粒子理论专题 手征对称性及其破缺

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QCD的手征对称性

线性σ模型

PGB质量

非线性σ模型

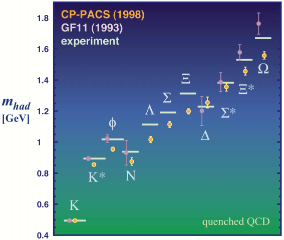
低能手征有效拉氏量

Wess-Zumino-Witten项

典型的强子质量尺度~1GeV, 但对0^-介子

$$m_{\pi^\pm} = 139\text{MeV} \qquad m_{\pi^0} = 135\text{MeV}$$

$$m_{K^\pm} = 494\text{MeV} \qquad m_{K^0,\overline{K}^0} = 498\text{MeV} \qquad m_\eta = 548\text{MeV}$$



为什么0^-赝标介子比其它强子轻的多？

将0^-介子解释成近似的Goldstone玻色子

- 严格连续对称性的自发破缺将产生零质量的Goldstone玻色子
- 近似的Goldstone玻色子可以允许有小的质量

$$\mathcal{L}_{\rm QCD} = -\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} + \overline{\psi}^{\alpha l}[i\partial\!\!\!/ + g(\frac{\lambda_a}{2})_{\alpha\beta}G\!\!\!/^a]\psi^{\beta l} - \overline{\psi}^{\alpha l}M_{ll'}\psi^{\alpha l'}$$

$$= -\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} + \overline{\psi}^{\alpha l}_L[i\partial\!\!\!/ + g(\frac{\lambda_a}{2})_{\alpha\beta}G\!\!\!/^a]\psi^{\beta l}_L + \overline{\psi}^{\alpha l}_R[i\partial\!\!\!/ + g(\frac{\lambda_a}{2})_{\alpha\beta}G\!\!\!/^a]\psi^{\beta l}_R - \overline{\psi}^{\alpha l}_L M_{ll'}\psi^{\alpha l'}_R - \overline{\psi}^{\alpha l}_R M_{ll'}\psi^{\alpha l'}_L$$

将轻夸克$N_f = 2,3$流质量项看成是小量，进行微扰

略去轻夸克流质量，$\mathcal{L}_{\rm QCD}$具有： $U(N_f)_L \otimes U(N_f)_R = SU(N_f)_L \otimes SU(N_f)_R \otimes U(1)_V \otimes U(1)_A$

QCD手征对称性的实现方式

将轻夸克$N_f=2,3$流质量项看成是小量，进行微扰

略去轻夸克流质量，$\mathcal{L}_{\rm QCD}$具有： $U(N_f)_L\otimes U(N_f)_R=SU(N_f)_L\otimes SU(N_f)_R\otimes U(1)_V\otimes U(1)_A$

$U(1)_V$ 重子数： $\mathcal{L}_{\rm QCD}$具有的对称性

$U(1)_A$ 轴$U(1)$手征对称性： $\mathcal{L}_0$略去轻夸克质量的QCD拉氏量具有的对称性

若$U(1)_A$不破缺: $\psi\to\psi'=i\gamma_5\psi=e^{i\gamma_5\frac{\pi}{2}}$ $\gamma_5\psi$与ψ宇称相反 物理谱中应该出现 宇称简并态 ！

若$U(1)_A$自发破缺: 物理谱中应该出现 第九个轻的赝标粒子 ！ $m_\eta=548{\rm MeV}$ $m_{\eta'}=958{\rm MeV}$ $U(1)_A$问题

$SU(N_f)_L\otimes SU(N_f)_R$ 手征对称性： $\mathcal{L}_0$具有的对称性

N_f个轻夸克略去流质量： $SU(N_f)_L\otimes SU(N_f)_R\overset{\langle\bar\psi\psi\rangle\neq0}{\Longrightarrow}SU(N_f)_V\overset{\text{连续对称性破缺}}{\Longrightarrow}N_f^2-1$个零质量赝标粒子

本节后面分别从不同方面论述轴部分的对称性要破缺,而矢量部分的对称性不破缺！

N_f个轻夸克考虑进流质量： N_f^2-1个原本零质量赝标粒子获得轻的质量 $m_{0^-}\sim m_{\text{流质量}}$

$SU(N_f)_L\otimes SU(N_f)_R\neq SU(N_f)_V\otimes SU(N_f)_A$ $SU(N_f)_A$不存在！ $SU(N_f)_L\otimes SU(N_f)_R/SU(N_f)_V$不形成群，是陪集！

若$SU(N_f)_L\otimes SU(N_f)_R$不破缺或恢复$\Rightarrow N_f^2-1$个赝标粒子将具有强作用的典型质量！

对称性和守恒流

$\phi_A(x)$ $\mathcal{L}[\phi_A(x), \partial_\mu\phi_A(x)]$ 拉格朗日方程： $\partial_\mu \frac{\delta\mathcal{L}}{\delta(\partial_\mu\phi_A)} - \frac{\delta\mathcal{L}}{\delta\phi_A} = 0$

考虑整体连续变换： $\phi_A(x) \to \phi'_A(x) = \phi_A(x) + \bar\delta\phi_A(x) = \phi_A(x) + iF_{ABC}\alpha_B\phi_C(x)$

$$\bar\delta\partial_\mu\phi_A(x) = \partial_\mu[\bar\delta\phi_A(x)] = iF_{ABC}\alpha_B\partial_\mu[\phi_C(x)]$$

$$\bar\delta\mathcal{L} = \frac{\delta\mathcal{L}}{\delta\phi_A}\bar\delta\phi_A + \frac{\delta\mathcal{L}}{\delta\partial_\mu\phi_A}\bar\delta\partial_\mu\phi_A = \frac{\delta\mathcal{L}}{\delta\phi_A}\bar\delta\phi_A + \partial_\mu\left[\frac{\delta\mathcal{L}}{\delta\partial_\mu\phi_A}\bar\delta\phi_A\right] - \partial_\mu[\frac{\delta\mathcal{L}}{\delta\partial_\mu\phi_A}]\bar\delta\phi_A$$

$$\xRightarrow{\text{运动方程}} \partial_\mu\left[\frac{\delta\mathcal{L}}{\delta\partial_\mu\phi_A}\bar\delta\phi_A\right] = \alpha_B\partial_\mu\left[iF_{ABC}\phi_C(x)\frac{\delta\mathcal{L}}{\delta\partial_\mu\phi_A}\right] \xRightarrow{\text{拉氏量具有整体连续对称性}} 0$$

$$j^\mu_B(x) \equiv -iF_{ABC}\frac{\delta\mathcal{L}}{\delta\partial_\mu\phi_A}\phi_C(x) \qquad \partial_\mu j^\mu_B(x) = 0$$

若将 $\alpha \Rightarrow \alpha(x)$： $\bar\delta\mathcal{L} = -[\partial_\mu\alpha_B(x)]j^\mu_B(x) - \alpha_B(x)\partial_\mu j^\mu_B(x) = \mathcal{L}' - \mathcal{L}$

$$j^\mu_B(x) = -\frac{\delta\mathcal{L}'}{\delta(\partial_\mu\alpha_B(x))} \qquad \partial_\mu j^\mu_B(x) = -\frac{\delta\mathcal{L}'}{\delta\alpha_B(x)}$$

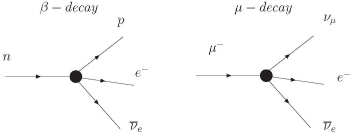
$$Q_B = \int d^3x\, j^0_B(x) \qquad j^\mu_B \xRightarrow{\bar\psi i\partial\!\!\!/\psi,\ \psi\to e^{i\alpha_B t_B}\psi} \bar\psi\gamma^\mu t_B\psi \qquad [Q_A, Q_B] \xlongequal{\text{硬算}} iC_{ABC}Q_C$$

Q_A是群生成元在场算符空间的表示！ $\Leftarrow$ 对比群的对易关系： $[t_A, t_B] = iC_{ABC}t_C$

矢量流守恒与轴矢流部分守恒

β衰变：$\mathcal{L}_\beta = -\frac{G_F}{\sqrt{2}}\overline{\psi}_p\gamma^\mu(1-\gamma_5)\psi_n\overline{\psi}_e\gamma_\mu(1-\gamma_5)\psi_{\nu_e}$

μ衰变：$\mathcal{L}_\mu = -\frac{G_F}{\sqrt{2}}\overline{\psi}_\mu\gamma^\mu(1-\gamma_5)\psi_{\nu_\mu}\overline{\psi}_e\gamma_\mu(1-\gamma_5)\psi_{\nu_e}$



考虑到β衰变可能的强作用修正：$\mathcal{L}_\beta^{\text{eff}} = -\frac{G_F}{\sqrt{2}}\overline{\psi}_p\gamma^\mu[g_V(q^2) - g_A(q^2)\gamma_5]\psi_n\overline{\psi}_e\gamma_\mu(1-\gamma_5)\psi_\nu$

$g_V(0) = 1 \qquad g_A(0) = 1.22 \pm 0.02$

为什么矢量流顶角不受强作用影响？ 而轴矢流顶角受部分强作用影响？

类似的问题：为什么实验测的质子电荷和正电子电荷完全一样？强作用修正哪去了？

强作用从初态绝热地引入,到终态再绝热地撤除$\Rightarrow$只要电荷守恒,质子电荷就不受强作用修正!

电流守恒: $\partial_\mu j^\mu = 0 \quad \Rightarrow \quad \frac{dQ}{dt} = 0 \qquad Q = \int d^3x\, j^0(x) \quad \Rightarrow$ 质子电荷就不受强作用修正

弱作用的矢量流守恒(CVC) 假设 保证矢量流顶角不受强作用修正 $g_V(0) = 1$

轴矢流部分守恒

Goldberger-Treiman关系

未减除虚部积分在无穷远不发散的色散关系 $\Rightarrow g_A(0) = \dfrac{f_\pi g_{\bar{N}N\pi}}{m_N} \xRightarrow{f_\pi=93\text{MeV}\ \ g_{\bar{N}N\pi}=13} g_A(0) = 1.2$

如何使色散积分在无穷远不发散?

Nambu; Chou; Bernstein; Fubini; Gell-Mann; Thiring $\Rightarrow \partial_\mu j^\mu_{5a}(x) = m_\pi^2 f_\pi \pi_a(x)$

轴矢流部分守恒 **PCAC**: 高动量时轴矢流守恒;低动量时轴矢流不守恒

现代理解: $\langle 0|j^\mu_{5a}(x)|\pi_b\rangle = i\delta_{ab} q^\mu f_\pi(q^2) e^{-iq\cdot x}$ f_π的定义 $\Rightarrow \langle 0|\partial_\mu j^\mu_{5a}|\pi_b\rangle = q^2 f_\pi(q^2)\delta_{ab} e^{-iq\cdot x}$

$\langle 0|\partial_\mu j^\mu_{5a}|\pi_b\rangle \xlongequal{\text{质壳条件 } q^2=m_\pi^2} m_\pi^2 f_\pi \delta_{ab} e^{-iq\cdot x} \Rightarrow \langle 0|\partial_\mu j^\mu_{5a}|\pi_b\rangle = \langle 0|m_\pi^2 f_\pi \pi_a|\pi_b\rangle$

从π介子在壳推广到不在壳: $\xRightarrow{\text{光滑性假设}} \partial_\mu j^\mu_{5a}(x) = m_\pi^2 f_\pi \pi_a(x) \xRightarrow{\text{或内插}\pi\text{场定义}} \pi_a(x) \equiv \dfrac{\partial_\mu j^\mu_{5a}(x)}{m_\pi^2 f_\pi}$

PCAC只在新粒子K介子出现的域以下适用! 以它为假设将色散积分推广到任意大动量区不合适

流代数

$$Q_V^a(t)=Q_R^a(t)+Q_L^a(t)=\int d^3x\, q^\dagger(\vec{x},t)\frac{\lambda^a}{2}q(\vec{x},t) \qquad Q_A^a(t)=Q_R^a(t)-Q_L^a(t)=\int d^3x\, q^\dagger(\vec{x},t)\frac{\lambda^a}{2}\gamma_5 q(\vec{x},t)$$

$$\{q_A(\vec{x},t),q_B^\dagger(\vec{y},t)\}=\delta(\vec{x}-\vec{y})\delta_{AB} \qquad \{q_A(\vec{x},t),q_B(\vec{y},t)\}=0 \qquad \{q_A^\dagger(\vec{x},t),q_B^\dagger(\vec{y},t)\}=0$$

$$[q^\dagger(\vec{x},t)\Gamma_1 q(\vec{x},t),q^\dagger(\vec{y},t)\Gamma_2 q(\vec{y},t)]=\delta(\vec{x}-\vec{y})[q^\dagger(\vec{x},t)\Gamma_1\Gamma_2 q(\vec{y},t)-q^\dagger(\vec{y},t)\Gamma_2\Gamma_1 q(\vec{x},t)]$$

$$[Q_V^a(t),Q_V^b(t)]=if_{abc}Q_V^c(t) \qquad [Q_A^a(t),Q_A^b(t)]=if_{abc}Q_V^c(t) \qquad [Q_V^a(t),Q_A^b(t)]=if_{abc}Q_A^c(t)$$

$$PQ_L^a(t)P^{-1}=Q_R^a(t) \qquad PQ_R^a(t)P^{-1}=Q_L^a(t) \qquad PQ_V^a(t)P^{-1}=Q_V^a(t) \qquad PQ_A^a(t)P^{-1}=-Q_A^a(t)$$

J^-介子多重态: $[Q_V^a(t),H_{\mathrm{QCD},0}]=[Q_A^a(t),H_{\mathrm{QCD},0}]=[P,H_{\mathrm{QCD},0}]=[Q_V^a(t),P]=0$

$H_{\mathrm{QCD}}|i,-\rangle=E_i|i,-\rangle \quad P|i,-\rangle=-|i,-\rangle \quad Q_V^a(t)|i,-\rangle=t_{ji}^a|j,-\rangle \quad H_{\mathrm{QCD},0}|i,-\rangle\approx E_i|i,-\rangle \quad t_{ji}^a$满足:

$[t^a,t^b]=if_{abc}t^c \quad H_{\mathrm{QCD},0}Q_V^a(t)|i,-\rangle=Q_V^a(t)H_{\mathrm{QCD},0}|i,-\rangle=E_iQ_V^a(t)|i,-\rangle \Rightarrow E_i=E_- \quad |i,-\rangle\equiv a_i^\dagger|0\rangle$

$$t_{ji}^a a_j^\dagger|0\rangle=t_{ji}^a|j,-\rangle=Q_V^a(t)a_i^\dagger|0\rangle=[Q_V^a(t),a_i^\dagger]|0\rangle+a_i^\dagger Q_V^a(t)|0\rangle \xRightarrow{Q_V^a(t)|0\rangle=0} [Q_V^a(t),a_i^\dagger]=t_{ji}^a a_j^\dagger$$

- J^-介子多重态近似具有同样的质量$E_i=E_-$! J^+也类似
- 它们构成$SU(3)_V$的表示! $[Q_V^a(t),a_i^\dagger]=t_{ji}^a a_j^\dagger$, $[t^a,t^b]=if_{abc}t^c$

$$Q_V^a(t)=Q_R^a(t)+Q_L^a(t)=\int d^3x\, q^\dagger(\vec{x},t)\frac{\lambda^a}{2}q(\vec{x},t) \qquad Q_A^a(t)=Q_R^a(t)-Q_L^a(t)=\int d^3x\, q^\dagger(\vec{x},t)\frac{\lambda^a}{2}\gamma_5 q(\vec{x},t)$$

$$[Q_V^a(t),Q_V^b(t)]=if_{abc}Q_V^c(t) \qquad [Q_A^a(t),Q_A^b(t)]=if_{abc}Q_V^c(t) \qquad [Q_V^a(t),Q_A^b(t)]=if_{abc}Q_A^c(t)$$

$$PQ_L^a(t)P^{-1}=Q_R^a(t) \qquad PQ_R^a(t)P^{-1}=Q_L^a(t) \qquad PQ_V^a(t)P^{-1}=Q_V^a(t) \qquad PQ_A^a(t)P^{-1}=-Q_A^a(t)$$

J^- 介子多重态: $Q_V^a(t)|i,-\rangle=t_{ji}^a|j,-\rangle \quad |i,-\rangle\equiv a_i^\dagger|0\rangle \quad [Q_V^a(t),a_i^\dagger]=t_{ji}^a a_j^\dagger$

$$Q_A^a(t)|i,-\rangle\equiv\tilde{t}_{si}^a|s,+\rangle \qquad P|s,+\rangle=|s,+\rangle \qquad H_{\mathrm{QCD},0}Q_A^a(t)|i,-\rangle=Q_A^a(t)H_{\mathrm{QCD},0}|i,-\rangle=E_-Q_A^a(t)|i,-\rangle$$

$H_{\mathrm{QCD},0}\tilde{t}_{si}^a|s,+\rangle=E_+\tilde{t}_{si}^a|s,+\rangle \quad \Rightarrow \quad E_+=E_-$ 宇称简并　　解释: 存在单粒子假设!

$$Q_A^a(t)|i,-\rangle=Q_A^a a_i^\dagger|0\rangle=\underbrace{[Q_A^a(t),a_i^\dagger]|0\rangle}_{a_i^\dagger\text{ 对称变换导致的单粒子态项}}+\underbrace{a_i^\dagger Q_A^a(t)|0\rangle}_{\text{多粒子态项}}$$

$Q_A^a(t)|0\rangle\neq0$ 即可解释 $Q_V^a(t)|0\rangle=0 \ \rightarrow\ \mathrm{SU}(3)_L\times\mathrm{SU}(3)_R\rightarrow\mathrm{SU}(3)_V$ 理论证明? $Q_A^a(t)|0\rangle=\tilde{b}_a^\dagger|0\rangle$

$$H_{\mathrm{QCD},0}\tilde{b}_a^\dagger|0\rangle=H_{\mathrm{QCD},0}Q_A^a(t)|0\rangle=Q_A^a(t)H_{\mathrm{QCD},0}|0\rangle=0$$

$$P\tilde{b}_a^\dagger|0\rangle=PQ_A^a(t)|0\rangle=PQ_A^a(t)P^{-1}P|0\rangle=-Q_A^a(t)|0\rangle \qquad \tilde{b}_a^\dagger|0\rangle\equiv|\phi^a(0)\rangle$$

$$Q_V^a(t)|\phi^b(0)\rangle=[Q_V^a(t),\tilde{b}_b^\dagger]|0\rangle=Q_V^a(t)Q_A^b(t)|0\rangle=[Q_V^a(t),Q_A^b(t)]|0\rangle=if_{abc}Q_A^c(t)|0\rangle=if_{abc}\tilde{b}_c^\dagger|0\rangle$$

$$[Q_V^a(t),\tilde{b}_b^\dagger]=if_{abc}\tilde{b}_c^\dagger \qquad Q_V^a(t)|\phi^b(0)\rangle=if_{abc}|\phi^c(0)\rangle \quad\Rightarrow\quad Q_V^a(t)|\phi^b(p)\rangle=if_{abc}|\phi^c(p)\rangle$$

赝标介子八重态: $T_3 = Q_V^3$ $Y = \frac{2}{\sqrt{3}} Q_V^8$ $Q_V^a(t)|\phi^b(p)\rangle = i f_{abc}|\phi^c(p)\rangle$

$$\mathbf{K}^0(-\frac{1}{2}, 1) \qquad \mathbf{K}^+(\frac{1}{2}, 1)$$

$$\phi^a(T_3, Y): \quad \pi^-(-1, 0) \quad \pi^0(0, 0) \quad \eta(0, 0) \quad \pi^+(1, 0)$$

$$\mathbf{K}^-(-\frac{1}{2}, -1) \qquad \bar{\mathbf{K}}^0(\frac{1}{2}, -1)$$

abc	123	147	156	246	257	345	367	458	678
f_{abc}	1	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{2}$

$$T_3(|\phi_1\rangle - i|\phi_2\rangle) = -(|\phi_1\rangle - i|\phi_2\rangle) \qquad Y(|\phi_1\rangle - i|\phi_2\rangle) = 0$$

$$T_3(|\phi_1\rangle + i|\phi_2\rangle) = |\phi_1\rangle + i|\phi_2\rangle \qquad Y(|\phi_1\rangle + i|\phi_2\rangle) = 0$$

$$T_3(|\phi_4\rangle - i|\phi_5\rangle) = -\frac{1}{2}(|\phi_4\rangle - i|\phi_5\rangle) \qquad Y(|\phi_4\rangle - i|\phi_5\rangle) = -(|\phi_4\rangle - i|\phi_5\rangle)$$

$$T_3(|\phi_4\rangle + i|\phi_5\rangle) = \frac{1}{2}(|\phi_4\rangle + i|\phi_5\rangle) \qquad Y(|\phi_4\rangle + i|\phi_5\rangle) = |\phi_4\rangle + i|\phi_5\rangle$$

$$T_3(|\phi_6\rangle - i|\phi_7\rangle) = \frac{1}{2}(|\phi_6\rangle - i|\phi_7\rangle) \qquad Y(|\phi_6\rangle - i|\phi_7\rangle) = -(|\phi_6\rangle - i|\phi_7\rangle)$$

$$T_3(|\phi_6\rangle + i|\phi_7\rangle) = -\frac{1}{2}(|\phi_6\rangle + i|\phi_7\rangle) \qquad Y(|\phi_6\rangle + i|\phi_7\rangle) = |\phi_6\rangle + i|\phi_7\rangle$$

$$T_3|\phi_3\rangle = Y|\phi_3\rangle = T_3|\phi_8\rangle = Y|\phi_8\rangle = 0$$

赝标介子八重态:

$$T_3 = Q_V^3 \qquad Y = \frac{2}{\sqrt{3}} Q_V^8 \qquad Q_V^a(t)|\phi^b(p)\rangle = if_{abc}|\phi^c(p)\rangle$$

$$\phi^a(T_3, Y): \quad \mathbf{K^0}(-\frac{1}{2}, 1) \quad \mathbf{K^+}(\frac{1}{2}, 1) \quad \pi^-(-1, 0) \quad \pi^0(0, 0) \quad \eta(0, 0) \quad \pi^+(1, 0) \quad \mathbf{K^-}(-\frac{1}{2}, -1) \quad \mathbf{\bar{K}^0}(\frac{1}{2}, -1)$$

$$\phi^T \equiv \sum_{a=1}^{8} \frac{\lambda^a}{\sqrt{2}} \phi^a \equiv \begin{pmatrix} \frac{\phi_3}{\sqrt{2}} + \frac{1}{\sqrt{6}}\phi_8 & \frac{\phi_1 - i\phi_2}{\sqrt{2}} & \frac{\phi_4 - i\phi_5}{\sqrt{2}} \\ \frac{\phi_1 + i\phi_2}{\sqrt{2}} & -\frac{\phi_3}{\sqrt{2}} + \frac{1}{\sqrt{6}}\phi_8 & \frac{\phi_6 - i\phi_7}{\sqrt{2}} \\ \frac{\phi_4 + i\phi_5}{\sqrt{2}} & \frac{\phi_6 + i\phi_7}{\sqrt{2}} & -\frac{\sqrt{2}}{\sqrt{3}}\phi_8 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{1}{\sqrt{6}}\eta & \pi^- & K^- \\ \pi^+ & -\frac{\pi^0}{\sqrt{2}} + \frac{1}{\sqrt{6}}\eta & \bar{K}^0 \\ K^+ & K^0 & -\frac{\sqrt{2}}{\sqrt{3}}\eta \end{pmatrix}$$

QCD的手征对称性:

- ♣ 手征极限的拉氏量具有对称性$SU(N_f)_L \otimes SU(N_f)_R \otimes U(1)_V \otimes U(1)_A$
- ♢ $U(1)_V$严格成立; $U(1)_A$被QCD反常破坏
- ♡ $SU(N_f)_L \otimes SU(N_f)_R$自发破缺到$SU(N_f)_V$
- ♠ 因为$\langle\overline{\psi}\psi\rangle \neq 0$
- ¶ 现象学: $SU(N)_V$保持由于CVC和粒子谱的$SU(N)_V$对称性
- ✠ 现象学: $SU(N_f)_L \otimes SU(N_f)_R/SU(N_f)_V$破坏由于
  - PCAC
  - 粒子谱无宇称简并
  - 存在 $N_f^2 - 1$ 个轻赝标介子

$O(2)$线性σ模型

π介子在低能长波区可以近似用局域场描述 高能将看到价夸克和海夸克和胶子

$$\mathcal{L}=\frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma+\frac{1}{2}\partial_\mu\pi\partial^\mu\pi-V(\sigma,\pi)$$

σ:标量 π:赝标量

$$V(\sigma,\pi)=\frac{\kappa}{2}(\sigma^2+\pi^2)+\frac{\lambda}{4}(\sigma^2+\pi^2)^2$$

拉氏量具有的$O(2)$不变性:
$$\begin{pmatrix}\sigma\\ \pi\end{pmatrix}\to\begin{pmatrix}\sigma'\\ \pi'\end{pmatrix}=\begin{pmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{pmatrix}\begin{pmatrix}\sigma\\ \pi\end{pmatrix}$$
$O(2)$的线性表示

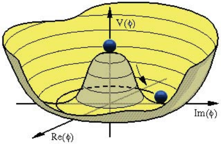
复场表示: $\phi=\sigma+i\pi$ σ宇称正，π宇称负。 讨论见$SU(2)_L\otimes SU(2)_R$模型 $\phi\xrightarrow{O(2)}\phi'=e^{i\alpha}\phi$ 也是$U(1)$对称性

经典真空: $\frac{\partial V}{\partial\sigma}=0\ \Rightarrow\ \langle\sigma\rangle[\kappa+\lambda(\langle\sigma\rangle^2+\langle\pi\rangle^2)]=0$ $\frac{\partial V}{\partial\pi}=0\ \Rightarrow\ \langle\pi\rangle[\kappa+\lambda(\langle\sigma\rangle^2+\langle\pi\rangle^2)]=0$

理论正定: $\lambda>0$ $\kappa\geq 0\ \Rightarrow\ \langle\sigma\rangle=\langle\pi\rangle=0$

$\kappa=-\mu^2<0$ **tachyon** $\Rightarrow\ \langle\sigma\rangle^2+\langle\pi\rangle^2=v^2\equiv-\frac{\kappa}{\lambda}\xRightarrow{\text{宇称不破缺}}\langle\sigma\rangle=v\ \langle\pi\rangle=0$

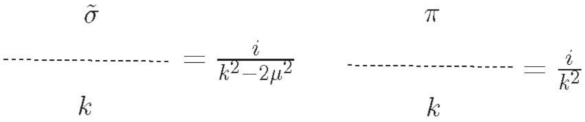
$\sigma=v+\tilde{\sigma}\ \Rightarrow\ m_{\tilde{\sigma}}^2=2\lambda v^2$ $m_{\tilde{\pi}}^2=0$

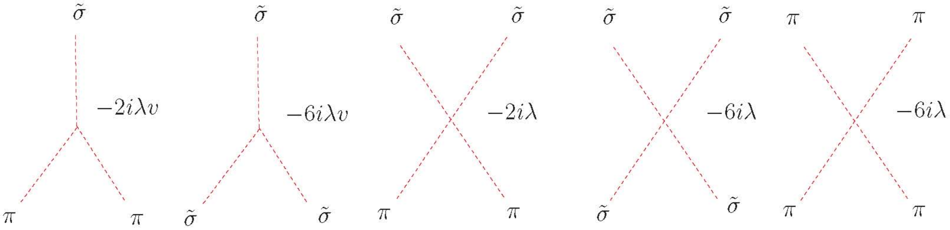


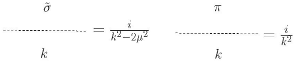
走山沟: 激发goldstone; **爬山: 激发σ**

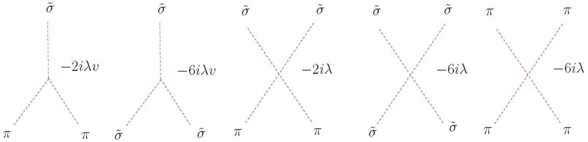
$O(2)$线性σ模型 $\quad \sigma = v + \tilde{\sigma} \qquad -\kappa = \mu^2 = v^2\lambda$

$$
\begin{aligned}
\mathcal{L} &= \frac{1}{2}\partial_\mu\tilde{\sigma}\partial^\mu\tilde{\sigma} + \frac{1}{2}\partial_\mu\pi\partial^\mu\pi - V(v+\tilde{\sigma},\pi) \\
V(v+\tilde{\sigma},\pi) &= -\frac{1}{2}\mu^2v^2 + \frac{\lambda}{4}v^4 + \lambda v^2\tilde{\sigma}^2 + v\lambda\tilde{\sigma}^3 + v\lambda\tilde{\sigma}\pi^2 + \frac{\lambda}{4}(\tilde{\sigma}^2+\pi^2)^2
\end{aligned}
$$

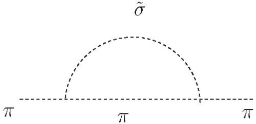




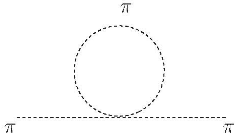


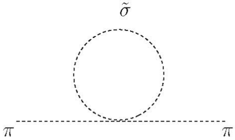


π自能$\Sigma_\pi(0)$计算: $I(m^2) \equiv \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2-m^2}$

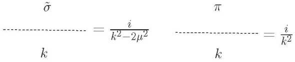


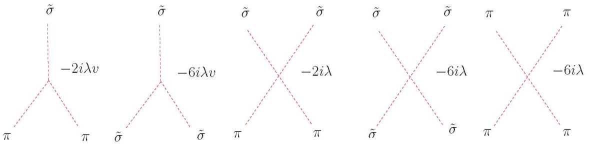
$$\Sigma_1(0) = \int \frac{d^4k}{(2\pi)^4} (-2iv\lambda) \frac{i}{k^2} (-2iv\lambda) \frac{i}{k^2-2\mu^2} = \frac{4v^2\lambda^2}{2\mu^2} \int \frac{d^4k}{(2\pi)^4} \left[\frac{1}{k^2-2\mu^2} - \frac{1}{k^2}\right] = 2\lambda[I(2\mu^2) - I(0)]$$



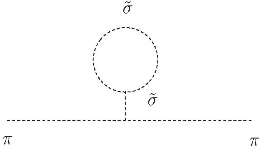


$$\Sigma_2(0) = \frac{1}{2}(-6i\lambda) \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2} = 3\lambda I(0) \qquad \Sigma_3(0) = \frac{1}{2}(-2i\lambda) \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2-2\mu^2} = \lambda I(2\mu^2)$$

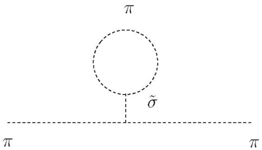




π自能$\Sigma_\pi(0)$计算:　　$I(m^2) \equiv \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2-m^2}$



$$\Sigma_4(0) = (-2iv\lambda)\frac{i}{-2\mu^2}\frac{1}{2}(-6iv\lambda)\int\frac{d^4k}{(2\pi)^4}\frac{i}{k^2-2\mu^2} = -3\lambda I(2\mu^2)$$

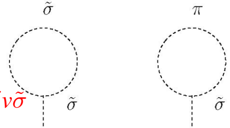


$$\Sigma_5(0) = (-2iv\lambda)\frac{i}{-2\mu^2}\frac{1}{2}(-2iv\lambda)\int\frac{d^4k}{(2\pi)^4}\frac{i}{k^2} = -\lambda I(0)$$

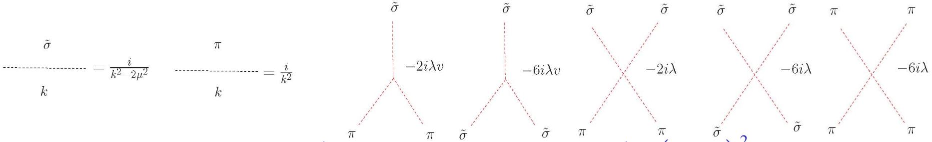
$\Sigma_1(0) = 2\lambda[I(2\mu^2) - I(0)]$　　$\Sigma_2(0) = 3\lambda I(0)$　　$\Sigma_3(0) = \lambda I(2\mu^2)$

$\Sigma_\pi(0) = \sum_{i=1}^{5}\Sigma_i(0) = 0$　**Goldstone 定理**

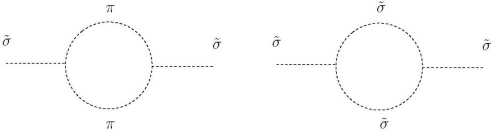
$\langle\tilde\sigma\rangle = \langle 0|\tilde\sigma|0\rangle = \langle\sigma\rangle - v = 0$　　$v \to v + \delta v$　抵消蝌蚪图



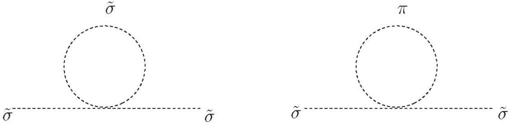
$$V \sim -\frac{\mu^2}{2}(\tilde\sigma+v+\delta v)^2 + \frac{\lambda}{4}(\tilde\sigma+v+\delta v)^4 \Rightarrow -\mu^2\delta v\tilde\sigma + 3\lambda\delta v v^2\tilde\sigma = 2\lambda v^2\delta v\tilde\sigma$$



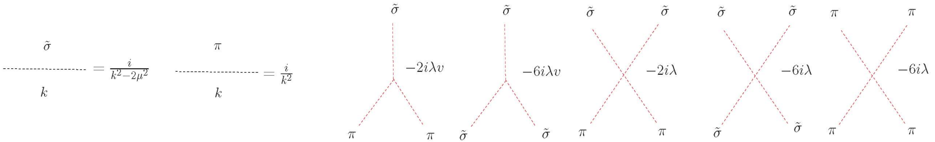
σ 自能 $\Sigma_\sigma(0)$ 计算: $I(m^2) \equiv \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2-m^2}$ 二次发散 $J(m^2) \equiv \int \frac{d^4k}{(2\pi)^4} \left(\frac{1}{k^2-m^2}\right)^2$ 对数发散



$$\Sigma_1'(0) = (-2iv\lambda)^2 \int \frac{d^4k}{(2\pi)^4} \frac{-1}{k^4} = 4v^2\lambda^2 J(0) \qquad \Sigma_2'(0) = (-2iv\lambda)^2 \int \frac{d^4k}{(2\pi)^4} \frac{-1}{(k^2-2\mu^2)^2} = 4v^2\lambda^2 J(2\mu^2)$$



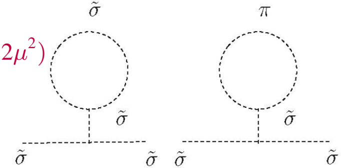
$$\Sigma_3'(0) = \frac{1}{2}(-6i\lambda) \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2-2\mu^2} = 3\lambda I(2\mu^2) \qquad \Sigma_4'(0) = \frac{1}{2}(-2i\lambda) \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2} = \lambda I(0)$$



σ 自能 $\Sigma_\sigma(0)$ 计算： $I(m^2)\equiv\int\frac{d^4k}{(2\pi)^4}\frac{1}{k^2-m^2}$ 二次发散 $J(m^2)\equiv\int\frac{d^4k}{(2\pi)^4}\left(\frac{1}{k^2-m^2}\right)^2$ 对数发散

$$\Sigma'_5(0)=(-6iv\lambda)\frac{i}{-2\mu^2}\frac{1}{2}(-6iv\lambda)\int\frac{d^4k}{(2\pi)^4}\frac{i}{k^2-2\mu^2}=-9\lambda I(2\mu^2)$$

$$\Sigma'_6(0)=(-6iv\lambda)\frac{i}{-2\mu^2}\frac{1}{2}(-2iv\lambda)\int\frac{d^4k}{(2\pi)^4}\frac{i}{k^2}=-3\lambda I(0)$$



$\Sigma'_1(0)=4v^2\lambda^2J(0)$ $\Sigma'_2(0)=4v^2\lambda^2J(2\mu^2)$ $\Sigma'_3(0)=3\lambda I(2\mu^2)$ $\Sigma'_4(0)=\lambda I(0$

$\Sigma_\sigma(0)=\sum_{i=1}^{6}\Sigma'_i(0)\neq 0$ 二次发散! **Higgs质量二次发散在目前的体现** $\Rightarrow$ 不自然性!

$SU(2)_L \otimes SU(2)_R$线性σ模型　为什么不直接研究$N_f=3$？ 简单；同位旋对称性好

QCD自由夸克项： $\mathcal{L}=\mathcal{L}_S+\mathcal{L}_{SB}$　$\mathcal{L}_S=\overline{\psi}_R i\partial\!\!\!/\psi_R+\overline{\psi}_L i\partial\!\!\!/\psi_L$　$\mathcal{L}_{SB}=-\overline{\psi}_L m\psi_R-\overline{\psi}_R m\psi_L$　$\psi_{\substack{L\\R}}=\begin{pmatrix} u_{\substack{L\\R}} \\ d_{\substack{L\\R}} \end{pmatrix}$

$$U_L\psi_{L,a}U_L^{-1}=\psi'_{L,a}=D_{L,ba}\psi_{L,b} \qquad D_{L,ba}=\left(e^{i\alpha_j\frac{\tau_j}{2}}\right)_{ba} \qquad U_L\psi_{R,a}U_L^{-1}=\psi_{R,a}$$

$$U_R\psi_{R,a}U_R^{-1}=\psi'_{R,a}=D_{R,ba}\psi_{R,b} \qquad D_{R,ba}=\left(e^{i\beta_j\frac{\tau_j}{2}}\right)_{ba} \qquad U_R\psi_{L,a}U_R^{-1}=\psi_{L,a}$$

$$U_RU_L\mathcal{L}_SU_L^{-1}U_R^{-1}=\mathcal{L}_S \qquad j^\mu_{Ri}\equiv\overline{\psi}\gamma^\mu\frac{\tau_i}{2}\frac{1+\gamma_5}{2}\psi \qquad j^\mu_{Li}\equiv\overline{\psi}\gamma^\mu\frac{\tau_i}{2}\frac{1-\gamma_5}{2}\psi$$

$$Q_{Li}\equiv\int d^3x\, j^0_{Li} \qquad Q_{Ri}\equiv\int d^3x\, j^0_{Ri} \qquad U_L=e^{i\alpha_jQ_{Lj}} \qquad U_R=e^{i\beta_jQ_{Rj}}$$

$$j^\mu_{Ri}+j^\mu_{Li}\equiv j^\mu_i \qquad j^\mu_{Ri}-j^\mu_{Li}\equiv j^\mu_{5i} \qquad Q_{Ri}+Q_{Li}=Q_i \qquad U=e^{i\gamma_jQ_j} \qquad Q_{Ri}-Q_{Li}=Q_{5i}$$

$$PQ_iP^{-1}=Q_i \qquad PQ_{5i}P^{-1}=-Q_{5i} \qquad PQ_{Li}P^{-1}=Q_{Ri} \qquad PQ_{Ri}P^{-1}=Q_{Li}$$

$$[Q_{\substack{L\\R},i},Q_{\substack{L\\R},j}]=i\epsilon_{ijk}Q_{\substack{L\\R},k} \qquad [Q_{L,i},Q_{R,j}]=0 \qquad [Q_i,Q_j]=i\epsilon_{ijk}Q_k \;\; SU(2)_V \qquad [Q_{5i},Q_{5j}]=i\epsilon_{ijk}Q_k$$

$$\phi\sim\psi\overline{\psi}=((\frac{1}{2}),(\frac{1}{2})^*)=\begin{pmatrix}\phi_{11} & \phi_{12}\\ \phi_{21} & \phi_{22}\end{pmatrix} \qquad \phi_{ab}=\sigma(1)_{ba}+i(\tau_j)_{ba}\pi_j \qquad \phi=\begin{pmatrix}\sigma+i\pi_3 & i\pi_1+\pi_2\\ i\pi_1-\pi_2 & \sigma-i\pi_3\end{pmatrix}$$

$$\sigma=\frac{1}{2}(\phi_{11}+\phi_{22}) \qquad \pi_3=-\frac{i}{2}(\phi_{11}-\phi_{22}) \qquad \pi_1=-\frac{i}{2}(\phi_{12}+\phi_{21}) \qquad \pi_2=\frac{1}{2}(\phi_{12}-\phi_{21})$$

$SU(2)_L \otimes SU(2)_R$线性σ模型

$$\phi \sim (\frac{1}{2},\frac{1}{2}) = \begin{pmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{pmatrix} \quad \phi_{a\dot{b}} = \sigma(1)_{\dot{b}a} + i(\tau_j)_{\dot{b}a}\pi_j \quad \phi = \begin{pmatrix} \sigma + i\pi_3 & i\pi_1 + \pi_2 \\ i\pi_1 - \pi_2 & \sigma - i\pi_3 \end{pmatrix}$$

$$\sigma = \frac{1}{2}(\phi_{11}+\phi_{22}) \quad \pi_3 = -\frac{i}{2}(\phi_{11}-\phi_{22}) \quad \pi_1 = -\frac{i}{2}(\phi_{12}+\phi_{21}) \quad \pi_2 = \frac{1}{2}(\phi_{12}-\phi_{21})$$

$$U_L(\alpha)\phi_{a\dot{b}}U_L(\alpha)^{-1} = \phi_{c\dot{b}}(e^{i\alpha_j\frac{\tau_j}{2}})_{ca} = [(e^{i\alpha_j\frac{\tau_j}{2}})^T\phi]_{a\dot{b}}$$
$$U_R(\beta)\phi_{a\dot{b}}U_R(\beta)^{-1} = (e^{-i\beta_j\frac{\tau_j}{2}})_{\dot{b}\dot{c}}\phi_{a\dot{c}} = [\phi(e^{-i\beta_j\frac{\tau_j}{2}})^T]_{a\dot{b}}$$

无穷小: $[Q_{Li},\phi_{a\dot{b}}] = \phi_{c\dot{b}}(\frac{\tau_i}{2})_{ca}$ $\quad [Q_{Ri},\phi_{a\dot{b}}] = -(\frac{\tau_i}{2})_{\dot{b}\dot{c}}\phi_{a\dot{c}}$ $\quad$ $\mathrm{tr}\phi\phi^\dagger = \mathrm{tr}\phi^\dagger\phi = \sigma^2 + \vec{\pi}\cdot\vec{\pi}$ 是不变量

$$[Q_{Li},\sigma] = \frac{i}{2}\pi_i \quad [Q_{Li},\pi_j] = -\frac{i}{2}\sigma\delta_{ij} + \frac{i}{2}\epsilon_{ijk}\pi_k \quad [Q_{Ri},\sigma] = -\frac{i}{2}\pi_i \quad [Q_{Ri},\pi_j] = \frac{i}{2}\sigma\delta_{ij} + \frac{i}{2}\epsilon_{ijk}\pi_k$$

$[Q_i,\sigma] = 0$ $\langle\sigma\rangle \neq 0$不破坏$SU(2)_V$ $\quad [Q_i,\pi_j] = i\epsilon_{ijk}\pi_k$ π是$SU(2)_V$伴随表示 $\quad [Q_{5i},\sigma] = -i\pi_i \quad [Q_{5i},\pi_j] = i\sigma\delta_{ij}$

$$P\phi_{c\dot{b}}P^{-1}(\frac{\tau_i}{2})_{ca} = P[Q_{Li},\phi_{a\dot{b}}]P^{-1} = [Q_{Ri},P\phi_{a\dot{b}}P^{-1}] \Rightarrow P\phi_{a\dot{b}}P^{-1} = \phi^\dagger_{a\dot{b}} \Rightarrow P\sigma P^{-1} = \sigma \quad P\pi_i P^{-1} = -\pi_i$$

$$\mathcal{L}_S = \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi}) - V(\sigma^2+\vec{\pi}\cdot\vec{\pi}) + \overline{\Psi}i\partial\!\!\!/\Psi + g\overline{\Psi}[\sigma + i\vec{\pi}\cdot\vec{\tau}\gamma_5]\Psi \quad V(z) = \frac{\kappa}{2}z + \frac{\lambda}{4}z^2$$

核子拉氏量

$$\mathcal{L}_S = \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma+\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi}) - V(\sigma^2+\vec{\pi}\cdot\vec{\pi}) + \overline{\Psi}i\partial\!\!\!/\Psi + g\overline{\Psi}[\sigma+i\vec{\pi}\cdot\vec{\tau}\gamma_5]\Psi \qquad V(z) = \frac{\kappa}{2}z+\frac{\lambda}{4}z^2$$

Wigner-Weyl实现: $\kappa > 0 \;\; \lambda > 0 \Rightarrow \langle\sigma\rangle = \langle\vec{\pi}\rangle = 0$

- 严格的$SU(2)_L \otimes SU(2)_R$对称性. j_i, j_{5i}守恒
- 费米子无质量 $m_N = 0$
- 玻色子等质量 $m_\sigma^2 = m_\pi^2 = \kappa$
- 正负宇称的玻色子简并

Nambu-Goldstone实现: $\kappa=-\mu^2 \;\; v^2=\langle\sigma\rangle^2+\langle\vec{\pi}\rangle\cdot\langle\vec{\pi}\rangle = \frac{\mu^2}{\lambda} \overset{\text{Parity}}{\Longrightarrow} \langle\sigma\rangle=v,\; \langle\vec{\pi}\rangle=0$

π是Goldstone Boson; 未破缺的对称性是同位旋

$U|0\rangle = e^{i\alpha_i Q_i}|0\rangle = |0\rangle \;\Rightarrow\; Q_i|0\rangle = 0 \qquad U_5|0\rangle = e^{i\alpha_i Q_{5i}}|0\rangle \neq |0\rangle \;\Rightarrow\; Q_{5i}|0\rangle \neq 0$ Goldstone

$PQ_{5i}|0\rangle = PQ_{5i}P^{-1}P|0\rangle = -Q_{5i}|0\rangle \qquad Q_iQ_{5j}|0\rangle = [Q_i, Q_{5j}]|0\rangle = i\epsilon_{ijk}Q_{5k}|0\rangle$ $SU(2)_V$三重态

- $SU(2)_L \otimes SU(2)_R \rightarrow SU(2)_V$ j_i, j_{5i}守恒
- 费米子获得质量 $m_N = -gv$
- 零质量Goldstone玻色子$m_\pi^2 = 0$; 有质量玻色子$m_{\tilde{\sigma}}^2 = 2v^2\lambda$
- 正负宇称的玻色子不简并

明显对称性破缺

$$\mathcal{L}_S = \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi}) - V(\sigma^2+\vec{\pi}\cdot\vec{\pi}) + \overline{\Psi}i\partial\!\!\!/\Psi + g\overline{\Psi}[\sigma + i\vec{\pi}\cdot\vec{\tau}\gamma_5]\Psi \qquad V(z) = \frac{\kappa}{2}z + \frac{\lambda}{4}z^2$$

- 夸克层次：$\mathcal{L}_{SB} = -\overline{\psi}M\psi$ $\qquad \langle\overline{\psi}\psi\rangle \neq 0$
- 强子层次：$\mathcal{L}_{SB} = -C\sigma$ σ项 $\qquad \langle\sigma\rangle \neq 0 \qquad \langle\sigma\rangle \sim \frac{\langle\overline{\psi}\psi\rangle}{M^2_{MS}}$

流守恒?

$$\bar{\delta}\mathcal{L}_S = -\partial_\mu\alpha_i(x)[\overline{\Psi}_L\gamma^\mu\frac{\tau_i}{2}\Psi_L + \frac{1}{2}(\pi_i\partial^\mu\sigma - \sigma\partial^\mu\pi_i) + \frac{1}{2}\epsilon_{ijk}\pi_k\partial^\mu\pi_j] \qquad \bar{\delta}\mathcal{L}_{SB} = \frac{C}{2}[\alpha_i(x) - \beta_i(x)]\pi_i$$

$$-\partial_\mu\beta_i(x)[\overline{\Psi}_R\gamma^\mu\frac{\tau_i}{2}\Psi_R - \frac{1}{2}(\pi_i\partial^\mu\sigma - \sigma\partial^\mu\pi_i) + \frac{1}{2}\epsilon_{ijk}\pi_k\partial^\mu\pi_j]$$

$$\partial_\mu j^\mu_{Li} = -\frac{\delta\mathcal{L}'_S}{\delta\alpha_i(x)} = -\frac{C}{2}\pi_i \qquad j^\mu_{Li} = -\frac{\delta\mathcal{L}'_S}{\delta\partial_\mu\alpha_i(x)} = \overline{\Psi}_L\gamma^\mu\frac{\tau_i}{2}\Psi_L + \frac{1}{2}(\pi_i\partial^\mu\sigma - \sigma\partial^\mu\pi_i) + \frac{1}{2}\epsilon_{ijk}\pi_k\partial^\mu\pi_j$$

$$\partial_\mu j^\mu_{Ri} = -\frac{\delta\mathcal{L}'_S}{\delta\beta_i(x)} = -\frac{C}{2}\pi_i \qquad j^\mu_{Ri} = -\frac{\delta\mathcal{L}'_S}{\delta\partial_\mu\beta_i(x)} = \overline{\Psi}_R\gamma^\mu\frac{\tau_i}{2}\Psi_R - \frac{1}{2}(\pi_i\partial^\mu\sigma - \sigma\partial^\mu\pi_i) + \frac{1}{2}\epsilon_{ijk}\pi_k\partial^\mu\pi_j$$

$$\partial_\mu j^\mu_i = 0 \qquad j^\mu_i = \overline{\Psi}\gamma^\mu\frac{\tau_i}{2}\Psi + \epsilon_{ijk}\pi_k\partial^\mu\pi_j$$

$$\partial_\mu j^\mu_{5i} = C\pi_i \qquad \underline{C = f_\pi m^2_\pi} \Leftrightarrow \text{PCAC} \qquad j^\mu_{5i} = \overline{\Psi}\gamma^\mu\gamma_5\frac{\tau_i}{2}\Psi + \pi_i\partial^\mu\sigma - \sigma\partial^\mu\pi_i$$

介子携带同位旋荷!

明显对称性破缺

$$\mathcal{L}_S = \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma+\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi}) - V(\sigma^2+\vec{\pi}\cdot\vec{\pi}) + \overline{\Psi}i\partial\!\!\!/\Psi + g\overline{\Psi}[\sigma+i\vec{\pi}\cdot\vec{\tau}\gamma_5]\Psi \qquad V(z) = \frac{\kappa}{2}z+\frac{\lambda}{4}z^2$$

$$\mathcal{L}_{SB} = -C\sigma \qquad V \to V + C\sigma \equiv V'$$

$$\frac{\delta V'}{\delta\pi_i} = \frac{\delta V}{\delta\pi_i} = 0 \qquad \frac{\delta V'}{\delta\sigma} = \frac{\delta V}{\delta\sigma} - C = 0 \Rightarrow \langle\sigma\rangle[\kappa+\lambda\langle\sigma\rangle^2] = -C \xRightarrow{\text{准到最低阶}} \langle\sigma\rangle^2 = -\frac{\kappa}{\lambda} - \frac{C}{\nu\lambda}$$

$$\sigma = \langle\sigma\rangle + \tilde{\sigma} \qquad m_{\tilde{\sigma}}^2 = \kappa + 3\lambda\langle\sigma\rangle^2 \qquad m_\pi^2 = \kappa + \lambda\langle\sigma\rangle^2 = -\frac{C}{\langle\sigma\rangle}$$ 近似的Goldstone $$m_N = -g\langle\sigma\rangle$$

$$-C = \langle\sigma\rangle[\kappa + \lambda\langle\sigma\rangle^2] = \langle\sigma\rangle m_\pi^2 \xRightarrow{C=f_\pi m_\pi^2} f_\pi = -\langle\sigma\rangle$$ 手征对称性破缺的标志!

$$m_N = gf_\pi$$ 比较G-T关系两边到$\langle\sigma\rangle$一级: $$f_\pi = \frac{m_N g_A(0)}{g} \Rightarrow g_A(0) = 1 + O(\langle\sigma\rangle)$$

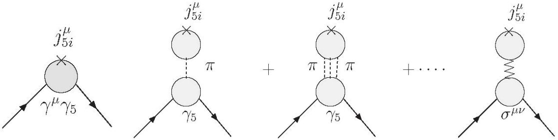
除了σ场尚有争议, $SU(2)_L \otimes SU(2)_R$线性σ模型的所有结果都与实验相符！

一般性讨论 H.Pagels Phys.Rep.16,219(1975)

G-T与PCAC没有关系！ CAC+SSB+smoothness⇒G-T

$$\langle 0|j^\mu_{5i}(x)|\pi_j\rangle = iq^\mu f_\pi \delta_{ij} e^{-iq\cdot x} \xrightarrow[]{\mathcal{L}_{SB}=0 \quad \partial_\mu j^\mu_{5i}=0} 0 = \langle 0|\partial_\mu j^\mu_{5i}(x)|\pi_j\rangle = m_\pi^2 f_\pi \delta_{ij} e^{-iq\cdot x} \Rightarrow m_\pi^2 f_\pi = 0$$

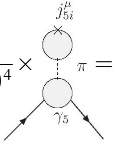
核子矩阵元$q \equiv p' - p$: $$\langle p'|j^\mu_{5i}(x)|p\rangle = \frac{e^{iq\cdot x}}{2E_N V}\bar{u}(p')\frac{\tau_i}{2}\left[\gamma^\mu\gamma_5 g_A(q^2) + q^\mu\gamma_5 g_P(q^2) + iq_\nu\sigma^{\mu\nu}g_T(q^2)\right]u(p)$$



$$\xrightarrow{\partial_\mu j^\mu_{5i}=0} 0 = \langle p'|\partial_\mu j^\mu_{5i}(x)|p\rangle = \frac{ie^{iq\cdot x}}{2E_N V}\bar{u}(p')\frac{\tau_i}{2}\left[\not{q}\gamma_5 g_A(q^2) + q^2\gamma_5 g_P(q^2)\right]u(p)$$

$$\xrightarrow{\text{Dirac Eq}} \frac{ie^{iq\cdot x}}{2E_N V}[2m_N g_A(q^2) + q^2 g_P(q^2)]\bar{u}(p')\frac{\tau_i}{2}\gamma_5 u(p) \Rightarrow 2m_N g_A(q^2) + q^2 g_P(q^2) = 0$$

$$\langle p'|j^\mu_{5i}(x)|p\rangle\Big|_{g_P} \xrightarrow{\text{单}\pi\text{为主}} \frac{i}{(2\pi)^4}\times$$



$$= \frac{i}{(2\pi)^4}(2\pi)^4(-iq^\mu f_\pi)\frac{i}{q^2-m_\pi^2}\left(2ig_{\bar{N}N\pi}\bar{u}(p')\frac{\tau_i}{2}\gamma_5 u(p)\right)\frac{e^{iq\cdot x}}{2E_N V}$$

$$\Rightarrow g_P(q^2) \approx -\frac{2g_{\bar{N}N\pi}f_\pi}{q^2-m_\pi^2}$$

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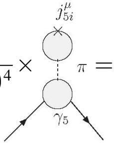


$$= \frac{i}{(2\pi)^4}(2\pi)^4(-iq^\mu f_\pi)\frac{i}{q^2-m_\pi^2}\Big(2ig_{\overline{N}N\pi}\bar{u}(p')\frac{\tau_i}{2}\gamma_5 u(p)\Big)\frac{e^{iq\cdot x}}{2E_N V}$$

$$\Rightarrow\; g_P(q^2)\approx -\frac{2g_{\overline{N}N\pi}f_\pi}{q^2-m_\pi^2}$$

$$m_\pi^2 f_\pi = 0 \qquad\qquad 2m_N g_A(q^2)+q^2 g_P(q^2)=0$$

i. $m_N=0,\ g_A$任意$\Rightarrow\ q^2 g_P(q^2)=0\Rightarrow\ g_P(q^2)\xrightarrow{q\to 0}$ 有限$\Rightarrow\ m_\pi^2\neq 0\Rightarrow\ f_\pi=0$ **Wigner-Weyl**

ii. $g_A(q^2)\neq 0,\ m_N\neq 0\Rightarrow\ q^2 g_P(q^2)\neq 0\Rightarrow\ m_\pi^2=0\ \ f_\pi$任意 **N-G** $m_N g_A(0)=g_{\overline{N}N\pi}f_\pi$ **G-T**

物理情形 没用PCAC！

iii. $g_A(q^2)=0,\ m_N$任意$\Rightarrow q^2 g_P(q^2)=0\Rightarrow m_\pi^2\neq 0\Rightarrow f_\pi=0\xrightarrow{g_P=g_A=0}\ \langle p'|j^\mu_{5i}(x)|p\rangle=0$

$$\Rightarrow\langle p'|Q_{5i}(x)|p\rangle=0 \quad [Q_{5i},Q_{5j}]|p\rangle=i\epsilon_{ijk}Q_k|p\rangle\neq 0\ \Rightarrow\ Q_{5i}(x)|p\rangle\neq 0\ \xrightarrow{PQ_{5i}|p\rangle=-Q_{5i}|p\rangle}$$ 存在宇称加倍

$$\mathcal{L} \xRightarrow{\langle\overline{\psi}\psi\rangle\neq 0} N_f^2 - 1 \text{ GBs} \xRightarrow{\mathcal{L}_{SB}\neq 0} \text{GB} \rightarrow \text{PGBs}$$

$$\mathcal{L}_{SB} = -\overline{\psi}^l_\alpha M_{lm} \psi^m_\alpha \qquad M = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}$$

$$U_{\substack{L\\R}} \psi^l_{\substack{L\\R}\alpha} U^{-1}_{\substack{L\\R}} = D^{ml}_{\substack{L\\R}} \psi^m_\alpha \quad \xRightarrow{D=e^{i\lambda_i\alpha_i}} \quad [Q^i_5, \psi^l_{\substack{L\\R}\alpha}] = \mp \left(\frac{\lambda_i}{2}\right)^{ml} \psi^m_{\substack{L\\R}\alpha} \qquad [Q^i_5, \overline{\psi}^l_{\substack{L\\R}\alpha}] = \pm \left(\frac{\lambda_i}{2}\right)^{ml} \overline{\psi}^m_{\substack{L\\R}\alpha}$$

$$\mathcal{L}_{SB} = 0 \qquad \Rightarrow \quad \langle 0 | j^\mu_{5i} | k, j\rangle_{\mathcal{L}_S} \overset{k_\mu k^\mu = 0}{=\!=\!=\!=\!=} i k_\mu f_i \delta_{ij} e^{-ik\cdot x}$$

$$\mathcal{L}_{SB} \neq 0 \qquad \Rightarrow \quad \langle 0 | j^\mu_{5i} | k, j\rangle_{\mathcal{L}} \overset{k_\mu k^\mu = \mu_j^2 \neq 0}{=\!=\!=\!=\!=\!=\!=} i k_\mu \left(f_i \delta_{ij} e^{-ik\cdot x} + O(\mathcal{L}_{SB}) \right) \qquad \mu_j^2 \sim O(\mathcal{L}_{SB})$$

$$\langle 0 | \partial_\mu j^\mu_{5i} | k, j\rangle_{\mathcal{L}} = \mu_j^2 f_i \delta_{ij} \; O(\mathcal{L}_{SB}) \; + O(\mathcal{L}^2_{SB})$$

通过此公式定义赝goldstone玻色子的质量

Dashen 公式 $\delta(x^0-y^0)[j^i_{5\mu}(x), j^j_{50}(y)] = i\delta^4(x-y)\epsilon_{ijk}j^k_\mu(y)$ $\quad \langle 0|j^k_\mu(y)|0\rangle = 0$

$$\frac{\partial}{\partial x_\mu}\frac{\partial}{\partial y_\nu}T\left(j^i_{5\mu}(x)j^j_{5\nu}(y)\right) = \frac{\partial}{\partial x_\mu}\frac{\partial}{\partial y_\nu}[\theta(x^0-y^0)j^i_{5\mu}(x)j^j_{5\nu}(y) + \theta(y^0-x^0)j^j_{5\nu}(y)j^i_{5\mu}(x)]$$

$$=\frac{\partial}{\partial x_\mu}\left[\theta(x^0-y^0)j^i_{5\mu}(x)\partial^\nu j^j_{5\nu}(y)+\theta(y^0-x^0)\partial^\nu j^j_{5\nu}(y)j^i_{5\mu}(x)-\delta(x^0-y^0)j^i_{5\mu}(x)j^j_{50}(y)+\delta(y^0-x^0)j^j_{50}(y)j^i_{5\mu}(x)\right]$$

$$= T\left(\partial^\mu j^i_{5\mu}(x)\partial^\nu j^j_{5\nu}(y)\right) + \delta(x^0-y^0)[j^i_{50}(x), \partial^\nu j^j_{5\nu}(y)] - \frac{\partial}{\partial x_\mu}\left(\delta(x^0-y^0)[j^i_{5\mu}(x), j^j_{50}(y)]\right)$$

$$q^\mu q^\nu \int d^4x d^4y\, e^{iq\cdot(x-y)}T\left(j^i_{5\mu}(x)j^j_{5\nu}(y)\right) = \int d^4x d^4y\, e^{iq\cdot(x-y)}\frac{\partial}{\partial x_\mu}\frac{\partial}{\partial y_\nu}T\left(j^i_{5\mu}(x)j^j_{5\nu}(y)\right)$$

$$=\int d^4x d^4y\, e^{iq\cdot(x-y)}\left[T\left(\partial^\mu j^i_{5\mu}(x)\partial^\nu j^j_{5\nu}(y)\right)+\delta(x^0-y^0)[j^i_{50}(x), \partial^\nu j^j_{5\nu}(y)] - \frac{\partial}{\partial x_\mu}\left(\delta(x^0-y^0)[j^i_{5\mu}(x), j^j_{50}(y)]\right)\right]$$

$$-\int d^4x d^4y\, e^{iq\cdot(x-y)}\frac{\partial}{\partial x_\mu}\left(\delta(x^0-y^0)\langle 0|[j^i_{5\mu}(x), j^j_{50}(y)]|0\rangle\right) = -q^\mu\epsilon_{ijk}\int d^4x d^4y\, e^{iq\cdot(x-y)}\delta^4(x-y)\langle 0|j^k_\mu(y)|0\rangle$$

$$q^\mu q^\nu \int d^4x d^4y e^{iq\cdot(x-y)}\langle 0|Tj^i_{5\mu}(x)j^j_{5\nu}(y)|0\rangle \qquad = -q^\mu\epsilon_{ijk}\int d^4x\, \langle 0|j^k_\mu(x)|0\rangle = 0$$

$$= \int d^4x d^4y e^{iq\cdot(x-y)}\left[\langle 0|T\partial^\mu j^i_{5\mu}(x)\partial^\nu j^j_{5\nu}(y)|0\rangle + \delta(x^0-y^0)\langle 0|[j^i_{50}(x), \partial^\nu j^j_{5\nu}(y)]|0\rangle\right]$$

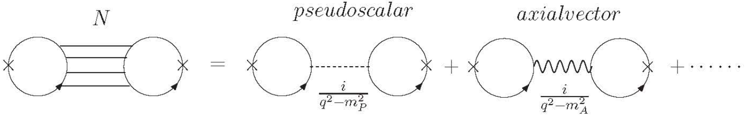
Dashen 公式 $\mu_i^2 f_i^2 \delta_{ij} = \langle 0|[Q_{5i}[Q_{5j}, \mathcal{H}]]|0\rangle_{\mathcal{L}_S}$

$$\partial^0 j_{50}^j(y) = \partial^\nu j_{5\nu}^j + \nabla \cdot \vec{j}_5^j$$

$$q^\mu q^\nu \int d^4x d^4y e^{iq\cdot(x-y)} \langle 0|T j_{5\mu}^i(x) j_{5\nu}^j(y)|0\rangle = \int d^4x d^4y e^{iq\cdot(x-y)} \Big[\langle 0|T\partial^\mu j_{5\mu}^i(x)\partial^\nu j_{5\nu}^j(y)|0\rangle$$

$$+\delta(x^0 - y^0)\langle 0|[j_{50}^i(x), \partial^\nu j_{5\nu}^j(y)]|0\rangle\Big]$$

$$\text{左边} = q^\mu q^\nu \int d^4x d^4y e^{iq(x-y)} \sum_N [\theta(x^0 - y^0)\langle 0|j_{5\mu}^i(x)|N\rangle\langle N|j_{5\nu}^j(y)|0\rangle + \theta(y^0 - x^0)\langle 0|j_{5\nu}^j(y)|N\rangle\langle N|j_{5\mu}^i(x)|0\rangle]$$



$\overset{q\to 0}{\Longrightarrow} 0$ 有贡献的中间态都有质量！

Dashen 公式 $\mu_i^2 f_i^2 \delta_{ij} = \langle 0|[Q_{5i}[Q_{5j}, \mathcal{H}]]|0\rangle_{\mathcal{L}_S}$ $\qquad \partial^0 j^i_{50}(y) = \partial^\nu j^i_{5\nu} + \nabla \cdot \vec{j}^{\,j}_5$

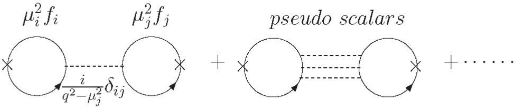
$$q^\mu q^\nu \int d^4x d^4y e^{iq\cdot(x-y)} \langle 0|T j^i_{5\mu}(x) j^j_{5\nu}(y)|0\rangle = \int d^4x d^4y e^{iq\cdot(x-y)} \Big[\langle 0|T\partial^\mu j^i_{5\mu}(x)\partial^\nu j^j_{5\nu}(y)|0\rangle$$

$$\text{左边} \xRightarrow{q\to 0} 0 \qquad\qquad + \delta(x^0-y^0)\langle 0|[j^i_{50}(x), \partial^\nu j^j_{5\nu}(y)]|0\rangle\Big]$$

$$\langle 0|\partial_\mu j^\mu_{5i}|k,j\rangle_{\mathcal{L}} = \mu_j^2 f_i \delta_{ij}$$

$$\text{右边第一项} = \int d^4x d^4y e^{iq\cdot(x-y)} \sum_N [\theta(x^0-y^0)\langle 0|\partial^\mu j^i_{5\mu}(x)|N\rangle\langle N|\partial^\nu j^j_{5\nu}(y)|0\rangle$$

$$+\theta(y^0-x^0)\langle 0|\partial^\nu j^j_{5\nu}(y)|N\rangle\langle N|\partial^\mu j^i_{5\mu}(x)|0\rangle] \xRightarrow{q\to 0\text{赝Goldstone贡献为主}} -i\mu_i^2 f_i^2 \delta_{ij}\int d^4x + O(\mathcal{L}_{SB}^2)$$



直接用PCAC得同样结果

$$\text{右边第一项} = \int d^4x d^4y e^{iq\cdot(x-y)} \langle 0|T\partial^\mu j^i_{5\mu}(x)\partial^\nu j^j_{5\nu}(y)|0\rangle$$

$$= \int d^4x d^4y e^{iq\cdot(x-y)} \langle 0|T \mu_i^2 f_i \phi_i(x) \mu_j^2 f_j \phi_j(y)|0\rangle = \mu_i^2 f_i \mu_j^2 f_j \frac{i\delta_{ij}}{q^2-\mu_j^2}\int d^4x = -i\mu_i^2 f_i^2 \delta_{ij}\int d^4x$$

Dashen 公式 $\mu_i^2 f_i^2 \delta_{ij} = \langle 0|[Q_{5i}[Q_{5j},\mathcal{H}]]|0\rangle_{\mathcal{L}_S}$

$\partial^0 j^j_{50}(y) = \partial^\nu j^j_{5\nu} + \nabla\cdot\vec{j}^j_5$

$$q^\mu q^\nu \int d^4x d^4y e^{iq\cdot(x-y)}\langle 0|Tj^i_{5\mu}(x)j^j_{5\nu}(y)|0\rangle = \int d^4x d^4y e^{iq\cdot(x-y)}\Big[\langle 0|T\partial^\mu j^i_{5\mu}(x)\partial^\nu j^j_{5\nu}(y)|0\rangle$$

$$+\delta(x^0-y^0)\langle 0|[j^i_{50}(x),\partial^\nu j^j_{5\nu}(y)]|0\rangle\Big]$$

左边 $\xRightarrow{q\to 0} 0$

右边第一项 $\xRightarrow{q\to 0\text{赝Goldstone贡献为主}} -i\mu_i^2 f_i^2\delta_{ij}\int d^4x + O(\mathcal{L}_{SB}^2)$

$$\int d^4x\,\mathcal{L}_{SB}\delta_{ij} \xlongequal{\sigma\to -i\pi\to\sigma} \int d^4x\,[Q_{5i}[Q_{5j},\mathcal{L}_{SB}]] = -\int d^4x\,[Q_{5i}[Q_{5j},\mathcal{H}_{SB}]] =$$

$$\xlongequal{Q_5\text{的海森堡方程}} i\int dx^0[Q_{5i},\partial^0 Q_{5j}] = i\int d^4x d^3y [j^i_{50}(x),\partial^0 j^j_{50}(y)]$$

$$= i\lim_{q\to 0}\int d^4x d^4y\, e^{iq\cdot(x-y)}\delta(x^0-y^0)[j^i_{50}(x),\partial^\nu j^j_{5\nu}(y)]$$

$$i\int d^4x\,\langle 0|[Q_{5i}[Q_{5j},\mathcal{H}]]|0\rangle = \lim_{q\to 0}\int d^4x d^4y\, e^{iq\cdot(x-y)}\delta(x^0-y^0)\langle 0|[j^i_{50}(x),\partial^\nu j^j_{5\nu}(y)]|0\rangle$$

Dashen 公式 $\mu_i^2 f_i^2 \delta_{ij} = \langle 0|[Q_{5i}[Q_{5j}, \mathcal{H}]]|0\rangle_{\mathcal{L}_S}$

$$[Q_{5j}, \mathcal{L}_{SB}] = -[Q_{5j}, (\overline{\psi}^l_{L\alpha} M_{ll} \psi^l_{R\alpha} + \overline{\psi}^l_{R\alpha} M_{ll} \psi^l_{L\alpha})]$$

$$= -\left(\frac{\lambda_j}{2}\right)^{lm} \overline{\psi}^m_{L\alpha} M_{ll} \psi^l_{R\alpha} - \overline{\psi}^l_{L\alpha} M_{ll} \left(\frac{\lambda_j}{2}\right)^{ml} \psi^m_{R\alpha} + \left(\frac{\lambda_j}{2}\right)^{lm} \overline{\psi}^m_{R\alpha} M_{ll} \psi^l_{L\alpha} + \overline{\psi}^l_{R\alpha} M_{ll} \left(\frac{\lambda_j}{2}\right)^{ml} \psi^m_{L\alpha}$$

$$= -\overline{\psi}^m_{L\alpha} \{\frac{\lambda_j}{2}, M\}^{lm} \psi^l_{R\alpha} + \overline{\psi}^m_{R\alpha} \{\frac{\lambda_j}{2}, M\}^{lm} \psi^l_{L\alpha}$$

$$[Q_{5i}, [Q_{5j}, \mathcal{L}_{SB}]] = -\overline{\psi}^m_{L\alpha} \{\frac{\lambda_i}{2}, \{\frac{\lambda_j}{2}, M\}\}^{lm} \psi^l_{R\alpha} - \overline{\psi}^m_{R\alpha} \{\frac{\lambda_i}{2}, \{\frac{\lambda_j}{2}, M\}\}^{lm} \psi^l_{L\alpha} \qquad \overline{\psi}^m_{L\alpha} \psi^l_{R\alpha} \propto \delta_{lm}$$

$$-\sum_{l=1}^{3} \langle \overline{\psi}^l_\alpha \psi^l_\alpha \rangle_{\mathcal{L}_S} \{\frac{\lambda_i}{2}, \{\frac{\lambda_i}{2}, M\}\}^{ll} = \mu_i^2 f_i^2 = \mu_i^2 f_{\mathcal{L}_S}^2 = \mu_i^2 f_\pi^2$$

$$\{\frac{\lambda_3}{2}, \{\frac{\lambda_3}{2}, M\}\} = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \mu_{\pi^0}^2 f_\pi^2 = -\langle \overline{u} u\rangle m_u - \langle \overline{d} d\rangle m_d$$

$$\sum_{i=1,2} \{\frac{\lambda_i}{2}, \{\frac{\lambda_i}{2}, M\}\} = \begin{pmatrix} \frac{m_u+m_d}{2} & 0 & 0 \\ 0 & \frac{m_u+m_d}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \mu_{\pi^\pm}^2 f_\pi^2 = -(\langle \overline{u} u\rangle + \langle \overline{d} d\rangle) \frac{m_u + m_d}{2}$$

Dashen 公式

$$-\sum_{l=1}^{3}\langle\overline{\psi}^l_\alpha\psi^l_\alpha\rangle_{\mathcal{L}_S}\{\frac{\lambda_i}{2},\{\frac{\lambda_i}{2},M\}\}^{ll}=\mu_i^2 f_\pi^2$$

$$\{\frac{\lambda_3}{2},\{\frac{\lambda_3}{2},M\}\}=\begin{pmatrix} m_u & 0 & 0\\ 0 & m_d & 0\\ 0 & 0 & 0\end{pmatrix}$$

$$\mu_{\pi^0}^2 f_\pi^2=-\langle\bar{u}u\rangle m_u-\langle\bar{d}d\rangle m_d$$

$$\sum_{i=1,2}\{\frac{\lambda_i}{2},\{\frac{\lambda_i}{2},M\}\}=\begin{pmatrix} \frac{m_u+m_d}{2} & 0 & 0\\ 0 & \frac{m_u+m_d}{2} & 0\\ 0 & 0 & 0\end{pmatrix}$$

$$\mu_{\pi^\pm}^2 f_\pi^2=-(\langle\bar{u}u\rangle+\langle\bar{d}d\rangle)\frac{m_u+m_d}{2}$$

$$\sum_{i=4,5}\{\frac{\lambda_i}{2},\{\frac{\lambda_i}{2},M\}\}=\begin{pmatrix} \frac{m_u+m_s}{2} & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & \frac{m_u+m_s}{2}\end{pmatrix}$$

$$\mu_{K^\pm}^2 f_\pi^2=-(\langle\bar{u}u\rangle+\langle\bar{s}s\rangle)\frac{m_u+m_s}{2}$$

$$\sum_{i=6,7}\{\frac{\lambda_i}{2},\{\frac{\lambda_i}{2},M\}\}=\begin{pmatrix} 0 & 0 & 0\\ 0 & \frac{m_d+m_s}{2} & 0\\ 0 & 0 & \frac{m_d+m_s}{2}\end{pmatrix}$$

$$\mu_{K^0}^2 f_\pi^2=-(\langle\bar{d}d\rangle+\langle\bar{s}s\rangle)\frac{m_d+m_s}{2}$$

$$\{\frac{\lambda_8}{2},\{\frac{\lambda_8}{2},M\}\}=\begin{pmatrix} \frac{m_u}{3} & 0 & 0\\ 0 & \frac{m_d}{3} & 0\\ 0 & 0 & \frac{4m_s}{3}\end{pmatrix}$$

$$\mu_\eta^2 f_\pi^2=-\frac{1}{3}\langle\bar{u}u\rangle m_u-\frac{1}{3}\langle\bar{d}d\rangle m_d-\frac{4}{3}\langle\bar{s}s\rangle m_s$$

$$\mu^2_{\pi^0} f^2_\pi = -\langle \bar{u}u \rangle m_u - \langle \bar{d}d \rangle m_d \qquad \mu^2_{\pi^\pm} f^2_\pi = -(\langle \bar{u}u \rangle + \langle \bar{d}d \rangle)\frac{m_u + m_d}{2}$$

$$\mu^2_{K^\pm} f^2_\pi = -(\langle \bar{u}u \rangle + \langle \bar{s}s \rangle)\frac{m_u + m_s}{2} \qquad \mu^2_{K^0} f^2_\pi = -(\langle \bar{d}d \rangle + \langle \bar{s}s \rangle)\frac{m_d + m_s}{2}$$

$$\mu^2_\eta f^2_\pi = -\frac{1}{3}\langle \bar{u}u \rangle m_u - \frac{1}{3}\langle \bar{d}d \rangle m_d - \frac{4}{3}\langle \bar{s}s \rangle m_s \qquad \langle \bar{u}u \rangle_{\mathcal{L}_S} = \langle \bar{d}d \rangle_{\mathcal{L}_S} = \langle \bar{s}s \rangle_{\mathcal{L}_S} \equiv -\Delta$$

$$\mu^2_\pi = f^{-1}_\pi \Delta(m_u + m_d) \qquad \mu^2_{K^\pm} = f^{-1}_\pi \Delta(m_u + m_s) \qquad \mu^2_{K^0} = f^{-1}_\pi \Delta(m_d + m_s)$$

$$\mu^2_\eta = f^{-1}_\pi \Delta(\frac{1}{3}m_u + \frac{1}{3}m_d + \frac{4}{3}m_s) \quad \Rightarrow \quad \mu^2_\eta = \frac{2}{3}(\mu^2_{K^\pm} + \mu^2_{K^0}) - \frac{1}{3}\mu^2_\pi$$

Gell-Mann Okubo公式

$$\frac{m_d}{m_u} = \frac{\mu^2_{K^0} - \mu^2_{K^\pm} + \mu^2_\pi}{\mu^2_{K^\pm} - \mu^2_{K^0} + \mu^2_\pi} \qquad \frac{m_s}{m_d} = \frac{\mu^2_{K^0} + \mu^2_{K^\pm} - \mu^2_\pi}{\mu^2_{K^0} - \mu^2_{K^\pm} + \mu^2_\pi}$$

考虑进电磁效应

$$\mu^2_{\pi^0} = f^{-1}_\pi \Delta(m_u + m_d) \qquad \mu^2_{\pi^\pm} = f^{-1}_\pi (\Delta(m_u + m_d) + \Delta_\gamma)$$

$$\mu^2_{K^0} = f^{-1}_\pi \Delta(m_d + m_s) \qquad \mu^2_{K^\pm} = f^{-1}_\pi (\Delta(m_u + m_s) + \Delta_\gamma)$$

$$\mu^2_\eta = f^{-1}_\pi \Delta(\frac{1}{3}m_u + \frac{1}{3}m_d + \frac{4}{3}m_s)$$

$$\frac{m_d}{m_u} = \frac{\mu^2_{K^\pm} - \mu^2_{K^0} - \mu^2_{\pi^\pm}}{\mu^2_{K^0} - \mu^2_{K^\pm} + \mu^2_{\pi^\pm} - \mu^2_{\pi^0}} \approx 1.83 \qquad \frac{m_s}{m_d} = \frac{\mu^2_{K^0} + \mu^2_{K^\pm} - \mu^2_{\pi^\pm}}{\mu^2_{K^0} - \mu^2_{K^\pm} + \mu^2_{\pi^\pm}} \approx 20.1$$

夸克质量

$$\frac{m_d}{m_u}=\frac{\mu^2_{K^\pm}-\mu^2_{K^0}-\mu^2_{\pi^\pm}}{\mu^2_{K^0}-\mu^2_{K^\pm}+\mu^2_{\pi^\pm}-\mu^2_{\pi^0}}\approx 1.83 \qquad \frac{m_s}{m_d}=\frac{\mu^2_{K^0}+\mu^2_{K^\pm}-\mu^2_{\pi^\pm}}{\mu^2_{K^0}-\mu^2_{K^\pm}+\mu^2_{\pi^\pm}}\approx 20.1$$

考虑含奇异夸克和不含奇异夸克的态之差:

- 1^-介子八重态: 120MeV
- 2^+介子八重态: 110MeV
- $\frac{1}{2}^+$重子八重态: 190MeV
- $\frac{3}{2}^+$重子八重态: 150MeV

考虑奇异夸克的动能贡献应该比较小, Weinberg最后取 $m_s\approx 150\text{MeV}\Rightarrow\ m_d=7.5\text{MeV},\ m_u=4.2\text{MeV}$

以上定的夸克质量都是 流夸克质量 !

流夸克与真空作用,穿上外衣形成组分夸克,相应的夸克质量叫 组分夸克质量

$\tilde{m}_{组分}=m_{流}+\Delta m_{QCD云}$ $m_N\approx 3\tilde{m}_{组分}$ $\mu_P\approx 2\tilde{m}_{组分}$ 轻夸克中QCD云($\sim 200-300$MeV)起主要作用

重夸克的组分质量通过势模型中的参数确定:

$\tilde{m}_c\cong 1.5\text{GeV}$ $\tilde{m}_c\cong 5\text{GeV}$ 重夸克中QCD云($\sim 200-300$MeV)的作用可以忽略, 组分夸克质量与流夸克质量基本相同

0^{++}**介子:** $m_{f_0(600)} = 400 - 1200\text{MeV}$ $\quad\Gamma_{\text{tot}}[f_0(600)] = 600 - 1000\text{MeV}$

$m_{f_0(980)} = 980 \pm 10\text{MeV}$ $\quad\Gamma_{\text{tot}}[f_0(980)] = 40 - 100\text{MeV}$

$$\Gamma(\tilde{\sigma} \to \pi^0\pi^0) = \frac{4\pi}{64\pi^2 m_{\tilde{\sigma}}}\sqrt{1-(\frac{2m_\pi}{m_{\tilde{\sigma}}})^3}\text{tr}|-iv\lambda\tau^3|^2 \xlongequal{v=-f_\pi,\ m_{\tilde{\sigma}}^2=2v^2\lambda} \frac{f_\pi^2\lambda^2}{8\pi m_{\tilde{\sigma}}}\sqrt{1-(\frac{2m_\pi}{m_{\tilde{\sigma}}})^3}$$

$$\simeq \frac{m_{\tilde{\sigma}}^3}{32\pi f_\pi^2}\sqrt{1-(\frac{3f_\pi}{m_{\tilde{\sigma}}})^3} \xlongequal{f_0(600)} 234\text{MeV} \qquad \xlongequal{f_0(980)} 1070\text{MeV}$$

能否构造一个不含σ只含π 的相互作用体系?

$$\langle\sigma\rangle^2 + \langle\vec{\pi}\rangle\cdot\langle\vec{\pi}\rangle = v^2 = f_\pi^2 \xRightarrow{\lambda\to\infty} \sigma^2 + \vec{\pi}\cdot\vec{\pi} \cong f_\pi^2$$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\pi_i\partial^\mu\pi_i) \xlongequal{\sigma=\sqrt{f_\pi^2-\vec{\pi}\cdot\vec{\pi}}} \frac{1}{2}\partial_\mu\pi_i\partial^\mu\pi_i + \frac{1}{2}\frac{(\pi_i\partial_\mu\pi_i)(\pi_j\partial^\mu\pi_j)}{f_\pi^2-\pi_i^2}$$ 在1 + 1维可重整

$$\phi = \sigma 1 + i\lambda_i\pi_i = \sqrt{f_\pi^2-(\pi_i)^2} + i\lambda_i\pi_i = f_\pi\left(\sqrt{1-\left(\frac{\lambda_i\pi_i}{f_\pi}\right)^2} + i\frac{\lambda_i\pi_i}{f_\pi}\right) \xlongequal{\frac{\lambda_i\pi_i}{f_\pi}=\sin\frac{\lambda_i\Pi_i}{f_\pi}} f_\pi e^{i\frac{\lambda_i\Pi_i}{f_\pi}}$$

$$U \equiv \frac{\phi}{f_\pi} = e^{i\frac{\lambda_i\Pi_i}{f_\pi}}$$ 非线性实现 $UU^\dagger = 1$ $\xRightarrow{SU(N_f)_L\otimes SU(N_f)_R} U' = V_L U V_R^\dagger \quad \mathcal{L} = \frac{f_\pi^2}{4}\text{tr}(\partial_\mu U\partial^\mu U^\dagger) + O(\partial^4)$

$\langle U\rangle \xlongequal{\text{N-G}} 1 \Leftrightarrow \pi_i = 0 \quad \langle V_L U V_R^\dagger\rangle = V_L\langle U\rangle V_R^\dagger = V_L V_R^\dagger = 1$ 真空具有$SU(N_f)_V$ 低能展开(p/Λ): p^2阶 p^4阶

关于非线性σ模型的评述

- ♣ 只包含Goldstone自由度! 经济
- ♠ 对称性非线性地实现! $U' = V_L U V_R^\dagger$ $U = e^{i\frac{\lambda_i \Pi_i}{f_\pi}}$
- ♡ 对称性自发破缺! $SU(N_f)_L \otimes SU(N_f)_R \to SU(N_f)_V$
- ♢ 独立于自发破缺的细节! $U^\dagger U = U U^\dagger = 1$
- ✠ 可以按低能展开一级级地研究!

S矩阵元 S.Coleman, J.Wess, and B.Zumino, Phys. Rev. 177,2239(1969)

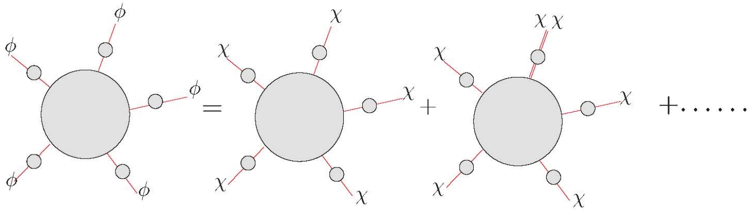
从线性σ模型的场π_i到非线性σ模型的场Π_i 的变换是否影响物理？

$$\mathcal{L}[\phi] = \mathcal{L}_0[\phi]+\mathcal{L}_1[\phi] \quad \phi = \chi F[\chi] = \chi[1+a\chi+\cdots] \quad F[0]=1 \quad \mathcal{L}[\chi F(\chi)] = \mathcal{L}_0(\chi)+\mathcal{L}_1(\chi)$$

关于外腿的差别： S矩阵是格林函数去掉外腿再补上外线波函数

- 对单独的ϕ或χ外腿有$G^{-1}G|_{\text{在壳}} = 1$
- 但对某个$\chi^n \geq 2$外腿，则有$G^{-1}G|_{\text{在壳}} = 0$

$F[0]=1 \Rightarrow$ 从线性σ模型的π_i到非线性σ模型的Π_i 的变换不影响S矩阵元！



重整化

$$\mathcal{L} = \underbrace{\frac{f_\pi^2}{4}\mathrm{tr}(\partial_\mu U \partial^\mu U)}_{O(p^2)} + \underbrace{(\partial^4)}_{O(p^4)}$$

通常理论不可重整的核心问题是会出现无穷多种发散，无法抵消!

核心问题是能否将这些发散的数目降下来!　$\Rightarrow$ **Weinberg 方案：**

结合低能展开，发散项的数目在低能展开的固定阶是有限的!

在低能展开的固定阶,可以引入有限数目的抵消项抵消发散!

在低能展开固定阶写下满足对称性的所有相互作用项，足以抵消所有发散!

低能展开可能会产生错误的高阶发散! C.P.Burgess,D.London, Phys.Rev.D48(1993)4337

$$\frac{1}{16\pi^2}[\ln(\frac{\Lambda^2}{m^2}+1) - \frac{\Lambda^2}{\Lambda^2+m^2}] = \frac{1}{16\pi^2}\int_0^{\Lambda^2} dp^2 \frac{p^2}{(p^2+m^2)^2} = -\frac{1}{16\pi^2}\sum_{n=0}^{\infty}\int_0^{\Lambda^2}\frac{dp^2}{m^2}(n+1)\left(-\frac{p^2}{m^2}\right)^{n+1} = \frac{1}{16\pi^2}\sum_{n=0}^{\infty}\frac{n+1}{n+2}\left(-\frac{\Lambda^2}{m^2}\right)^{n+2}$$

Gasser-Leutwyler拉氏量

Gasser and Leutwyler, Ann. Phys. (N.Y.)158,142(1984); Nucl. Phys. B250,2165(1985)

$$Z[s,p,v,a,\theta] = e^{iW[s,p,v,a,\theta]} \equiv \int \mathcal{D}G\mathcal{D}\Psi\mathcal{D}\overline{\Psi}\mathcal{D}\psi\mathcal{D}\overline{\psi}\; e^{i\int d^4x[\mathcal{L}_{\rm QCD}(\theta,\psi,\Psi,G)+\overline{\psi}J\psi]}$$

$$\mathcal{L}_{\rm QCD}(\theta,\psi,\Psi,G) = \overline{\psi}(i\partial\!\!\!/ + g G\!\!\!/^a\frac{\lambda^a}{2})\psi + \mathcal{L}'_{\rm QCD}(\theta,\Psi,G)$$

$$\mathcal{L}'_{\rm QCD}(\theta,\Psi,G) = -\frac{1}{4}G^a_{\mu\nu}G^{\mu\nu a} + \overline{\Psi}(i\partial\!\!\!/ - M_h + g G\!\!\!/^a\frac{\lambda^a}{2})\Psi - \frac{\theta}{16\pi^2}G^a_{\mu\nu}\tilde{G}^{\mu\nu a}$$

$$J(x) \equiv -s(x) + i\gamma_5 p(x) + v\!\!\!/(x) + a\!\!\!/(x)\gamma_5$$

实际物理态： $s = M$ 负号被吸到s定义中， $p=0,\ v^\mu = a^\mu = 0$

"局域"手征对称性 $SU(N_f)_L \otimes SU(N_f)_R$

$$\psi(x) \to \psi'(x) = [R(x)P_R + L(x)P_L]\psi(x) \qquad P_{\substack{R\\L}} \equiv \frac{1\pm\gamma_5}{2}$$

$$J(x) \to J'(x) = [R(x)P_L + L(x)P_R][J(x) + i\partial\!\!\!/][R^\dagger(x)P_R + L^\dagger(x)P_L]$$

$$v_\mu + a_\mu \to v'_\mu + a'_\mu = R(v_\mu + a_\mu)R^\dagger + Ri\partial_\mu R^\dagger \qquad v_\mu - a_\mu \to v'_\mu - a'_\mu = L(v_\mu - a_\mu)L^\dagger + Li\partial_\mu L^\dagger$$

$$s - ip \to s' - ip' = L(s-ip)R^\dagger \qquad s + ip \to s' + ip' = R(s+ip)L^\dagger$$

$$\mathcal{L}_{\rm QCD}(\theta,\psi',\Psi,G) + \overline{\psi}'J'\psi' = \mathcal{L}_{\rm QCD}(\theta,\psi,\Psi,G) + \overline{\psi}J\psi$$

生成泛函在手征对称性变换下的行为

$$Z[s,p,v,a,\theta] = e^{iW[s,p,v,a,\theta]} \equiv \int \mathcal{D}G\mathcal{D}\Psi\mathcal{D}\overline{\Psi}\mathcal{D}\psi\mathcal{D}\overline{\psi}\; e^{i\int d^4x[\mathcal{L}_{\rm QCD}(\theta,\psi,\Psi,G)+\overline{\psi}J\psi]}$$

$$\mathcal{L}_{\rm QCD}(\theta,\psi,\Psi,G) = \overline{\psi}(i\partial\!\!\!/ + gG\!\!\!/^a\frac{\lambda^a}{2})\psi + \mathcal{L}'_{\rm QCD}(\theta,\Psi,G)$$

$$J(x) \equiv -s(x)+i\gamma_5 p(x)+v\!\!\!/(x)+a\!\!\!/(x)\gamma_5 \quad J'(x) = [R(x)P_L+L(x)P_R][J(x)+i\partial\!\!\!/][R^\dagger(x)P_R+L^\dagger(x)P_L]$$

$$Z[s,p,v,a,\theta] = \int \mathcal{D}G\mathcal{D}\Psi\mathcal{D}\overline{\Psi}\; e^{{\rm Trln}(i\partial\!\!\!/+gG\!\!\!/^a\frac{\lambda^a}{2}+J)+i\int d^4x\;\mathcal{L}'_{\rm QCD}(\theta,\Psi,G)}$$

$$R(x) = 1 + i\alpha(x) + i\beta(x) \qquad\qquad L(x) = 1 + i\alpha(x) - i\beta(x)$$

$$\delta J(x) = i[\alpha(x) - \beta(x)\gamma_5]J(x) - i[J(x) + i\partial\!\!\!/][\alpha(x) + \beta(x)\gamma_5]$$

$$\delta{\rm Trln}(i\partial\!\!\!/ + gG\!\!\!/^a\frac{\lambda^a}{2} + J) \equiv {\rm Trln}(i\partial\!\!\!/ + gG\!\!\!/^a\frac{\lambda^a}{2} + J') - {\rm Trln}(i\partial\!\!\!/ + gG\!\!\!/^a\frac{\lambda^a}{2} + J)$$

$$= {\rm Tr}[(i\partial\!\!\!/ + gG\!\!\!/^a\frac{\lambda^a}{2} + J)^{-1}\delta(i\partial\!\!\!/ + gG\!\!\!/^a\frac{\lambda^a}{2} + J)]$$

$$= i{\rm Tr}[(i\partial\!\!\!/ + gG\!\!\!/^a\frac{\lambda^a}{2} + J)^{-1}[(\alpha - \beta\gamma_5)(J + i\partial\!\!\!/ + gG\!\!\!/^a\frac{\lambda^a}{2}) - (J + i\partial\!\!\!/ + gG\!\!\!/^a\frac{\lambda^a}{2})(\alpha + \beta\gamma_5)]$$

$$= -2i{\rm Tr}(\beta\gamma_5) = -2i\lim_{\Lambda\to\infty}{\rm Tr}[\beta\gamma_5 e^{\frac{(i\partial\!\!\!/+gG\!\!\!/^a\frac{\lambda^a}{2}+J)^2}{\Lambda^2}}] = -i\int d^4x\,{\rm tr}\{\beta(x)[\tilde{\Omega}(x) + \frac{g^2}{32\pi^2}G^a_{\mu\nu}\tilde{G}^{\mu\nu a}]\}$$

$$Z[s,p,v,a,\theta]=\int \mathcal{D}G\mathcal{D}\Psi\mathcal{D}\overline{\Psi}\; e^{\mathrm{Trln}(i\partial\!\!\!/+g G\!\!\!/^a\frac{\lambda^a}{2}+J)+i\int d^4x\;\mathcal{L}'_{\rm QCD}(\theta,\Psi,G)}$$

$$\mathcal{L}'_{\rm QCD}(\theta,\Psi,G)=-\frac{1}{4}G^a_{\mu\nu}G^{\mu\nu a}++\overline{\Psi}(i\partial\!\!\!/-M_h+gG\!\!\!/^a\frac{\lambda^a}{2})\Psi-\frac{\theta}{16\pi^2}G^a_{\mu\nu}\tilde{G}^{\mu\nu a}$$

$$\delta\mathrm{Trln}(i\partial\!\!\!/+gG\!\!\!/^a\frac{\lambda^a}{2}+J)\equiv\mathrm{Trln}(i\partial\!\!\!/+gG\!\!\!/^a\frac{\lambda^a}{2}+J')-\mathrm{Trln}(i\partial\!\!\!/+gG\!\!\!/^a\frac{\lambda^a}{2}+J)$$

$$=-2i\mathrm{Tr}(\beta\gamma_5)=-2i\lim_{\Lambda\to\infty}\mathrm{Tr}[\beta\gamma_5 e^{\frac{(i\partial\!\!\!/+gG\!\!\!/^a\frac{\lambda^a}{2}+J)^2}{\Lambda^2}}]=-i\int d^4x\;\mathrm{tr}\{\beta(x)[\tilde{\Omega}(x)+\frac{g^2}{8\pi^2}G^a_{\mu\nu}\tilde{G}^{\mu\nu a}]\}$$

$$\tilde{\Omega}=\frac{N_c\epsilon^{\mu\nu\mu'\nu'}}{16\pi^2}[v_{\mu\nu}v_{\mu'\nu'}+\frac{4}{3}d_\mu a_\nu d_{\mu'}a_{\nu'}+\frac{2i}{3}\{v_{\mu\nu},a_{\mu'}a_{\nu'}\}+\frac{8i}{3}a_\mu v_{\mu'\nu'}a_\nu+\frac{4}{3}a_\mu a_\nu a_{\mu'}a_{\nu'}]$$

$$Z[s',p',v',a',\theta]=\int \mathcal{D}G\mathcal{D}\Psi\mathcal{D}\overline{\Psi}\; e^{\mathrm{Trln}(i\partial\!\!\!/+gG\!\!\!/^a\frac{\lambda^a}{2}+J')+i\int d^4x\;\mathcal{L}'_{\rm QCD}(\theta,\Psi,G)}$$

$$=e^{-i\int d^4x\;\mathrm{tr}[\beta(x)\tilde{\Omega}(x)]}\int \mathcal{D}G\mathcal{D}\Psi\mathcal{D}\overline{\Psi}\; e^{\mathrm{Trln}(i\partial\!\!\!/+gG\!\!\!/^a\frac{\lambda^a}{2}+J)+i\int d^4x\;[\mathcal{L}'_{\rm QCD}(\theta,\Psi,G)-\mathrm{tr}(\beta)\frac{g^2}{8\pi^2}G^a_{\mu\nu}\tilde{G}^{\mu\nu a}]}$$

$$=e^{-i\int d^4x\;\mathrm{tr}[\beta(x)\tilde{\Omega}(x)]}Z[s,p,v,a,\theta+2\mathrm{tr}\beta]$$

$R=1+i\alpha+i\beta \quad L=1+i\alpha-i\beta$

$$W[s',p',v',a',\theta']=W[s,p,v,a,\theta]-\int d^4x\mathrm{tr}[\beta(x)\tilde{\Omega}(x)]$$

$$\theta'=\theta-2\mathrm{tr}\beta=\theta+i\ln\det(RL^\dagger)$$

对有限大变换： $R(x)=e^{i[\beta(x)+\alpha(x)]}$ $L(x)=e^{-i[\beta(x)-\alpha(x)]}$ $J'=e^{\delta_\beta}J$ $\theta'=e^{\delta_\beta}\theta=\theta-2\mathrm{tr}\beta$

$$W[J',\theta']=e^{\delta_\beta}W[J,\theta]=W[J,\theta]+\frac{e^{\delta_\beta}-1}{\delta_\beta}\delta_\beta W[J,\theta]=W[J,\theta]-\int_0^1 dt\int d^4x\; e^{t\delta_\beta}\;\mathrm{tr}[\beta(x)\tilde{\Omega}(x)]$$

Goldstone场

$\pi^a \xrightarrow{g} \phi^a[g,\pi]$ $\quad \pi^a \overset{\text{真空}}{=\!=\!=} 0$ $\qquad \phi^a[I,\pi] = \pi^a$ $\qquad \phi^a[g_1,\phi[g_2,\pi]] = \phi^a[g_1 g_2,\pi]$

$\phi^a[I,\phi[g,0]] = \phi^a[g,0]$ $\qquad \phi[g_1,\phi[g_2,\phi[g,0]]] = \phi[g_1 g_2 g,0] = \phi[g_1 g_2,\phi[g,0]]$

$\phi^a[g,0]$可以用来描述π^a： $\phi^a[I,0] = 0$

所有使$\phi(h,0)=0$的h ⊂SU(3)$_L$×SU(3)$_R$形成子群H

单位元：$\pi = \mathbf{0} \rightarrow \phi(\mathbf{I},\mathbf{0}) = \mathbf{0}$

乘法: $\phi(\mathbf{h_1},\mathbf{0}) = \mathbf{0} \quad \phi(\mathbf{h_2},\mathbf{0}) = \mathbf{0} \rightarrow \phi(\mathbf{h_1 h_2},\mathbf{0}) = \phi[\mathbf{h_1},\phi(\mathbf{h_2},\mathbf{0})] = \phi[\mathbf{h_1},\mathbf{0}] = \mathbf{0}$

由于此乘法就是原来群的乘法，自然是满足结合律

逆元：$\mathbf{0} = \phi(\mathbf{h}^{-1}\mathbf{h},\mathbf{0}) = \phi[\mathbf{h}^{-1},\phi(\mathbf{h},\mathbf{0})] = \phi[\mathbf{h}^{-1},\mathbf{0}]$

H的物理含义在于保持真空不变（即从$\pi=0$变到$\phi=0$）

手征极限下发生对称性自发破缺后剩下对称性是SU(3)$_V$ → H=SU(3)$_V$!

对所有的g ⊂SU(3)$_L$×SU(3)$_R$和h ⊂H有，$\phi(g,0) = \phi(gh,0)$!

对所有的g_1, g_2 ⊂SU(3)$_L$×SU(3)$_R$,若$\phi(g_1,0) = \phi(g_2,0)$则$g_1^{-1}g_2$ ⊂H!

$$\phi(\bar{g},0) \overset{g_2=g_1\bar{g}}{=\!=\!=\!=} \phi[g_1^{-1},\phi[g_1,\phi(\bar{g},0)]] = \phi[g_1^{-1},\phi(g_1\bar{g},0)] = \phi[g_1^{-1},\phi(g_2,0)] = \phi[g_1^{-1},\phi(g_1,0)]$$

$$= \phi(I,0) = 0 \;\rightarrow\; g_1^{-1}g_2 = \bar{g} \subset \mathrm{H}$$

- $\phi^a[g,0]$可以用来描述π^a
- 所有使$\phi(h,0)=0$的$h\subset$SU(3)$_L\times$SU(3)$_R$形成子群H
- 对所有的$g\subset$SU(3)$_L\times$SU(3)$_R$和$h\subset$H有，$\phi(g,0)=\phi(gh,0)$
- 对所有的$g_1,g_2\subset$SU(3)$_L\times$SU(3)$_R$,若$\phi(g_1,0)=\phi(g_2,0)$则$g_1^{-1}g_2\subset$H

赝标Goldstone粒子场$\phi(g,0)$同构于SU(3)$_L\times$SU(3)$_R$/SU(3)$_V$！
它可以看在是定义在SU(3)$_L\times$SU(3)$_R$的左陪集上（准到$g'=gh$）。
可以在每个陪集上选一个代表元素n，用它来代表赝标Goldstone粒子场。
每个陪集上都选定代表元素后，任意的一个群元都可唯一地分解为nh
对任意的群元g及陪集上的代表元n，我们有$gn=n'h$决定了n在操作g下到的n'变换行为，其中n'也是陪集上的代表元。

标准选择： $n=(1,U)\qquad U(x)=e^{i\phi(x)/F_0}\qquad \phi(x)=\sum\frac{\lambda^a}{\sqrt{2}}\phi^a(x)\qquad \phi\overset{\text{真空}}{=\!=\!=}0\qquad U\overset{\text{真空}}{=\!=\!=}1$

$$gn=(L,R)(1,U)=(L,RU)=(1,RUL^\dagger)(L,L)=(1,U')(L,L)=n'h$$

$$U\to U'=RUL^\dagger\qquad U_{ll'}(x)\sim q_{R,l\alpha s}(x)\bar{q}_{L,l'\alpha s}(x)\qquad \det U(x)=e^{-i\bar\theta}$$

$$\det U'=\det(RUL^\dagger)=(\det U)e^{\ln\det(RL^\dagger)}=e^{-i[\theta+i\ln\det(RL^\dagger)]}=e^{-i\theta'}$$

$$\phi[(L,R),U(x)]=RU(x)L^\dagger\qquad \phi[(V,V),1]=V1V^\dagger=1\qquad \phi[(A,A^\dagger),1]=A^\dagger 1A^\dagger\neq 1$$

- $\phi^a[g,0]$可以用来描述π^a
- 所有使$\phi(h,0)=0$的$h\subset$SU(3)$_L\times$SU(3)$_R$形成子群H
- 对所有的$g\subset$SU(3)$_L\times$SU(3)$_R$和$h\subset$H有，$\phi(g,0)=\phi(gh,0)$
- 对所有的$g_1,g_2\subset$SU(3)$_L\times$SU(3)$_R$,若$\phi(g_1,0)=\phi(g_2,0)$则$g_1^{-1}g_2\subset$H

赝标Goldstone粒子场$\phi(g,0)$同构于SU(3)$_L\times$SU(3)$_R$/SU(3)$_V$！
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每个陪集上都选定代表元素后，任意的一个群元都可唯一地分解为nh
对任意的群元g及陪集上的代表元n，我们有$gn=n'h$决定了n在操作g下到的n'变换行为，其中n'也是陪集上的代表元。

另类选择：$n=(\Omega^\dagger,\Omega)\qquad gn=(L,R)(\Omega^\dagger,\Omega)=(L\Omega^\dagger,R\Omega)=(L\Omega^\dagger\tilde{h}^\dagger,R\Omega\tilde{h}^\dagger)(\tilde{h}\tilde{h})=n'h$

$$\xRightarrow{\text{存在}h=(\tilde{h}\tilde{h})\text{使}} R\Omega\tilde{h}^\dagger=[L\Omega^\dagger\tilde{h}^\dagger]^\dagger=\tilde{h}\Omega L^\dagger \qquad \Omega\to\Omega'=R\Omega\tilde{h}^\dagger=\tilde{h}\Omega^\dagger L^\dagger$$

$$\phi[(L,R),\Omega(x)]=R\Omega\tilde{h}^\dagger=\tilde{h}\Omega^\dagger L^\dagger \qquad \phi[(V,V),1]=V1V^\dagger=1 \qquad \phi[(A,A^\dagger),1]=A^\dagger 1\tilde{h}^\dagger=\tilde{h}1A^\dagger\neq 1$$

$$\Omega^2(x)\to\Omega^{2\prime}(x)=R\Omega^2(x)L^\dagger \qquad \Rightarrow \quad U=\Omega^2 \qquad \Omega(x)=e^{i\phi(x)/(2F_0)}$$

$$\phi\overset{\text{真空}}{=\!=\!=}0 \qquad \Omega\overset{\text{真空}}{=\!=\!=}1$$

手征有效拉氏量

$$W[s',p',v',a',\theta'] = W[s,p,v,a,\theta] - \int d^4x \mathrm{tr}[\beta(x)\tilde{\Omega}(x)] \qquad \theta' = \theta - 2\mathrm{tr}\beta = \theta + i\ln\det(RL^\dagger)$$

$$Z[s,p,v,a,\theta] = e^{iW[s,p,v,a,\theta]} = \int \mathcal{D}\mu_U\, e^{i\int d^4x\, \mathcal{L}_{\text{eff}}[U,s,p,v,a,\theta]}$$

$$e^{iW[s',p',v',a',\theta']} = \int \mathcal{D}\mu_{U'}\, e^{i\int d^4x\, \mathcal{L}_{\text{eff}}[U',s',p',v',a',\theta']} \overset{\mathcal{D}\mu_{U'}=\mathcal{D}\mu_U}{=\!=\!=\!=\!=\!=} \int \mathcal{D}\mu_U\, e^{i\int d^4x\, \mathcal{L}_{\text{eff}}[U',s',p',v',a',\theta']}$$

$$= e^{iW[s,p,v,a,\theta]-\int d^4x\, \mathrm{tr}[\beta\tilde{\Omega}]} = \int \mathcal{D}\mu_U\, e^{i\int d^4x\, \{\mathcal{L}_{\text{eff}}[U,s,p,v,a,\theta]-\mathrm{tr}[\beta\tilde{\Omega}]\}}$$

$$\mathcal{L}_{\text{eff}}[U',s',p',v',a',\theta'] = \mathcal{L}_{\text{eff}}[U,s,p,v,a,\theta] - \mathrm{tr}[\beta\tilde{\Omega}]$$

反常部分留待下节讨论

对有限大变换： $R = e^{i[\beta+\alpha]}$ $L = e^{-i[\beta-\alpha]}$ $J' = e^{\delta_\beta} J$ $\theta' = e^{\delta_\beta}\theta = \theta - 2\mathrm{tr}\beta$

$$\mathcal{L}_{\text{eff}}[U',J'] = e^{\delta_\beta}\mathcal{L}_{\text{eff}}[U,J] = \mathcal{L}_{\text{eff}}[U,J] + \frac{e^{\delta_\beta}-1}{\delta_\beta}\delta_\beta\mathcal{L}_{\text{eff}}[U,J] = \mathcal{L}_{\text{eff}}[U,J] - \int_0^1 dt \int d^4x\, e^{t\delta_\beta}\, \mathrm{tr}[\beta\tilde{\Omega}]$$

一般地： $U' = RUL^\dagger$ $\quad s' - ip' = L(s-ip)R^\dagger$ $\quad s' + ip' = R(s+ip)L^\dagger$

$$v'_\mu + a'_\mu = R(v_\mu + a_\mu)R^\dagger + Ri\partial_\mu R^\dagger \qquad v'_\mu - a'_\mu = L(v_\mu - a_\mu)L^\dagger + Li\partial_\mu L^\dagger$$

$$\nabla_\mu U \equiv \partial_\mu U - i(v_\mu + a_\mu)U + iU(v_\mu - a_\mu) \qquad (\nabla_\mu U)' = \partial_\mu U' - i(v'_\mu + a'_\mu)U' + iU'(v'_\mu - a'_\mu) = R(\nabla^\mu U)L^\dagger$$

$$\det U = e^{-i\theta} \qquad \mathrm{tr}(U^\dagger\nabla_\mu U) = -i\nabla_\mu\theta \qquad \nabla_\mu\theta = \partial_\mu + 2\mathrm{tr}a_\mu$$

低能展开的数幂

- U取为p^0 包含进所有的信息
- ∂_μ取为p^1
- v_μ，a_μ取为p^1 协变微商的整体性
- $s \sim M \xRightarrow{\text{Dashen公式}} M_\pi^2$取为$p^2$ 保证π介子的极点
- p取为p^2

$$\mathcal{L}_{\rm eff}^{(0)} = 0$$

$$\mathcal{L}_{\rm eff}^{(2)} = \frac{F_0^2}{4}\left[{\rm tr}[\nabla_\mu U \nabla^\mu U^\dagger] + 2B_0{\rm tr}[(s-ip)U + (s+ip)U^\dagger]\right] + \frac{H_0}{12}{\rm tr}(\nabla_\mu\theta\nabla^\mu\theta)$$

$$\begin{aligned}\mathcal{L}_{\rm eff}^{(4)} = & L_1\{{\rm tr}[\nabla_\mu U(\nabla^\mu U)^\dagger]\}^2 + L_2{\rm tr}[\nabla_\mu U(\nabla_\nu U)^\dagger{\rm tr}[\nabla^\mu U(\nabla^\nu U)^\dagger] \\ & + L_3{\rm tr}[\nabla_\mu U(\nabla^\mu U)^\dagger\nabla_\nu U(\nabla^\nu U)^\dagger] + L_4{\rm tr}[\nabla_\mu U(\nabla^\mu U)^\dagger]{\rm tr}(\chi U^\dagger + U\chi^\dagger) \\ & + L_5{\rm tr}[\nabla_\mu U(\nabla^\mu U)^\dagger(\chi U^\dagger + U\chi^\dagger)] + L_6[{\rm tr}(\chi U^\dagger + U\chi^\dagger)]^2 + L_7[{\rm tr}(\chi U^\dagger - U\chi^\dagger)]^2 \\ & + L_8{\rm tr}(U\chi^\dagger U\chi^\dagger + \chi U^\dagger\chi U^\dagger) - iL_9{\rm tr}[f_{\mu\nu}^R\nabla^\mu U(\nabla^\nu U)^\dagger + f_{\mu\nu}^L(\nabla^\mu U)^\dagger\nabla^\nu U] \\ & + L_{10}{\rm tr}(Uf_{\mu\nu}^L U^\dagger f_R^{\mu\nu}) + H_1{\rm tr}(f_{\mu\nu}^R f_R^{\mu\nu} + f_{\mu\nu}^L f_L^{\mu\nu}) + H_2{\rm tr}(\chi\chi^\dagger)\end{aligned}$$

For $O(p^4)$: $\theta=1$; ${\rm det}U=1$; ${\rm tr}v_\mu={\rm tr}a_\mu=0$; $\chi\equiv 2B_0(s+ip)$; $f_{\mu\nu}^R\equiv\partial_\mu r_\nu-\partial_\nu r_\mu-i[r_\mu,r_\nu]$; $f_{\mu\nu}^L\equiv\partial_\mu l_\nu-\partial_\nu l_\mu-i[l_\mu,l_\nu]$

另一种方式参数化手征拉氏量

取特定的手征转动$L = R^\dagger = \Omega \quad U = \Omega^2 \;\Rightarrow\; U_\Omega = RUL^\dagger = \Omega^\dagger U \Omega^\dagger = 1$

$$J_\Omega \equiv \not{v}_\Omega + \not{a}_\Omega(x)\gamma_5 - s_\Omega + i p_\Omega \gamma_5 = [\Omega P_R + \Omega^\dagger P_L]\,[J + \not{\partial}_x]\,[\Omega P_R + \Omega^\dagger P_L]$$

$$s_\Omega = \frac{1}{2}[\Omega(s - ip)\Omega + \Omega^\dagger(s + ip)\Omega^\dagger] \qquad p_\Omega = \frac{i}{2}[\Omega(s - ip)\Omega - \Omega^\dagger(s + ip)\Omega^\dagger]$$

$$v_\Omega^\mu = \frac{1}{2}[\Omega^\dagger(v^\mu + a^\mu + i\partial^\mu)\Omega + \Omega(v^\mu - a^\mu + i\partial^\mu)\Omega^\dagger] \quad a_\Omega^\mu = \frac{1}{2}[\Omega^\dagger(v^\mu + a^\mu + i\partial^\mu)\Omega - \Omega(v^\mu - a^\mu + i\partial^\mu)\Omega^\dagger]$$

$\Omega \equiv e^{-i\beta} \quad \beta = -\frac{\phi}{2f_\pi} \quad \mathcal{L}_{\rm eff}[U, J] = \mathcal{L}_{\rm eff}[1, J_\Omega] + \int_0^1 dt\; e^{t\delta_\beta}\; {\rm tr}_f[\beta(x)\tilde{\Omega}(x)]$

$$\mathcal{L}_{\rm eff}^{(2)} = F_0^2 {\rm tr}[a_\Omega^2 + B_0 s_\Omega]$$

$$\begin{aligned}\mathcal{L}_{\rm eff}^{(4)} = {\rm tr}\{&-\mathcal{K}_1[d_\mu a_\Omega^\mu]^2 - \mathcal{K}_2(d^\mu a_\Omega^\nu - d^\nu a_\Omega^\mu)(d_\mu a_{\Omega,\nu} - d_\nu a_{\Omega,\mu}) + \mathcal{K}_3[a_\Omega^2]^2\\ &+\mathcal{K}_4 a_\Omega^\mu a_\Omega^\nu a_{\Omega,\mu} a_{\Omega,\nu} + \mathcal{K}_5 a_\Omega^2 {\rm tr}_f[a_\Omega^2] + \mathcal{K}_6 a_\Omega^\mu a_\Omega^\nu {\rm tr}[a_{\Omega,\mu} a_{\Omega,\nu}] + \mathcal{K}_7 s_\Omega^2\\ &+\mathcal{K}_8 s_\Omega {\rm tr}[s_\Omega] + \mathcal{K}_9 p_\Omega^2 + \mathcal{K}_{10} p_\Omega {\rm tr}[p_\Omega] + \mathcal{K}_{11} s_\Omega a_\Omega^2 + \mathcal{K}_{12} s_\Omega {\rm tr}[a_\Omega^2]\\ &-\mathcal{K}_{13} V_\Omega^{\mu\nu} V_{\Omega,\mu\nu} + i\mathcal{K}_{14} V_\Omega^{\mu\nu} a_{\Omega,\mu} a_{\Omega,\nu} + \mathcal{K}_{15} p_\Omega d^\mu a_{\Omega,\mu}\}\end{aligned}$$

$$d^\mu a_\Omega^\nu = [\partial^\mu - i v_\Omega^\mu,\; a_\Omega^\nu] \qquad V_\Omega^{\mu\nu} = i[\bar{v}_\Omega^\mu, \bar{v}_\Omega^\nu]$$

另一种方式参数化手征拉氏量

$$L_1 = \frac{1}{32}\mathcal{K}_4 + \frac{1}{16}\mathcal{K}_5 + \frac{1}{16}\mathcal{K}_{13} - \frac{1}{32}\mathcal{K}_{14} \qquad L_2 = \frac{1}{16}(\mathcal{K}_4 + \mathcal{K}_6) + \frac{1}{8}\mathcal{K}_{13} - \frac{1}{16}\mathcal{K}_{14}$$

$$L_3 = \frac{1}{16}(\mathcal{K}_3 - 2\mathcal{K}_4 - 6\mathcal{K}_{13} + 3\mathcal{K}_{14}) \qquad L_4 = \frac{\mathcal{K}_{12}}{16B_0} \qquad L_5 = \frac{\mathcal{K}_{11}}{16B_0} \qquad L_6 = \frac{\mathcal{K}_8}{16B_0^2}$$

$$L_7 = -\frac{\mathcal{K}_1}{16N_f} - \frac{\mathcal{K}_{10}}{16B_0^2} - \frac{\mathcal{K}_{15}}{16B_0N_f} \qquad L_8 = \frac{1}{16}[\mathcal{K}_1 + \frac{1}{B_0^2}\mathcal{K}_7 - \frac{1}{B_0^2}\mathcal{K}_9 + \frac{1}{B_0}\mathcal{K}_{15}]$$

$$L_9 = \frac{1}{8}(4\mathcal{K}_{13} - \mathcal{K}_{14}) \qquad L_{10} = \frac{1}{2}(\mathcal{K}_2 - \mathcal{K}_{13}) \qquad H_1 = -\frac{1}{4}(\mathcal{K}_2 + \mathcal{K}_{13})$$

$$H_2 = \frac{1}{8}[-\mathcal{K}_1 + \frac{1}{B_0^2}\mathcal{K}_7 + \frac{1}{B_0^2}\mathcal{K}_9 - \frac{1}{B_0}\mathcal{K}_{15}]$$

$$d^\mu a_\Omega^\nu - d^\nu a_\Omega^\mu = \frac{1}{2}[\Omega^\dagger f_R^{\mu\nu}\Omega - \Omega f_L^{\mu\nu}\Omega^\dagger] \qquad a_\Omega^\mu = \frac{i}{2}\Omega^\dagger[\nabla^\mu U]\Omega^\dagger$$

$$s_\Omega = \frac{1}{2}[\Omega(s - ip)\Omega + \Omega^\dagger(s + ip)\Omega^\dagger] \qquad p_\Omega = \frac{i}{2}[\Omega(s - ip)\Omega - \Omega^\dagger(s + ip)\Omega^\dagger]$$

$$V_\Omega^{\mu\nu} = \frac{i}{4}\Omega^\dagger[-(\nabla^\mu U)U^\dagger(\nabla^\nu U) + (\nabla^\nu U)U^\dagger(\nabla^\mu U)]\Omega^\dagger + \frac{1}{2}[\Omega^\dagger f_R^{\mu\nu}\Omega + \Omega f_L^{\mu\nu}\Omega^\dagger]$$

$$d_\mu a_\Omega^\mu = -B_0[p_\Omega - \frac{1}{3}\mathrm{tr}_f(p_\Omega)]$$

$$f_\pi = F_0\left[1 - 2\mu_\pi - \mu_K + 2\hat{m}K_6 + K_7\right] \qquad \mu_P \equiv \frac{1}{32\pi^2}\frac{m_P^2}{F_0^2}\ln\frac{m_P^2}{\mu^2} \qquad \mu = m_\eta$$

93MeV

$$K_6 \equiv \frac{4B_0}{F_0^2}L_5 \qquad L_7 \equiv (m_u + m_d + m_s)\frac{8B_0}{F_0^2}L_6$$

$L_1 = (0.9 \pm 0.3) \times 10^{-3}$ $\quad L_2 = (1.7 \pm 0.7) \times 10^{-3}$ $\quad L_3 = (-4.4 \pm 2.5) \times 10^{-3}$

$L_4 = (0 \pm 0.5) \times 10^{-3}$ $\quad L_5 = (2.2 \pm 0.5) \times 10^{-3}$ $\quad L_6 = (0 \pm 0.3) \times 10^{-3}$

$L_7 = (-0.4 \pm 0.15) \times 10^{-3}$ $\quad L_8 = (1.1 \pm 0.3) \times 10^{-3}$ $\quad L_9 = (7.4 \pm 0.7) \times 10^{-3}$

$L_{10} = (-6.0 \pm 0.7) \times 10^{-3}$

如何引入电弱相互作用？ 考虑$N_f = 2$情形:

$$\mathcal{L} = (\bar{u}\ \bar{d})[i\partial\!\!\!/ + g\frac{\lambda_a}{2}G\!\!\!/^a]\begin{pmatrix}u\\d\end{pmatrix} + (\bar{u}\ \bar{d})[g_2\frac{\tau^a}{2}W\!\!\!\!/^a P_L + \frac{g_1}{6}B\!\!\!/ P_L]\begin{pmatrix}u\\d\end{pmatrix} + \bar{u}\frac{2}{3}g_1 B\!\!\!/ P_R u - \bar{d}\frac{1}{3}g_1 B\!\!\!/ P_R d$$

$$= (\bar{u}\ \bar{d})[i\partial\!\!\!/ + g\frac{\lambda_a}{2}G\!\!\!/^a + g_2\frac{\tau^a}{2}W\!\!\!\!/^a P_L + \frac{g_1}{6}B\!\!\!/ P_L + g_1 B\!\!\!/(\frac{1}{6} + \frac{\tau^3}{2})P_R]\begin{pmatrix}u\\d\end{pmatrix}$$

$$= (\bar{u}\ \bar{d})[i\partial\!\!\!/ + g\frac{\lambda_a}{2}G\!\!\!/^a + g_2\frac{\tau^a}{2}W\!\!\!\!/^a P_L + \frac{g_1}{6}B\!\!\!/ + g_1 B\!\!\!/\frac{\tau^3}{2}P_R]\begin{pmatrix}u\\d\end{pmatrix}$$

$N_f = 3$怎么办？

$$(\bar{u}\ \bar{d})[v\!\!\!/ + a\!\!\!/\gamma_5]\begin{pmatrix}u\\d\end{pmatrix} \Leftrightarrow v_\mu = g_2\frac{\tau^a}{4}W_\mu^a + (\frac{1}{6} + \frac{\tau^3}{4})g_1 B_\mu \qquad a^\mu = -g_2\frac{\tau^a}{4}W_\mu^a + (\frac{1}{6} + \frac{\tau^3}{4})g_1 B_\mu$$

Weinberg数幂规则

J.F.Donoghue, E.Golowich and B.R.Holestein 《Dynamics of the Standard Model》pp.107-109

无外源的Goldstone反应振幅 $T \sim m^{\mathcal{D}_m} E^{\mathcal{D}_E} H(\ln \frac{E}{\mu})$ $\qquad \mathcal{D}_m + \mathcal{D}_E = \mathcal{D}_T = 4 - N_E$ 外线数

v_n个第n类型顶角，其有b_n条外线d_n个微商： $\Rightarrow$ 这些顶角的耦合常数量纲 $\sim m^{\sum v_n(4-d_n-b_n)}$

$\sum v_n b_n = N_E + 2N_I$ 外线数 $\qquad N_I = N_L$ 圈数 $+ \sum v_n - 1$

$$\mathcal{D}_m = \sum v_n(4-d_n) - N_E - 2N_I = \sum v_n(2-d_n) + 2 - N_E - 2N_L$$

$$\mathcal{D}_E = 4 - N_E - \mathcal{D}_m = \sum v_n(d_n - 2) + 2N_L + 2 \qquad T \sim \frac{E^2}{f_\pi^{N_E-2}}\left[1 + O\left(\frac{1}{16\pi^2}\frac{E^2}{f_\pi^2}\right) + \ldots\ldots\right]$$

- p^2阶： 只能考虑p^2顶角的树图
- p^4阶： a. p^2顶角的一圈图 b. 含一个p^4顶角其它都是p^2顶角的树图 回上页f_π

  p^4顶角系数可抵消p^2顶角一圈图产生的发散！ $\Rightarrow$ p^4顶角系数是p^2顶角一圈图贡献的量级$\sim 1/(16\pi^2)$

  理解上页实验定出的L_i系数的量级！ 低能展开的上限$\Lambda = 4\pi f_\pi$
- p^6阶： a. p^2顶角的二圈图 b. 含一个p^4顶角其它都是p^2顶角的一圈图
  c. 含两个p^4顶角其它都是p^2顶角的树图
  d. 含一个p^6顶角其它都是p^2顶角的树图

从QCD反常理解Gasser-Leutwyler拉氏量

- D.Espriu, E.De Rafael, and J.Taron, Nucl.Phys.B345,22(1990).
- A.A. Andrianov et al, Phys. Lett. B186, 401 (1987); A.A. Andrianov, Phys. Lett. B157, 425 (1985);
- N.I.Karchev and A.A.Slavnov, Theor.Mat.Phys.65,192(1985);
- L.-H. Chan, Phys. Rev. Lett. 55, 21 (1985);
- A. Zaks, Nucl. Phys. B 260, 241 (1985);
- J. Balog. Phys. Lett. B 149, 197 (1984);
- A.A. Andrianov and L. Bonora, Nucl. Phys. B233, 232(1984);

$$\int \mathcal{D}\mu_U\, e^{i\int d^4x\, \mathcal{L}_{\rm eff}[U,J]} = e^{iW[J]} = \int \mathcal{D}G\mathcal{D}\Psi\mathcal{D}\overline{\Psi}\mathcal{D}\psi\mathcal{D}\overline{\psi}\, e^{i\int d^4x[\mathcal{L}_{\rm QCD}(\theta,\psi,\Psi,G)+\overline{\psi}J\psi]}$$

$$\psi' = [RP_R + LP_L]\psi \qquad J' = [RP_L + LP_R][J + i\partial\!\!\!/][R^\dagger P_R + L^\dagger P_L] \qquad U' = RUL^\dagger$$

$$U = \Omega^2 \qquad \psi_\Omega = [\Omega^\dagger P_R + \Omega P_L]\psi \qquad J_\Omega = [\Omega P_R + \Omega^\dagger P_L][J + i\partial\!\!\!/][\Omega P_R + \Omega^\dagger P_L] \qquad U_\Omega = 1$$

$$e^{iW[J]} \overset{W[J']=W[J]+\text{反常}}{=\!=\!=\!=\!=\!=\!=\!=\!=} \int \mathcal{D}\mu_U\, e^{iW[J_\Omega]} = \int \mathcal{D}\mu_U \int \mathcal{D}G\mathcal{D}\Psi\mathcal{D}\overline{\Psi}\mathcal{D}\psi\mathcal{D}\overline{\psi}\, e^{i\int d^4x[\mathcal{L}_{\rm QCD}(\theta,\psi,\Psi,G)+\overline{\psi}J_\Omega\psi]}$$

$$e^{i\int d^4x\, \mathcal{L}_{\rm eff}[U,J]} = \frac{\int \mathcal{D}G\mathcal{D}\Psi\mathcal{D}\overline{\Psi}\mathcal{D}\psi_\Omega\mathcal{D}\overline{\psi}_\Omega\, e^{i\int d^4x[\mathcal{L}_{\rm QCD}(\theta,\psi,\Psi,G)+\overline{\psi}J\psi]}}{\int \mathcal{D}G\mathcal{D}\Psi\mathcal{D}\overline{\Psi}\mathcal{D}\psi\mathcal{D}\overline{\psi}\, e^{i\int d^4x[\mathcal{L}_{\rm QCD}(\theta,\psi,\Psi,G)+\overline{\psi}J\psi]}} = \text{QCD反常}$$

$$8L_1 = 4L_2 = -2L_3 = 24L_7 = -8L_8 = L_9 = -2L_{10} = N_c/(48\pi^2) \qquad F_0 = 0, \propto \Lambda \quad \Leftarrow \text{略去QCD动力学}$$

或手放进理论： $-M_Q(\overline{\psi}_R U\psi_L + \overline{\psi}_L U^\dagger \psi_R) \Rightarrow f_\pi^2 = M_Q^2/(4\pi^4)[1 + \ldots\ldots]$

从QCD动力学理解Gasser-Leutwyler拉氏量

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- Q.Wang, Y.P.Kuang, H.Yang, Q.Lu,J. Phys. G28, L55(2002)
- H.Yang, Q.Wang, Y.P.Kuang and Q.Lu, Phys. Rev. D66 014019(2002)
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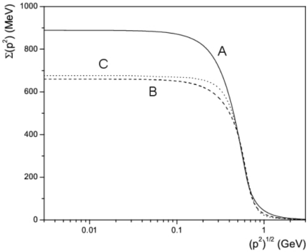
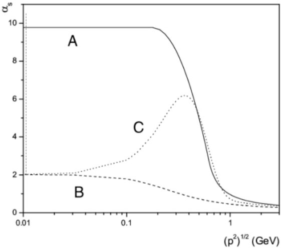
- 从QCD出发，形式地严格积掉夸克和胶子，积进赝标介子，得到CL
- 所有手征拉氏量中的参数都可严格表为QCD的一些多点夸克格林函数
- 理论中的SCSB与否要由理论本身的自恰动力学方程SDE的解决定
- 如果发生了SCSB，赝标介子就是Goldstone玻色子
- 近似计算表明手征对称性确实发生自发破缺
- QCD反常的贡献被严格抵消了!
- CL中的参数完全依赖QCD的SCSB夸克自能$\Sigma(p^2)$，数值与实验结果一致

对探索新物理的应用!



Exp(Old)：Gasser&Leutwyler 84 Exp(New)：A.Pich MENU2004

	L_1	L_2	L_3	L_4	L_5
A	1.10	2.20	-7.82	0	1.62
B	0.921	1.84	-6.73	0	1.43
C	0.948	1.90	-6.90	0	1.29
Anomaly	0.79	1.58	-3.17	0	0
Exp (Old)	0.9 ± 0.3	1.7 ± 0.7	-4.4 ± 2.5	0 ± 0.5	2.2 ± 0.5
Exp (New)	0.4 ± 0.3	1.4 ± 0.3	-3.5 ± 1.1	-0.3 ± 0.5	1.4 ± 0.5



$\Leftarrow$Kernel of SD eq is proportional to α_s with its low energy behavior not known, we model its possible low energy behavior by : A(solid line), B(dash line), C(dot line)

	L_6	L_7	L_8	L_9	L_{10}
A	0	-0.70	1.75	5.07	-7.06
B	0	-0.673	1.64	3.80	-6.22
C	0	-0.632	1.56	3.95	-6.21
Amomaly	0	0.26	-0.79	6.33	-3.17
Exp (Old)	0 ± 0.3	-0.4 ± 0.15	1.1 ± 0.3	7.4 ± 0.7	-6.0 ± 0.7
Exp (New)	-0.2 ± 0.3	-0.4 ± 0.2	0.9 ± 0.3	6.9 ± 0.7	-5.5 ± 0.7

orders and signs are correct !

E.Witten, Nucl.Phys.B223,422(1983)

分析不含外源的手征拉氏量的对称性$\mathcal{L} = \frac{f_\pi^2}{4}\mathrm{tr}(\partial_\mu U \partial^\mu U^\dagger)$

- $U \to U^T \xLeftrightarrow{\text{电荷共轭变换}} \pi^+ \leftrightarrow \pi^- \quad K^+ \leftrightarrow K^- \quad K^0 \leftrightarrow \bar{K}^0$

$$U = e^{i\frac{\phi}{f_\pi}} \qquad \phi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{\sqrt{2}}{\sqrt{3}}\eta \end{pmatrix}$$

- Naive Parity P_0 不是物理的宇称变换: $x \to -x,\ t \leftrightarrow t,\ U \leftrightarrow U$
- $U \leftrightarrow U^{-1} \xLeftrightarrow{\text{内部宇称变换}} \phi \leftrightarrow -\phi$ 对N_B个Goldstone的顶角$(-1)^{N_B}$ 不是物理的宇称变换

物理上的宇称: $P_0(-1)^{N_B}$

QCD只具有电荷共轭和物理的宇称对称性，不分别具有P_0或$(-1)^{N_B}$对称性！

寻找不分别具有P_0和$(-1)^{N_B}$但有$P_0(-1)^{N_B}$的物理过程

有效理论中应有不分别具有P_0和$(-1)^{N_B}$但有$P_0(-1)^{N_B}$对称性的项！

寻找不分别具有P_0和$(-1)^{N_B}$但有$P_0(-1)^{N_B}$的物理过程

$$\phi \rightarrow \underbrace{K^+K^-}\ \xrightarrow{\text{末态相互作用}}\ \pi^0\pi^+\pi^-$$

$J^P=1^-$ $J^P=1^-$ $J^P=1^-$

K^+K^- : $(-1)^{N_B}=1$ 轨道角动量$l \xlongequal{\text{两个无自旋赝标介子形成}J=1\text{的态}} 1$ $P_0 \xlongequal{\text{两个赝标介子形成}J=1\text{为矢量态}} -1$

$J=1$由$l=1$实现；$P=-$由$P_0=-1$实现

$\pi^0\pi^+\pi^-$: $(-1)^{N_B}=-1$ 轨道角动量$l \xlongequal{\text{三个无自旋赝标介子形成}J=1\text{的态}} 1$ $P_0 \xlongequal{\text{三个赝标介子形成}J=1\text{为轴矢量态}} 1$

$J=1$由$l=1$实现；$P=-$由$(-1)^{N_B}=-1$实现

$K^+K^- \rightarrow \pi^0\pi^=\pi^-$的过程不分别具有$P_0$和$(-1)^{N_B}$但有$P_0(-1)^{N_B}$对称性！

它不能用我们已建立的手征拉氏量描述因它不分别具有P_0和$(-1)^{N_B}$对称性

手征拉氏量的$U=e^{i\frac{\phi}{f_\pi}}$的$(-1)^{N_B}$对称性在按$\phi$的幂次展开后只允许存在偶数个$\phi$。

需要在建立的手征拉氏量中加入新的相互作用项描述这类物理过程！

原本手征拉氏量中就还应该有反常项 在外源变化下改变的项

分析运动方程：$\partial_\mu(\frac{1}{2}f_\pi^2 U^{-1}\partial^\mu U)=0$

$$\partial_\mu(\frac{1}{2}f_\pi^2 U^{-1}\partial^\mu U)+\lambda\epsilon^{\mu\nu\sigma\rho}[U^{-1}(\partial_\mu U)U^{-1}(\partial_\nu U)U^{-1}(\partial_\sigma U)U^{-1}(\partial_\rho U)]=0$$

P_0 : 1 $\quad -1$

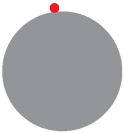
$(-1)^{N_B}$: -1 $\quad 1$ $\quad U^{-1}\partial_\mu U \xRightarrow{U\leftrightarrow U^{-1}} U\partial_\mu U^{-1}=-(\partial_\mu U)U^{-1}$

运动方程式中的两项所具有的物理宇称对称性$P=P_0(-1)^{N_B}$是一样的！

对应在拉氏量中应该加入：$\lambda\epsilon^{\mu\nu\sigma\rho}\mathrm{tr}[U^{-1}(\partial_\mu U)U^{-1}(\partial_\nu U)U^{-1}(\partial_\sigma U)U^{-1}(\partial_\rho U)]$

但这项恒为零！ 为什么非平庸的场方程项没有非平庸的拉氏量对应？

考虑一个简化的问题： 自由质点在单位球上运动



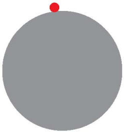
$L=\frac{1}{2}m\dot{\vec{r}}^2$ $\quad \vec{r}\cdot\vec{r}=1$ $\quad$ 拉氏量和约束条件分别在$\vec{r}\leftrightarrow-\vec{r}$和$t\leftrightarrow -t$ 下都保持不变

能否在拉氏量中加一项使$\vec{r}\leftrightarrow-\vec{r}$和$t\leftrightarrow -t$ 分别不成立，但联合变换成立？

$S=\int dt\,[\frac{1}{2}m\dot{x}_i^2+\lambda(x_i^2-1)] \xRightarrow{\text{运动方程}} m\ddot{x}_i-2\lambda x_i=0 \Rightarrow \lambda=-\frac{m}{2}\dot{x}_i^2 \Leftarrow \dot{x}_i^2+x_i\ddot{x}_i=0 \xLeftarrow{\text{约束条件微商两次}}$

$\Rightarrow m(\ddot{x}_i+x_i\dot{x}_j^2)=0$ 空间反射变号、时间反演不变号 $\Rightarrow m(\ddot{x}_i+x_i\dot{x}_j^2)=\alpha\epsilon^{ijk}x_j\dot{x}_k \xRightarrow{\text{对应拉氏量}} \alpha\epsilon^{ijk}x_i x_j\dot{x}_k=0$

考虑一个简化的问题： 自由质点在单位球上运动



$$L=\frac{1}{2}m\dot{\vec{r}}^2 \qquad \vec{r}\cdot\vec{r}=1$$

拉氏量和约束条件分别在 $\vec{r}\leftrightarrow-\vec{r}$ 和 $t\leftrightarrow -t$ 下都保持不变

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重写新加入的力项： $\alpha\epsilon^{ijk}x_j\dot{x}_k=-\alpha(\dot{r}\times\frac{\vec{r}}{r^3})^i$ 洛伦兹力 $q\vec{v}\times\vec{B}$

以库伦势形式出现的磁场: $\vec{B}=\frac{\vec{r}}{r^3}$ $\Leftarrow$ 磁场只能由位于球心的单位点磁荷产生！

所需加的外力实际上就是在球心加一单位磁荷的磁单极，并认为质点带电荷 $-\alpha$

因而受洛伦兹力 $-\alpha\vec{v}\times\vec{B}$

但磁单极势不能在全空间定义,存在Dirac弦 像无穷长无穷细螺线管 $\nabla\cdot\vec{B}\neq 0,\ \vec{B}=\nabla\times\vec{A}\Rightarrow\nabla\cdot\vec{B}=0$

此种问题对应的拉氏量不能在连续可微的函数中寻找！

$$S=\int dt\,[\frac{1}{2}m\dot{x}_i^2+\alpha\dot{x}_iA_i]$$ A_i 有奇异性

点磁荷在球心有磁单极单位球上运动： $\int \mathcal{D}x_i\, e^{iS}$ 路径积分

$$S = \int dt\, [\frac{1}{2}m\dot{x}_i^2 + \alpha \dot{x}_i A_i] \qquad \int dt \alpha \dot{x}_i A_i = \int_\gamma \alpha A_i dx_i$$

γ封闭路径： $x_i(t=\infty) = x_i(t=-\infty)$

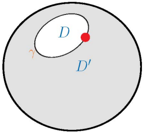
势A_i是奇异的，如何表达它？

路径积分轨道到二维球面的所有可能映射都是拓扑等价的 $\Pi_1(S^2) = 0$

路径积分轨道映射到二维球面的曲线所包围的面积 无孔 可应用格林公式

$$e^{i\alpha \int_\gamma A_i dx_i} = e^{i\alpha \iint_D F_{ij} d\Sigma^{ij}}$$

D: γ所包围的面积　　磁场F_{ij}在球面上是没有奇异性的



面积D有两种取法D, D',若要求理论自洽 $\oint_D F_{ij} d\Sigma^{ij} = -\int d\vec{S}\cdot\nabla\frac{1}{r} = -\int d\tau \nabla^2 \frac{1}{r} = 4\pi$

$$e^{i\alpha \iint_D F_{ij} d\Sigma^{ij}} = e^{-i\alpha \iint_{D'} F_{ij} d\Sigma^{ij}} \Rightarrow 1 = e^{i\alpha \oint F_{ij} d\Sigma^{ij}} \xlongequal{\text{单位磁荷磁通量}} e^{4i\pi\alpha} \Rightarrow \alpha = \frac{n}{2}$$

Dirac磁荷量子化

所引入磁力的作用量在一维空间中无法定义，但在二维球面上确可很好地定义！

通过升高空间维数，可解决作用量无定义问题！ $S = \frac{1}{2}m\int dt\, \dot{x}_i^2 + \frac{n}{2}\iint_D F_{ij} d\Sigma^{ij}$

回来讨论反常部分的手征拉氏量

在四维欧氏时空E_4讨论：将无穷远看成同一点$\Rightarrow$ S_4映射到$SU(3)$ $\Pi_4(SU(3))=0$

反常作用量应定义在四维空间S_4的五维盘Q或Q'上：

$$\Gamma=\int_Q \omega_{ijklm}d\Sigma^{ijklm} \qquad \Gamma'=-\int_{Q'}\omega_{ijklm}d\Sigma^{ijklm} \xRightarrow{e^{i\Gamma}=e^{i\Gamma'}} \oint_{S_5=Q+Q'}\omega_{ijklm}d\Sigma^{ijklm}=2n\pi$$

$n=1$的结果已由数学家给出： R.Seeley, Comm.Math.Phys.62,235(1978)

$$\omega_{ijklm}d\Sigma^{ijklm}=-\frac{i}{240\pi^2}\mathrm{tr}(U^{-1}\frac{\partial U}{\partial y_i}U^{-1}\frac{\partial U}{\partial y_j}U^{-1}\frac{\partial U}{\partial y_k}U^{-1}\frac{\partial U}{\partial y_l}U^{-1}\frac{\partial U}{\partial y_m})d\Sigma^{ijklm}$$

无外源的反常部分作用量： Wess-Zumino研究过Γ,Witten加上系数n

$$S_{反常}=n\Gamma \quad \Leftarrow \text{Wess-Zumino-Witten项} \qquad \Gamma=\int_Q \omega_{ijklm}d\Sigma^{ijklm}$$

最低阶的物理过程： $U^{-1}\partial_\mu U=\frac{i}{\sqrt{2}f_\pi}\phi+O(\phi^2)$

$$240\pi^2 f_\pi^5\int\omega_{ijklm}d\Sigma^{ijklm}=\int\mathrm{tr}(\partial_i\phi\partial_j\phi\partial_k\phi\partial_l\phi\partial_m\phi)d\Sigma^{ijklm}=\int\partial_i\mathrm{tr}(\phi\partial_j\phi\partial_k\phi\partial_l\phi\partial_m\phi)d\Sigma^{ijklm}$$

$$=\epsilon^{\mu\nu\sigma\rho}\int d^4x\,\mathrm{tr}(\phi\partial_\mu\phi\partial_\nu\phi\partial_\sigma\phi\partial_\rho\phi)+O(\phi^6)$$

加入电磁作用 为了计算n

考虑三个味道: $Q = \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{pmatrix}$

$$\Gamma \Rightarrow \tilde{\Gamma} = \Gamma(U) - e\int j^\mu A_\mu d^4x$$
$$+\frac{ie^2}{24\pi^2}\int \epsilon^{\mu\nu\sigma\rho}(\partial_\mu A_\nu)A_\sigma \mathrm{tr}[Q^2(\partial_\rho U)U^{-1} + Q^2U^{-1}\partial_\rho U + QUQU^{-1}(\partial_\rho U)U^{-1}]d^4x$$

$$j^\mu \equiv \frac{1}{48\pi}\epsilon^{\mu\nu\sigma\rho}\mathrm{tr}[Q(\partial_\nu U)U^{-1}(\partial_\sigma U)U^{-1}(\partial_\rho U)U^{-1} + QU^{-1}(\partial_\nu U)U^{-1}(\partial_\sigma U)U^{-1}(\partial_\rho U)]$$

上式会导致过程$\pi^0 \to \gamma\gamma$,振幅为

$$A = \frac{ne^2}{96\pi^2 f_\pi}\pi^0\epsilon^{\mu\nu\sigma\rho}F_{\mu\nu}F_{\sigma\rho} \xRightarrow{\text{反常导出的公式对比}} n = N_c$$

系统地建立规范化反常作用量的方法(Abel和非Abel): 周光召、郭汉英、宋行长

从变换性质积分得到反常作用量

$$R = e^{i[\beta+\alpha]} \qquad L = e^{-i[\beta-\alpha]} \qquad J' = e^{\delta_\beta} J \qquad \theta' = e^{\delta_\beta}\theta = \theta - 2\mathrm{tr}\beta$$

$$\mathcal{L}_{\text{eff}}[U', J'] = \mathcal{L}_{\text{eff}}[U, J] - \int_0^1 dt \int d^4x\, e^{t\delta_\beta}\ \mathrm{tr}[\beta(x)\tilde{\Omega}(x)]$$

实行特殊的手征转动 $L = R^\dagger = \Omega = e^{-i\beta} \qquad \beta = -\frac{\phi}{2f_\pi} \qquad U_\Omega = 1$

$$\mathcal{L}_{\text{eff}}[U, J] = \mathcal{L}_{\text{eff}}[1, J_\Omega] + \Gamma \qquad \Gamma \equiv \int_0^1 dt \int d^4x\, e^{t\delta_\beta}\ \mathrm{tr}[\beta(x)\tilde{\Omega}(x)]$$

$$\Gamma = (1 - e^{\delta_\beta})Z[J] = Z[J] - Z[J_\Omega] \overset{\Omega_t = e^{-it\beta(x)}}{=\!=\!=\!=\!=\!=} Z[J_{\Omega_0}] - Z[J_{\Omega_1}] = -\int_0^1 dt\, \frac{dZ[J_{\Omega_t}]}{dt}$$

$$= \int_0^1 dt \int d^4x\, \mathrm{tr}[\beta(x)\tilde{\Omega}(x)]\Big|_{J\to J_{\Omega_t}} = \frac{i}{2}\int_0^1 dt \int d^4x\, \mathrm{tr}[\frac{\partial U_t(x)}{\partial t} U_t^\dagger(x)\tilde{\Omega}(x)]\Big|_{J\to J_{\Omega_t}}$$

$$\overset{J=0}{=\!=\!=} -\frac{N_c i}{48\pi^2}\int_0^1 dt \int d^4x\, \epsilon^{\mu\nu\mu'\nu'}\mathrm{tr}\left[U_t^\dagger(x)\frac{\partial U_t(x)}{\partial t} L_\mu(t,x) L_\nu(t,x) L_{\mu'}(t,x) L_{\nu'}(t,x)\right]$$

$$L_\mu \equiv U^\dagger \partial_\mu U \qquad d^\mu a_\Omega^\nu\Big|_{J=0} = d^\nu a_\Omega^\mu\Big|_{J=0} \qquad V_\Omega^{\mu\nu}\Big|_{J=0} = i[a_\Omega^\mu, a_\Omega^\nu]\Big|_{J=0}$$

实行特殊的手征转动 $L = R^\dagger = \Omega = e^{-i\beta}$ $\beta = -\frac{\phi}{2f_\pi}$ $U_\Omega = 1$

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$$\overset{J=0}{===} -\frac{N_c i}{48\pi^2}\int_0^1 dt \int d^4x\, \epsilon^{\mu\nu\mu'\nu'}{\rm tr}\left[U_t^\dagger(x)\frac{\partial U_t(x)}{\partial t}L_\mu(t,x)L_\nu(t,x)L_{\mu'}(t,x)L_{\nu'}(t,x)\right]$$

$$L_\mu \equiv U^\dagger\partial_\mu U \quad d^\mu a^\nu_\Omega\Big|_{J=0} = d^\nu a^\mu_\Omega\Big|_{J=0} \quad V^{\mu\nu}_\Omega\Big|_{J=0} = i[a^\mu_\Omega, a^\nu_\Omega]\Big|_{J=0} \quad \int d^4x\, \epsilon^{\mu\nu\sigma\rho} = \int_Q d\Sigma^{ijklm}\frac{\partial}{\partial y^m}$$

$$\frac{\partial}{\partial t}{\rm tr}[L_i(t,y)L_j(t,y)L_k(t,y)L_l(t,y)L_m(t,y)]d\Sigma^{ijklm} = 5\frac{\partial}{\partial y^m}{\rm tr}[U^\dagger(t,y)\frac{\partial U(t,y)}{\partial t}L_i(t,y)L_j(t,y)L_k(t,y)L_l(t,y)]d\Sigma^{ijklm}$$

$$\int_Q d\Sigma^{ijklm}{\rm tr}[L_i(y)L_j(y)L_k(y)L_l(y)L_m(y)] = \int_Q d\Sigma^{ijklm}\int_0^1 dt\frac{\partial}{\partial t}{\rm tr}[L_i(t,y)L_j(t,y)L_k(t,y)L_l(t,y)L_m(t,y)]$$

$$= 5\int d^4x \int_0^1 dt\, \epsilon^{\mu\nu\sigma\rho}{\rm tr}[U^\dagger(t,x)\frac{\partial U(t,x)}{\partial t}L_\mu(t,x)L_\nu(t,x)L_\sigma(t,x)L_\rho(t,x)]$$

$$\Gamma \overset{J=0}{==} -\frac{N_c i}{48\pi^2}\int_0^1 dt\int d^4x\, \epsilon^{\mu\nu\mu'\nu'}{\rm tr}[U^\dagger\frac{\partial U}{\partial t}L_\mu L_\nu L_{\mu'}L_{\nu'}] = -\frac{N_c i}{240\pi^2}\int_Q d\Sigma^{ijklm}{\rm tr}[L_i(y)L_j(y)L_k(y)L_l(y)L_m(y)]$$

Wess-Zumino-Witten项

$$\Gamma \overset{J=0}{==} -\frac{N_c i}{48\pi^2}\int_0^1 dt \int d^4x\, \epsilon^{\mu\nu\mu'\nu'} \mathrm{tr}[U^\dagger \frac{\partial U}{\partial t} L_\mu L_\nu L_{\mu'} L_{\nu'}] = -\frac{N_c i}{240\pi^2}\int_Q d\Sigma^{ijklm} \mathrm{tr}[L_i(y)L_j(y)L_k(y)L_l(y)L_m(y)]$$

$$\Gamma = \Gamma\Big|_{J=0} + \frac{1}{48\pi^2}\int d^4x \varepsilon_{\mu\nu\alpha\beta} W_{\mu\nu\alpha\beta}$$

$$U_{\mu L} = \partial_\mu U \cdot U^{-1} \qquad U_{\mu R} = U^{-1}\partial_\mu U$$

$$\begin{aligned}
W_{\mu\nu\alpha\beta} = \mathrm{tr}\Big\{ & [-l_\mu U_{\nu L} U_{\alpha L} U_{\beta L} + \partial_\mu l_\nu l_\alpha U_{\beta L} + l_\mu \partial_\nu l_\alpha U_{\beta L} + (L \to R)] \\
& + \partial_\mu l_\nu U r_\alpha U^{-1} U_{\beta L} + U \partial_\mu r_\nu U^{-1} l_\alpha U_{\beta L} - \frac{1}{2}[l_\mu U_{\nu L} l_\alpha U_{\beta L} - L \to R] \\
& + l_\mu U r_\nu U^{-1} U_{\alpha L} U_{\beta L} - U r_\mu U^{-1} l_\nu U_{\alpha L} U_{\beta L} - l_\mu \partial_\nu l_\alpha U r_\beta U^{-1} - \partial_\mu l_\nu l_\alpha U r_\beta U^{-1} \\
& + r_\mu \partial_\nu r_\alpha U^{-1} l_\beta U + \partial_\mu r_\nu r_\alpha U^{-1} l_\beta U + l_\mu U r_\nu U^{-1} l_\alpha U_{\beta L} + r_\mu U^{-1} l_\nu U r_\alpha U_{\beta R} \\
& + [l_\mu l_\nu l_\alpha U_{\beta L} + (L \to R)] - l_\mu l_{nu} l_\alpha U r_\beta U^{-1} + r_\mu r_\nu r_\alpha U^{-1} l_\beta U_{\beta L} \\
& - \frac{1}{2} l_\mu U r_\nu U^{-1} l_\alpha U r_\beta U^{-1} \Big\}
\end{aligned}$$

高阶反常项在对称性变换下是不变的！

从QCD动力学理解反常项 Y.L.Ma, Q.Wang,Phys. Lett. B560, 188 (2003)

$$S_{\text{eff},反常} = -\frac{24}{\pi^2}\epsilon_{\mu\nu\alpha\beta}\int_0^1 dt\int d^4x\,\mathbf{tr_f}\Big\{\frac{\partial U}{\partial t}U^\dagger\Big[C(2\bar\nabla^\mu\bar\nabla^\nu\bar\nabla^\alpha\bar\nabla^\beta + 2a_\Omega^\mu a_\Omega^\nu\bar\nabla^\alpha\bar\nabla^\beta$$
$$-2\bar\nabla^\mu a_\Omega^\nu\bar\nabla^\alpha a_\Omega^\beta + 2\bar\nabla^\mu a_\Omega^\nu a_\Omega^\alpha\bar\nabla^\beta + 2a_\Omega^\mu\bar\nabla^\nu\bar\nabla^\alpha a_\Omega^\beta - 2a_\Omega^\mu\bar\nabla^\nu a_\Omega^\alpha\bar\nabla^\beta$$
$$+2\bar\nabla^\mu\bar\nabla^\nu a_\Omega^\alpha a_\Omega^\beta + 2a_\Omega^\mu a_\Omega^\nu a_\Omega^\alpha a_\Omega^\beta) + C'(4\bar\nabla^\mu\bar\nabla^\nu\bar\nabla^\alpha\bar\nabla^\beta + 2a_\Omega^\mu a_\Omega^\nu\bar\nabla^\alpha\bar\nabla^\beta$$
$$-2\bar\nabla^\mu a_\Omega^\nu\bar\nabla^\alpha a_\Omega^\beta + 4a_\Omega^\mu\bar\nabla^\nu\bar\nabla^\alpha a_\Omega^\beta - 2a_\Omega^\mu\bar\nabla^\nu a_\Omega^\alpha\bar\nabla^\beta + 2\bar\nabla^\mu\bar\nabla^\nu a_\Omega^\alpha a_\Omega^\beta)$$
$$+\frac{C"}{3}(2\bar\nabla^\mu\bar\nabla^\nu\bar\nabla^\alpha\bar\nabla^\beta - 2\bar\nabla^\mu a_\Omega^\nu a_\Omega^\alpha\bar\nabla^\beta + 2a_\Omega^\mu\bar\nabla^\nu\bar\nabla^\alpha a_\Omega^\beta - 2a_\Omega^\mu a_\Omega^\nu a_\Omega^\alpha a_\Omega^\beta)\Big]\Big\} + O(p^6)$$
$$\overset{\Sigma\neq 0}{=\!=\!=} \Gamma + O(p^6)$$

$$C = \frac{3}{\pi^2}\int d^4k\Big\{\frac{4\Sigma^6(k^2)}{[k^2+\Sigma^2(k^2)]^5} - \frac{8k^2\Sigma'(k^2)\Sigma^5(k^2)}{[k^2+\Sigma^2(k^2)]^5}\Big\} \quad C' = \frac{3}{\pi^2}\int d^4k\Big\{\frac{4k^2\Sigma^4(k^2)}{[k^2+\Sigma^2(k^2)]^5} - \frac{8k^4\Sigma'(k^2)\Sigma^3(k^2)}{[k^2+\Sigma^2(k^2)]^5}\Big\}$$

$$C" = \frac{3}{\pi^2}\int d^4k\Big\{\frac{4k^4\Sigma^2(k^2)}{[k^2+\Sigma^2(k^2)]^5} - \frac{8k^6\Sigma'(k^2)\Sigma(k^2)}{[k^2+\Sigma^2(k^2)]^5}\Big\} \quad C = C' = C" = \begin{cases} 0 & \Sigma(k^2)=0 \\ 1 & \Sigma(k^2)\neq 0 \end{cases} \quad \frac{\Sigma(k^2)}{k^2} \to \begin{cases} \infty & k^2 = 0 \\ 0 & k^2\to\infty \end{cases}$$

- 直接计算得到 $n = N_c$
- 若发生手征对称性自发破缺 $S_{\text{eff},反常} = \Gamma + O(p^6)$
- 若不发生手征对称性自发破缺 $S_{\text{eff},反常} = 0$