A Course for Advanced Quantum Field Theory *

— Gauge Field Theories —

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(Updated for Spring, 2010)

My Overall Advice:

No Business is Too Small.

No Problem is Too Big.

No real boundary between different branches of Physics/Science.

†**Note: The contents of each chapter will be updated every spring.**

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Chapter 1. Why Gauge Field Theories (4-6hrs)

1.1. Why Quantum Field Theories: A Modern View

1. A Brief History about QFT
2. Effective Field Theory as the Foundation of QFT

1.2. Why Gauge Field Theories

1. All Fundamental Forces in Nature
2. Gauge Revolution

1.3. Vacuum Energy, Cosmological Constant and Dark Energy

1. Vacuum Energy from Quantum Field Theory
2. Einstein Equation and Effective Cosmological Constant
3. Friedman Equation and Connection to Dark Energy

1.4. Inflation as an Effective Theory

1.5. A Condensed Matter Application: Order Parameter

1. Order Parameter, Phase Transition, Spontaneous Symmetry Breaking
2. Landau-Ginzburg Theory, Universality, Renormalization Group

Chapter 2. Symmetries and Conservation Laws (4-6hrs)

2.1. Symmetries and Currents

1. Action Principle
2. Noether Theorem
3. Energy-Momentum Tensor

2.2. Lorentz and Poincare Symmetries

1. Lorentz Group and Irreducible Representations for Quantum Fields
2. Poincare Group and its Casimir Operators
3. Massless Particles and Helicity
4. Master Group, No-Go Theorem and Supersymmetry

2.3. Weyl, Majorana and Dirac Fermions

1. Definitions, Mass terms and Relations
2. Majorana Neutrino and Seesaw Mechanism

2.4. Finite Temperature Field Theory

2.5. Brief Review of Lie Groups*

Chapter 3. Symmetries and Their Breaking (8-9hrs)

3.1. Global and Local Symmetries

1. Concept of Local Gauge Symmetry
2. Abel Gauge Symmetry: Maxwell Theory and QED
3. Non-Abel Gauge Symmetry: Gauge Sector and Fermion Sector

3.2. Gauge Invariance and Geometry

1. Gauge Field as a Phase Field
2. Gauge Field and Parallel Transport
3. Gauge Field Strength and Curvature Tensor
4. Nonintegrable Phase Factor and Global Formulation of Gauge Fields

3.3. Gravity as a Gauge Theory

1. Equivalence Principle
2. Principle of General Covariance vs. Gauge Invariance
3. Recipe for Constructing Generally Covariant Action
    1. Example-1: Scalar Fields in Curved Spacetime
    2. Example-2: Gauge Fields in Curved Spacetime
    3. Example-3: Spinor Fields in Curved Spacetime
    4. Example-4: Einstein-Hilbert Action of Graviton Fields

Chapter 3. Symmetries and Their Breaking (8-9hrs)

3.4. Spontaneous Global Symmetry Breaking

1. Explicit Symmetry Breaking vs. Spontaneous Symmetry Breaking (SSB)
2. Physical Vacuum vs. Spontaneous Symmetry Breaking
3. Goldstone Theorem and Three Ways of Proving It
4. An Explicit Model of SSB and Nambu-Goldstone Bosons
5. Pseudo-Nambu-Goldstone Bosons
6. Goldstone Theorem vs. Spacetime Dimensions

3.5. Spontaneous Gauge Symmetry Breaking

1. Anderson-Higgs Mechanism
    1. Explicit Example-1: An Abel Higgs Model
    2. Explicit Example-1: An Non-Abel Higgs Model
    3. The Formal Proof
2. Vacuum Energy from Spontaneous Symmetry Breaking

3.6. Superconductivity as a Higgs Phenomenon

1. Cooper Pair and Bose-Einstein Condensation
2. Photon Mass, London Penetration Depth, and Meissner Effect
3. Flux Quantization and Abel Goldstone Boson

Chap.4. Path Integral Quantization: Gauge Fields (6-8hrs)

4.1. Faddeev-Popov Quantization Method

1. Redundant Gauge Orbits and Consistent Gauge Fixing
2. Faddeev-Popov Determinant and Introduction of Ghosts
3. Explicit Examples of Faddeev-Popov Quantization and Feynman Rules in Lorentz Gauge, Axial Gauge and R_ξ Gauge
4. Gribov Copies
    1. Example of Coulomb Gauge
    2. Nonzero Topological Charge and Universality of Gribov Copies

4.2. BRST Symmetry and BRST Quantization

1. BRST Transformations and Nilpotency
2. Proof of BRST Symmetry for Quantized Lagrangian

4.3. Ward-Takahashi and Slavnov-Taylor Identities

1. Ward-Takahashi Identities from BRST: A Review for QED
2. Slavnov-Taylor Identities from BRST: General Gauge Theories

Chapter 5. Renormalization of Gauge Theories (11-13hr)

5.1. Renormalization Program

1. Renormalization Procedures
2. Power Counting and Superficial Divergences
3. Renormalization Counter Terms
4. Multiplicative Renormalization vs. BPHZ Renormalization

5.2. Renormalization Types and Regularization Schemes

1. Renormalization Types:
   1. Non-renormalizable Theories (Effective Theories)
   2. Renormalizable, Super-renormalizable and Finite Theories
2. Regularization Schemes:
   1. Dimensional Regularization
   2. Pauli-Villars Regularization
   3. Lattice Regularization

5.3. Renormalizability and Gauge Invariance

1. Lee-Zinn-Justin Equations
2. Proof of Renormalizability by Induction: *
   1. Pure Gauge Theories
   2. Gauge Theories with Spontaneous Symmetry Breaking
3. Proving Gauge Invariance of S-Matrix

Chapter 5. Renormalization of Gauge Theories (10-12hrs)

5.4. Renormalization Group (RG)

1. Concept of Renormalization Group
2. Renormalization Group Equation
3. Running Coupling Constant
4. Ultraviolet and Infrared Fixed Points
5. Callan-Symanzik Equation
6. Weinberg Theorem and Asymptotic Solution of RG Equation
7. Minimal Subtraction Scheme
8. Bjorken Scaling and Scaling Violation
9. Altarelli-Parisi Equation *

5.5. Renormalization of Non-Abelian Gauge Theory at One-Loop

1. Lagrangian and Counter Terms
2. Gauge Boson Self-Energy at One-Loop
3. Fermion Self-Energy at One-Loop
4. Vertex Corrections at One-Loop

Chapter 5. Renormalization of Gauge Theories (10-12hrs)

5.6. Asymptotic Freedom of Non-Abelian Gauge Theory

1. Computation of β-Function at One-Loop
2. Asymptotic Freedom of QCD: Physical Interpretation
3. Asymptotic Freedom and Spontaneous Symmetry Breaking

5.7. Background Field Method and Application to β-Function

1. Background Field Method
2. One-Loop Correction to the Effective Action
3. Computation of Functional Determinants

Chapter 6. Anomalies (4-6hrs)

6.1. Chiral Anomalies

1. Concept of Chiral Anomaly: Global vs. Gauge Anomalies
2. Abelian Chiral Anomaly
3. General Properties of Chiral Anomaly
4. Non-Abelian Chiral Anomaly and Application to $\pi^0 \to 2\gamma$

6.2. Path Integral Formulation of Chiral Anomalies

1. Chiral Symmetry and Path Integral Measure
2. Chiral Anomaly from Regularizing Jacobian of Quantum Measure
3. Chiral Anomaly and Atiyah-Singer Index Theorem

6.3. Gauge Anomaly Cancellation Condition

6.4. Scale Anomaly

Chapter 7. Electroweak Standard Model & Beyond (12hr)

7.1. Structure of the Standard Model

1. Overview
2. Gauge Anomaly Cancellation

7.2. The Standard Model Lagrangian

1. Gauge-Higgs Sector and Weak Gauge Boson Masses
2. Fermion-Gauge Sector: Charged and Neutral Currents
3. Fermion-Higgs Yukawa Sector and Fermion Mass Generation
4. Quark Mixing in Charged Currents: CKM Matrix
    1. CKM Parametrization
    2. Wolfenstein Parametrization
    3. Jarlskog Invariant: Measure of CP-Violation
    4. Unitarity Triangle
5. Neutrino/Lepton Mixing in Charged Currents: MNSP Matrix
    1. Neutrino Masses from Weinberg dim-5 Operator
    2. Neutrino/Lepton Mixing and MNSP Matrix
6. Neutrino Masses, Neutrino Oscillations and Neutrinoless Double-β Decay

7.3. R_ξ Gauge Quantization and Feynman Rules

1. R_ξ Gauge-Fixing, Propagators and Feynman Rules
2. SM Gauge Transformations and Ghost Lagrangian
3. SM Feynman Rules in R_ξ Gauge

7.4. Higgs Mechanism and Equivalence Theorem

1. Equivalence Theorem: Mathematical Formulation
2. Equivalence Theorem as a Formulation of Higgs Mechanism
3. Kaluza-Klein Equivalence Theorem and Geometric Higgs Mechanism
    1. Kaluza-Klein Compactification in 5d
    2. Geometric Higgs Mechanism and KK Equivalence Theorem
4. Ideas of Dynamical Electroweak Symmetry Breaking

7.5. WW Scattering and Unitarity Bound

1. WW Scattering and Probing the Electroweak Symmetry Breaking
2. Partial Wave Analysis and Unitarity Condition
3. Universal Scales of New Physics for All SM Particles
    1. Unitary Limit on Higgs Mass
    2. Unitary Limit on the Scale of Electroweak Symmetry Breaking
    3. Unitary Limit on the Scales of Mass Generations for Quarks, Leptons and Neutrinos

7.6. Radiative Corrections

1. Higgs Mass, Radiative Corrections, Fine-Tuning and Trivality
2. Oblique Corrections and Screening Theorem
    1. Oblique Corrections at Z-Pole and W-Pole
    2. WT Identities and their Approximation in Gaugeless Limit
    3. One-Loop Oblique Corrections and Screening Theorem
3. Coleman-Weinberg Potential *

7.7. Dark Matter and Particle Physics

1. Evidence of Dark Matter
2. WIMP Model for Dark Matter and CERN LHC

The Goal:

1. Master Fundamental Concepts
2. Master Major Calculation Techniques
3. Explore unsolved/new Puzzles → Get into Research!

My advice on how to learn QFT:

- Think deeply (using your mind) → QFT is still an open book !
- Derive/Compute extensively (using your "hands").
- Ask questions, Discuss with classmates.
- Feel free to contact TA and me if you have any suggestion.
- Have Fun and Enjoy it !

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Basic knowledge: (Required)

- Gauge Symmetries: Abelian and non-Abelian
- Quantization of Gauge Theories
- Renormalization of Gauge Theories and 1-loop Calculation
- Spontaneous Gauge Symmetry Breaking

Advanced Topics and Applications:[‡]

- Standard Model: Electroweak Interaction, Neutrino Masses
- Nonperturbative Aspects
  - Anomalies
  - Nonperturbative Solutions and Methods [†]
- Supersymmetry and Gauge Unification [†]
- Perturbative Gravity [†]
- Application to Cosmologies [*]
- Applications to Condensed Matter Physics [*]

[‡] Items marked by [*] are not required, and items marked by [†] will be covered in my Fall Semester Course (see Part-II below). In Chapter-7, the Sec. 7.6-7.7 may be moved to Part-II according to the actual progress of the lectures.

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Note:

- The topics marked with * may be taken as after-class reading under my guidance.
- Final Exam is in the 17th week.
- For future update of this List of Content, please check our Web site under “Gauge Field Theories”: http://hep.tsinghua.edu.cn/training/index.html

Contact Info and Time/Location

Contact Information:

- My email address: hjhe@tsinghua.edu.cn, Tel: 6277-3919(o)
- Web: http://hep.tsinghua.edu.cn/~hjhe
- Teaching Assistant: Tel: 6277-3916(o)
  Shao-Feng Ge (gesf02@mails.tsinghua.edu.cn)

Time and Location:

- Every Friday afternoon (1:30-4:55pm).
- Teaching Building No.6, Room 6B201, Tsinghua University.

Part-II: Advanced Topics in Particle Theory

Chapter 8. Supersymmetry (SUSY) † (20hrs)§

8.1. SUSY and SUSY Algebra

1. What is SUSY?
2. SUSY Algebra
3. Vacuum Energy and SUSY
4. Structure of the Supersymmetric SM
5. The Wess-Zumino Model

8.2. Supersymmetric Lagrangian

1. SUSY Lagrangian for Chiral Supermultiplets
2. SUSY Lagrangian for Gauge Supermultiplets
3. Summary: SUSY Interactions

8.3. Superspace and Superfields

1. Review of Dotted and Undotted Indices of Weyl Spinors
2. Superspace and Superfields

§Chapters marked by † will be covered by my fall semester class "Advanced Topics in Particle Theory".

Chapter 8. Supersymmetry (SUSY) † (20hr)

8.4. Soft SUSY Breaking

8.5. Minimal Supersymmetric SM (MSSM)

1. MSSM Superpotential with and without R-Parity
2. MSSM Higgs Sector: Higgs Masses and Problem of Little Fine-Tuning

8.6. Gauge Unifications

1. Gauge Unification Problem in the SM
2. Gauge Unification in Multi-Higgs Extension of the SM
3. Gauge Unification in the MSSM

8.7. MSSM Spectrum, R-Parity and SUSY Dark Matter

8.8. Mechanism of Supersymmetry Breaking

Chapter 9. Perturbative Gravity † (12-16hr)

9.1. Einstein Gravity as an Effective Field Theory

9.2. Gravity Coupled to the Standard Model

9.3. Kaluza-Klein Compactification of 5D

9.4. Effective Theory for Black Hole

Language:

- Read and Write in English.
- Master all terminologies in English. (Try to speak English as much as you can.)
- Important for International Communication (Conferences, Exchanges, Cooperations with USA/Europe/Japan).

Major Reference Books: (Modern)

R1. An Introduction to Quantum Field Theory
M.E. Peskin & D.V. Schroeder

R2. The Quantum Theory of Fields (I+II), S. Weinberg

R3. Quantum Field Theory in a Nutshell, A. Zee

R4. Field Theory: A Modern Primer, P. Ramond

R5. Quantum Field Theory – A Modern Introduction, M. Kaku

R6. Gauge Theory of Elementary Particle Physics, T.P. Cheng, L.F. Li

R7. Particle Physics and Field Theory, T. D. Lee

R8. Quantum Field Theory, C. Itzykson and J.B. Zuber

R9. Gauge Theories for Interactions, Y. B. Dai (in Chinese)

R10. Quantum Field Theory (III): Supersymmetry, S. Weinberg

Major Reference Books: (Modern)

R10. Dynamics of the Standard Model, J. Donoghue, et al.

R12. Quantum Fields in Curved Space, N.D. Birrell and P.C.W. Davies

R13. Cosmology, (Oxford Press, 2008), Steven Weinberg

R14. An Introduction to Modern Cosmology, Andrew Liddle

R15. Modern Cosmology, Scott Dodelson

R16. The Early Universe, E. W. Kolb and M. S. Turner

R17. Quantum Field Theory and Critical Phenomena, J. Zinn-Justin

R18. Statistical Field Theory, G. Parisi

R19. Statistical Field Theory, (I+II), C. Itzykson and J. Drouffe

R20. Finite Temperature Field Theory – Principles and Applications, J. I. Kapusta and C. Gale

R21. Quantum Field Theory in Condensed Matter Physics, A. Tsvelik