粒子理论专题 动 力 学 对 称 性 自 发 破 缺

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圈图展开与有效势

Coleman-Weinberg理论

Gross-Neveu模型

局域复合算符的有效势

圈图展开 Y.Nambu,Phys.Lett.B26,626(1968) 普通微扰论破坏规范对称性: $\mathcal{L}=-\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu}$

$$\mathcal{L}_0=-\frac{1}{4}(\partial_\mu G^a_\nu-\partial_\nu G^a_\mu)^2 \qquad \mathcal{L}_{\rm int}=-gC_{abc}(\partial_\mu G^a_\nu)G^b_\mu G^c_\nu-\frac{1}{4}g^2C_{abc}C_{ade}G^b_\mu G^c_\nu G^d_\mu G^e_\nu$$

在微扰论中，规范对称性的破坏与否只能准确到微扰论计算所达到的精度！
无法准确地判断自发破缺的发生与否

构造不破坏对称性的展开方法: 按$\hbar$幂次展开 = 半经典展开

当量子涨落效应比较小时: $Z[J]=e^{\frac{i}{\hbar}W[J]}=\int\mathcal{D}\phi\, e^{\frac{i}{\hbar}\int d^4x(\mathcal{L}+\hbar J\phi)}$ 可按$\hbar$的正幂次进行展开

$$\phi=\sqrt{\hbar}\phi' \qquad Z[J]=e^{\frac{i}{\hbar}W[J]}=\Pi\hbar\int\mathcal{D}\phi'\, e^{i\int d^4x\left[\mathcal{L}_0(\phi')\ \text{二次项}+\frac{1}{\hbar}\mathcal{L}_{\rm int}(\sqrt{\hbar}\phi')+\sqrt{\hbar}J\phi'\right]}$$

$$=e^{i\int d^4x\frac{1}{\hbar}\mathcal{L}_{\rm int}(\frac{1}{i}\frac{\delta}{\delta J})}e^{\frac{i}{2}\iint d^4xd^4y\ \hbar J(x)\Delta'(x-y)J(y)} \qquad \Delta'(x-y)=\langle 0|\mathbf{T}\phi'(x)\phi'(y)|0\rangle$$

分析以ϕ'构造的费曼图, E条外线, I条内线, V个顶角

费曼图的$\hbar$幂次: $\hbar^{\frac{E}{2}+I-V}\ \overset{L=I-(V-1)}{=\!=\!=\!=\!=\!=\!=}\ \hbar^{\frac{E}{2}+L-1}$

给定外线的费曼图 按圈的数目展开等价于按$\hbar$的幂次展开！量子涨落是以圈的数目衡量的!

有些理论 圈展开=微扰展开: $\lambda\phi^4$ $L\ \overset{4V=E+2I}{=\!=\!=\!=\!=\!=}\ V+1-E/2$ 有多种顶角(真空期望值)时除外

有效势

$$Z[J]=e^{iW[J]}=\int \mathcal{D}\phi\, e^{i\int d^4x(\mathcal{L}+J\phi)}$$

定义经典场: $\hat{\phi}(x)\equiv\frac{\delta W[J]}{\delta J(x)}=-i\frac{1}{Z[J]}\frac{\delta Z[J]}{\delta J(x)}$

有效作用量: $\Gamma[\hat{\phi}]\equiv W[J]-\int d^4x\, J(x)\hat{\phi}(x)$

$$\frac{\delta\Gamma[\hat{\phi}]}{\delta\hat{\phi}(x)}=\int d^4y\frac{\delta W}{\delta J(y)}\frac{\delta J(y)}{\delta\hat{\phi}(x)}-\int d^4y\,[\frac{\delta J(y)}{\delta\hat{\phi}(x)}\hat{\phi}(y)+J(y)\delta(x-y)]=-J(x)$$

对比经典的含外源的作用量所满足的场方程:

$$S_J(\phi)\equiv\int d^4x[\mathcal{L}(\phi)+J\phi]$$

场方程: $\frac{\delta S_J(\phi)}{\delta\phi(x)}=0\ \Rightarrow\ \frac{\delta\int d^4y\,\mathcal{L}[\phi]}{\delta\phi(x)}=-J(x)$

$$\Gamma^{(n)}(x_1,\cdots,x_n)\equiv\frac{\delta^n\Gamma}{\delta\hat{\phi}(x_1)\cdots\delta\hat{\phi}(x_n)}$$

$\Gamma^{(n)}(x_1,\cdots,x_n)\Big|_{J=0}=$ **n点一粒子不可约顶角**

$$\hat{\phi}(x)\overset{J=0}{=\!=\!=}v$$

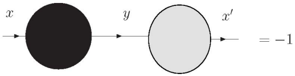
$$\Gamma[\hat{\phi}]=\sum_n\int d^4x_1\cdots d^4x_n\frac{1}{n!}\Gamma^{(n)}(x_1,\cdots,x_n)[\hat{\phi}(x_1)-v]\cdots[\hat{\phi}(x_n)-v]$$

一粒子不可约顶角生成泛函

两线顶角:

$$\delta(x-x') = \frac{\delta\hat{\phi}(x)}{\delta\hat{\phi}(x')} = \int d^4y \frac{\delta\hat{\phi}(x)}{\delta J(y)} \frac{\delta J(y)}{\delta\hat{\phi}(x')} = -\int d^4y \frac{\delta^2 W[J]}{\delta J(x)\delta J(y)} \frac{\delta^2\Gamma[\hat{\phi}]}{\delta\hat{\phi}(y)\delta\hat{\phi}(x')}$$

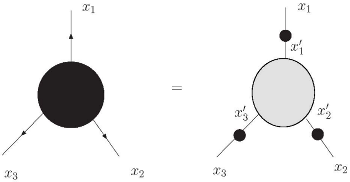
三线顶角:



$$0 = \frac{\delta}{\delta J(x_3)} \int d^4x_2 \frac{\delta^2 W[J]}{\delta J(x_1)\delta J(x_2)} \frac{\delta\Gamma[\hat{\phi}]}{\delta\hat{\phi}(x_2)\delta\hat{\phi}(x')}$$

$$= \int d^4x_2 \left[\frac{\delta^3 W[J]}{\delta J(x_1)\delta J(x_2)\delta J(x_3)} \frac{\delta^2\Gamma[\hat{\psi}]}{\delta\hat{\phi}(x_2)\delta\hat{\phi}(x')} + \frac{\delta^2 W[J]}{\delta J(x_1)\delta J(x_2)} \int d^4y \frac{\delta^3\Gamma[\hat{\phi}]}{\delta\hat{\phi}(y)\delta\hat{\phi}(x_2)\delta\hat{\phi}(x')} \frac{\delta\hat{\phi}(y)}{\delta J(x_3)} \right]$$

$$\frac{\delta^3 W[J]}{\delta J(x_1)\delta J(x_2)\delta J(x_3)} = \int d^4x_1' d^4x_2' d^4x_3' \frac{\delta^2 W[J]}{\delta J(x_1)\delta J(x_1')} \frac{\delta^2 W[J]}{\delta J(x_2)\delta J(x_2')} \frac{\delta^2 W[J]}{\delta J(x_3)\delta J(x_3')} \frac{\delta^3\Gamma[\hat{\psi}]}{\delta\hat{\phi}(x_1')\delta\hat{\phi}(x_2')\delta\hat{\phi}(x_3')}$$



四线顶角： $\dfrac{\delta^4 W[J]}{\delta J(x_1)\delta J(x_2)\delta J(x_3)\delta J(x_4)}$

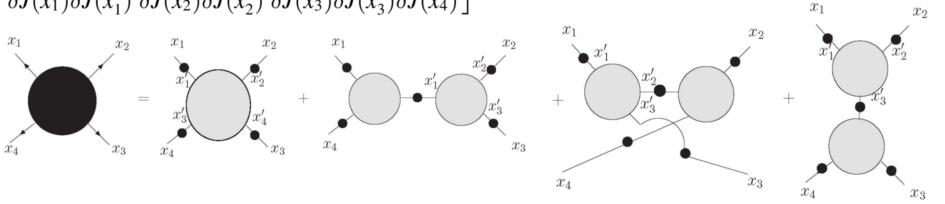
$$= \frac{\delta}{\delta J(x_4)}\left[\int d^4x_1' d^4x_2' d^4x_3' \frac{\delta^2 W[J]}{\delta J(x_1)\delta J(x_1')}\frac{\delta^2 W[J]}{\delta J(x_2)\delta J(x_2')}\frac{\delta^2 W[J]}{\delta J(x_3)\delta J(x_3')}\frac{\delta^3\Gamma[\hat\phi]}{\delta\hat\phi(x_1')\delta\hat\phi(x_2')\delta\hat\phi(x_3')}\right]$$

$$= \int d^4x_1' d^4x_2' d^4x_3' d^4x_4' \frac{\delta^2 W[J]}{\delta J(x_1)\delta J(x_1')}\frac{\delta^2 W[J]}{\delta J(x_2)\delta J(x_2')}\frac{\delta^2 W[J]}{\delta J(x_3)\delta J(x_3')}\frac{\delta^2 W[J]}{\delta J(x_4)\delta J(x_4')}$$

$$\times \frac{\delta^4\Gamma[\hat\phi]}{\delta\hat\phi(x_1')\delta\hat\phi(x_2')\delta\hat\phi(x_3')\delta\hat\phi(x_4')} + \int d^4x_1' d^4x_2' d^4x_3' \frac{\delta^3\Gamma[\hat\phi]}{\delta\hat\phi(x_1')\delta\hat\phi(x_2')\delta\hat\phi(x_3')}$$

$$\times\left[\frac{\delta^3 W[J]}{\delta J(x_1)\delta J(x_1')\delta J(x_4)}\frac{\delta^2 W[J]}{\delta J(x_2)\delta J(x_2')}\frac{\delta^2 W[J]}{\delta J(x_3)\delta J(x_3')} + \frac{\delta^2 W[J]}{\delta J(x_1)\delta J(x_1')}\frac{\delta^3 W[J]}{\delta J(x_2)\delta J(x_2')\delta J(x_4)}\frac{\delta^2 W[J]}{\delta J(x_3)\delta J(x_3')}\right.$$

$$\left. + \frac{\delta^2 W[J]}{\delta J(x_1)\delta J(x_1')}\frac{\delta^2 W[J]}{\delta J(x_2)\delta J(x_2')}\frac{\delta^3 W[J]}{\delta J(x_3)\delta J(x_3')\delta J(x_4)}\right]$$



真空态的平移不变性: $\hat{\phi}(x) \overset{J=0}{=\!=\!=} \hat{\phi}(x+a)$ 真空期望值是与坐标无关的常数!

研究具有平移不变性的$\hat{\phi}(x) = \hat{\phi}$ $\quad \Gamma^{(n)}(x_1, \cdots, x_n) \equiv \dfrac{\delta^n \Gamma}{\delta\hat{\phi}(x_1)\cdots\delta\hat{\phi}(x_n)}$ 只依赖$n-1$个坐标

$$\Gamma[\hat{\phi}] = \sum_n \int d^4x_1 \cdots d^4x_n \, \frac{1}{n!} \Gamma^{(n)}(x_1, \cdots, x_n)[\hat{\phi}(x_1) - v] \cdots [\hat{\phi}(x_n) - v]$$ 对平移不变$\hat{\phi}$含一整体$\int d^4x$因子

$$\Gamma[\hat{\phi}] = -V_{\rm eff}(\hat{\phi}) \int d^4x$$ 包含所有有效作用量中不含场的微商的相互作用项 $\xrightarrow{\left.\frac{\delta\Gamma}{\delta\hat{\phi}}\right|_{J=0}=0}$ $\left.\dfrac{\partial V_{\rm eff}[\hat{\phi}]}{\partial\hat{\phi}}\right|_{\hat{\phi}=v} = 0$

$\phi(x) = \hat{\phi} + \tilde{\phi}(x)$, $\mathcal{D}\phi = \mathcal{D}\tilde{\phi}$, 若略去量子涨落$\tilde{\phi}(x)$ 圈图的贡献

$$Z[J] = e^{iW[J]} = e^{i\int d^4x[-V(\hat{\phi})+J(x)\hat{\phi}]} \Rightarrow W[J] = \int d^4x[-V(\hat{\phi}) + J(x)\hat{\phi}]$$

$$\Gamma[\hat{\phi}] = W[J] - \int d^4x \, J(x)\hat{\phi} = -V(\hat{\phi}) \int d^4x \xrightarrow{\text{略去}\tilde{\phi}(x)} V_{\rm eff}(\hat{\phi}) = V(\hat{\phi})$$ $\tilde{\phi}(x)$的修正需要计算圈图!

标准模型对真空的非微扰估计主要就是在此水平上进行的! 另加小的圈图修正

$$V_{\rm eff}(\hat{\phi}) = -\sum_n \int d^4x_2 \cdots d^4x_n \frac{1}{n!} \Gamma^{(n)}(0, x_2 - x_1, \cdots, x_n - x_1)(\hat{\phi} - v)^n = -\sum_n \frac{1}{n!} \Gamma^{(n)}(p_1 = 0, \cdots, p_n = 0)(\hat{\phi} - v)^n$$

有效势的泛函计算

- R.Jackiw, Phys.Rev.D9,1686(1974)
- J.Iliopolous, C.Itzykson, and A.Martin, Rev. Mod. Phys.47, 165(1975)

$$S=\int d^4x\,\mathcal{L}\qquad S_J=\int d^4x\,(\mathcal{L}+J\phi)$$

经典解$\phi_{\rm cl}$满足：$\frac{\delta S_J[\phi_{\rm cl}]}{\delta\phi_{\rm cl}}=0\ \Rightarrow\ \frac{\delta S[\phi_{\rm cl}]}{\delta\phi_{\rm cl}(x)}=-J(x)$

$$\phi=\phi_{\rm cl}+\tilde\phi\qquad S_J(\phi_{\rm cl}+\tilde\phi)=S_J(\phi_{\rm cl})+\frac{1}{2}\int d^4xd^4y\tilde\phi(x)\frac{\delta S[\phi_{\rm cl}]}{\delta\phi_{\rm cl}(x)\delta\phi_{\rm cl}(y)}\tilde\phi(y)+I_{\rm int}[\phi_{\rm cl},\tilde\phi]$$

$$J(x)\tilde\phi(x)+\frac{\delta S(\phi_{\rm cl})}{\delta\phi_{\rm cl}(x)}\tilde\phi(x)=0\qquad i\mathcal{D}^{-1}(\phi_{\rm cl};x,y)\equiv\frac{\delta S[\phi_{\rm cl}]}{\delta\phi_{\rm cl}(x)\delta\phi_{\rm cl}(y)}$$

$$S_J(\phi_{\rm cl}+\tilde\phi)=S_J(\phi_{\rm cl})+\frac{1}{2}\int d^4xd^4y\,\tilde\phi(x)i\mathcal{D}^{-1}(\phi_{\rm cl};x,y)\tilde\phi(y)+I_{\rm int}[\phi_{\rm cl},\tilde\phi]$$

$$Z[J]=e^{iW[J]}=\int\mathcal{D}\phi\,e^{iS_J[\phi_{\rm cl}+\tilde\phi]}=e^{iS_J(\phi_{\rm cl})}\int\mathcal{D}\phi\,e^{i\{\frac{1}{2}\int d^4xd^4y\tilde\phi(x)i\mathcal{D}^{-1}(\phi_{\rm cl};x,y)\tilde\phi(y)+I_{\rm int}[\phi_{\rm cl},\tilde\phi]\}}$$

$$=e^{iS_J(\phi_{\rm cl})}\int\mathcal{D}\phi\,e^{i\{\frac{1}{2}\int d^4xd^4y\tilde\phi(x)i\mathcal{D}^{-1}(\phi_{\rm cl};x,y)\tilde\phi(y)\}}Z_2[J]=e^{iS_J(\phi_{\rm cl})-\frac{1}{2}{\rm Tr}\ln\mathcal{D}^{-1}(\phi_{\rm cl})+iW_2[J]}$$

$O(\hbar^0)$ $O(\hbar^1)$ $O(\hbar^2)$

$$Z_2[J]=e^{iW_2[J]}=\frac{e^{i\{\frac{1}{2}\int d^4xd^4y\tilde\phi(x)i\mathcal{D}^{-1}(\phi_{\rm cl};x,y)\tilde\phi(y)+I_{\rm int}[\phi_{\rm cl},\tilde\phi]\}}}{e^{i\{\frac{1}{2}\int d^4xd^4y\tilde\phi(x)i\mathcal{D}^{-1}(\phi_{\rm cl};x,y)\tilde\phi(y)\}}}\qquad {\rm Det}^{-\frac{1}{2}}i\mathcal{D}^{-1}(\phi_{\rm cl})=e^{-\frac{1}{2}{\rm Tr}\ln i\mathcal{D}^{-1}(\phi_{\rm cl})}$$

$$W[J] = W_0[J] + W_1[J] + W_2[J] \qquad W_0[J] = S_J(\phi_{\rm cl}) = S(\phi_{\rm cl}) + \int d^4x\, J\phi_{\rm cl} \qquad W_1[J] = \frac{i}{2}{\rm Tr}\ln i\mathcal{D}^{-1}(\phi_{\rm cl})$$

$$\hat\phi = \frac{\delta W}{\delta J} \qquad \frac{\delta W_0}{\delta J(x)} = \int d^4y \frac{\delta S[\phi_{\rm cl}]}{\delta\phi_{\rm cl}(y)}\frac{\delta\phi_{\rm cl}(y)}{\delta J(x)} + \int d^4y \left[\frac{\delta\phi_{\rm cl}(y)}{\delta J(x)}J(y) + \phi_{\rm cl}(y)\delta(x-y)\right] = \phi_{\rm cl}(x)$$

$$\hat\phi = \phi_{\rm cl} + \phi_1 \qquad \phi_1 = \frac{\delta W_L}{\delta J(x)} \qquad W_L = W_1 + W_2$$

$$\Gamma[\hat\phi] = W[J] - \int d^4x J\hat\phi = S[\phi_{\rm cl}] + \int d^4x\, J[\phi_{\rm cl}](\phi_{\rm cl} - \hat\phi) + W_L[\phi_{\rm cl}] = S[\hat\phi - \phi_1] + \int d^4x\, J\phi_1 + W_L[\hat\phi - \phi_1]$$

$$= S[\hat\phi] - \int d^4x \frac{\delta S[\hat\phi]}{\delta\hat\phi(x)}\phi_1(x) + \frac{1}{2}\int d^4x d^4y\, \phi_1(x)\frac{\delta S[\hat\phi]}{\delta\hat\phi(x)\delta\hat\phi(y)}\phi_1(y) - \int d^4x\, J[\phi_{\rm cl}]\phi_1$$

$$+ W_L[\hat\phi] - \int d^4x \frac{\delta W_L[\hat\phi]}{\delta\hat\phi(x)}\phi_1(x) + O(\hbar^3)$$

$$\int d^4x \left[\frac{\delta S[\hat\phi]}{\delta\hat\phi(x)} + J[\phi_{\rm cl}]\right]\phi_1(x) = \int d^4x \left[\frac{\delta S[\phi_{\rm cl}]}{\delta\phi_{\rm cl}(x)} + \int d^4y \frac{\delta^2 S[\phi_{\rm cl}]}{\delta\phi_{\rm cl}(x)\delta\phi_{\rm cl}(y)}\phi_1(y) + J[\phi_{\rm cl}]\right]\phi_1(x)$$

$$\Gamma[\hat\phi] = S[\hat\phi] - \frac{1}{2}\int d^4x d^4y\, \phi_1(x)\frac{\delta S[\hat\phi]}{\delta\hat\phi(x)\delta\hat\phi(y)}\phi_1(y) + W_L[\hat\phi] - \int d^4x \frac{\delta W_L[\hat\phi]}{\delta\hat\phi(x)}\phi_1(x) + O(\hbar^3)$$

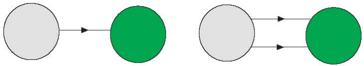
$$= \Gamma_0[\hat\phi] + \Gamma_1[\hat\phi] + \Gamma_2[\hat\phi] + O(\hbar^3)$$

$$\Gamma[\hat\phi]=\Gamma_0[\hat\phi]+\Gamma_1[\hat\phi]+\Gamma_2[\hat\phi]+O(\hbar^3)\qquad \Gamma_0[\hat\phi]=S[\hat\phi]\qquad \Gamma_1[\hat\phi]=W_1[\hat\phi]=\frac{i}{2}\mathrm{Tr}\ln i\mathcal{D}^{-1}(\hat\phi)$$

$$\Gamma_2[\hat\phi]=-\frac{1}{2}\int d^4x d^4y\,\phi_1(x)\frac{\delta S[\hat\phi]}{\delta\hat\phi(x)\delta\hat\phi(y)}\phi_1(y)+W_2[\hat\phi]-\int d^4x\,\frac{\delta W_2[\hat\phi]}{\delta\hat\phi(x)}\phi_1(x)$$

$\overset{\text{实际例子}}{=\!=\!=\!=\!=}$ 以$\mathcal{D}^{-1}(\phi_{\rm cl};x,y)$为传播子, $I_{\rm int}[\hat\phi,\tilde\phi]$为相互作用顶角的所有一粒子不可约两圈图

$$\Gamma[\hat\phi]=S[\hat\phi]+\frac{i}{2}\mathrm{Tr}\ln i\mathcal{D}^{-1}(\hat\phi)+\overline{\Gamma}_2\qquad \overline{\Gamma}_2=\sum_{n=2}^{\infty}\Gamma_n[\hat\phi]$$



$\Gamma_n[\hat\phi]=$ **以**$\mathcal{D}(\phi_{\rm cl};x,y)$**为传播子,** $I_{\rm int}[\hat\phi,\tilde\phi]$**为相互作用顶角的所有一粒子不可约n圈图**

$$e^{i\Gamma[\hat\phi]}=e^{iW[J]-i\int d^4x\,J\hat\phi}=\int\mathcal{D}\phi\;e^{iS[\phi]+i\int d^4x\,J(\phi-\hat\phi)}=\int\mathcal{D}\tilde\phi\;e^{iS[\hat\phi+\tilde\phi]+i\int d^4x\,J\tilde\phi}$$

$$\Gamma_1+\overline{\Gamma}_2=\Gamma-\Gamma_0=-i\ln\int\mathcal{D}\tilde\phi\;e^{i\{S[\hat\phi+\tilde\phi]-S[\hat\phi]+J\tilde\phi\}}=-i\ln\int\mathcal{D}\tilde\phi\;e^{i\int d^4x\,[\frac{1}{2}\tilde\phi i\mathcal{D}^{-1}\tilde\phi+\mathcal{L}_{\rm int}(\hat\phi,\tilde\phi)]}$$

$$\frac{\partial\overline{\Gamma}_2}{\partial\mathcal{D}}=-\mathcal{D}^{-1}\frac{\partial(\Gamma_1+\overline{\Gamma}_2)}{\partial\mathcal{D}^{-1}}\mathcal{D}^{-1}+\frac{i}{2}\mathcal{D}^{-1}=\frac{i}{2}\mathcal{D}^{-1}\frac{\int\mathcal{D}\tilde\phi\;\tilde\phi\tilde\phi\;e^{i\cdots}}{\int\mathcal{D}\tilde\phi\;e^{i\cdots}}\mathcal{D}^{-1}+\frac{i}{2}\mathcal{D}^{-1}=\frac{i}{2}\mathcal{D}^{-1}[-G+\mathcal{D}]\mathcal{D}^{-1}$$

$\overset{G=\mathcal{D}+\mathcal{D}\Pi\mathcal{D}+\mathcal{D}\Pi\mathcal{D}\Pi\mathcal{D}+\cdots}{=\!=\!=\!=\!=\!=\!=\!=\!=\!=}-\frac{i}{2}[\Pi+\Pi\mathcal{D}\Pi+\cdots]$ 连接图 $\Rightarrow$ $\overline{\Gamma}_2$是一粒子不可约的

利用有效作用的圈图展开计算路径积分

$e^{iW[J]} = \int \mathcal{D}\phi \, e^{i\int d^4x(\mathcal{L}+J\phi)}$ 有效作用量：$\Gamma[\hat{\phi}] \equiv W[J] - \int d^4x \, J(x)\hat{\phi}(x)$ $\quad \frac{\delta\Gamma[\hat{\phi}]}{\delta\hat{\phi}(x)} = -J(x)$

$$\Gamma[\hat{\phi}] = S[\hat{\phi}] + \frac{i}{2}\mathrm{Tr}\ln i\mathcal{D}^{-1}(\hat{\phi}) + \overline{\Gamma}_2 \qquad \overline{\Gamma}_2 = \sum_{n=2}^{\infty}\Gamma_n[\hat{\phi}]$$

$\Gamma_n[\hat{\phi}] =$ 以$\mathcal{D}(\phi_{\rm cl};x,y)$为传播子，$I_{\rm int}[\hat{\phi},\tilde{\phi}]$为相互作用顶角的所有一粒子不可约**n**圈图

路径积分的圈图计算： $\hat{\phi} \overset{J=0}{=\!=\!=\!=} \langle\phi\rangle$

$$e^{iW[0]} = e^{i\Gamma[\langle\phi\rangle]} = \int \mathcal{D}\phi \, e^{iS[\phi]} \qquad \frac{\delta\Gamma[\langle\phi\rangle]}{\delta\langle\phi(x)\rangle} = 0$$

$$\Gamma[\langle\phi\rangle] = S[\langle\phi\rangle] + \frac{i}{2}\mathrm{Tr}\ln\frac{\delta^2 S[\langle\phi\rangle]}{\delta\langle\phi(x)\rangle\delta\langle\phi(y)\rangle} + \overline{\Gamma}_2 \qquad \overline{\Gamma}_2 = \sum_{n=2}^{\infty}\Gamma_n[\langle\phi\rangle]$$

$\Gamma_n[\hat{\phi}] =$ 以$\left[\frac{\delta^2 S[\langle\phi\rangle]}{\delta\langle\phi(x)\rangle\delta\langle\phi(y)\rangle}\right]^{-1}$为传播子，$I_{\rm int}[\langle\phi\rangle,\tilde{\phi}]$为相互作用顶角的所有一粒子不可约**n**圈图

$I_{\rm int}[\langle\phi\rangle,\tilde{\phi}]$ 是 $S[\langle\phi\rangle+\tilde{\phi}]$ 中按 $\tilde{\phi}$ 幂次展开三次和三次以上项之和

有效势 $\Gamma[\hat\phi] = S[\hat\phi] + \frac{i}{2}\mathrm{Tr}\ln i\mathcal{D}^{-1}(\hat\phi) + \overline{\Gamma}_2 \overset{\text{平移不变}}{=\!=\!=\!=} -V_{\text{eff}}[\hat\phi]\int d^4x$

$V_{\text{eff}} = V(\hat\phi) + V_1(\hat\phi) + V_2(\hat\phi) + \cdots \quad V_1(\hat\phi) = -\frac{i}{2}\int\frac{d^4p}{(2\pi)^4}\mathrm{tr}\ln i\mathcal{D}^{-1}(\hat\phi;p) \quad V_2(\hat\phi) = i\left\langle e^{i\int d^4x\mathcal{L}_{\text{int}}(\hat\phi,\tilde\phi)}\right\rangle_{1\text{PI}}$

$$i\mathcal{D}^{-1}(\hat\phi;x,y) \overset{\text{平移不变}}{=\!=\!=\!=} \int\frac{d^4p}{(2\pi)^4}e^{-ip\cdot(x-y)}i\mathcal{D}^{-1}(\hat\phi;p) = i\mathcal{D}^{-1}(\hat\phi;i\partial_x)\delta(x-y)$$

$$\int d^4z\,[A(i\partial_x)\delta(x-z)][B(i\partial_z)\delta(z-y)] = \int d^4z\,[A(i\partial_x)\delta(x-z)][B(-i\partial_y)\delta(z-y)] = A(i\partial_x)B(i\partial_x)\delta(x-y)$$

$$\int d^4z_1\cdots d^4z_{n-1}[A_1(i\partial_x)\delta(x-z_1)][A_2(i\partial_{z_1})\delta(z_1-z_2)]\cdot\cdot[A_n(i\partial_{z_{n-1}})\delta(z_{n-1}-y)] = A_1(i\partial_x)\cdot\cdot\cdot A_n(i\partial_x)\delta(x-y)$$

$$[\ln i\mathcal{D}^{-1}(\hat\phi)](x,y) = [\ln i\mathcal{D}^{-1}(\hat\phi;i\partial_x)]\delta(x-y)$$

$$\mathrm{Tr}\ln i\mathcal{D}^{-1}(\hat\phi) = \int d^4x\,\mathrm{tr}[\ln i\mathcal{D}^{-1}(\hat\phi)](x,x) = \int d^4x\int\frac{d^4p}{(2\pi)^4}\,\mathrm{tr}\ln i\mathcal{D}^{-1}(\hat\phi;p)$$

有效势可以相差一个与$\hat\phi$无关的常数！ $V_1(\hat\phi) = -\frac{i}{2}\int\frac{d^4p}{(2\pi)^4}\ln\frac{\det[i\mathcal{D}^{-1}(\hat\phi;p)]}{\det[i\mathcal{D}^{-1}(0;p)]}$

对费米子V_1中的系数 $-\frac{i}{2}$ 应该被换为 $+i$！因自由玻色和费米场的积分分别给出 Det$^{-\frac{1}{2}}$ 和 Det

$$i\mathcal{D}^{-1}(\hat\phi;x,y) \Rightarrow i\mathcal{S}^{-1}(\hat{\overline{\psi}},\hat\psi) \equiv \frac{\delta S[\hat{\overline{\psi}},\hat\psi]}{\delta\hat{\overline{\psi}}(x)\delta\hat\psi(y)}$$

关于玻色子与费米子对有效势贡献的讨论

♣ 玻色子与费米子对有效势的一阶贡献的符号相反

◇ 若玻色子倾向产生自发破缺,则费米子倾向破坏自发破缺,反之亦然！

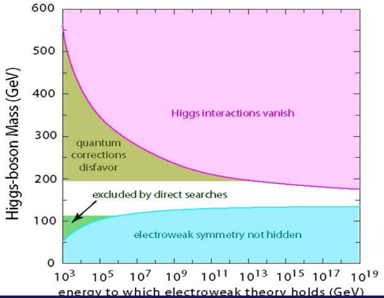
♡ 由此从势的稳定性给出对标准模型中的Higgs质量的限制！两圈图计算

♠ 若玻色子与费米子之间达到某种平衡, 也就是具有超对称性:

¶ 有效势有可能不受高阶量子修正影响！

✠ 导致体系无动力学自发破坏！

ℜ 还导致理论的有效势可精确计算！



S.Coleman and E.Weinberg, Phys.Rev.D7,1888(1973)

考虑纯由量子修正产生的对称性自发破缺当初未用Jackiw的方法

- 任意性小,预言性高
- 量子修正的重要性

$O(2)$ σ**模型:** $S = \int d^4x \, \mathcal{L} = \int d^4x [\frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\pi\partial^\mu\pi) - \frac{\lambda}{4!}(\sigma^2+\pi^2)^2]$

对应原来的无自发破缺情形: $\kappa = -\mu^2 \xRightarrow{\frac{\kappa}{2}(\sigma^2+\pi^2)} \kappa = 0$

$$V_{\rm eff} = V_0 + V_1 \qquad V_0 = -\frac{\lambda}{4!}(\sigma^2+\pi^2)^2 \qquad V_1 = -\frac{i}{2}\int\frac{d^4p}{(2\pi)^4}\ln\frac{\det[i\mathcal{D}^{-1}(\hat\sigma,\hat\pi;p)]}{\det[i\mathcal{D}^{-1}(0,0;p)]}$$

$$i\mathcal{D}^{-1}_{\sigma\sigma}(\hat\sigma,\hat\pi;x,y) = \frac{\delta^2 S}{\delta\sigma(x)\delta\sigma(y)} = -(\partial_x^2 + \frac{\lambda}{2}\hat\sigma^2 + \frac{\lambda}{6}\hat\pi^2)\delta(x-y)$$

$$i\mathcal{D}^{-1}_{\sigma\pi}(\hat\sigma,\hat\pi;x,y) = \frac{\delta^2 S}{\delta\sigma(x)\delta\pi(y)} = i\mathcal{D}^{-1}_{\pi\sigma}(\hat\sigma,\hat\pi;x,y) = -\frac{\lambda}{3}\hat\sigma\hat\pi\delta(x-y)$$

$$i\mathcal{D}^{-1}_{\pi\pi}(\hat\sigma,\hat\pi;x,y) = \frac{\delta^2 S}{\delta\pi(x)\delta\pi(y)} = -(\partial_x^2 + \frac{\lambda}{2}\hat\pi^2 + \frac{\lambda}{6}\hat\sigma^2)\delta(x-y)$$

$O(2)$ σ模型: $S=\int d^4x\,\mathcal{L}=\int d^4x[\frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma+\partial_\mu\pi\partial^\mu\pi)-\frac{\lambda}{4!}(\sigma^2+\pi^2)^2]$

$$V_{\rm eff}=V_0+V_1 \qquad V_0=-\frac{\lambda}{4!}(\sigma^2+\pi^2)^2 \qquad V_1=-\frac{i}{2}\int\frac{d^4p}{(2\pi)^4}\ln\frac{\det[i\mathcal{D}^{-1}(\hat\sigma,\hat\pi;p)]}{\det[i\mathcal{D}^{-1}(0,0;p)]}$$

$$i\mathcal{D}^{-1}(\hat\sigma,\hat\pi;p)=\begin{pmatrix} i\mathcal{D}^{-1}_{\sigma\sigma}(\hat\sigma,\hat\pi;p) & i\mathcal{D}^{-1}_{\sigma\pi}(\hat\sigma,\hat\pi;p)\\ i\mathcal{D}^{-1}_{\pi\sigma}(\hat\sigma,\hat\pi;p) & i\mathcal{D}^{-1}_{\pi\pi}(\hat\sigma,\hat\pi;p)\end{pmatrix}=\begin{pmatrix} p^2-\frac{\lambda}{2}\hat\sigma^2-\frac{\lambda}{6}\hat\pi^2 & -\frac{\lambda}{3}\hat\sigma\hat\pi\\ -\frac{\lambda}{3}\hat\sigma\hat\pi & p^2-\frac{\lambda}{2}\hat\pi^2-\frac{\lambda}{6}\hat\sigma^2\end{pmatrix}$$

$$\det[i\mathcal{D}^{-1}(\hat\sigma,\hat\pi;p)]=(p^2-\frac{\lambda}{2}\hat\sigma^2-\frac{\lambda}{6}\hat\pi^2)(p^2-\frac{\lambda}{2}\hat\pi^2-\frac{\lambda}{6}\hat\sigma^2)-\frac{\lambda^2}{9}\hat\sigma^2\hat\pi^2=[p^2-\frac{\lambda}{2}(\hat\sigma^2+\hat\pi^2)][p^2-\frac{\lambda}{6}(\hat\pi^2+\hat\sigma^2)]$$

$$V_1=-\frac{i}{2}\int\frac{d^4p}{(2\pi)^4}\left[\ln(1-\frac{\lambda}{2p^2}\hat\phi^2)+\ln(1-\frac{\lambda}{6p^2}\hat\phi^2)\right] \qquad \hat\phi\equiv\hat\sigma^2+\hat\pi^2$$

$$\overset{\text{Wick Rotation}}{=\!=\!=\!=\!=\!=\!=}-\frac{i}{2}\frac{i\pi^2}{(2\pi)^4}\lim_{\Lambda\to\infty}\int_0^{\Lambda^2}p_E^2dp_E^2\left[\ln(1+\frac{\lambda}{2p_E^2}\hat\phi^2)+\ln(1+\frac{\lambda}{6p_E^2}\hat\phi^2)\right]$$

在有平移不变的情形下,应用动量截断正规化并不破坏规范对称性!

$$\int_0^{\Lambda^2}xdx\ln(1+\frac{a}{x})=\frac{x^2}{2}\ln(1+\frac{a}{x})\Big|_0^{\Lambda^2}+\frac{1}{2}\int_0^{\Lambda^2}\frac{a}{1+\frac{a}{x}}dx=a\Lambda^2+\frac{a^2}{2}(\ln\frac{a}{\Lambda^2}-\frac{1}{2})$$

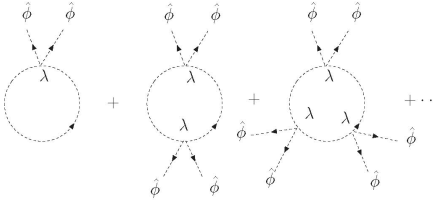
$$V_1=\frac{1}{32\pi^2}\left[\frac{\lambda}{2}\hat\phi^2\Lambda^2+\frac{\lambda^2\hat\phi^4}{8}(\ln\frac{\frac{\lambda}{2}\hat\phi^2}{\Lambda^2}-\frac{1}{2})+\frac{\lambda}{6}\hat\phi^2\Lambda^2+\frac{\lambda^2\hat\phi^4}{72}(\ln\frac{\frac{\lambda}{6}\hat\phi^2}{\Lambda^2}-\frac{1}{2})\right]$$

$O(2)$ σ模型: $S=\int d^4x\,\mathcal{L}=\int d^4x[\frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma+\partial_\mu\pi\partial^\mu\pi)-\frac{\lambda}{4!}(\sigma^2+\pi^2)^2]$

$V_{\rm eff}=V_0+V_1\quad V_0=-\frac{\lambda}{4!}(\sigma^2+\pi^2)^2\quad V_1=\frac{1}{32\pi^2}\left[\frac{2\lambda}{3}\hat\phi^2\Lambda^2+\frac{\lambda^2\hat\phi^4}{8}(\ln\frac{\frac{\lambda}{2}\hat\phi^2}{\Lambda^2}-\frac{1}{2})+\frac{\lambda^2\hat\phi^4}{72}(\ln\frac{\frac{\lambda}{6}\hat\phi^2}{\Lambda^2}-\frac{1}{2})\right]$

早期的费曼图求和 每个图都有红外发散,将所有阶图求和可消去

$\ln(1-\frac{\frac{\lambda}{2}}{p^2}\hat\phi^2)=-\frac{\frac{\lambda}{2}}{p^2}\hat\phi^2-\frac{1}{2}\frac{\frac{\lambda}{2}}{p^2}\hat\phi^2\frac{\frac{\lambda}{2}}{p^2}\hat\phi^2-\frac{1}{3}\frac{\frac{\lambda}{2}}{p^2}\hat\phi^2\frac{\frac{\lambda}{2}}{p^2}\hat\phi^2\frac{\frac{\lambda}{2}}{p^2}\hat\phi^2+\cdots$



Jackiw的公式实现了对无穷多费曼图的求和!

重整化: 采用抵消项

$$V_1\overset{\text{重整化}}{=\!=\!=\!=}\frac{1}{32\pi^2}\left[\frac{2\lambda}{3}\hat\phi^2\Lambda^2+\frac{\lambda^2\hat\phi^4}{8}(\ln\frac{\frac{\lambda}{2}\hat\phi^2}{\Lambda^2}-\frac{1}{2})+\frac{\lambda^2\hat\phi^4}{72}(\ln\frac{\frac{\lambda}{6}\hat\phi^2}{\Lambda^2}-\frac{1}{2})\right]-\frac{1}{2}C_1\hat\phi^2-\frac{1}{4!}C_2\hat\phi^4$$

归一化条件: $\xrightarrow[\quad]{V_0=\frac{\lambda}{4!}\hat\phi^4\ \text{裸常数不需量子修正}}$ $\left.\frac{d^2V_{\rm eff}}{d\hat\phi^2}\right|_{\hat\phi=0}=0\quad\left.\frac{d^4V_{\rm eff}}{d\hat\phi^4}\right|_{\hat\phi=M}=\lambda\equiv\lambda(M)$ ⇐无量纲理论通过归一化引入量纲

$$C_1=\frac{\lambda\Lambda^2}{24\pi^2}\qquad C_2=\frac{55\lambda^2}{144\pi^2}+\frac{3\lambda^2}{32\pi^2}\ln\frac{\frac{\lambda}{2}M^2}{\Lambda^2}+\frac{\lambda^2}{96\pi^2}\ln\frac{\frac{\lambda}{6}M^2}{\Lambda^2}\qquad V_{\rm eff}=\frac{\lambda}{4!}\hat\phi^4+\frac{5\lambda^2\hat\phi^4}{1152\pi^2}(\ln\frac{\hat\phi^2}{M^2}-\frac{25}{6})$$

展开收敛条件: $|\lambda|<1\quad|\lambda\ln\frac{\hat\phi^2}{M^2}|<1$

$O(2)$ σ模型: $S=\int d^4x\,\mathcal{L}=\int d^4x[\frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma+\partial_\mu\pi\partial^\mu\pi)-\frac{\lambda}{4!}(\sigma^2+\pi^2)^2]$

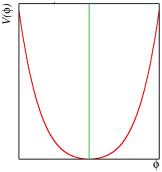
归一化条件: $\xrightarrow[\hspace{3cm}]{V_0=\frac{\lambda}{4!}\hat\phi^4\ \text{裸常数不需量子修正}} \left.\frac{d^2V_{\rm eff}}{d\hat\phi^2}\right|_{\hat\phi=0}=0 \quad \left.\frac{d^4V_{\rm eff}}{d\hat\phi^4}\right|_{\hat\phi=M}=\lambda\equiv\lambda(M)$ ⇐无量纲理论通过归一化引入量纲

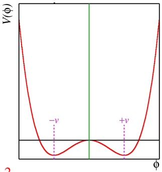
$$V_{\rm eff}=\frac{\lambda}{4!}\hat\phi^4+\frac{5\lambda^2\hat\phi^4}{1152\pi^2}\left(\ln\frac{\hat\phi^2}{M^2}-\frac{25}{6}\right)$$

展开收敛条件: $|\lambda|<1 \quad |\lambda\ln\frac{\hat\phi^2}{M^2}|<1$

尺度变换性质: $V_{\rm eff}=\frac{\lambda'}{4!}\hat\phi^4+\frac{5\lambda'^2\hat\phi^4}{1152\pi^2}(\ln\frac{\hat\phi^2}{M'^2}-\frac{25}{6})$

$$\lambda(M)\xRightarrow{M\to M'}\lambda'\equiv\lambda(M')=\left.\frac{d^4V_{\rm eff}}{d\hat\phi^4}\right|_{\hat\phi=M'}=\lambda+\frac{5\lambda^2}{48\pi^2}\ln\frac{M'^2}{M^2}$$





圈图对数项在小大 $\hat\phi$区降低升高有效势

$$0=\left.\frac{dV_{\rm eff}}{d\hat\phi}\right|_{\hat\phi=\langle\phi\rangle}=\lambda\langle\phi\rangle^3\left[\frac{1}{6}+\frac{5\lambda}{288\pi^2}\ln\frac{\langle\phi\rangle^2}{M^2}-\frac{55\lambda}{964\pi^2}\right]\Rightarrow\langle\phi\rangle=0,\ Me^{-\frac{24\pi^2}{5\lambda}+\frac{11}{6}}$$

与M无关

$$V_{\rm eff}(\hat\phi=0)=0 \qquad V_{\rm eff}(\hat\phi=Me^{-\frac{24\pi^2}{5\lambda}+\frac{11}{6}})=-\frac{5\lambda^2\langle\phi\rangle^4}{2304\pi^2}$$

耦合常数的负幂次: 非微扰

判断一个理论是否发生对称性自发破缺, 只考虑数图可能会出错, 应该计算圈图!

维数转移: $\lambda(M=\langle\phi\rangle)=\frac{144}{55}\pi^2$ 不合理, 数值并不可靠!

标量QED

$\phi = \begin{pmatrix} \sigma \\ \pi \end{pmatrix}$ 两个无量纲参数

$$S = \int d^4x\, \mathcal{L} = \int d^4x\, [-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\alpha}(\partial_\mu A^\mu)^2 + \frac{1}{2}(\partial_\mu\sigma + eA_\mu\pi)^2 + \frac{1}{2}(\partial_\mu\pi - eA_\mu\sigma)^2 - \frac{\lambda}{4!}(\sigma^2+\pi^2)^2]$$

$\frac{\delta W}{\delta J} = \hat{\phi}$ $\frac{\delta W}{\delta J_\mu} \overset{J_\mu A^\mu}{=\!=\!=\!=} \hat{A}^\mu$ 洛伦兹对称性禁戒其有真空期望值 $V_{\rm eff}(\hat{\phi}) \equiv V_{\rm eff}(\hat{\phi}, \hat{A}^\mu = 0)$

$$i\mathcal{D}^{-1}_{\mu\nu}(\hat\sigma, \hat\pi; x, y) = \left.\frac{\delta^2 S}{\delta A^\mu(x)\delta A^\nu(y)}\right|_{\substack{\sigma=\hat\sigma \\ \pi=\hat\pi}} = [g_{\mu\nu}\partial_x^2 - (1-\frac{1}{\alpha})\partial_\mu\partial_\nu + e^2 g_{\mu\nu}(\hat\sigma^2 + \hat\pi^2)]\delta(x-y)$$

$$i\mathcal{D}^{-1}_{\mu\sigma}(\hat\sigma, \hat\pi; x, y) = \left.\frac{\delta^2 S}{\delta A^\mu(x)\delta \sigma(y)}\right|_{\substack{\sigma=\hat\sigma \\ \pi=\hat\pi}} = 2e\hat\pi\partial_\mu\delta(x-y)$$

$i\mathcal{D}^{-1}_{\sigma\pi}$, $i\mathcal{D}^{-1}_{\pi\pi}$, $i\mathcal{D}^{-1}_{\sigma\pi}$ 与前一样

$$i\mathcal{D}^{-1}_{\mu\pi}(\hat\sigma, \hat\pi; x, y) = \left.\frac{\delta^2 S}{\delta A^\mu(x)\delta \pi(y)}\right|_{\substack{\sigma=\hat\sigma \\ \pi=\hat\pi}} = -2e\hat\sigma\partial_\mu\delta(x-y)$$

为方便计算取特殊坐标系: $p^\mu = (p^0, 0, 0, 0)$

标量QED

$$\phi = \begin{pmatrix} \sigma \\ \pi \end{pmatrix}$$ 两个无量纲参数

$$S = \int d^4x\, \mathcal{L} = \int d^4x\, [-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\alpha}(\partial_\mu A^\mu)^2 + \frac{1}{2}(\partial_\mu\sigma + eA_\mu\pi)^2 + \frac{1}{2}(\partial_\mu\pi - eA_\mu\sigma)^2 - \frac{\lambda}{4!}(\sigma^2+\pi^2)^2]$$

$$\det[i\mathcal{D}^{-1}(\hat\sigma,\hat\pi;p)] \overset{p^\mu=(p^0,0,0,0)}{=\!=\!=\!=\!=\!=\!=} \begin{vmatrix} i\mathcal{D}_{11}^{-1} & 0 & 0 & 0 & 0 & 0 \\ 0 & i\mathcal{D}_{22}^{-1} & 0 & 0 & 0 & 0 \\ 0 & 0 & i\mathcal{D}_{33}^{-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & i\mathcal{D}_{00}^{-1} & i\mathcal{D}_{0\sigma}^{-1} & i\mathcal{D}_{0\pi}^{-1} \\ 0 & 0 & 0 & i\mathcal{D}_{\sigma 0}^{-1} & i\mathcal{D}_{\sigma\sigma}^{-1} & i\mathcal{D}_{\sigma\pi}^{-1} \\ 0 & 0 & 0 & i\mathcal{D}_{\pi 0}^{-1} & i\mathcal{D}_{\pi\sigma}^{-1} & i\mathcal{D}_{\pi\pi}^{-1} \end{vmatrix} = i\mathcal{D}_{11}^{-1} i\mathcal{D}_{22}^{-1} i\mathcal{D}_{33}^{-1}$$

$$\times[i\mathcal{D}_{00}^{-1}(i\mathcal{D}_{\sigma\sigma}^{-1}i\mathcal{D}_{\pi\pi}^{-1} - i\mathcal{D}_{\sigma\pi}^{-1}i\mathcal{D}_{\pi\sigma}^{-1}) - i\mathcal{D}_{0\sigma}^{-1}(i\mathcal{D}_{\sigma 0}^{-1}i\mathcal{D}_{\pi\pi}^{-1} - i\mathcal{D}_{\pi 0}^{-1}i\mathcal{D}_{\sigma\pi}^{-1}) + i\mathcal{D}_{0\pi}^{-1}(i\mathcal{D}_{\sigma 0}^{-1}i\mathcal{D}_{\pi\sigma}^{-1} - i\mathcal{D}_{\pi}^{-1}i\mathcal{D}_{\sigma\sigma}^{-1})]$$

$$\overset{(p^0)^2\Rightarrow p^2}{=\!=\!=\!=\!=} (-p^2 + e^2\hat\phi^2)^3[(-\frac{1}{\alpha}p^2 + e^2\hat\phi^2)(-p^2 + \frac{\lambda}{2}\hat\phi^2)(-p^2 + \frac{\lambda}{6}\hat\phi^2) - 4e^2\hat\phi^2 p^2(-p^2 + \frac{\lambda}{2}\hat\phi^2)]$$

$$\frac{\det[i\mathcal{D}^{-1}(\hat\sigma,\hat\pi;p)]}{\det[i\mathcal{D}^{-1}(0,0;p)]} = (1 - \frac{e^2\hat\phi^2}{p^2})^3[(1 - \alpha\frac{e^2\hat\phi^2}{p^2})(1 - \frac{\frac{\lambda}{2}\hat\phi^2}{p^2})(1 - \frac{\frac{\lambda}{6}\hat\phi^2}{p^2}) - 4\alpha\frac{e^2\hat\phi^2}{p^2}(1 - \frac{\frac{\lambda}{2}\hat\phi^2}{p^2})]$$

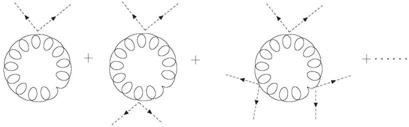
有效势一般说是规范依赖的,只在真空极小点处是规范无关的！

R.Fukuda and T.Kugo, Phys. Rev. D13,3469(1974)

标量QED

$$\phi = \begin{pmatrix} \sigma \\ \pi \end{pmatrix}$$ 两个无量纲参数

$$\frac{\det[i\mathcal{D}^{-1}(\hat{\sigma},\hat{\pi};p)]}{\det[i\mathcal{D}^{-1}(0,0;p)]} \overset{\text{Landau规范}\alpha=0}{=\!=\!=\!=\!=\!=\!=} (1-\frac{e^2\hat{\phi}^2}{p^2})^3(1-\frac{\frac{\lambda}{2}\hat{\phi}^2}{p^2})(1-\frac{\frac{\lambda}{6}\hat{\phi}^2}{p^2})$$ $e=0$ 回到$O(2)$ σ模型



$$V_1(\hat{\phi}) = \frac{2\lambda+9e^2}{96\pi^2}\hat{\phi}^2\Lambda^2 - \frac{\frac{5}{18}\lambda^2+3e^4}{128\pi^2}\hat{\phi}^4 + \frac{\lambda^2\hat{\phi}^4}{256\pi^2}\ln\frac{\frac{\lambda}{2}\hat{\phi}^2}{\Lambda^2} + \frac{\lambda^2\hat{\phi}^4}{2304\pi^2}\ln\frac{\frac{\lambda}{6}\hat{\phi}^2}{\Lambda^2} + \frac{3e^4\hat{\phi}^4}{64\pi^2}\ln\frac{e^2\hat{\phi}^2}{\Lambda^2} - \frac{1}{2}C_1\hat{\phi}^2 - \frac{\lambda}{4!}C_2\hat{\phi}^4$$

$$\overset{\text{裸常数不需量子修正}}{=\!=\!=\!=\!=\!=\!=\!=\!\Rightarrow} \left.\frac{d^2V_{\text{eff}}}{d\hat{\phi}^2}\right|_{\hat{\phi}=0} = 0 \quad \left.\frac{d^4V_{\text{eff}}}{d\hat{\phi}^4}\right|_{\hat{\phi}=M} = \lambda \equiv \lambda(M) \quad \Rightarrow \quad C_1 = \frac{2\lambda+9e^2}{48\pi^2}\Lambda^2$$

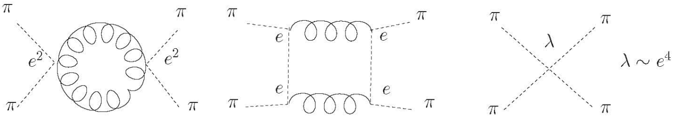
$$C_2 = \frac{11}{8\pi^2}(\frac{5}{18\pi^2}\lambda^2+3e^4) + \frac{3\lambda^2}{32\pi^2}\ln\frac{\frac{\lambda}{2}M^2}{\Lambda^2} + \frac{\lambda^2}{96\pi^2}\ln\frac{\frac{\lambda}{6}M^2}{\Lambda^2} + \frac{9e^4}{8\pi^2}\ln\frac{e^2M^2}{\Lambda^2}$$

$$V_{\text{eff}}(\hat{\phi}) = \frac{\lambda}{4!}\hat{\phi}^4 + (\frac{5\lambda^2}{1152\pi^2} + \frac{3e^4}{64\pi^2})\hat{\phi}^4(\ln\frac{\hat{\phi}^2}{M^2} - \frac{25}{6})$$

标量QED

$\phi = \begin{pmatrix} \sigma \\ \pi \end{pmatrix}$ 两个无量纲参数

$\pi\pi$散射告诉我们:



$$V_{\rm eff}(\hat\phi) = \frac{\lambda}{4!}\hat\phi^4 + \left(\frac{5\lambda^2}{1152\pi^2} + \frac{3e^4}{64\pi^2}\right)\hat\phi^4\left(\ln\frac{\hat\phi^2}{M^2} - \frac{25}{6}\right) \cong \frac{\lambda}{4!}\hat\phi^4 + \frac{3e^4}{64\pi^2}\hat\phi^4\left(\ln\frac{\hat\phi^2}{M^2} - \frac{25}{6}\right)$$

$$0 = \frac{dV_{\rm eff}}{d\hat\phi}\bigg|_{\hat\phi=\langle\phi\rangle} = \langle\phi\rangle^3\left[\frac{\lambda}{6} - \frac{11e^4}{16\pi^2} + 3\ln\frac{\langle\phi\rangle^2}{M^2}\right] \Rightarrow \lambda \overset{\langle\phi\rangle=M\neq 0}{=\!=\!=\!=\!=} \frac{33}{8\pi^2}e^4 \qquad V_{\rm eff}(\hat\phi) = \frac{3e^4}{64\pi^2}\hat\phi^4\left(\ln\frac{\hat\phi^2}{\langle\phi\rangle^2} - \frac{1}{2}\right)$$

两个解: $V_{\rm eff}[\,\langle\phi\rangle = 0\,] = 0 \qquad V_{\rm eff}[\,\langle\phi\rangle = M \neq 0\,] = -\frac{3e^4}{128\pi^2}\langle\phi\rangle^4$

$$\hat\phi \neq 0 \overset{\text{宇称不破坏}}{=\!=\!=\!=\!=\!\Rightarrow} \langle\sigma\rangle = \langle\phi\rangle \neq 0,\ \langle\pi\rangle = 0 \quad \Rightarrow \quad M_A^2 = e^2\langle\sigma\rangle^2$$ 电磁规范对称性自发破缺,光子获得质量

$$\sigma = \langle\sigma\rangle + \tilde\sigma \qquad m_{\tilde\sigma}^2 = \frac{d^2V_{\rm eff}}{d\hat\phi^2}\bigg|_{\hat\phi=\langle\phi\rangle} = \frac{3e^4}{8\pi^2}\langle\sigma\rangle^2 = \frac{3}{2\pi}\left(\frac{e^2}{4\pi}\right)M_A^2$$

现在的自发破缺解不限制一定要取大的耦合常数!

进一步推广到 $SU(2) \times U(1)$理论

期望重现标量QED的结果:

- 规范对称性被自发破缺
- 给出耦合常数之间的关联
- 给出由自发破缺而获得的标量粒子质量和规范粒子质量之间的关联

经过复杂计算得到:

$$m_H^2 = \frac{3}{32\pi^2}[2g^2 M_W^2 + (g^2 + g'^2) M_Z^2] \xrightarrow{\tan\theta_W = \frac{g'}{g},\ \ e = g\sin\theta_W} 9.4 \sim 10.4\ \text{GeV}$$

实际 LEPII: $m_H > 114$ GeV

此质量应是标准模型中的Higgs质量最小值！ $\mu^2 = 0$仍有此结果

虽然对标准模型并不成功,但理论想法会有很多应用！

- 大统一 **Gell-mann,Slansky：**考虑一圈修正后,破缺倾向于保持最大子对称性
- 宇宙学
- 确定精细结构常数, ...

D.J.Gross and A.Nevue, Phys. Rev. D10,3235(1974)

寻找与QCD类似又严格可解的模型：$1+1$ 维时空

- 可重整
- 色自由度直接关系相互作用
- 渐进自由
- 手征对称性自发破缺

$$\mathcal{L} = \bar{\psi} i \partial\!\!\!/ \psi + \frac{1}{2} g^2 (\bar{\psi}\psi)^2$$ 好像势无下界？

$$\psi = \begin{pmatrix} \psi_1 \\ \vdots \\ \psi_N \end{pmatrix}$$ 2N个分量

$$\bar{\psi}\psi \equiv \sum_a \bar{\psi}_a \psi_a$$

它对应积掉胶子场的QCD理论！ $\dim(\psi) = \frac{1}{2} \quad \dim(g) = 0$

拉氏量具有的对称性$U(N)$对应QCD的色对称性联系动力学,不应发生破缺!

Coleman定理：$D \leq 2$ 维时空中不存在Goldstone玻色子！

存在分立对称性 对应QCD的手征对称性：$\psi \rightarrow \gamma_5 \psi = e^{i\frac{\pi}{2}\gamma_5}\psi$

$$\bar{\psi}\psi \rightarrow (\gamma_5\psi)^\dagger \gamma^0 \gamma_5 \psi = -\bar{\psi}\psi \qquad \bar{\psi}\gamma^\mu\psi \rightarrow (\gamma_5\psi)^\dagger \gamma^0 \gamma^\mu \gamma_5 \psi = \bar{\psi}\gamma^\mu\psi$$

存在分立对称性 对应QCD的手征对称性: $\psi \to \gamma_5\psi = e^{i\frac{\pi}{2}\gamma_5}\psi$

$$\mathcal{L} = \overline{\psi}i\partial\!\!\!/\psi + \frac{1}{2}g^2(\overline{\psi}\psi)^2 \qquad \psi = \begin{pmatrix}\psi_1 \\ \vdots \\ \psi_N\end{pmatrix}$$ 2N个分量 $$\overline{\psi}\psi \equiv \sum_a \overline{\psi}_a\psi_a$$

$$\overline{\psi}\psi \to (\gamma_5\psi)^\dagger\gamma^0\gamma_5\psi = -\overline{\psi}\psi \qquad \overline{\psi}\gamma^\mu\psi \to (\gamma_5\psi)^\dagger\gamma^0\gamma^\mu\gamma_5\psi = \overline{\psi}\gamma^\mu\psi$$

$$\gamma^0 = \begin{pmatrix}0 & 1 \\ 1 & 0\end{pmatrix} \qquad \gamma^1 = \begin{pmatrix}0 & -1 \\ 1 & 0\end{pmatrix} \qquad \gamma_5 = \gamma^0\gamma^1 = \begin{pmatrix}1 & 0 \\ 0 & -1\end{pmatrix}$$

$$P_L \equiv \frac{1-\gamma_5}{2} = \begin{pmatrix}1 & 0 \\ 0 & 1\end{pmatrix} \qquad P_R \equiv \frac{1+\gamma_5}{2} = \begin{pmatrix}1 & 0 \\ 0 & 0\end{pmatrix}$$

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \qquad \{\gamma_5, \gamma_\mu\} = 0 \qquad \mu = 0, 1$$

需要研究描述$\overline{\psi}(x)\psi(x)$的有效势理论

描述$\overline{\psi}(x)\psi(y)$的有效势理论: Cornwell,Jackiw,Tomboulis,Phys.Rev.D10,2428(1974)

它导致积分方程!

存在分立对称性 对应QCD的手征对称性: $\psi \to \gamma_5\psi = e^{i\frac{\pi}{2}\gamma_5}\psi$

$$\mathcal{L} = \overline{\psi}i\partial\!\!\!/\psi + \frac{1}{2}g^2(\overline{\psi}\psi)^2 \qquad \psi = \begin{pmatrix}\psi_1\\ \vdots \\ \psi_N\end{pmatrix} \quad \text{2N个分量} \qquad \overline{\psi}\psi \equiv \sum_a \overline{\psi}_a\psi_a$$

$$\overline{\psi}\psi \to (\gamma_5\psi)^\dagger\gamma^0\gamma_5\psi = -\overline{\psi}\psi \qquad \overline{\psi}\gamma^\mu\psi \to (\gamma_5\psi)^\dagger\gamma^0\gamma^\mu\gamma_5\psi = \overline{\psi}\gamma^\mu\psi$$

建立生成泛函: $Z[K] = e^{W[K]} = \int \mathcal{D}\psi\mathcal{D}\overline{\psi}\; e^{i\int d^2x\,[\mathcal{L}+Kg\overline{\psi}\psi]}$

$$\int \mathcal{D}\sigma\; e^{-\frac{i}{2}\int d^2x\,(\sigma-g\overline{\psi}\psi)^2} = \int \mathcal{D}(\sigma-g\overline{\psi}\psi)\; e^{-\frac{i}{2}\int d^2x\,(\sigma-g\overline{\psi}\psi)^2} = \int \mathcal{D}\sigma\; e^{-\frac{i}{2}\int d^2x\,\sigma^2} = \text{常数}$$

$$Z[K] = \int \mathcal{D}\psi\mathcal{D}\overline{\psi}\mathcal{D}\sigma\; e^{i\int d^2x\,[\mathcal{L}-\frac{1}{2}(\sigma-g\overline{\psi}\psi)^2+gK\overline{\psi}\psi]} = \int \mathcal{D}\psi\mathcal{D}\overline{\psi}\mathcal{D}\sigma\; e^{i\int d^2x\,[\overline{\psi}(i\partial\!\!\!/+g\sigma+gK)\psi-\frac{1}{2}\sigma^2]}$$

$\mathcal{L}_\sigma = \overline{\psi}(i\partial\!\!\!/ + g\sigma)\psi - \frac{1}{2}\sigma^2$ σ: 辅助场 无动能项 $\xRightarrow{\text{经典解}}$ $\sigma = g\overline{\psi}\psi$ $\quad \mathcal{L}_\sigma\Big|_{\sigma=g\overline{\psi}\psi} = \mathcal{L}$

$$Z[K] = \int \mathcal{D}\psi\mathcal{D}\overline{\psi}\mathcal{D}\sigma\; e^{i\int d^2x\,[\overline{\psi}(i\partial\!\!\!/+g\sigma)\psi-\frac{1}{2}(\sigma-K)^2]} = e^{-i\frac{K^2}{2}}\int \mathcal{D}\psi\mathcal{D}\overline{\psi}\mathcal{D}\sigma\; e^{i\int d^2x\,[\overline{\psi}(i\partial\!\!\!/+g\sigma)\psi-\frac{1}{2}\sigma^2+K\sigma]}$$

$$\langle g\overline{\psi}(x)\psi(x)\rangle_{\mathcal{L}} = \frac{\delta W[K]}{\delta K(x)}\bigg|_{K=0} = \langle\sigma(x)\rangle_{\mathcal{L}_\sigma} \xRightarrow{\langle\sigma\rangle\neq 0} m_f = -g\langle\sigma\rangle$$

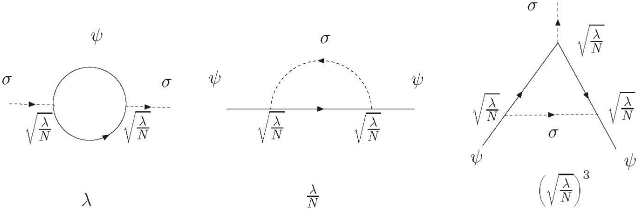
$1/N$展开的费曼图计算

$$\mathcal{L}=\underbrace{\overline{\psi}i\partial\!\!\!/\psi}_{N\text{ 项}}+\frac{1}{2}\underbrace{g^2(\overline{\psi}\psi)^2}_{N^2\text{ 项}}\ \xRightarrow{\text{为平衡动能与相互作用项}}\ g^2N\equiv\lambda\ \Leftarrow\ \text{与}N\text{无关}$$

存在分立对称性 对应**QCD**的手征对称性：$\psi\to\gamma_5\psi=e^{i\frac{\pi}{2}\gamma_5}\psi\quad \sigma\to-\sigma\quad \overline{\psi}\psi\to-\overline{\psi}\psi$

$$\mathcal{L}_\sigma=\overline{\psi}i\partial\!\!\!/\psi+\sqrt{\frac{\lambda}{N}}\sigma\overline{\psi}\psi-\frac{1}{2}\sigma^2 \qquad g\langle\overline{\psi}(x)\psi(x)\rangle_{\mathcal{L}}=\frac{1}{g}\langle\sigma(x)\rangle_{\mathcal{L}_\sigma}$$

一圈图：



所有两圈以上的一粒子不可约图都在 $N\to 0$ 时趋于零，在大 N 极限只须算一个一圈图！

$1/N$展开的路径积分计算

$$e^{iW[K]} = e^{-i\frac{K^2}{2}} \int \mathcal{D}\psi \mathcal{D}\overline{\psi} \mathcal{D}\sigma \; e^{i\int d^2x \, [\overline{\psi}(i\partial\!\!\!/+g\sigma)\psi - \frac{1}{2}\sigma^2 + K\sigma]} = e^{-i\frac{K^2}{2}} \int \mathcal{D}\sigma \; e^{\mathrm{Tr}\ln(i\partial\!\!\!/+g\sigma) + i\int d^2x \, [-\frac{1}{2}\sigma^2 + K\sigma]}$$

$$e^{iW[K]} \overset{\sigma=\sqrt{N}\bar{\sigma},\; K=\sqrt{N}\bar{K}}{=\!=\!=\!=\!=\!=\!=\!=\!=} e^{-iN\frac{\bar{K}^2}{2}} \int \mathcal{D}\bar{\sigma} \; e^{N\mathrm{Tr}'\,\ln(i\partial\!\!\!/+\sqrt{\lambda}\bar{\sigma}) + i\int d^2x \; N(-\frac{1}{2}\bar{\sigma}^2 + \bar{K}\bar{\sigma})} = e^{-iN\frac{\bar{K}^2}{2}} \int \mathcal{D}\bar{\sigma} \; e^{iNS[\bar{\sigma},\bar{K}]}$$

$$S[\bar{\sigma},\bar{K}] \equiv -i\mathrm{Tr}'\,\ln(i\partial\!\!\!/ + \sqrt{\lambda}\bar{\sigma}) + \int d^2x \,(-\frac{1}{2}\bar{\sigma}^2 + \bar{K}\bar{\sigma})$$

应用Jackiw的路径积分算法： $\frac{\partial W[K]}{\partial\langle\bar{\sigma}\rangle} = 0$

$$W[K] = \underbrace{-N\frac{\bar{K}^2}{2} + NS[\langle\bar{\sigma}\rangle,\bar{K}]}_{O(N)} + \frac{i}{2}\underbrace{\mathrm{Tr}\ln\frac{\delta^2 S[\langle\bar{\sigma}\rangle,\bar{K}]}{\delta\langle\bar{\sigma}(x)\rangle\delta\langle\bar{\sigma}(y)\rangle}}_{O(\frac{1}{N})} + \overline{\Gamma}_2 \qquad \overline{\Gamma}_2 = \sum_{n=2}^{\infty}\underbrace{\Gamma_n}_{O(\frac{1}{N^n})}$$

Γ_n = 以$[\frac{\delta^2 S[\langle\bar{\sigma}\rangle,\bar{K}]}{\delta\langle\bar{\sigma}(x)\rangle\delta\langle\bar{\sigma}(y)\rangle}]^{-1}$ 为传播子, $I_{\rm int}[\langle\bar{\sigma}\rangle,\tilde{\sigma}]$为相互作用顶角的所有一粒子不可约n圈图

$I_{\rm int}[\langle\bar{\sigma}\rangle,\tilde{\sigma}]$ 是 $S[\langle\bar{\sigma}\rangle+\tilde{\sigma},\bar{K}]$ 中按 $\tilde{\sigma}$ 幂次展开三次和三次以上项之和

渐进自由

$$\beta(g)=\mu\frac{\partial g}{\partial\mu}\Big|_{g_0,\Lambda}\qquad \gamma_\sigma(g)=\frac{1}{2}\mu\frac{\partial}{\partial\mu}\ln Z_\sigma\Big|_{g_0,\Lambda}=\frac{1}{\sqrt{Z_\sigma}}\mu\frac{\partial}{\partial\mu}\sqrt{Z_\sigma}\Big|_{g_0,\Lambda}\qquad \sigma_0=\sqrt{Z_\sigma}\sigma$$

$$g_0\bar\psi_0\psi_0\sigma_0\overset{\text{费米子无修正}}{=\!=\!=\!=\!=\!=\!=\!=}g_0\bar\psi\psi\sigma_0=g_0\sqrt{Z_\sigma}\,\bar\psi\psi\sigma\ \Rightarrow\ g=g_0\sqrt{Z_\sigma}$$

$$\beta(g)=\mu\frac{\partial g}{\partial\mu}\Big|_{g_0,\Lambda}=g_0\mu\frac{\partial}{\partial\mu}\sqrt{Z_\sigma}\Big|_{g_0,\Lambda}=\frac{g}{\sqrt{Z_\sigma}}\mu\frac{\partial}{\partial\mu}\sqrt{Z_\sigma}\Big|_{g_0,\Lambda}=g\gamma_\sigma(g)$$

计算一圈σ场自能: $\Pi(p)=-g^2N\int\frac{d^2k}{(2\pi)^2}\frac{\mathrm{tr}[\not k(\not k-\not p)]}{k^2(k-p)^2}=-\lambda\int_0^1 d\alpha\int\frac{d^2k}{(2\pi)^2}\frac{\mathrm{tr}[\not k(\not k-\not p)]}{[(k-p)^2\alpha+k^2(1-\alpha)]^2}$

$$\overset{q\equiv k-\alpha p}{=\!=\!=\!=\!=}-\lambda\int_0^1 d\alpha\int\frac{d^2q}{(2\pi)^2}\frac{\mathrm{tr}\{(\not q+\alpha\not p)[\not q+(\alpha-1)\not p]\}}{[q^2+\alpha(1-\alpha)p^2]^2}=\lambda\int_0^1 d\alpha\int\frac{i\pi dq^2}{(2\pi)^2}\frac{2[q^2+\alpha(1-\alpha)p^2]-2(2\alpha-1)q\cdot p}{[q^2-\alpha(1-\alpha)p^2]^2}$$

$$=\frac{i\lambda}{2\pi}\int_0^1 d\alpha\Big[\ln\frac{-\Lambda^2}{\alpha(1-\alpha)p^2}-2\Big]=\frac{i\lambda}{2\pi}\ln\frac{-\Lambda^2}{p^2}\qquad \Pi_r=\Pi(p)-\Pi_c\qquad D_0=-i\qquad D_r^{-1}=D_0^{-1}-\Pi_r$$

归一化条件: $D_r(p^2=-\mu^2)=-i\ \Rightarrow\ \Pi_c=\frac{\lambda}{2\pi}\ln\frac{\mu^2}{\Lambda^2}\ \Rightarrow\ D_r^{-1}=i\Big[1+\frac{\lambda}{2\pi}\ln\frac{-p^2}{\mu^2}\Big]$

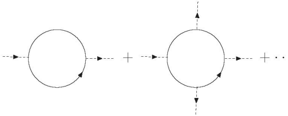
$$\Big[\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}-2\gamma_\sigma(g)\Big]D_r^{-1}(p)=0\Rightarrow\beta(g)=g\gamma_\sigma(g)=-\frac{g\lambda}{2\pi}\ \underline{\text{渐进自由}}\Rightarrow\bar\lambda(t)\overset{t=\frac{1}{2}\ln\frac{-p^2}{\mu^2}}{=\!=\!=\!=\!=\!=}\frac{\lambda}{1+\frac{\lambda t}{\pi}}$$

动力学对称性破缺

$$iS^{-1}(\hat{\sigma};x,y)=\frac{\delta^2 S}{\delta\bar{\psi}(x)\delta\psi(y)}=(i\partial\!\!\!/+g\hat{\sigma})\delta(x-y)\quad\Rightarrow\quad iS^{-1}(\hat{\sigma},p)=-p\!\!\!/+g\hat{\sigma}$$

$$V_{\rm eff}(\hat{\sigma})=\frac{1}{2}\sigma^2+i\int\frac{d^2p}{(2\pi)^2}\ln\frac{\det iS^{-1}(\hat{\sigma},p)}{\det iS^{-1}(0,p)}=\frac{1}{2}\sigma^2+iN\int\frac{d^2p}{(2\pi)^2}\ln\frac{\begin{vmatrix}g\hat{\sigma} & -p_0+p_1\\ -p_0-p_1 & g\hat{\sigma}\end{vmatrix}}{\begin{vmatrix}0 & -p_0+p_1\\ -p_0-p_1 & 0\end{vmatrix}}$$

$$=\frac{1}{2}\sigma^2+iN\int\frac{d^2p}{(2\pi)^2}\ln(1-\frac{g^2\hat{\sigma}^2}{p^2})$$



$$=\frac{1}{2}\hat{\sigma}^2+\frac{\lambda\hat{\sigma}^2}{4\pi}[\ln\frac{g^2\hat{\sigma}^2}{\Lambda^2}-1]$$

重整化： 在有效势中加入抵消项 $-\frac{1}{2}C\hat{\sigma}^2$

$$1=\frac{\partial^2 V_{\rm eff}}{\partial\hat{\sigma}^2}\Big|_{\hat{\sigma}=M}\quad\Rightarrow\quad C=\frac{\lambda}{2\pi}[\ln\frac{g^2M^2}{\Lambda^2}+2]$$

$$V_{\rm eff}(\hat{\sigma})=\frac{1}{2}\hat{\sigma}^2+\frac{\lambda\hat{\sigma}^2}{4\pi}[\ln\frac{\hat{\sigma}^2}{M^2}-3]$$

$$0=\frac{dV_{\rm eff}}{d\hat{\sigma}}\Big|_{\hat{\sigma}=\langle\sigma\rangle}=\langle\sigma\rangle[1+\frac{\lambda}{2\pi}(\ln\frac{\langle\sigma\rangle^2}{M^2}-2)]$$

$$\langle\sigma\rangle=\begin{cases}0\\ \pm Me^{1-\frac{\pi}{\lambda}}\end{cases}\quad\Rightarrow\lambda\overset{M=\langle\sigma\rangle}{=\!=\!=\!=\!=}\pi$$

$$V_{\rm eff}(0)=0\qquad V_{\rm eff}(\pm Me^{1-\frac{\pi}{\lambda}})=-\frac{\lambda}{4\pi}M^2e^{2-\frac{2\pi}{\lambda}}$$

考虑在Gross-Neveu模型中求$\langle[\overline{\psi}(x)\psi(x)]^2\rangle$,在原来拉氏量中插入积分

$$\int \mathcal{D}\sigma\, e^{-\frac{i}{2}\int d^2x\,[\sigma-\frac{g^2}{M}(\overline{\psi}\psi)^2]^2} \Rightarrow \mathcal{L}_\sigma = \overline{\psi}i\partial\!\!\!/\psi - \frac{1}{2}\sigma^2 + \frac{g^2}{M}(\overline{\psi}\psi)^2\sigma + \frac{1}{2}g^2(\overline{\psi}\psi)^2 - \frac{1}{2}\frac{g^4}{M^2}(\overline{\psi}\psi)^4$$

出现了形式上不可重整的相互作用项,但Gross-Neveu模型原本是可重整的

需要建立直接描述 $\langle[\overline{\psi}(x)\psi(x)]^2\rangle$ 的有效势理论

- Kuang, Li, Zhao, Commun. Theor. Phys. 3, 251(1984)
- He, Wang, Kuang, Z.Phys. C45,407(1990)
- He, Kuang, Z.Phys. C47,565(1990)

$$\begin{aligned}
Z[K] &= \int \mathcal{D}\psi\mathcal{D}\overline{\psi}\mathcal{D}\sigma\, e^{i\int d^2x\,[\overline{\psi}(i\partial\!\!\!/+g\sigma)\psi-\frac{1}{2}\sigma^2+K\sigma]} \\
&= \int \mathcal{D}\psi\mathcal{D}\overline{\psi}\mathcal{D}\sigma\, e^{i\int d^2x\,[\overline{\psi}i\partial\!\!\!/\psi-\frac{1}{2}(\sigma-g\overline{\psi}\psi-K)^2+\frac{1}{2}g^2(\overline{\psi}\psi)^2+\frac{1}{2}K^2]} \\
&= \int \mathcal{D}\psi\mathcal{D}\overline{\psi}\, e^{i\int d^2x\,[\overline{\psi}i\partial\!\!\!/\psi+\frac{1}{2}g^2(\overline{\psi}\psi)^2+Kg\overline{\psi}\psi+\frac{1}{2}K^2]}
\end{aligned}$$

经验: 构造局域复合算符的有效势需要 加入纯外源项 !

如何决定纯外源项？

$$W[I,\bar{I},K]=-i\ln Z[I,\bar{I},K] \qquad Z[I,\bar{I},K]=\int\mathcal{D}\psi\mathcal{D}\overline{\psi}\, e^{i\int d^2x\,[\overline{\psi}i\partial\!\!\!/\psi+\frac{1}{2}g^2(\overline{\psi}\psi)^2+\bar{I}\psi+\overline{\psi}I+Kg\overline{\psi}\psi+P(K)]}$$

$$\frac{\delta W}{\delta\bar{I}}=\hat{\psi} \qquad \frac{\delta W}{\delta I}=-\hat{\overline{\psi}} \qquad \frac{\delta W}{\delta K}=g\hat{\overline{\psi}}\hat{\psi}+\Sigma$$ ⇐连接部分 $$\hat{\psi}\Big|_{I=\bar{I}=0}=\hat{\overline{\psi}}\Big|_{I=\bar{I}=0}=0$$

$$\frac{\delta^2W}{\delta K^2}=-i\frac{\delta}{\delta K}Z^{-1}\frac{\delta Z}{\delta K}=-i\left(Z^{-1}\frac{\delta^2Z}{\delta K^2}-Z^{-1}\frac{\delta Z}{\delta K}Z^{-1}\frac{\delta Z}{\delta K}\right)=-iZ^{-1}\frac{\delta^2Z}{\delta K^2}\Big|_{\text{连接}} \quad\cdots\quad \frac{\delta^nW}{\delta K^n}=-iZ^{-1}\frac{\delta^nZ}{\delta K^n}\Big|_{\text{连接}}$$

自洽条件： $$\frac{\delta^nW_c}{\delta K^n}\Big|_{I=\bar{I}=K=0}=0 \qquad n\geq 2$$

经典方程： $$g\overline{\psi}_{\rm cl}[g\overline{\psi}_{\rm cl}\psi_{\rm cl}+K]=0 \quad\Rightarrow\quad g\overline{\psi}_{\rm cl}\psi_{\rm cl}=-K$$

$$W_c[0,0,K]=\int d^2x[\frac{1}{2}g^2(\overline{\psi}_{\rm cl}\psi_{\rm cl})^2+Kg\overline{\psi}_{\rm cl}\psi_{\rm cl}+P(K)]=\int d^2x[-\frac{1}{2}K^2+P(K)] \;\Rightarrow\; P(K)=\frac{1}{2}K^2$$

纯外源项由自洽条件决定！

直接一圈图计算：

$$W[I,\bar{I},K] = -i\ln Z[I,\bar{I},K] \qquad Z[I,\bar{I},K] = \int \mathcal{D}\psi\mathcal{D}\bar{\psi}\; e^{i\int d^2x\,[\bar{\psi}i\partial\!\!\!/\psi+\frac{1}{2}g^2(\bar{\psi}\psi)^2+\bar{I}\psi+\bar{\psi}I+Kg\bar{\psi}\psi+P(K)]}$$

$$W = W_c + W_1 \qquad W_1 = -\frac{\lambda}{4\pi}\int d^2x\, F^2(\ln\frac{g^2F^2}{\Lambda^2}-1) \qquad F \equiv g\bar{\psi}_{\rm cl}\psi_{\rm cl}+K$$

需加入抵消项：

$$-\frac{1}{2}C_1(g\bar{\psi}_{\rm cl}\psi_{\rm cl})^2 - C_2 g\bar{\psi}_{\rm cl}\psi_{\rm cl}K - \frac{1}{2}C_3K^2 \qquad \Gamma^{\text{部分}}[\psi_{\rm cl},\bar{\psi}_{\rm cl},K] = W[I,\bar{I},K] - \int d^2x\,[\bar{I}\psi_{\rm cl}+\bar{\psi}_{\rm cl}I]$$

归一化条件：

$$\left.\frac{\delta^4\Gamma^{\text{部分}}}{\delta\bar{\psi}_{\rm cl}\delta\psi_{\rm cl}\delta\bar{\psi}_{\rm cl}\delta\psi_{\rm cl}}\right|_{\text{减除点}} = g^2 \qquad \left.\frac{\delta^3\Gamma^{\text{部分}}}{\delta\bar{\psi}_{\rm cl}\delta\psi_{\rm cl}\delta K}\right|_{\text{减除点}} = g \qquad \left.\frac{\delta^2\Gamma^{\text{部分}}}{\delta K^2}\right|_{\text{减除点}} = 1$$

$$C_1 = C_2 = C_3 = -\frac{\lambda}{2\pi}(\ln\frac{g^2M^2}{\Lambda^2}+2) \qquad M \equiv F\Big|_{\text{减除点}}$$

$$W_1 = \int d^2x\,[\frac{1}{2}(g\bar{\psi}_{\rm cl}\psi_{\rm cl})^2 + \bar{\psi}_{\rm cl}I + \bar{I}\psi_{\rm cl} + g\bar{\psi}_{\rm cl}\psi_{\rm cl}K + \frac{1}{2}K^2 - \frac{\lambda}{4\pi}F^2(\ln\frac{F^2}{M^2}-3)]$$

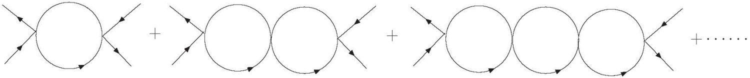
$$\Gamma[\hat{\psi},\hat{\bar{\psi}},\Sigma] = W[I,\bar{I},K] - \int d^2x[\bar{I}\hat{\psi}+\hat{\bar{\psi}}I+K\Sigma] = V_{\rm eff}[\hat{\psi},\hat{\bar{\psi}},\Sigma]\int d^2x$$

$$= \int d^2x\left[\frac{1}{2}(g\hat{\bar{\psi}}\hat{\psi})^2 - \frac{1}{2}\Sigma^2 - \frac{\lambda}{4\pi}(g\hat{\bar{\psi}}\hat{\psi}+\Sigma)^2[\ln\frac{(g\hat{\bar{\psi}}\hat{\psi}+\Sigma)^2}{M^2}-3]\right] \xRightarrow{\Sigma\leftrightarrow\sigma} V_{\rm eff}[0,0,\sigma]$$

回到上节结果！

目前的结果只准到费米子的一圈图 ！

一圈图的结果被证明在**大N极限**下到任意阶圈图都严格成立的！



建立了类似**Jackiw**给出的一般计算规则— 两粒子不可约图

在计算 $\langle\overline{\psi}\psi\rangle$ 的同时也计算 $(\langle\overline{\psi}\psi\rangle)^2$

引入外源项： $K_1\overline{\psi}\psi + K_2(\overline{\psi}\psi)^2 + P(K_1, K_2)$

首次系统计算了 四费米算符的真空凝聚 ！

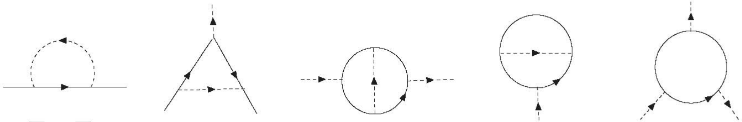
D维($2<D<4$)四费米理论

D维四费米理论当$D>2$时在微扰论中是不可重整的！

但发现$D=3$的四费米理论在$1/N$展开中是可重整的！

B.Rosenstein, B.Warr, and S.H.Park, Phys. Rev. Lett. 62, 1433(1989)

$O(\frac{1}{N})$



$\overline{\psi}\psi,\ \overline{\psi}i\partial\!\!\!/\psi$ $\overline{\psi}\psi\sigma$ σ^2 σ σ^3

理论具有对称性: $\overline{\psi}\psi \xRightarrow{\psi\to\gamma_5\psi} -\overline{\psi}\psi$, $\sigma \xRightarrow{\psi\to\gamma_5\psi} -\sigma$

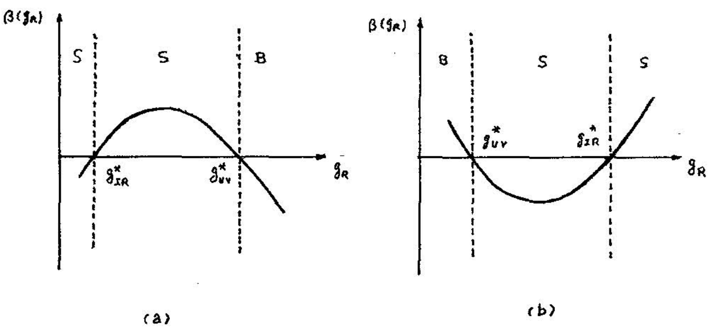
使得$O(\frac{1}{N})$六个图中只要计算方法不破坏对称性只有三个存在！ 动力学自发破缺不增加新发散!

剩下三个图在拉氏量中分别有对应的抵消项！此分析可以被推广到任意阶

探索非微扰理论中的可重整性更一般维数的四费米理论在$1/N$展开中可重整行为！

H.J.He, Y.P.Kuang, Q.Wang and Y.P.Yi, Phys. Rev. D45, 4610(1992)

探索非微扰理论中的可重整性更一般维数的四费米理论在$1/N$展开中可重整行为！



理论存在非零的紫外固定点 $\lambda^* = \frac{(4\pi)^{D/2}(D-2)}{8(D-1)\Gamma(2-\frac{D}{2})}[1+\frac{2^{D-3}(D-1)}{N}]$

$$\beta(\lambda) = (D-2)\lambda(1-\frac{\lambda}{\lambda^*})[1-\frac{1}{NDB(\frac{D}{2}-1,2-\frac{D}{2})B(\frac{D}{2},\frac{D}{2})}]$$

大的反常量纲： $\gamma_{\overline{\psi}\psi}(\lambda^*) = D-2+\frac{1}{NDB(\frac{D}{2}-1,2-\frac{D}{2})B(\frac{D}{2},\frac{D}{2})}$

$B(p,q) \equiv \int_0^1 dt\, t^{p-1}(1-t)^{q-1}$