Dynamical Electroweak Symmetry Breaking

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1. Review: The Standard Model Higgs Boson

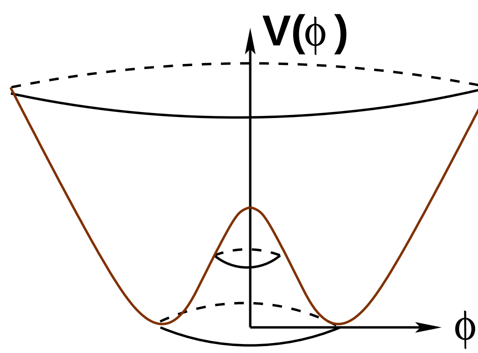
The Higgs Sector

A Fundamental Scalar Doublet:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad ,$$

with potential:

$$V(\phi) = \lambda \left(\phi^\dagger \phi - \frac{v^2}{2} \right)^2 \quad .$$



is employed both to break the electroweak symmetry and to generate masses for the fermions in the Standard Model.

Define $\tilde{\phi} = i\sigma_2\phi^*$ and

$$\Phi = \left(\tilde{\phi}\ \phi\right) \quad \Rightarrow \quad \Phi^\dagger\Phi = \Phi\Phi^\dagger = (\phi^\dagger\phi)\,\mathcal{I}\ .$$

Under $SU(2)_L \times U(1)_Y$, $\Phi \to L\Phi R^\dagger$,

$$L = \exp\left(\frac{iw^a(x)\sigma^a}{2}\right)\ ,\ R = \exp\left(\frac{ib(x)\sigma^3}{2}\right)\ .$$

The Higgs-sector Lagrangian becomes

$$\frac{1}{2}\mathrm{Tr}\left(D^\mu\Phi D_\mu\Phi^\dagger\right) + \frac{\lambda}{4}\left(\mathrm{Tr}\left(\Phi\Phi^\dagger\right) - v^2\right)^2\ ,$$

$$D_\mu\Phi = \partial_\mu\Phi + \mathrm{i}g\mathbf{W}_\mu\Phi - \mathrm{i}\Phi g'\mathbf{B}_\mu\ .$$

The potential manifests the symmetry $SU(2)_L \times SU(2)_R \to SU(2)_V$

A "Polar decomposition" of Φ

$$\Phi(x) = \frac{1}{\sqrt{2}}\left(H(x) + v\right)\Sigma(x)\ ,$$

$$\Sigma(x) = \exp(i\pi^a(x)\sigma^a/v)\ .$$

By gauge choice, $\langle\Sigma\rangle = \mathcal{I}$.

Broken Symmetries $\Rightarrow$ Nambu-Goldstone Bosons

Gauge $SU(2)_W \times U(1)_Y \Rightarrow$ Higgs Mechanism

$$\pi^{\pm},\, \pi^0 \;\rightarrow\; W_L^{\pm},\, Z_L$$

$$M_W = \frac{gv}{2} \rightarrow v \approx 250\text{GeV} \ .$$

Due to residual $SU(2)_V$ "custodial symmetry" for $g' \rightarrow 0$, the $SU(2)_L$ gauge bosons are degenerate.

This, plus $m_\gamma = 0$, tells us

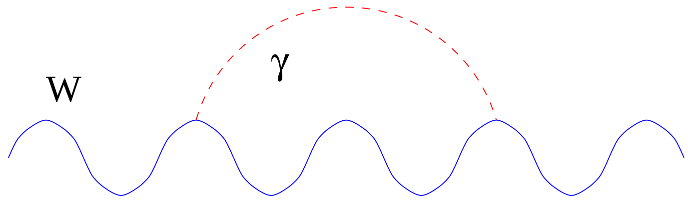
$$M^2 = \frac{v^2}{2}\begin{pmatrix} g^2 & & & \\ & g^2 & & \\ & & g^2 & -gg' \\ & & -gg' & g'^2 \end{pmatrix} ,$$

and hence

$$\rho \equiv \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = 1 \ .$$

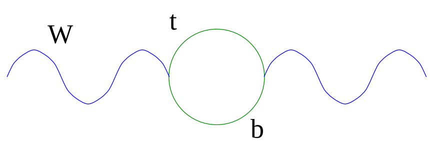
Violations of Custodial Symmetry

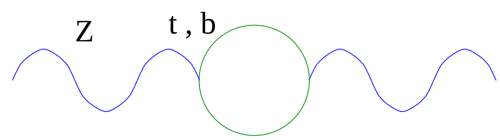
Electromagnetism: $\mathcal{O}(\alpha)$ corrections to $\Delta\rho$ from



Yukawa Couplings:

$$\bar{\psi}_L \Phi \begin{pmatrix} y_t & \\ & y_b \end{pmatrix} \begin{pmatrix} t_R \\ b_R \end{pmatrix} \ .$$





$$\Delta\rho \approx \frac{3y_t^2}{32\pi^2} \ .$$

Nota Bene: Custodial symmetry is accidental. It applies to $SU(2)_L \times U(1)_Y$ invariant terms of <u>dimension 4 or less</u> ($g' \to 0$). Can be violated by terms of higher dimension, *e.g.*

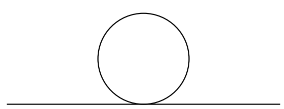
$$(\phi^\dagger D^\mu \phi)(\phi^\dagger D_\mu \phi) = \frac{1}{4}\left(\mathrm{Tr}\,\sigma_3 \Phi^\dagger D^\mu \Phi\right)\left(\mathrm{Tr}\,\sigma_3 \Phi^\dagger D_\mu \Phi\right) \ .$$

Problems with the Higgs Model

- No fundamental scalars observed in nature

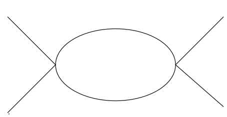
- No explanation of dynamics responsible for Electroweak Symmetry Breaking

- Hierarchy and Naturalness Problem



$$\Rightarrow \; m_H^2 \propto \Lambda^2 \; .$$

- Triviality Problem ...



$$\Rightarrow \beta = \frac{3\lambda^2}{2\pi^2} > 0$$

$$\lambda(\mu) < \frac{3}{2\pi^2 \log \frac{\Lambda}{\mu}} \quad .$$

The Triviality of the Standard Higgs Model

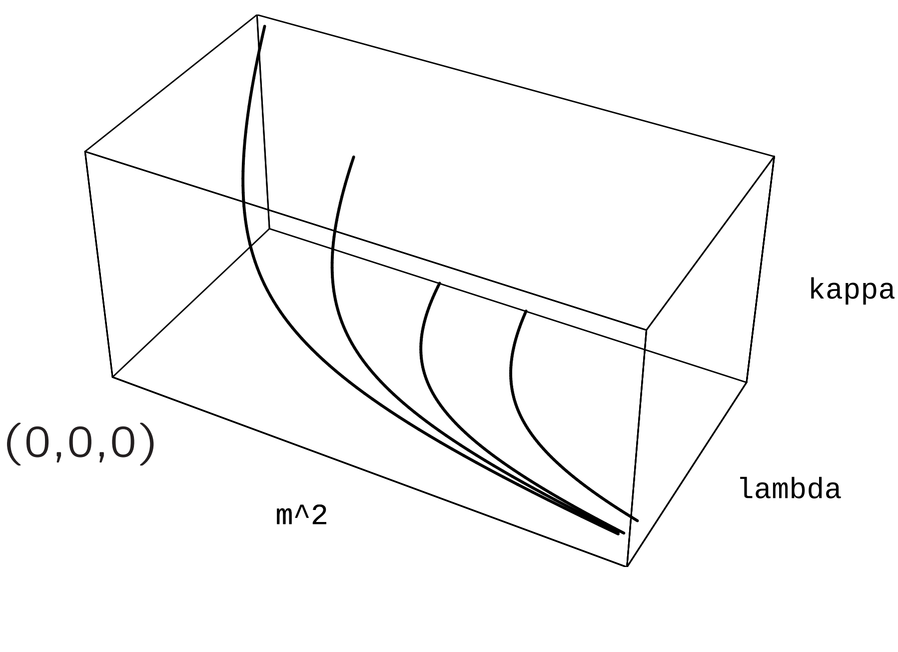
Define theory with a fixed UV-cutoff:

$$\mathcal{L}_\Lambda = D^\mu\phi^\dagger D_\mu\phi + m^2(\Lambda)\phi^\dagger\phi + \frac{\lambda(\Lambda)}{4}(\phi^\dagger\phi)^2 + \frac{\kappa(\Lambda)}{36\Lambda^2}(\phi^\dagger\phi)^3 + \ldots$$

Integrate out states with $\Lambda' < k < \Lambda$:

$$\begin{aligned} \mathcal{L}_\Lambda &\Rightarrow \mathcal{L}_{\Lambda'} \\ m^2(\Lambda) &\rightarrow m^2(\Lambda') \\ \lambda(\Lambda) &\rightarrow \lambda(\Lambda') \\ \kappa(\Lambda) &\rightarrow \kappa(\Lambda') \end{aligned}$$

Consider evolution of couplings in the IR-limit....



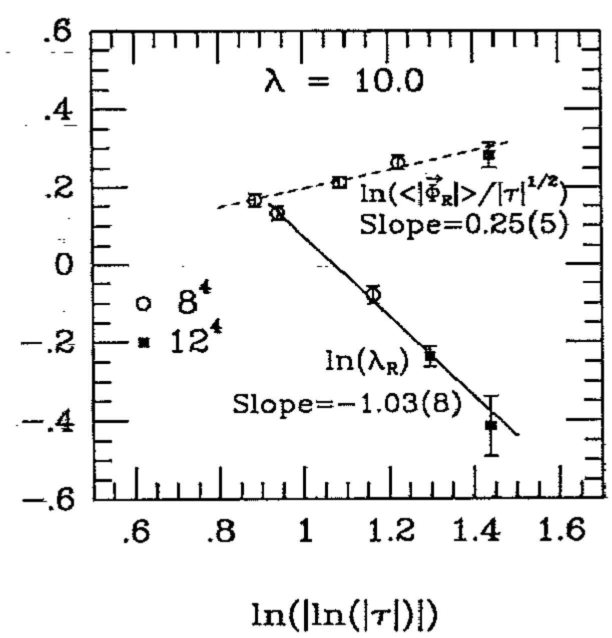
Consequences of perturbative analysis:

- $\kappa \longrightarrow 0$
  "Renormalizability", if $m_H \ll \Lambda$.
- $m^2 \longrightarrow \infty$
  Naturalness/Hierarchy Problem:

$$\frac{\Delta m^2(\Lambda)}{m^2(\Lambda)} \propto \frac{v^2}{\Lambda^2}$$

- $\lambda \longrightarrow 0$
  Triviality ...

Moreover, nonperturbative analysis
yields same* behavior:



*J. Kuti, *et. al.*, PRL 61 (1988) 678

Implications of Triviality

- The Standard Higgs model is, at best, a low-energy effective theory valid below a scale Λ, characteristic of the underlying physics.

- Dashen & Neuberger:
  Given $m_H^2 = 2v^2\lambda(m_H)$, there is an *upper* bound on Λ.
  An *estimate* of this bound can be obtained by integrating the one-loop β-function, which yields

$$\Lambda \gtrsim m_H \exp\left(\frac{4\pi^2 v^2}{3m_H^2}\right) \ ,$$

- Constraints on the underlying physics will result in a lower bound on Λ and will give rise to an upper bound on m_H.

2. Chiral Symmetry Breaking

Technicolor

$SU(N_{TC})$ gauge theory,

$$\Psi_L = \begin{pmatrix} U \\ D \end{pmatrix}_L \qquad U_R, D_R$$

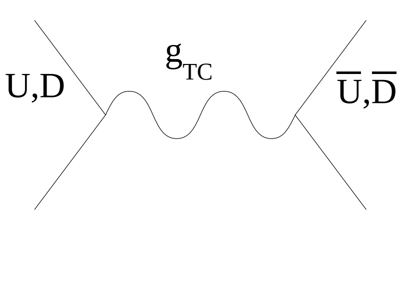
with massless fermions

$$\begin{aligned} \mathcal{L} &= \bar{U}_L i \not{D} U_L + \bar{U}_R i \not{D} U_R + \\ &\quad \bar{D}_L i \not{D} D_L + \bar{D}_R i \not{D} D_R \,. \end{aligned}$$

Like QCD in m_u, $m_d \to 0$ limit:

- Chiral $SU(2)_L \times SU(2)_R$ symmetry
- Dynamically broken symmetry

$$SU(2)_L \times SU(2)_R \to SU(2)_V$$



$$\to \langle \bar{U}_L U_R \rangle = \langle \bar{D}_L D_R \rangle \neq 0 \,.$$

Now gauge $SU(2)_L$ (to be $SU(2)_W$) and the T_3 piece of $SU(2)_R$ (to be $U(1)_Y$).

The condensate of EW-charged technifermions $\langle \bar{U}_L U_R \rangle = \langle \bar{D}_L D_R \rangle \neq 0$ now breaks the EW symmetry.

Broken Chiral Symmetries $\Rightarrow$ Nambu-Goldstone Bosons once again... and this triggers the Higgs Mechanism

$$\pi_T^{\pm}, \pi_T^0 \rightarrow W_L^{\pm}, Z_L$$

$$M_W = \frac{g F_{TC}}{2} \rightarrow F_{TC} \approx 250\text{GeV}$$

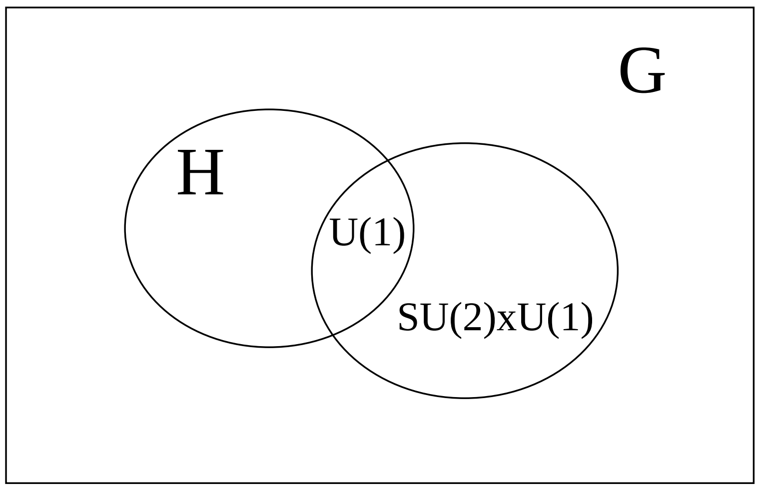
Properties of the technicolor dynamics follow if one scales up QCD by

$$\frac{F_{TC}}{f_\pi} \approx 2500\,.$$

Generalizations

Any strongly interacting gauge theory with

$$SU(2)_W \times U(1)_Y \subseteq G \to H \supseteq SU(2)_V \supset U(1)_{em}$$



where "custodial" $SU(2)_V$ symmetry insures

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = 1$$

can be used to construct a "technicolor" model.

Common Chiral Symmetry Breaking Patterns

- Complex Representations: $N_F \times (R + R^*)$
  $G = SU(N_F)_L \times SU(N_F)_R \times U(1)_V$
  $H = SU(N_F)_V \times U(1)_V$

- Real Representations: $N_F \times R$
  $G = SU(N_F)$
  $H = SO(N_F)$

- Pseudoreal Represenations: $2N_F \times \tilde{R}$
  $G = SU(2N_F)$
  $H = Sp(2N_F)$

Note, however that Extended Symmetry-Breaking Sectors lead to

Larger Symmetries (G/H)

Pseudo Nambu-Goldstone Bosons (PNGBs)

3. The Chiral Lagrangian

Chiral Theory for $SU(2)_L \times SU(2)_R \rightarrow SU(2)_V$:

$$\Sigma = \exp(iw^a\sigma^a/F_{TC}) \rightarrow L\Sigma R^\dagger$$

$$D_\mu\Sigma = \partial_\mu\Sigma + \mathrm{i}g\mathbf{W}_\mu\Sigma - \mathrm{i}\Sigma g'\mathbf{B}_\mu$$

Allowing for custodial $SU(2)$ violation:

$$\frac{F_{TC}^2}{4}\mathrm{Tr}\left[D^\mu\Sigma^\dagger D_\mu\Sigma\right] + \frac{F_{TC}^2}{2}\left(\frac{1}{\rho} - 1\right)\left[\mathrm{Tr}T_3\Sigma^\dagger D^\mu\Sigma\right]^2$$

plus corrections of $\mathcal{O}(p^4/\Lambda_\chi^2)$, and:

$$-\frac{1}{2}\mathrm{Tr}\left[\mathbf{W}^{\mu\nu}\mathbf{W}_{\mu\nu}\right] - \frac{1}{2}\mathrm{Tr}\left[\mathbf{B}^{\mu\nu}\mathbf{B}_{\mu\nu}\right] .$$

$\Sigma = 1$ in Unitary Gauge:

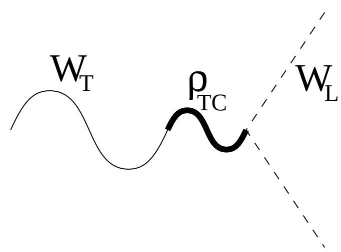
$$\frac{g^2F_{TC}^2}{4}W_-^\mu W_{+\mu} + \frac{g^2F_{TC}^2}{8\rho\cos^2\theta}Z^\mu Z_\mu \quad .$$

$$\Rightarrow M_{W,Z} \neq 0,\ \rho \approx 1$$

Low-Energy Phenomenology

For energies lower than $M_{\rho_{TC}}$, at O(p^4), following Gasser and Leutwyler, we also have corrections to

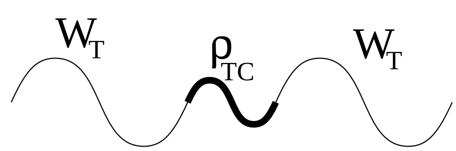
3-pt functions:



$$- ig\frac{l_{9L}}{16\pi^2}\,\mathrm{Tr}\mathbf{W}^{\mu\nu}D_\mu\Sigma D_\nu\Sigma^\dagger$$

$$- ig'\frac{l_{9R}}{16\pi^2}\,\mathrm{Tr}\mathbf{B}^{\mu\nu}D_\mu\Sigma^\dagger D_\nu\Sigma$$

and 2-pt functions:



$$+ gg'\frac{l_{10}}{16\pi^2}\,\mathrm{Tr}\Sigma\mathbf{B}^{\mu\nu}\Sigma^\dagger\mathbf{W}_{\mu\nu} \quad .$$

The l's are normalized to be O(1).

3-pt functions ⇒ Gauge-Boson Self-Couplings

From the work of Hagiwara†, *et. al.*:

$$\frac{i}{e\cot\theta}\ \mathcal{L}_{WWZ} = g_1(W^\dagger_{\mu\nu}W^\mu Z^\nu - W^\dagger_\mu Z_\nu W^{\mu\nu}) + \kappa_Z W^\dagger_\mu W_\nu Z^\mu\nu + \frac{\lambda_Z}{M_W^2} W^\dagger_{\lambda\mu} W^\mu_\nu Z^{\nu\lambda}$$

$$\frac{i}{e}\ \mathcal{L}_{\mathcal{W}\mathcal{W}\gamma} = (W^\dagger_{\mu\nu}W^\mu A^\nu - W^\dagger_\mu A_\nu W^{\mu\nu}) + \kappa_\gamma W^\dagger_\mu W_\nu F^\mu\nu + \frac{\lambda_\gamma}{M_W^2} W^\dagger_{\lambda\mu} W^\mu_\nu F^{\nu\lambda}$$

In terms of $\mathcal{L}_{p^4}$ we expect

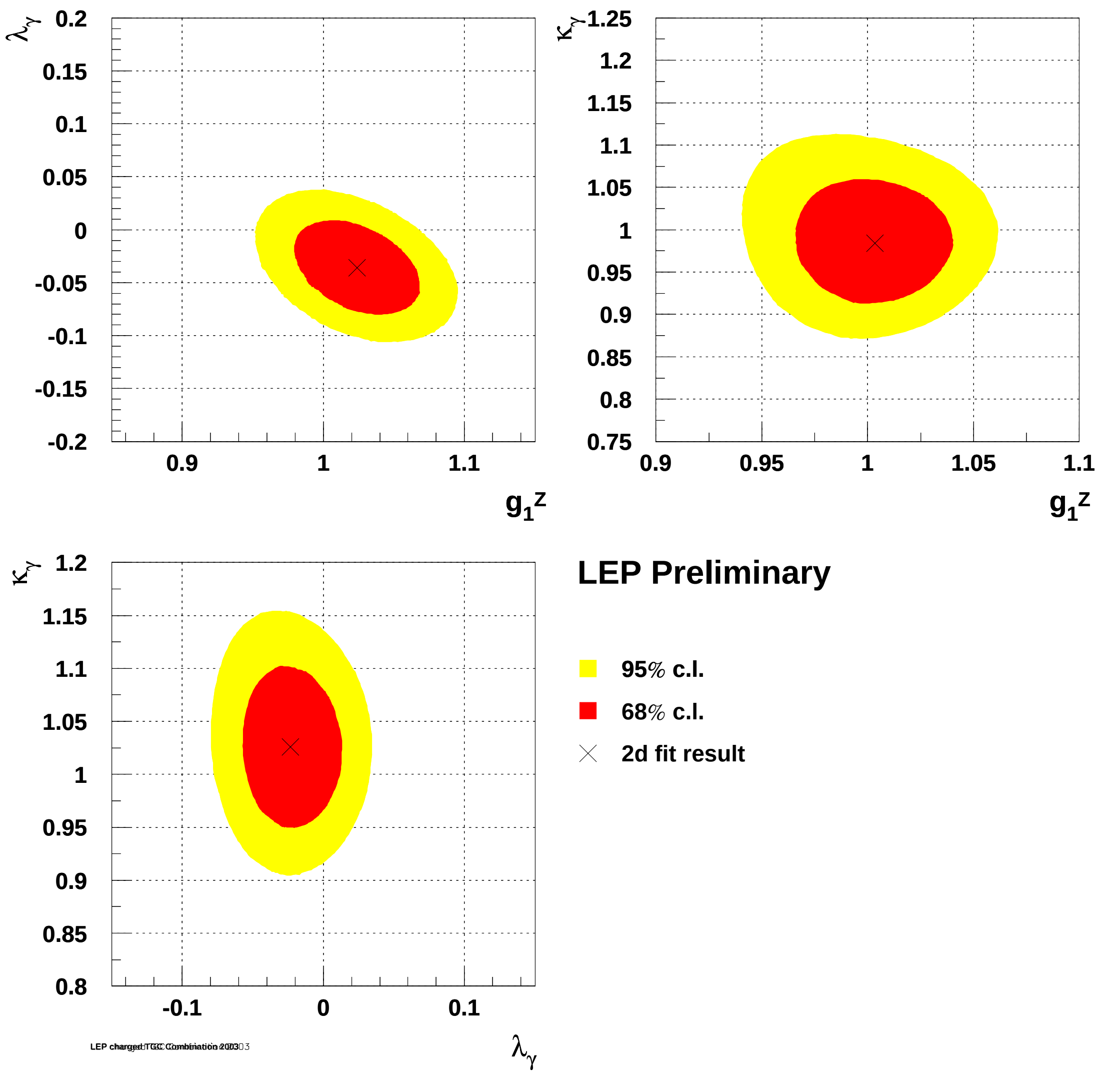
$$\left.\begin{array}{c} g_1 - 1 \\ \kappa_Z - 1 \\ \kappa_\gamma - 1 \end{array}\right\} \approx \frac{\alpha_* l_i}{4\pi\sin^2\theta} = \mathcal{O}(10^{-2} - 10^{-3})$$

Similarly, $\lambda_{Z,\gamma}$ arise in $\mathcal{L}_{p^6}$ so that

$$\lambda_{Z,\gamma} = \mathcal{O}(10^{-4} - 10^{-5})\,.$$

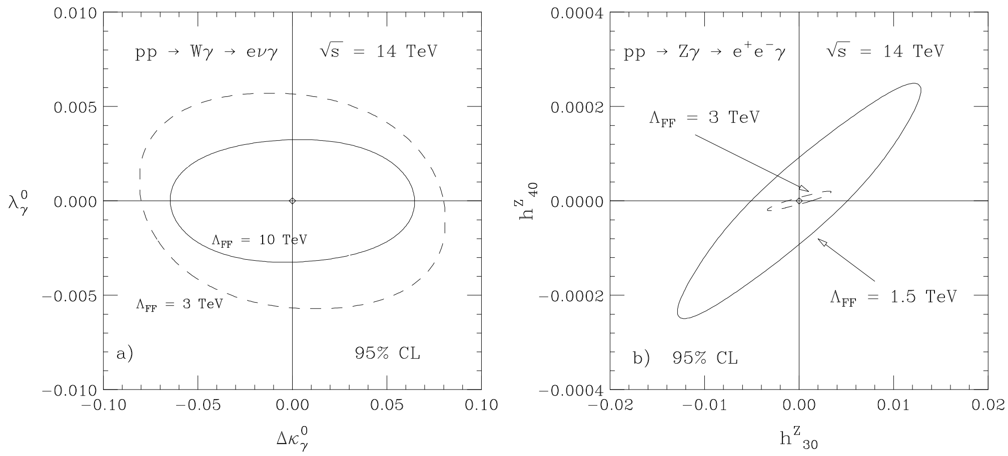
† Hagiwara, *et. al.*, NPB 282 (1987) 253

Current Limits†:



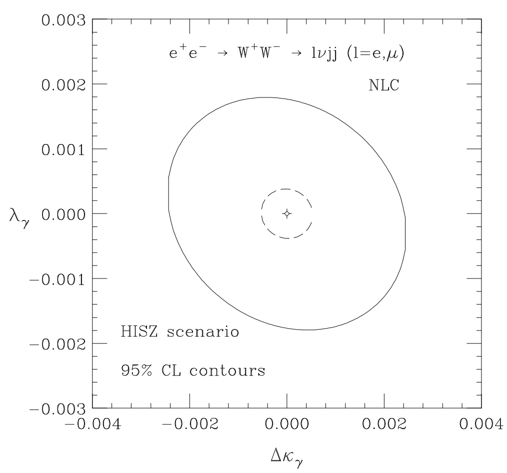
† LEPEWWG, hep-ex/0312023

Experimental Prospects at the LHC†:



$\sqrt{s} = 14\,\mathrm{TeV},\ 100\ \mathrm{fb}^{-1}$

and NLC†:



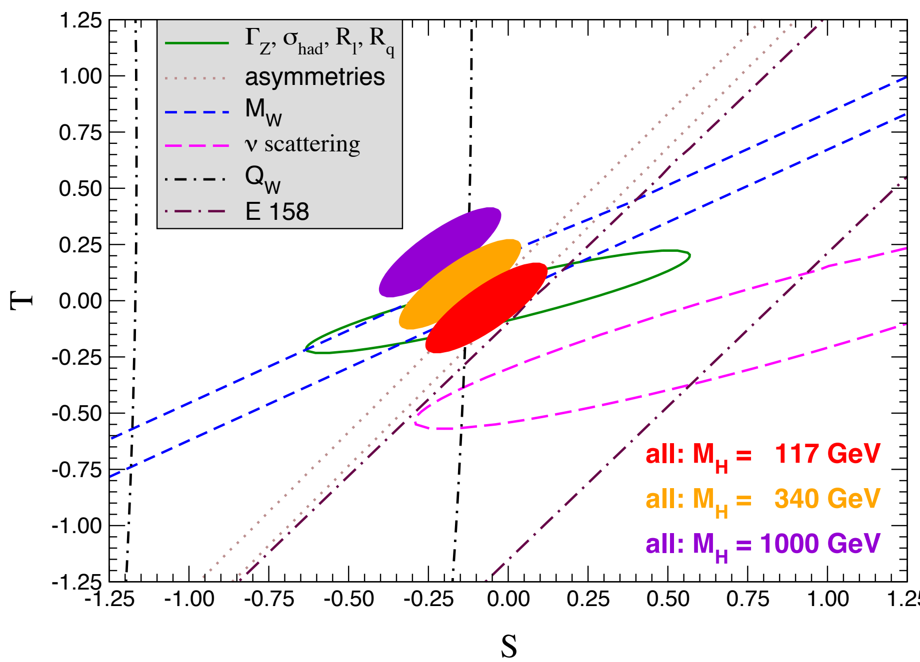
$\sqrt{s} = 500, 1500\,\mathrm{GeV},\ 80\ \&\ 190\ \mathrm{fb}^{-1}$

† Aihara *et. al.*, `hep-ph/9503425`

2-pt functions ⇒ Oblique Parameters[†]

$$S \equiv 16\pi \left[\Pi'_{33}(0) - \Pi'_{3Q}(0) \right]$$
$$= -\frac{l_{10}}{\pi} \approx 4\pi \left(\frac{F^2_{\rho_{TC}}}{M^2_{\rho_{TC}}} - \frac{F^2_{A_{TC}}}{M^2_{A_{TC}}} \right) N_D$$

$$\alpha T \equiv \frac{g^2}{\cos^2\theta_W M_Z^2} \left[\Pi_{11}(0) - \Pi_{33}(0) \right] = \rho - 1$$



Scaling from QCD ⇒ $S \approx 0.28 N_D (N_{TC}/3)$

[†] Particle Data Group 2006

4. Resonances and Unitarity

Gauge-Boson Scattering

At High-Energies, use Equivalence Theorem

$$\mathcal{A}(W_L W_L) = \mathcal{A}(ww) + \mathcal{O}(\frac{M_W}{E}) .$$

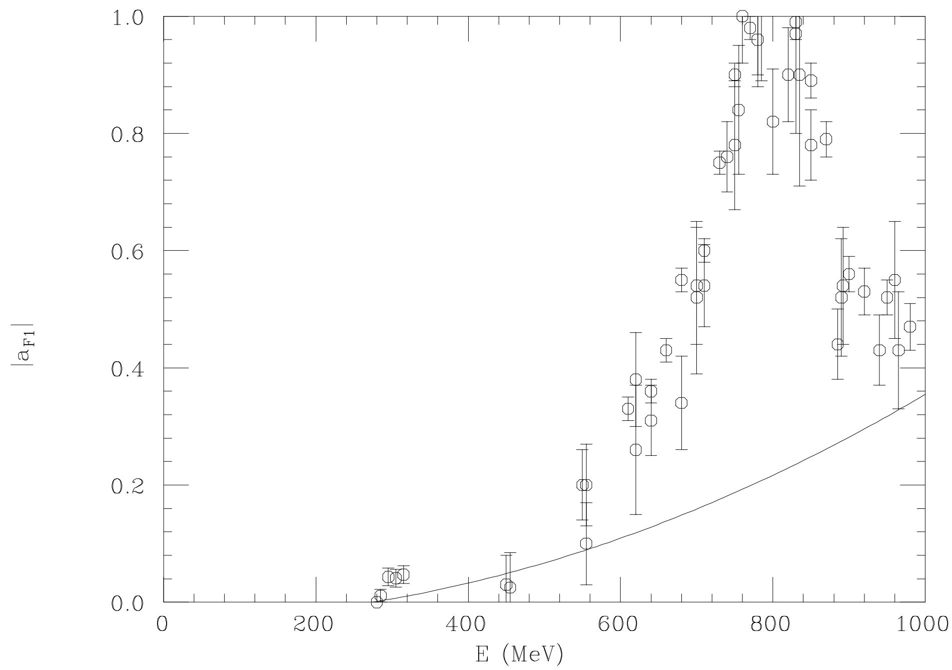
Results in **Universal** "Low-Energy Theorems":

$$\begin{aligned}
\mathcal{A}[W_L^+ W_L^- \to W_L^+ W_L^-] &= \frac{iu}{v^2 \rho} \\
\mathcal{A}[W_L^+ W_L^- \to Z_L Z_L] &= \frac{is}{v^2}\left(4 - \frac{3}{\rho}\right) \\
\mathcal{A}[Z_L Z_L \to Z_L Z_L] &= 0 \quad .
\end{aligned}$$

What dynamics cuts off growth in amplitude?

- New particles ?
- Born approximation fails $\Rightarrow$ electroweak interactions become strong ?
- Both ?

QCD Data† and low-energy theorem prediction for the spin-1/isospin-1 pion scattering amplitude:

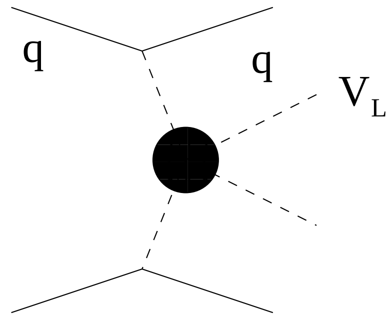


To get predictions for QCD-like technicolor, scale by $v/f_\pi \approx 2600$. That is,

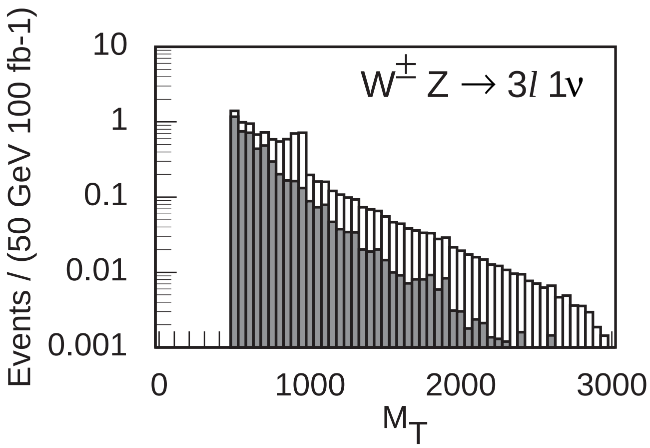
$$M_{\rho_{TC}} \approx 2\,\text{TeV}\sqrt{\frac{3}{N_{TC}}}$$

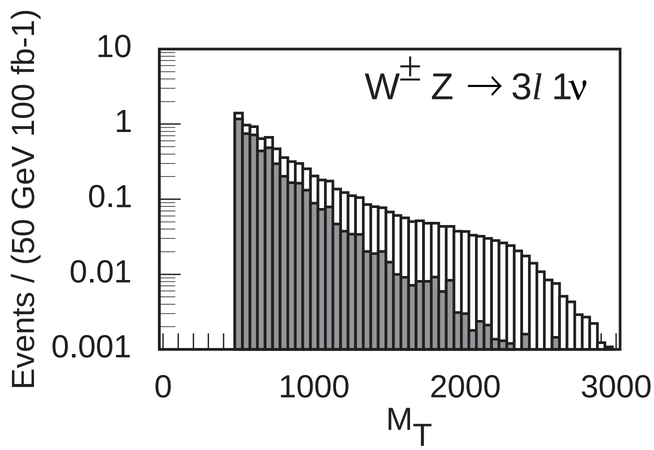
†Donoghue, *et. al.*, PRD 38 (1988) 2195.

Gauge-Boson Scattering at the LHC*



For $M_{\rho_{TC}} = 1.0$ TeV, 2.5 TeV:

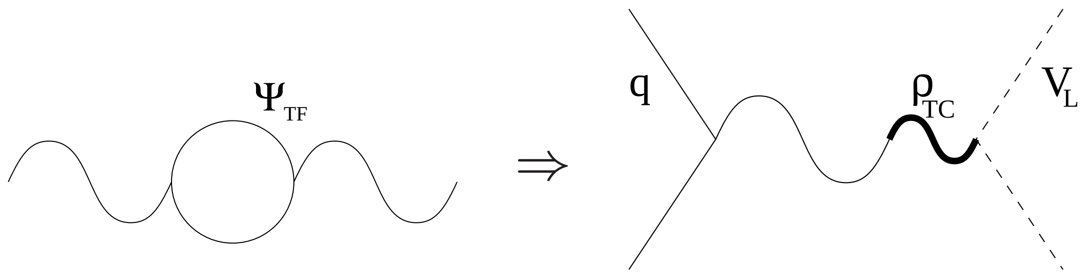


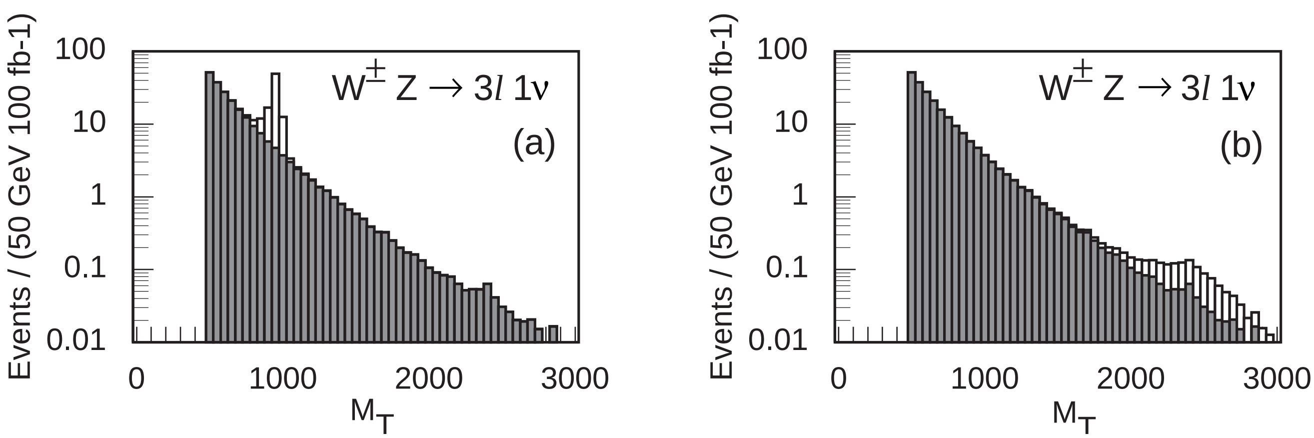


leptonic cuts	jet cuts
$\lvert y(\ell)\rvert < 2.5$	$E(j_{tag}) > 0.8$ TeV
$p_T(\ell) > 40$ GeV	$3.0 < \lvert y(j_{tag})\rvert < 5.0$
$p_T^{miss} > 50$ GeV	$p_T(j_{tag}) > 40$ GeV
$p_T(Z) > \frac{1}{4} M_T$	$p_T(j_{veto}) > 60$ GeV
$M_T > 500$ GeV	$\lvert y(j_{veto})\rvert < 3.0$

*J. Bagger *et. al.*, hep-ph/9306256, 9504426

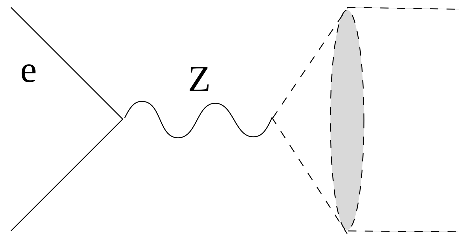
Gauge Boson — Vector Meson Mixing at LHC*





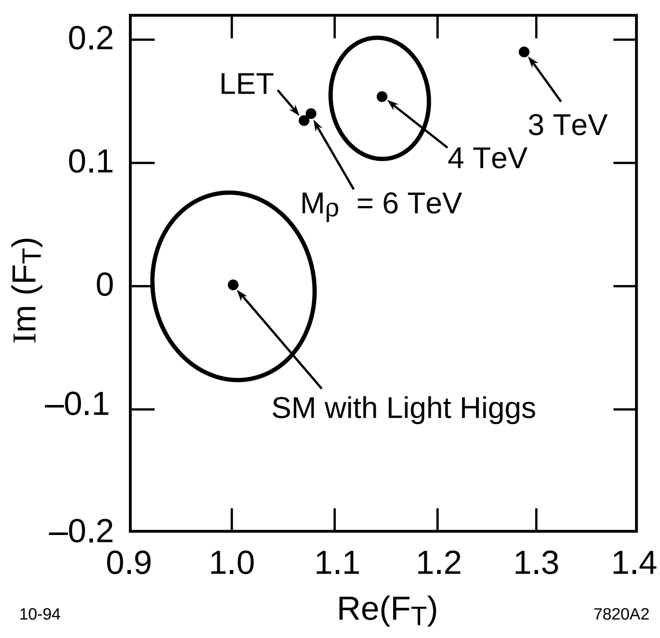
*M. Golden, *et. al.*, `hep-ph/9511206`

Gauge-Boson Re-Scattering at the NLC*



$$F_T = \exp[\frac{1}{\pi}\int_0^\infty ds'\delta(s', M_\rho, \Gamma_\rho)\{\frac{1}{s'-s-i\epsilon} - \frac{1}{s'}\}]$$

$$\delta(s) = \frac{1}{96\pi}\frac{s}{v^2} + \frac{3\pi}{8}\left[\tanh(\frac{s-M_\rho^2}{M_\rho\Gamma_\rho}) + 1\right]$$



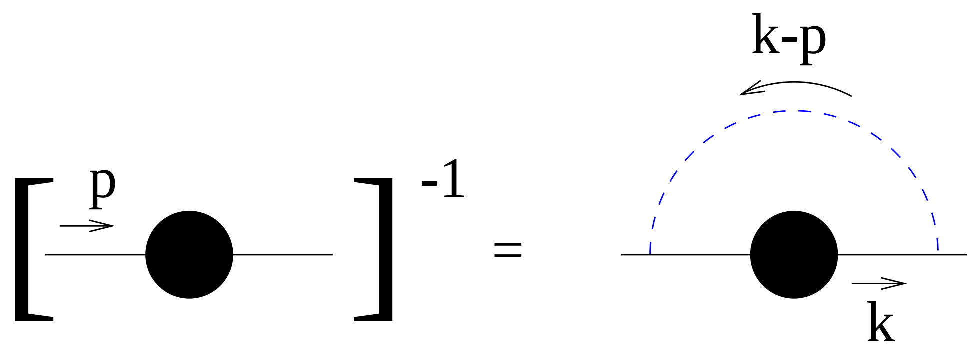
$$\sqrt{s} = 1.5\,\text{TeV},\ 200\ \text{fb}^{-1}$$

*T. Barklow, Physics and Expts. with Linear Colliders

5. Chiral Symmetry Breaking Dynamics

The Gap-Equation

Nonperturbative approximation to chiral-symmetry breaking dynamics: "rainbow" approximation to Schwinger-Dyson equation for $\Sigma(p)$.



Linearized Form:

$$\Sigma(p) = 3C_2(R) \int \frac{d^4k}{(2\pi)^4} \frac{\alpha_{TC}((k-p)^2)}{(k-p)^2} \frac{\Sigma(k)}{k^2}$$

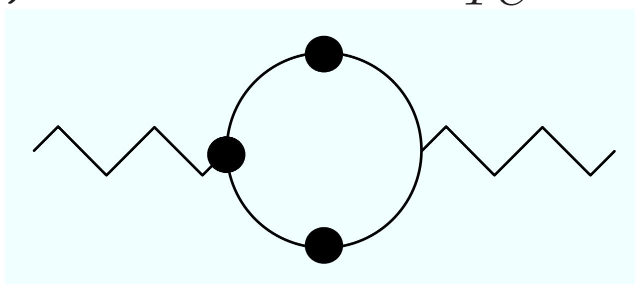
"WKB" Solution(s):

$$\Sigma(p) \propto p^{-\gamma_m(\mu)}\,,\ p^{\gamma_m(\mu)-2}$$

$$\gamma_m(\mu) = 1 - \sqrt{1 - \frac{\alpha_{TC}(\mu)}{\alpha_C}}\,;\ \alpha_C \equiv \frac{\pi}{3C_2(R)}\,.$$

The Pagels-Stokar Formula*

How is $\Sigma(p)$ connected to F_{TC}?



Approximating full vertex function...

...using $\Sigma(p)$ from gap equation...

...and assuming $Z(p) \equiv 1$, yields

$$F_{TC}^2 = \frac{N_{TC}}{16\pi^2} \int_0^\infty dk^2 \, \frac{4k^2\Sigma^2 + \Sigma^4}{(k^2 + \Sigma^2)^2} \ .$$

Size of F_{TC} determined by $\Sigma(0)$.

*Jackiw and Johnson, PRD 8 (1973) 2386

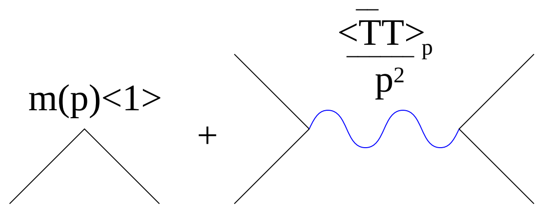
Pagels and Stokar, PRD 20 (1979) 2947

Appelquist *et. al.*, PRD 41 (1990) 3192.

OPE Analysis of Chiral Symmetry Breaking

Physical interpretation of two solutions:

$$\lim_{p \to \infty} \Sigma(p) \quad \propto$$



First solution $\Rightarrow$ "hard mass", explicit χSB.

Second solution $\Rightarrow$ "soft mass".

Dynamical symmetry breaking requires

$$\lim_{m_0 \to 0} \Sigma(p) \neq 0 \, .$$

Analysis of Gap Equation implies this happens iff α_{TC} reaches α_C. Chiral symmetry breaking scale Λ_{TC}

$$\alpha_{TC}(\Lambda_{TC}) = \alpha_C \qquad \Longleftrightarrow \qquad \gamma_m(\Lambda_{TC}) = 1.$$

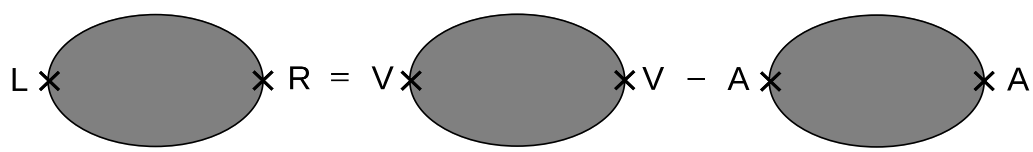
χSB occurs when "hard" and "soft" masses scale the same way; generalizes to all orders?

OPE Analysis of
Current Correlation Functions

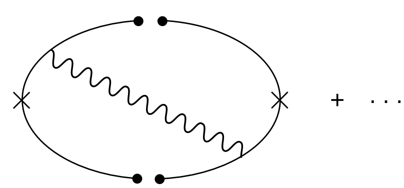
$$\Pi^{\mu\nu}_{LR}(q) = 4i\int d^4x e^{iq\cdot x}\langle 0|T(j^{\mu}_{L}(s)j^{\nu}_{R}(0)^{\dagger}|0\rangle$$

Transversality implies:

$$\Pi^{\mu\nu}_{LR}(Q^2) = (q^\mu q^\nu - g^{\mu\nu}q^2)\Pi_{LR}(Q^2 = -q^2)$$



$$\Pi_{LR}(Q^2) = \Pi_{VV}(Q^2) - \Pi_{AA}(Q^2)$$



In large-N_C approximation:

$$\Pi_{LR}(Q^2) = \frac{1}{Q^6}\left[-4\pi^2\left(\frac{\alpha_s}{\pi} + \mathcal{O}(\alpha_s^2)\right)\langle\bar{\psi}\psi\rangle^2\right] + \ldots$$

Weinberg Sum Rules

"Duality": saturate spectral functions with resonances:

$$\Pi_{VV}(Q^2) = \sum_{\hat{l}} \frac{g^2_{\hat{l}V}}{m^2_{\hat{l}}(m^2_{\hat{l}} + Q^2)}$$

$$\Pi_{AA}(Q^2) = \frac{F^2}{Q^2} + \sum_{\hat{l}} \frac{g^2_{\hat{l}A}}{m^2_{\hat{l}}(m^2_{\hat{l}} + Q^2)}$$

$$\langle 0 | J^a_{V,A\mu}(0) | A^{b\hat{l}} \rangle = g_{\hat{l}V,A} \delta^{ab} \varepsilon_\mu$$

$$\lim_{Q^2 \to \infty} Q^2 \, \Pi_{LR}(Q^2) = \lim_{Q^2 \to \infty} Q^4 \, \Pi_{LR}(Q^2) = 0 \Rightarrow$$

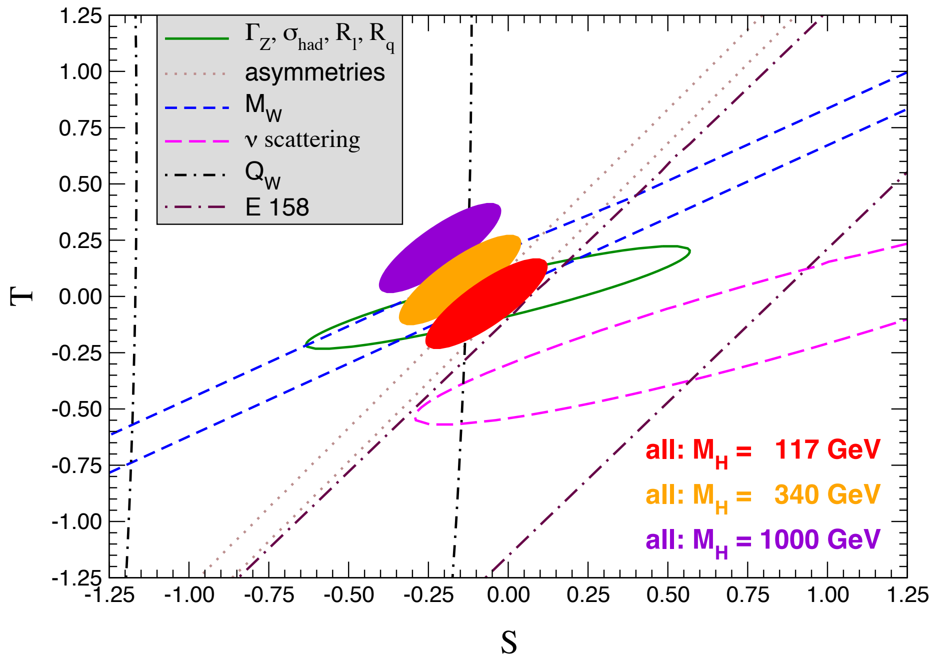
$$\sum_{\hat{l}} \left(\frac{g^2_{\hat{l}V}}{m^2_{\hat{l}}} - \frac{g^2_{\hat{l}A}}{m^2_{\hat{l}}} \right) = F^2 \quad \& \quad \sum_{\hat{l}} \left(g^2_{\hat{l}V} - g^2_{\hat{l}A} \right) \equiv 0$$

Constraint on spectrum of resonances!

RECALL: 2-pt functions ⇒ Oblique Parameters†

$$S \equiv 16\pi\left[\Pi'_{33}(0) - \Pi'_{3Q}(0)\right]$$
$$= -\frac{l_{10}}{\pi} \approx 4\pi\left(\frac{F^2_{\rho_{TC}}}{M^2_{\rho_{TC}}} - \frac{F^2_{A_{TC}}}{M^2_{A_{TC}}}\right) N_D$$

$$\alpha T \equiv \frac{g^2}{\cos^2\theta_W M_Z^2}\left[\Pi_{11}(0) - \Pi_{33}(0)\right] = \rho - 1$$



Scaling from QCD ⇒ $S \approx 0.28 N_D (N_{TC}/3)$

† Particle Data Group 2006

6. Interim Conclusions

Strong Electroweak Symmetry Breaking
Eliminates Fundamental Scalars

Electroweak gauge symmetry is broken by the nonzero expectation value of a fermion bilinear, driven by new strong interactions.

Understanding of strongly-interacting gauge theories is extremely limited. Theories often constructed by analogy!

Next: how to move beyond analogy and how to include fermions...

Flavor Symmetry Breaking and ETC

7. Fermions and Extended Technicolor

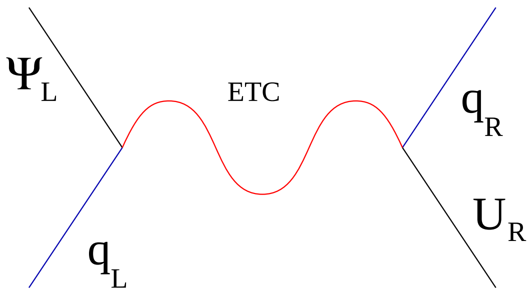
8. The Details

9. Walking Technicolor

10. Conclusions

7. Fermion Masses & ETC Interactions

Extended Technicolor Interactions — Connect chiral-symmetries of TFs to quarks & leptons.



$$\Rightarrow \quad \frac{g^2_{ETC}}{M^2_{ETC}} (\overline{\Psi}_L U_R)(\overline{q}_R q_L)$$

$$m_q \approx \frac{g^2_{ETC}}{M^2_{ETC}} \langle \overline{U} U \rangle_{ETC}$$

$$\langle \overline{U} U \rangle_{ETC} = \langle \overline{U} U \rangle_{TC} \exp\left(\int_{\Lambda_{TC}}^{M_{ETC}} \frac{d\mu}{\mu} \gamma_m(\mu) \right)$$

For QCD-like TC (“precociously” asymptotically free), γ_m is small over this range:

$$\langle \overline{U} U \rangle_{ETC} \approx \langle \overline{U} U \rangle_{TC} \approx 4\pi F^3_{TC}$$

$$\frac{M_{ETC}}{g_{ETC}} \approx 40\,\text{TeV} \left(\frac{F_{TC}}{250\,\text{GeV}} \right)^{\frac{3}{2}} \left(\frac{100\,\text{MeV}}{m_q} \right)^{\frac{1}{2}}$$

"Toy" Model : $SU(N_{ETC})$ $N_{ETC} = N_{TC} + N_F$

$$Q_L = (N_{ETC}, 3, 2)_{1/6} \quad L_L = (N_{ETC}, 1, 2)_{-1/2}$$
$$U_R = (N_{ETC}, 3, 1)_{2/3} \quad E_R = (N_{ETC}, 1, 1)_{-1}$$
$$D_R = (N_{ETC}, 3, 1)_{-1/3} \quad N_R = (N_{ETC}, 1, 1)_0$$

$$\begin{array}{ccc} & SU(N_{TC}+3) & \\ \Lambda_1 & \downarrow & m_1 \approx \frac{4\pi F^3}{\Lambda_1^2} \\ & SU(N_{TC}+2) & \\ \Lambda_2 & \downarrow & m_2 \approx \frac{4\pi F^3}{\Lambda_2^2} \\ & SU(N_{TC}+1) & \\ \Lambda_3 & \downarrow & m_3 \approx \frac{4\pi F^3}{\Lambda_3^2} \\ & SU(N_{TC}) & \end{array}$$

$$[G_{ETC}, SU(3)_C] = [G_{ETC}, SU(2)_W] = 0$$

"One-Family" possesses chiral symmetry
$SU(8)_L \times SU(8)_R \to SU(8)_V$

Yields three isospin-symmetric families of degenerate quarks and leptons, $m_1 < m_2 < m_3$.

Moral: ETC interactions necessarily incorporate a dynamical theory of flavor!

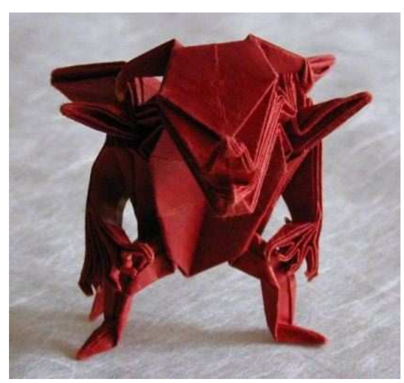
Shortcomings of this toy model:

- What breaks ETC?
- Need a separate scale for each family.
- All quark (& lepton) mixing angles zero.
- $T_3 = \pm\frac{1}{2}$ fermions have equal masses.
  u_R & d_R must be in different representations of ETC.
- RH-technineutrinos $\Rightarrow$ RH-ν's, $m_\nu \neq 0$.

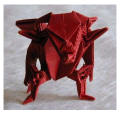
The devil is in the details

...

no complete theory exists



8. The Details

 Flavor-Changing Neutral Currents

Quark mixing implies transitions between different generations: $q \to \Psi \to q'$.

ETC algebra:

$$[\overline{q}\gamma\Psi, \overline{\Psi}\gamma q'] \supset \overline{q}\gamma q' \,.$$

This is true of charged-current weak interactions as well. But $SU(2)_W$ respects a global $(SU(3) \times U(1))^5$ chiral symmetry: GIM!

ETC interactions cannot respect GIM (exactly): they must distinguish between the various generations to provide a hierarchy of masses.

$|\Delta S| = 2$ interactions:

$$\mathcal{L}_{|\Delta S|=2} = \frac{g_{ETC}^2\,\theta_{sd}^2}{M_{ETC}^2}\,\overline{s}\Gamma^{\mu} d\;\overline{s}\Gamma'_{\mu} d + \text{h.c.}$$

$$(\Delta M_K^2)_{ETC} = \frac{g_{ETC}^2\,\theta_{sd}^2}{M_{ETC}^2}\,\langle\overline{K^0}|\overline{s}\Gamma^{\mu} d\,\overline{s}\Gamma'_{\mu} d|K^0\rangle + \text{c.c}$$

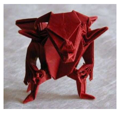
Using vacuum insertion approximation, one finds

$$(\Delta M_K^2)_{ETC} \simeq \frac{g_{ETC}^2\,\text{Re}(\theta_{sd}^2)}{2M_{ETC}^2}\,f_K^2 M_K^2$$

$$\text{Then}\quad \Delta M_K < 3.5\times 10^{-12}\ \text{MeV}\quad\text{implies}$$
$$\frac{M_{ETC}}{g_{ETC}\sqrt{\text{Re}(\theta_{sd}^2)}} > 600\ \text{TeV}$$

$$m_{q,\ell} \simeq \frac{g_{ETC}^2}{M_{ETC}^2}\langle\overline{T}T\rangle_{ETC} < \frac{0.5\ \text{MeV}}{N_D^{3/2}\,\theta_{sd}^2}$$

Hard to reach s & c masses, let alone b & t!

 Pseudo-Nambu-Goldstone Bosons

One-Family of Technifermions

$SU(8)_L \times SU(8)_R \to SU(8)_V \Rightarrow$

63 PGBs/Mesons:

$\mathrm{SU}(3)_C$	$\mathrm{SU}(2)_V$	Particle
1	1	$P^{0\prime}$, ω_T
1	3	$P^{0,\pm}$, $\rho_T^{0,\pm}$
3	1	$P_3^{0\prime}$, $\rho_{T3}^{0\prime}$
3	3	$P_3^{0,\pm}$, $\rho_{T3}^{0,\pm}$
8	1	$P_8^{0\prime}(\eta_T)$, $\rho_{T8}^{0\prime}$
8	3	$P_8^{0,\pm}$, $\rho_{T8}^{0,\pm}$

Gauge interactions break chiral symmetry: color octets and triplets PGBs get masses of order 200 — 300 GeV, in analogy to $m_{\pi^+} - m_{\pi^0}$ in QCD. Others massless to O(α)!

ETC-contributions to PGB Masses

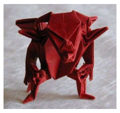
Dashen's Formula $\Rightarrow$

$$F_{TC}^2 M_{\pi_T}^2 \propto \frac{g_{ETC}^2}{M_{ETC}^2} \langle (\overline{T}T)^2 \rangle_{ETC}$$

Assuming QCD-like dynamics, meaning small γ_m, one finds

$$M_{\pi_T} \simeq 55\,\mathrm{GeV} \sqrt{\frac{m_f}{1\,\mathrm{GeV}}} \sqrt{\frac{250\,\mathrm{GeV}}{F_{TC}}}$$

Is this large enough?

 Other model-building constraints on a realistic TC/ETC Theory:

- ETC should be asymptotically free.
- There can be no gauge anomalies.
- Neutrino masses, if nonzero, must be small.
- There should be no extra massless or light gauge bosons.
- Weak CP-violation, without strong CP-violation.
- Isospin-violation in fermion mass without large $\Delta\rho$.
- Accomodate a large m_t.
- Small corrections to $Z \to \bar{b}b$ and $b \to s\gamma$.

The rest of this lecture and tomorrow's lecture will attack these challenges.

9. “Walking” Technicolor

We have assumed (following QCD) that $\gamma_m(\mu)$ is small for $\Lambda_{TC} < \mu < M_{ETC}$.

However, if β_{TC} is *small*, α_{TC} may be large ... and therefore γ_m may be large.

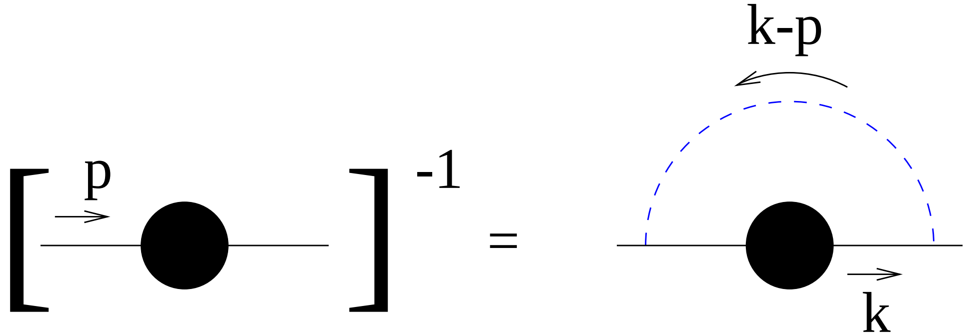
$$\langle \overline{T}T \rangle_{ETC} = \langle \overline{T}T \rangle_{TC} \exp\left(\int_{\Lambda_{TC}}^{M_{ETC}} \frac{d\mu}{\mu} \gamma_m(\mu) \right)$$

$\Rightarrow$ could enhance $\langle \overline{T}T \rangle_{ETC}$ and fermion masses.

Question: How large can γ_m be and how does this affect TC χ-symmetry breaking dynamics?

Recall: The Gap-Equation

Nonperturbative approximation to chiral-symmetry breaking dynamics: “rainbow” approximation to Schwinger-Dyson equation for $\Sigma(p)$.



Linearized Form:

$$\Sigma(p) = 3C_2(R) \int \frac{d^4k}{(2\pi)^4} \frac{\alpha_{TC}((k-p)^2)}{(k-p)^2} \frac{\Sigma(k)}{k^2}$$

“WKB” Solution(s):

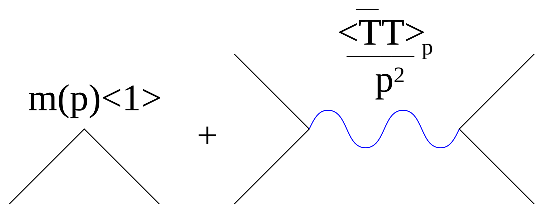
$$\Sigma(p) \propto p^{-\gamma_m(\mu)}, \; p^{\gamma_m(\mu)-2}$$

$$\gamma_m(\mu) = 1 - \sqrt{1 - \frac{\alpha_{TC}(\mu)}{\alpha_C}}; \; \alpha_C \equiv \frac{\pi}{3C_2(R)}.$$

OPE Analysis of Chiral Symmetry Breaking

Physical interpretation of two solutions:

$$\lim_{p\to\infty} \Sigma(p) \quad \propto$$



First solution $\Rightarrow$ "hard mass", explicit χSB.

Second solution $\Rightarrow$ "soft mass".

Dynamical symmetry breaking requires

$$\lim_{m_0\to 0} \Sigma(p) \neq 0\,.$$

Analysis of Gap Equation implies this happens iff α_{TC} reaches α_C. Chiral symmetry breaking scale Λ_{TC}

$$\alpha_{TC}(\Lambda_{TC}) = \alpha_C \qquad \Longleftrightarrow \qquad \gamma_m(\Lambda_{TC}) = 1.$$

χSB occurs when "hard" and "soft" masses scale the same way; generalizes to all orders?

Implications of Walking: Fermion Masses

If $\beta(\alpha_{TC}) \simeq 0$ all the way from Λ_{TC} to M_{ETC}, *i.e.* if the TC-coupling "walks" $\Rightarrow \gamma_m(\mu) \cong 1$

$$m_{q,l} = \frac{g^2_{ETC}}{M^2_{ETC}} \times \left(\langle \overline{T}T \rangle_{ETC} \cong \langle \overline{T}T \rangle_{TC} \frac{M_{ETC}}{\Lambda_{TC}} \right)$$

FCNCs $\Rightarrow M_{ETC}/\Lambda_{TC} \gtrsim 100 - 1000$

$$m_{q,l} \gtrsim \frac{50 \;-\; 500\,\text{MeV}}{N_D^{3/2} \theta^2_{sd}}$$

enough to accommodate s and c quarks.

Caveat: Estimates for "extreme walking" limit.

Query: What about top and bottom masses?

Implications of Walking: PGB Masses

Dashen's formula, revisited:

$$
\begin{aligned}
F^2_{TC} M^2_{\pi_T} &\propto \frac{g^2_{ETC}}{M^2_{ETC}} \langle \left(\overline{T}T\right)^2 \rangle)_{ETC} \\
&\approx \frac{g^2_{ETC}}{M^2_{ETC}} \left(\langle \overline{T}T \rangle_{ETC}\right)^2 \\
&\simeq \frac{g^2_{ETC}}{M^2_{ETC}} \frac{M^2_{ETC}}{\Lambda^2_{TC}} \left(\langle \overline{T}T \rangle_{TC}\right)^2
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
M_{\pi_T} &\simeq g_{ETC} \left(\frac{4\pi F^2_{TC}}{\Lambda_{TC}}\right) \\
&\simeq g_{ETC} \left(\frac{750\,\text{GeV}}{N_D}\right) \left(\frac{1\,\text{TeV}}{\Lambda_{TC}}\right)
\end{aligned}
$$

What about S?

Assumptions of previous calculation:

- Techni-isospin is a good symmetry.
- Technicolor is QCD-like.
  - •• Weinberg's sum rules are valid.
  - •• Spectral functions saturated by lowest resonances.
  - •• Masses/couplings of resonances scaled from QCD.

A "realistic" walking technicolor theory may be very unlike QCD.

- Walking $\Rightarrow$ different behavior of spectral functions.
- Many flavors/PGBs and non-fundamental representations makes scaling from QCD suspect.

10. Conclusions

Strong Electroweak Symmetry Breaking Eliminates Fundamental Scalars

Dynamical generation of fermion masses requires introducing ETC interactions – and creating a dynamical theory of flavor!

Challenges include

- Lack of GIM implies FCNCs
- Large chiral symmetry yields light PNGBs
- QCD-like dynamics makes S too large

Walking TC Theory ($\gamma_m \approx 1$)

- may create c quark mass w/o FCNCs
- helps boost masses of PNGBs
- can no longer predict S by scaling

Next time: the top quark.