Threshold Resummation in Direct Top Quark Production at Hadron Colliders

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Based on our recent work (with Chong Sheng Li, Yang Gao and Jian Jun Liu) Phys. Rev. D 73, 074017 Outline

Introduction

Resummation and Soft-Collinear Effective Theory

Threshold Resummation in Direct Top Quark Production

Summary

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- ▶ The production and decay of top quark through flavor changing neutral current (FCNC) couplings are very sensitive to new physics contributions.

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- ▶ The production and decay of top quark through flavor changing neutral current (FCNC) couplings are very sensitive to new physics contributions.
- The new physics effects can be studied in a model-independent way by an effective FCNC Lagrangian

$$\mathcal{L}_{\text{eff}} = -g_s \sum_{q=u,c} \frac{\kappa_{tq}^g}{\Lambda_{\text{NP}}} \bar{t} \sigma^{\mu\nu} T^a (f_{tq}^g + ih_{tq}^g \gamma_5) q G^a_{\mu\nu} + \text{h.c.} + (Z,\gamma)$$

 κ : anomalous couplings; $\Lambda_{\rm NP}$: new physics scale.

Direct top quark production



The most sensitive process to t-q-g anomalous couplings

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 - ► In general, a next-to-leading order (NLO) QCD correction is capable to reduce the scale dependence significantly.
 - Last year, Liu, Li, Jin and I carried out the NLO calculations (Phys. Rev. D 72, 074018). The results were surprising: the scale dependence was NOT improved for direct top quark production at LHC, and even became WORSE in the region μ_r = μ_f < m_t.

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 - ► It is thus worthwhile to consider the higher order corrections, especially the soft gluon effects through threshold resummation.

Threshold Resummation

Gluon emission at every order in the perturbative expansion:



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Gluon emission at every order in the perturbative expansion:



- As the partonic center-of-mass energy \sqrt{s} approaches the top quark mass m_t (the production threshold), the emitted gluons are constrained to be soft.
- ▶ Two hard scales appear in this problem: $Q \sim \sqrt{s} \sim m_t$ and $Q' \sim Q(1-z)$ with $Q \gg Q'$. Large double logarithms $\ln^2(1-z)$ appear with each soft gluon attached $(z = m_t^2/s)$.
- ▶ The convergence of the perturbative series will be spoiled if $\alpha_s \ln^2(1-z) \lesssim 1 \implies$ need to resum the logs to all orders.

How to Resum: Conventional Approach

► The difficuty here is that with the two distinct scales present, the straightforward application of the renormalization group equations (RGE) can not eliminate the two logarithms $\ln(Q^2/\mu^2)$ and $\ln(Q'^2/\mu^2)$ simultaneously.

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- ▶ Axial gauge invariance is utilized to separate the two scales by the evolution equations with respect to the axial parameter (Collins, Soper, Sterman, *et. al*).

Resummation is closely related to factorization. In the threshold region, the cross section can be factorized in the axial gauge as

$$\sigma = H\left(\frac{p_1 \cdot \xi}{\mu}, \frac{p_2 \cdot \xi}{\mu}\right) \otimes J_1\left(\frac{p_1 \cdot \xi}{\mu}, \frac{p_1^2}{\mu^2}\right) \otimes J_2\left(\frac{p_2 \cdot \xi}{\mu}, \frac{p_2^2}{\mu^2}\right)$$
$$\sim H\left(\frac{Q}{\mu}, \xi\right) \otimes J_1\left(\frac{Q}{\mu}, \frac{Q'}{\mu}, \xi\right) \otimes J_2\left(\frac{Q}{\mu}, \frac{Q'}{\mu}, \xi\right),$$

where ξ is the axial gauge parameter. Here $p_1 \cdot \xi \sim p_1^+ \xi^- \sim Q\xi^-$ and $p_2 \cdot \xi \sim p_2^- \xi^+ \sim Q\xi^+$, while $p_1^2 \sim p_2^2 \sim Q'^2 \sim Q^2(1-z)^2$.

Now H only depends on the higher scale Q, and the logs are present in the jet functions J_1 and J_2 .

To further factorize the phase space, use the Mellin transformation

$$f(z) o \widetilde{f}(N) = \int_0^1 f(z) z^{N-1} dz.$$

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In moment space the cross section becomes

$$\tilde{\sigma}(N) = H\left(\frac{p_1^+}{\mu}, \frac{p_2^-}{\mu}\right) \times J_1\left(\frac{p_1^+}{\mu}, \frac{Q}{N\mu}\right) \times J_2\left(\frac{p_2^-}{\mu}, \frac{Q}{N\mu}\right) + \mathcal{O}\left(\frac{1}{N}\right).$$

Note that

- ▶ The convolutions have become mulplications.
- ▶ N appears only in the combination $Q/N\mu$.

Now the independece of $\tilde{\sigma}$ on ξ immediately gives

$$\frac{\partial}{\partial \ln p_2^-} \ln \tilde{J}_2\left(\frac{p_2^-}{\mu}, \frac{Q}{N\mu}, \alpha_s(\mu)\right) = K\left(\frac{Q}{N\mu}, \alpha_s(\mu)\right) + G\left(\frac{p_2^-}{\mu}, \alpha_s(\mu)\right)$$

and similar for \tilde{J}_1 . Here I have made the dependences of these functions on $\alpha_s(\mu)$ explicit.

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Note that now the contributions from the higher scale Q and the lower scale Q/N have been separated! The remaining things are to make use of these equations together with the RGE of K and G

$$\mu \frac{d}{d\mu} K = -\gamma_K(\alpha_s) = -\mu \frac{d}{d\mu} G$$

to derive the resummation formula for \tilde{J} .

Making Use of Effective Theories

As seen above, the key to resummation is the separation of scales. And we know that effective theories are good at this!

Basic ideas of effective theories:

- ▶ Concentrate on one scale at a time, from high to low;
- Construct effective fields and effective operators at this scale, all contributions from higher scales are encoded in the Wilson coefficients by a procedure called matching;
- Sum logarithms between different scales by running, i.e., solving the renormalization group equations (RGE).

Which Effective Theories?

► The nearly static top quark: described by the heavy quark effective theory (HQET)

$$p^{\mu} \sim m_t v^{\mu} + k^{\mu}, \quad v^2 = 1, \quad k \sim \Lambda \ll m_t$$

 $\psi(x) = \sum_v e^{-im_t v \cdot x} h_v(x)$

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▶ The initial collinear gluon and quark: described by the soft-collinear effective theory (SCET) (Bauer, Beneke, Fleming, Neubert, Pirjol, Rothstein, Stewart, *et. al*)

$$p^{\mu} \sim \frac{1}{2}Qn^{\mu} + p^{\mu}_{\perp} + k^{\mu} \equiv \tilde{p}^{\mu} + k^{\mu}, \quad n^2 = 0$$

$$\phi_n(x) = \sum_{\tilde{p}} e^{-i\tilde{p}\cdot x} \phi_{n,p}(x), \quad k \sim \Lambda \ll p_{\perp} \ll Q$$

Fields in SCET

Using light cone coordinate $(n^2 = \bar{n}^2 = 0, n \cdot \bar{n} = 2)$

$$p^{\mu} = (p^+, p^-, p_{\perp}) = (n \cdot p, \bar{n} \cdot p, p_{\perp})$$

SCET_I: $\lambda \sim \sqrt{\Lambda/Q}$ \blacktriangleright collinear: $p \sim Q(\lambda^2, 1, \lambda), p^2 \sim Q^2 \lambda^2$ \blacktriangleright usoft: $p \sim Q(\lambda^2, \lambda^2, \lambda^2), p^2 \sim Q^2 \lambda^4$ SCET_{II}: $\eta \sim \Lambda/Q$ \blacktriangleright collinear: $p \sim Q(\eta^2, 1, \eta), p^2 \sim Q^2 \eta^2$ \blacktriangleright soft: $p \sim Q(\eta, \eta, \eta), p^2 \sim Q^2 \eta^2$

Lagrangian of $\mathrm{SCET}_{\mathrm{I}}$

At leading order in λ

$$\begin{aligned} \mathcal{L}_{us} &= \bar{\psi}_{us} \, i \mathcal{D}_{us} \, \psi_{us} \\ \mathcal{L}_{cq} &= \bar{\xi}_n \left\{ in \cdot D_c + i \mathcal{D}_c^{\perp} \frac{1}{i\bar{n} \cdot D_c} i \mathcal{D}_c^{\perp} + \underbrace{gn \cdot A_{us}} \right\} \frac{\vec{p}}{2} \xi_n \\ \mathcal{L}_{cg} &= \frac{1}{2g^2} \operatorname{Tr} \left\{ [i \mathcal{D}^{\mu} + g A_n^{\mu}, i \mathcal{D}^{\nu} + g A_n^{\nu}] \right\}^2 \\ &\quad + \text{gauge fixing + ghost} \\ i \mathcal{D}^{\mu} &= \frac{n^{\mu}}{2} \bar{\mathcal{P}} + \mathcal{P}_{\perp}^{\mu} + \frac{\bar{n}^{\mu}}{2} \left(in \cdot \partial + \underbrace{gn \cdot A_{us}} \right) \end{aligned}$$

Note that the usoft & collinear sectors interact only through the circled terms.

Soft-Collinear Decoupling

Introducing a usoft Wilson line

$$Y_n(x) = P \exp\left(ig \int dsn \cdot A_{us}(ns+x)\right)$$

and making the field redefinition

$$\xi_n \to Y_n^{\dagger} \xi_n, \qquad A_n \to Y_n^{\dagger} A_n Y_n,$$

the usoft-collinear interactions are eliminated:

$$\mathcal{L}_{cq}
ightarrow ar{\xi}_n \left\{ in \cdot D_c + i D_c^{\perp} rac{1}{i ar{n} \cdot D_c} i D_c^{\perp}
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and similar for \mathcal{L}_{cg} .

Threshold Resummation Using SCET

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The first matching: QCD \rightarrow SCET_I at $\mu \sim Q$



The matching condition at $\mu \sim Q$ gives the Wilson coefficient

$$C_{gq}(Q^2,\mu) = 1 + \frac{\alpha_s}{12\pi} \left[-12\ln\frac{\mu^2}{Q^2} - \frac{13}{2}\ln^2\frac{\mu^2}{Q^2} - 23 + \frac{55\pi^2}{12} \right],$$

as well as the anomalous dimension of the effective operator \mathcal{T}_{gq}

$$\gamma_1(\mu) = rac{lpha_s}{6\pi} \left[13 \ln rac{\mu^2}{Q^2} + (6eta_0 + 10)
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With this anomalous dimension, we can run the operator down to the intermediate scale $\sim Q\lambda$ and match onto SCET_{II} after performing the field redefinition. Here $\lambda \sim \sqrt{1-z}$ is the expansion parameter of SCET_I. Now we can calculate the NLO cross section in SCET_{II} at the scale $\mu \sim Q\eta$ where no large logs are present ($\eta \sim 1 - z$ is the expansion parameter of SCET_{II}), and then match the result onto the product of two parton distribution functions (PDFs), which means

$$\sigma_{\mathrm{II}}(z) = \sigma_0 \mathcal{M}(z,\mu) \otimes [f_g(z,\mu) \otimes f_q(z,\mu)]$$

$$\equiv \mathcal{M}(z,\mu) \otimes \mathcal{F}(z,\mu),$$

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The convolution here can again be factorized by Mellin transformation:

$$ilde{\sigma}_{\mathrm{II}}(N) = ilde{\mathcal{M}}(N,\mu) imes ilde{\mathcal{F}}(N,\mu).$$

The moment of the matching coefficient \mathcal{M} is

$$\tilde{\mathcal{M}}(N,\mu) = \frac{\alpha_s}{6\pi} \left[8 + \frac{13\pi^2}{12} + 26\ln^2\frac{\bar{N}\mu}{Q} + 8\ln\frac{\bar{N}\mu}{Q} \right],$$

where $\bar{N} \equiv N e^{\gamma_E}$. To avoid the Euler constant γ_E appearing in the coefficient, we can choose the matching scale $\mu = Q/\bar{N}$ rather than Q/N.

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The running of the moments of ${\mathcal F}$ is governed by it anomalous dimension

$$\gamma_2(\mu) = rac{lpha_s}{3\pi} [-26 \ln \bar{N} + 6eta_0 + 6].$$

The Resummed Cross Section

Combining the above results, we can write down the resummed cross section in moment space

$$\begin{split} \tilde{\sigma}^{\text{SCET}}(N) &= \sigma_0(\mu) \left| C_{gq}(Q^2,\mu) \right|^2 \left[1 + \tilde{\mathcal{M}}(N,\mu) \right] \tilde{\mathcal{F}}(N,\mu) \\ &= \sigma_0(\mu_r) \left| C_{gq}(Q^2,\mu_r) \right|^2 e^{I_1} \\ &\times \left[1 + \tilde{\mathcal{M}}(N,Q/\bar{N}) \right] e^{I_2} \tilde{\mathcal{F}}(N,\mu_f). \end{split}$$

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Now the large logs $\ln N$ have been resummed into the exponents:

$$I_1 = \int_{Q/\bar{N}}^{\mu_r} \frac{d\mu}{\mu} 2\gamma_1(\mu), \qquad I_2 = \int_{\mu_f}^{Q/N} \frac{d\mu}{\mu} \gamma_2(\mu).$$

Here μ_r and μ_f correspond to the renomalization and factorization scales in the full theory, respectively.

Resummation at NLL accuracy

The two anomalous dimensions can in general be written as

$$\gamma_1(\mu) = A_1(\alpha_s) \ln \frac{\mu^2}{Q^2} + A_0(\alpha_s),$$

$$\gamma_2(\mu) = B_1(\alpha_s) \ln \bar{N} + B_0(\alpha_s),$$

and $4A_1(\alpha_s) = -B_1(\alpha_s)$.

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and $4A_1(\alpha_s) = -B_1(\alpha_s)$.

From the NLO matching, we can only extract these coefficients to the first order in α_s :

$$A_1^{(1)} = -\frac{1}{4}B_1^{(1)} = \frac{13}{6},$$

$$A_0^{(1)} = \beta_0 + \frac{5}{3}, \quad B_0^{(1)} = 2\beta_0 + 2$$

However, to reach the accuracy of next-to-leading logs (NLL) in the exponents, we need the coefficients A_1 and B_1 to the second order in α_s .

Fortunately, these two coefficients are process-independent and can be extracted from the two-loop DGLAP splitting kernels:

$$A_1^{(2)} = -\frac{1}{4}B_1^{(2)} = \frac{A_1^{(1)}}{2} \left[C_A \left(\frac{67}{18} - \frac{\pi^2}{6} \right) - \frac{5}{9}n_f \right].$$

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Now we can perform the inverse Mellin transformation for the NLL cross section:

$$\sigma^{\mathrm{NLL}}(\tau) = rac{1}{2\pi i} \int_C dN \tau^{-N} \sigma^{\mathrm{NLL}}(N).$$

The NLL cross section collects only the logarithms in the NLO cross section, and the total resummed cross section should also include the remaining terms in the NLO result, i.e,

$$\sigma^{\text{Resum}} = \sigma^{\text{NLL}} + \sigma^{\text{NLO}} - \sigma^{\text{NLL}} \bigg|_{\alpha_s = \mathbf{0}} - \alpha_s \left(\frac{\partial \sigma^{\text{NLL}}}{\partial \alpha_s} \right)_{\alpha_s = \mathbf{0}}.$$

This is our final expression for numerical evaluation.

Numerical Results

subprocess	PDF	LHC $\left(\frac{\kappa/\Lambda}{0.01 \text{TeV}^{-1}}\right)^2 \text{pb}$		Tevatron $\left(\frac{\kappa/\Lambda}{0.01 \text{TeV}^{-1}}\right)^2 \text{fb}$	
1		LO NLO	Resum	LO NLO	Resum
$gu \to t$	CTEQ	12.9 17.0	23.7	268 425	547
	MRST	$12.2 \ 16.3$	19.5	262 426	520
$gc \rightarrow t$	CTEQ	$1.71 \ 2.53$	3.71	13.1 28.1	38.2
	MRST	$1.68 \ \ 2.38$	2.92	$17.0 \ \ 30.3$	38.6

Here $\mu_r = \mu_f = m_t$.

- ▶ The resummation effects further increase the NLO cross sections.
- ▶ The discrepancies between the different PDF sets are still large. These have to be improved by the fitting groups.

Scale Dependence at the Tevatron



Here $\mu_r = \mu_f = \mu$ and $R(\mu) \equiv \sigma(\mu)/\sigma(m_t)$. CTEQ6 PDF sets are used.

The resummation effects further decrease the scale dependence of the NLO cross sections remarkably.

Scale Dependence at the LHC



The NLO corrections do not improve the scale dependence of the LO cross sections, while the effects of threshold resummation are significant.

Summary

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- ▶ The resummation effects increase the total cross sections and significantly reduce the dependence of the cross sections on the renomalization and factorization scales.
- Our results are useful for the current and future experiments at the Tevatron and the LHC for searching new physics, and are essential for extracting the anomalous couplings if signals of the direct top quark production are discovered in the future.

Thank you!