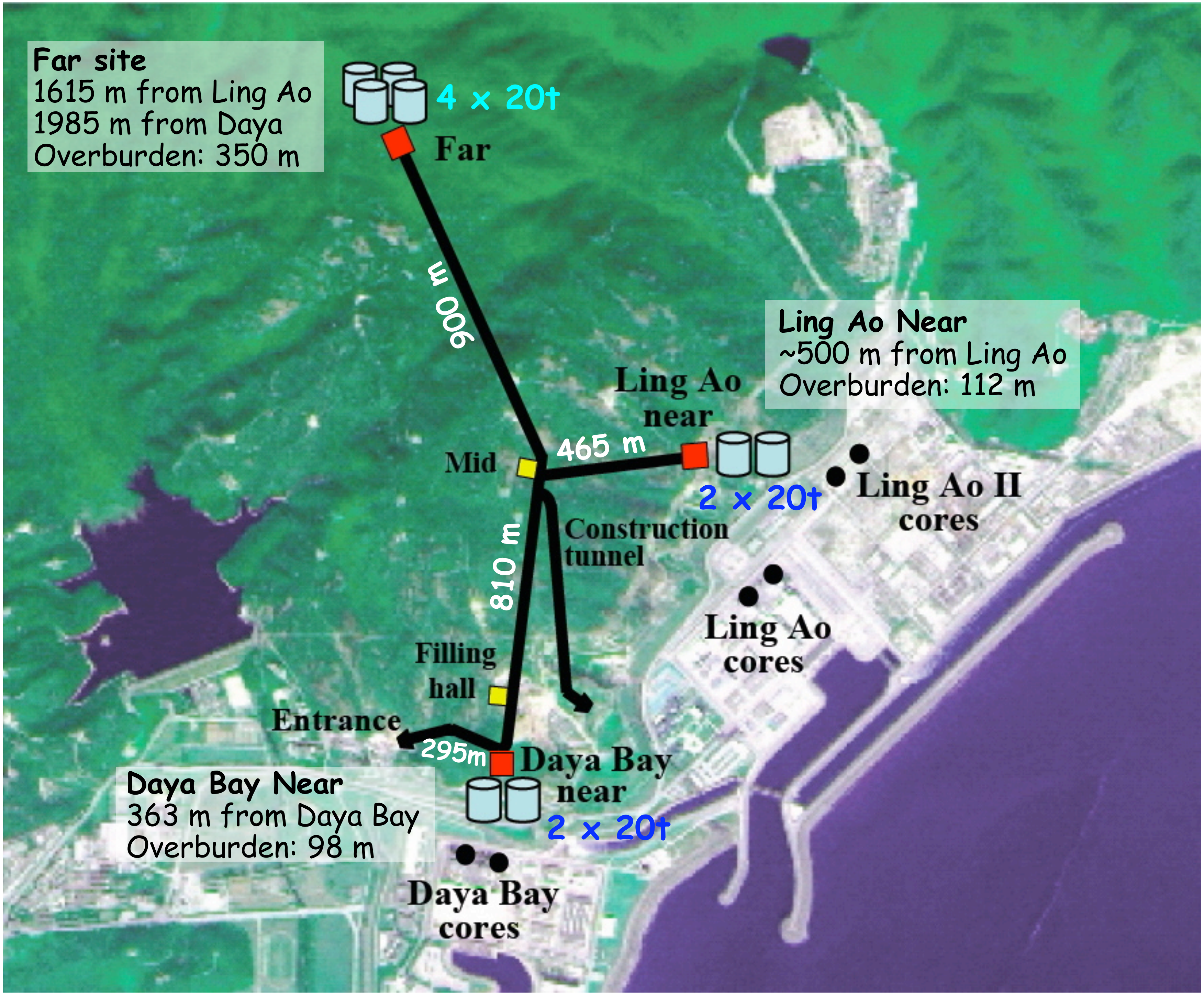
- Since reactor $\bar{\nu}_e$ are low-energy, it is a disappearance experiment:

$$P(\bar{\nu}_e \to x) \approx \sin^2 2\theta_{13} \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right) - \cos^4\theta_{13} \sin^2 2\theta_{12} \sin^2\left(\frac{\Delta m^2_{21} L}{4E}\right)$$

- Place near detector(s) close to reactor(s) to measure flux and spectrum of $\bar{\nu}_e$ for normalization, hence reducing reactor-related systematic

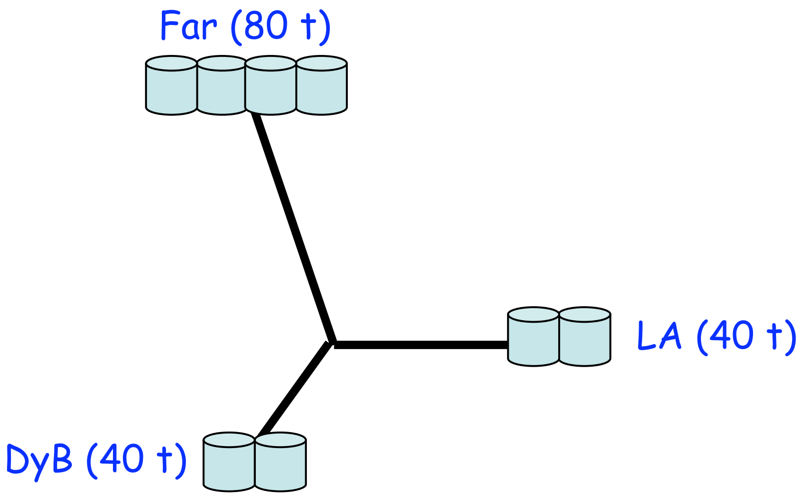
- Position a far detector near the first oscillation maximum to get the highest sensitivity, and also be less affected by θ_{12}



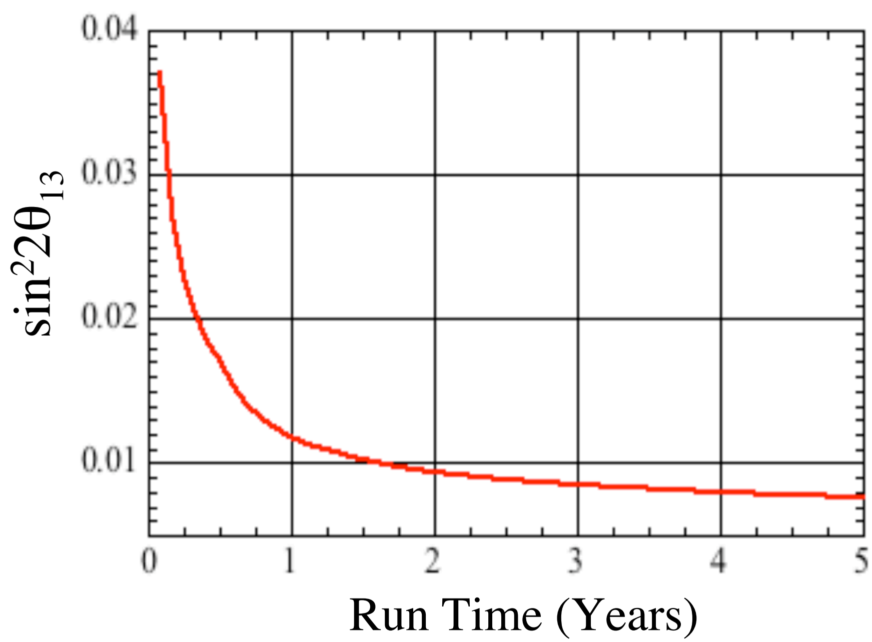


Far site
1615 m from Ling Ao
1985 m from Daya
Overburden: 350 m
4 x 20t
Far
900 m
Ling Ao Near
~500 m from Ling Ao
Overburden: 112 m
Ling Ao
near
465 m
Mid
2 x 20t
Ling Ao II
cores
810 m
Construction
tunnel
Ling Ao
cores
Filling
hall
Entrance
295m
Daya Bay
near
Daya Bay Near
363 m from Daya Bay
Overburden: 98 m
2 x 20t
Daya Bay
cores

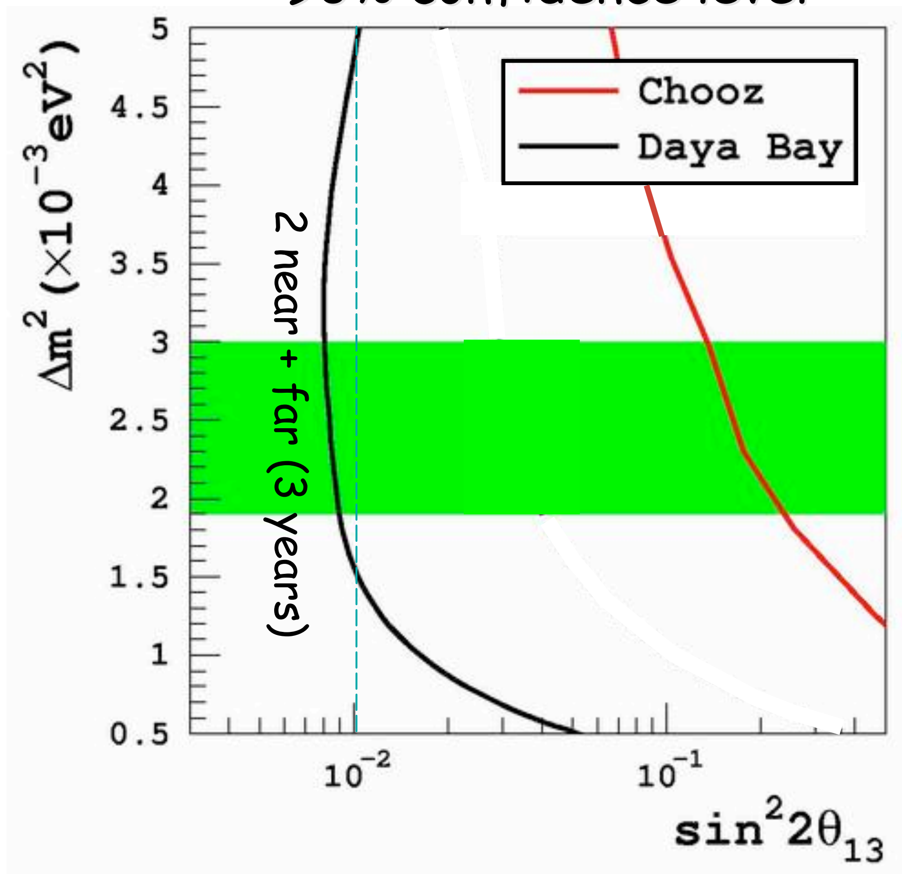
Sensitivity in $\sin^2 2\theta_{13}$



- Use rate and spectral shape
- input relative detector syst. error of 0.38%/detector



90% confidence level



Summary

- The basic set of mixing parameters describing neutrino oscillation is $\{\theta_{12}, \theta_{23}, \theta_{13}, \Delta m_{21}^2, \Delta m_{32}^2, \delta\}$
- The remaining unknowns are $\{\theta_{13}$, sign of $\Delta m_{32}^2, \delta\}$.
- The value of θ_{13} will determine whether we can study *CP* violation in neutrino oscillation in the future.
- Run out of time to cover:
  - many experiments
  - measuring θ_{13}, resolving the sign of Δm_{32}^2, and matter effect with accelerator neutrino beams
  - approaches in tackling leptonic *CP* violation
  - neutrinoless double beta-decay
  - measuring the absolute mass of ν_e
  - role of neutrino in astrophysics and cosmology