A Course for Advanced Quantum Field Theory

— Gauge Field Theories —

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Spring, 2009

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No Business is Too Small.

No Problem is Too Big.

No real boundary between different branches of Physics/Science.

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1. **Why Quantum Field Theories: A Modern View**
   1. A Brief History about QFT
   2. Effective Field Theory as the Foundation of QFT
2. **Why Gauge Field Theories**
   1. All Fundamental Forces in Nature
   2. Gauge Revolution
3. **Vacuum Energy, Cosmological Constant and Dark Energy**
   1. Vacuum Energy from Quantum Field Theory
   2. Einstein Equation and Effective Cosmological Constant
   3. Friedman Equation and Connection to Dark Energy
4. **A Condensed Matter Application: Order Parameter**
   1. Order Parameter, Phase Transition, Spontaneous Symmetry Breaking
   2. Landau-Ginzburg Theory, Universality vs. Renormalization Group

1. **Global and Local Symmetries**
   1. Concept of Local Gauge Symmetry
   2. Abel Gauge Symmetry: Maxwell Theory and QED
   3. Non-Abel Gauge Symmetry: Gauge Sector and Fermion Sector
2. **Gauge Invariance and Geometry**
   1. Gauge Field as a Phase Field
   2. Gauge Field and Parallel Transport
   3. Gauge Field Strength and Curvature Tensor
   4. Nonintegrable Phase Factor and Global Formulation of Gauge Fields
3. **Gravity as a Gauge Theory**
   1. Equivalence Principle
   2. Principle of General Covariance vs. Gauge Invariance
   3. Recipe for Constructing Generally Covariant Action
      1. Example-1: Scalar Fields in Curved Spacetime
      2. Example-2: Gauge Fields in Curved Spacetime
      3. Example-3: Einstein-Hilbert Action of Graviton Fields

4. **Spontaneous Global Symmetry Breaking**
   1. Explicit Symmetry Breaking vs. Spontaneous Symmetry Breaking (SSB)
   2. Physical Vacuum vs. Spontaneous Symmetry Breaking
   3. Goldstone Theorem and Three Ways of Proving It
   4. An Explicit Model of SSB and Nambu-Goldstone Bosons
   5. Pseudo-Nambu-Goldstone Bosons
   6. Goldstone Theorem vs. Spacetime Dimensions
5. **Spontaneous Gauge Symmetry Breaking**
   1. Anderson-Higgs Mechanism
      1. Explicit Example-1: An Abel Higgs Model
      2. Explicit Example-1: An Non-Abel Higgs Model
      3. The Formal Proof
   2. Vacuum Energy from Spontaneous Symmetry Breaking
6. **Superconductivity as a Higgs Phenomenon**

1. Bose-Einstein Condensation and Cooper Pair
2. Photon Mass, London Penetration Depth, and Meissner Effect
3. Flux Quantization and Abel Goldstone Boson

1. **Faddeev-Popov Quantization Method**
   1. Redundant Gauge Orbits and Consistent Gauge Fixing
   2. Faddeev-Popov Determinant and Introduction of Ghosts
   3. Explicit Examples of Faddeev-Popov Quantization and Feynman Rules in Lorentz Gauge, Axial Gauge and R_ξ Gauge
   4. Gribov Copies
      1. Example of Coulomb Gauge
      2. Nonzero Topological Charge and Universality of Gribov Copies
2. **BRST Symmetry and BRST Quantization**
   1. BRST Transformations and Nilpotency
   2. Proof of BRST Symmetry for Quantized Lagrangian
3. **Ward-Takahashi and Slavnov-Taylor Identities**
   1. Ward-Takahashi Identities from BRST: A Review for QED
   2. Slavnov-Taylor Identities from BRST: General Gauge Theories

1. Renormalization Program
    1. Renormalization Procedures
    2. Power Counting and Superficial Divergences
    3. Renormalization Counter Terms
    4. Multiplicative Renormalization vs. BPHZ Renormalization
2. Renormalization Types and Regularization Schemes
    1. Renormalization Types:
        1. Non-renormalizable Theories (Effective Theories)
        2. Renormalizable, Super-renormalizable and Finite Theories
    2. Regularization Schemes:
        1. Dimensional Regularization
        2. Pauli-Villars Regularization
        3. Lattice Regularization
3. Renormalizability and Gauge Invariance
    1. Lee-Zinn-Justin Equations
    2. Proof of Renormalizability by Induction:
        1. Pure Gauge Theories

   2. Gauge Theories with Spontaneous Symmetry Breaking
   3. Proving Gauge Invariance of S-Matrix
4. Renormalization Group (RG)
   1. Concept of Renormalization Group
   2. Renormalization Group Equation
   3. Running Coupling Constant
   4. Ultraviolet and Infrared Fixed Points
   5. Callan-Symanzik Equation
   6. Weinberg Theorem and Asymptotic Solution of RG Equation
   7. Minimal Subtraction Scheme
   8. Bjorken Scaling and Scaling Violation
   9. Altarelli-Parisi Equation *
5. Renormalization of Non-Abelian Gauge Theory at One-Loop
   1. Lagrangian and Counter Terms
   2. Gauge Boson Self-Energy at One-Loop
   3. Fermion Self-Energy at One-Loop
   4. Vertex Corrections at One-Loop
6. Asymptotic Freedom of Non-Abelian Gauge Theory

1. Computation of β-Function at One-Loop
2. Asymptotic Freedom of QCD: Physical Interpretation
3. Asymptotic Freedom and Spontaneous Symmetry Breaking

7. Background Field Method and Application to β-Function
   1. Background Field Method
   2. One-Loop Correction to the Effective Action
   3. Computation of Functional Determinants

Chapter 7. Electroweak Standard Model and Beyond (10-12hr) I

6. Neutrino Masses, Neutrino Oscillations and Neutrinoless Double-β Decay

3. R_ξ Gauge Quantization and Feynman Rules
   1. R_ξ Gauge-Fixing, Propagators and Feynman Rules
   2. SM Gauge Transformations and Ghost Lagrangian
   3. SM Feynman Rules in R_ξ Gauge

4. Higgs Mechanism and Equivalence Theorem
   1. Equivalence Theorem: Mathematical Formulation
   2. Equivalence Theorem as a Formulation of Higgs Mechanism
   3. Kaluza-Klein Equivalence Theorem and Geometric Higgs Mechanism
      1. Kaluza-Klein Compactification in 5d
      2. Geometric Higgs Mechanism and KK Equivalence Theorem

5. WW Scattering and Unitarity Bound
   1. WW Scattering and Probing the Electroweak Symmetry Breaking
   2. Partial Wave Analysis and Unitarity Condition

3. Unitary Limits on Higgs Mass and on Scales of New Physics
   1. Unitary Limit on Higgs Mass
   2. Unitary Limit on the Scale of Electroweak Symmetry Breaking
   3. Unitary Limit on the Scales of Mass Generations for Quarks, Leptons and Neutrinos

6. Radiative Corrections
   1. Higgs Mass, Radiative Corrections and Fine-Tuning Problem
   2. Oblique Corrections and Screening Theorem
      1. Oblique Corrections at Z-Pole and W-Pole
      2. WT Identities and their Approximation in Gaugeless Limit
      3. One-Loop Oblique Corrections and Screening Theorem
   3. Coleman-Weinberg Potential *

7. Electroweak Chiral Lagrangian *

Chapter 9. Nonperturbative Aspects of Gauge Theories † [5-7hr]

1. Aharonov-Bohm Effect and Berry Phase
2. Vortices and Monopoles
   1. Vortices
   2. Dirac Monopoles, ’t Hooft-Polyakov Monopoles, BPS Monopoles
3. Instanton, Vacuum Tunnelling and Strong CP Problem
4. Large-N Expansion
5. Lattice Gauge Theory

1. $SU(5)$ GUT
2. Coupling Constant Unification
3. Proton Decay
4. $SO(10)$ GUT *
5. Seesaw Mechanism and Cosmic Baryon Asymmetry

Chapter 11. Perturbative Gravity † [3-4hr]

1. Einstein Gravity as an Effective Field Theory
2. Gravity Coupled to the Standard Model
3. Kaluza-Klein Compactification of 5D
4. Inflation, Cosmic Coincidence Problem

Chapter 12. Field Theory and Collective Phenomena † [3hr]

1. Field Theory at Finite Temperature
2. Critical Phenomena and Mean Field Approximation
3. Quantum Hall Effect
4. Fractional Statistics, Chern-Simmons, and Topological Field Theory *
5. DNA and Quantum Field Theory *

Note:

- The topics marked with * may be taken as after-class reading under my guidance.
- Chapters marked by † will be moved to Tsinghua's fall semester class "Selected Topics in Particle Theory"
- I will continue to teach them for the last 1/3 of this course (about 20hrs).
- The first week of May is holiday, so the actual time for the course is $16 - 1 = 15$ weeks.
- Final Exam is in the 17th week.
- For future update of this List of Content, please check our Web site under "Gauge Field Theories": http://hep.tsinghua.edu.cn/training/index.html

Contact Info and Time/Location

Contact Information:

- My email address: hjhe@tsinghua.edu.cn, Tel: 6277-3919(o)
- Web: http://hep.tsinghua.edu.cn/˜hjhe
- Teaching Assistant: Tel: 6277-3916(o)
  Xu-Feng Wang (xf-wang03@mails.tsinghua.edu.cn)

Time and Location:

- Every Wednesday afternoon (1:30-4:55pm).
- Teaching Building No.6, Room-6B306, Tsinghua University.

The Goal:

1. Master Fundamental Concepts
2. Master Major Calculation Techniques
3. Explore unsolved/new Puzzles + Get into Research!

My advice on how to learn QFT:

- Think deeply (using your mind) $\rightarrow$ QFT is still an open book !
- Derive/Compute extensively (using your "hands").
- Ask questions, Discuss with classmates.
- Feel free to contact TA and me if you have any suggestion.
- Have Fun and Enjoy it !

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Contents of this Course:

Basic knowledge:

- Gauge Symmetries: Abel and non-Abel
- Quantization of Gauge Theories
- Renormalization of Gauge Theories and 1-loop Calculation
- Gauge Symmetry Breaking

Advanced Topics and Applications:

- Standard Model: Electroweak Interaction, Neutrino Masses
- Nonperturbative Aspects
  - Anomalies
  - Nonperturbative Solutions and Methods
- SUSY, Grand Unification, Perturbative Gravity
- Application to Cosmologies
- Application to Condensed Matter Physics

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Language:

- Read & Write in English.
- Master all terminologies in English.
- Important for International Communication. (Conferences, Exchanges, Cooperations with USA/Europe/Japan)
- Eventually I may try English for oral presentation.

Major Reference Books: (Modern)

1. An Introduction to Quantum Field Theory
   M.E. Peskin & D.V. Schroeder
2. The Quantum Theory of Fields (I+II), S. Weinberg
3. Quantum Field Theory in a Nutshell, A. Zee
4. Field Theory: A Modern Primer, P. Ramond
5. Guage Theory of Elementary Particle Physics, T.P. Cheng, L.F. Li
6. Quantum Field Theory, C. Itzykson and J.B. Zuber
7. Gauge Theories for Interactions, Y.B. Dai (in Chinese)
8. Quantum Field Theory and Critical Phenomena, J. Zinn-Justin
9. Quantum Field Theory in Condensed Matter Physics, A. Tsvelik
10. An Introduction to Modern Cosmology, Andrew Liddle

Homeworks:

The homework of my Gauge Field Theory (GFT) course is classified into 3 categories below.

1A. **Regular Exercises (REx)**
You have to derive some Key Formulas presented in my Lectures or extend some of them to more general cases.

1B. **Non-Regular Exercises (NREx)**
You have to derive some Important Results with further thinking on the basis of what I taught; you need to study some major reference books on my list. Some of them may not have been fully worked out in the reference books, so your independent thinking and derivations are needed.

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N1. Homeworks:

1C. Advanced Homework (AHW)

Answers or results cannot be found in the existing books and references, and these topics may develop into research projects worth of journal-publication. For those students with good results, I will arrange presentations at the seminars of our Center for High Energy Physics on Wednesday or Friday.

N1. Homeworks:

Important Notes:

The **REx** is required and will amount to 20% of your Final Score. **NREx** is not required, but depending on your performance it can contribute up to 30% of your Final Score. The **AHW** is also not required, but depending on your outcome it can amount up to 60% of your Final Score. These homework will be published and updated during the course at the following Web:

http://hep.tsinghua.edu.cn/training/courses/gauge.html/exercises/

Your homeworks should be sent to my TA for record, as soon as you finish for each class.

N2. Term Papers: I

1 Term Paper is required and will amount to 20% of your Final Score. For Term Paper above the normal level, depending on the quality of your results, it can amount up to 60% of your Final Score.

— My goal is to bring you into the exciting research frontiers of QFT.
— I strongly encourage you to explore some of the Nature's deep puzzles with the knowledge you have just learnt in the class.

2A. Write 1 Term Paper, which is due by 10th week (after the "Labor Day/May 1" week).

N2. Term Papers: II

2B. The topics of the Term Papers should be chosen based on the "Further Reading" materials I listed after each chapter. (For exceptional case, choosing a Topic outside my List may be allowed.) Each Term Paper should contain 2 parts: Part-1 is to summarize the key points and derivations of the paper you have read and reproduced; Part-2 (Major Part) is to describe your further new thinking and analysis based on the original paper.

2C. For excellent results you may have obtained from the above N1C and N2A+N2B, I will arrange you to give a Presentation at the Seminars of our HEP-Center.

N3. Final Exam:

The Final Exam (17th week) amounts to 40% (at most) of your Final Score.

By the 16th week, I will announce those students who need not to attend the Final Exam due to their excellent performance in "Homeworks" and "Term Paper".