

Deconstruction and Higgsless Models

R. Sekhar Chivukula
Michigan State University

Topical Seminar on Frontier of Particle Physics 2006:

Beyond the Standard Model

Beijing, China, August 7-11, 2006

w/He, Kurachi, Tanabashi,
and Simmons

Marcel Duchamp
1912



Recipe for a Higgsless Model:

- Choose “bulk” gauge group, location of fermions, and boundary conditions
- Choose $g(x_5)$
- Choose metric/manifold: $g_{MN}(x_5)$
- Calculate spectrum & eigenfunctions
- Calculate fermion couplings
- Compare to Standard Model: S, T, U, ...

Electroweak Parameters I

EW corrections (S , T , $\Delta\rho$, δ) defined from amplitudes for “on-shell” 4-fermion processes

$$\begin{aligned}
 -\mathcal{A}_{NC} = & e^2 \frac{QQ'}{Q^2} + \frac{(I_3 - s^2 Q)(I'_3 - s^2 Q')}{\left(\frac{s^2 c^2}{e^2} - \frac{S}{16\pi}\right) Q^2 + \frac{1}{4\sqrt{2}G_F} \left(1 + \frac{\alpha\delta}{4s^2 c^2} - \alpha T\right)} \\
 & + \sqrt{2}G_F \frac{\alpha\delta}{s^2 c^2} I_3 I'_3 + 4\sqrt{2}G_F (\Delta\rho - \alpha T) (Q - I_3)(Q' - I'_3)
 \end{aligned}$$

$$-\mathcal{A}_{CC} = \frac{(I_+ I'_- + I_- I'_+)/2}{\left(\frac{s^2}{e^2} - \frac{S}{16\pi}\right) Q^2 + \frac{1}{4\sqrt{2}G_F} \left(1 + \frac{\alpha\delta}{4s^2 c^2}\right)} + \sqrt{2}G_F \frac{\alpha\delta}{s^2 c^2} \frac{(I_+ I'_- + I_- I'_+)}{2}.$$

Electroweak Parameters II

Alternative formulation defined at zero momentum

$$\hat{S} = \frac{1}{4s^2} \left(\alpha S + 4c^2(\Delta\rho - \alpha T) + \frac{\alpha\delta}{c^2} \right)$$

$$\hat{T} = \Delta\rho$$

$$Y = \frac{c^2}{s^2} (\Delta\rho - \alpha T)$$

$$W = \frac{\alpha\delta}{4s^2c^2}$$

Recipe for a Higgsless Model:

- Choose “bulk” gauge group, location of fermions, and boundary conditions
- Choose $g(x_5)$
- Choose metric/manifold: $g_{MN}(x_5)$
- Calculate spectrum & eigenfunctions
- Calculate fermion couplings
- Compare to Standard Model : S, T, U
- Declare model viable or not



Can we do better?

Yes ...

Deconstruction

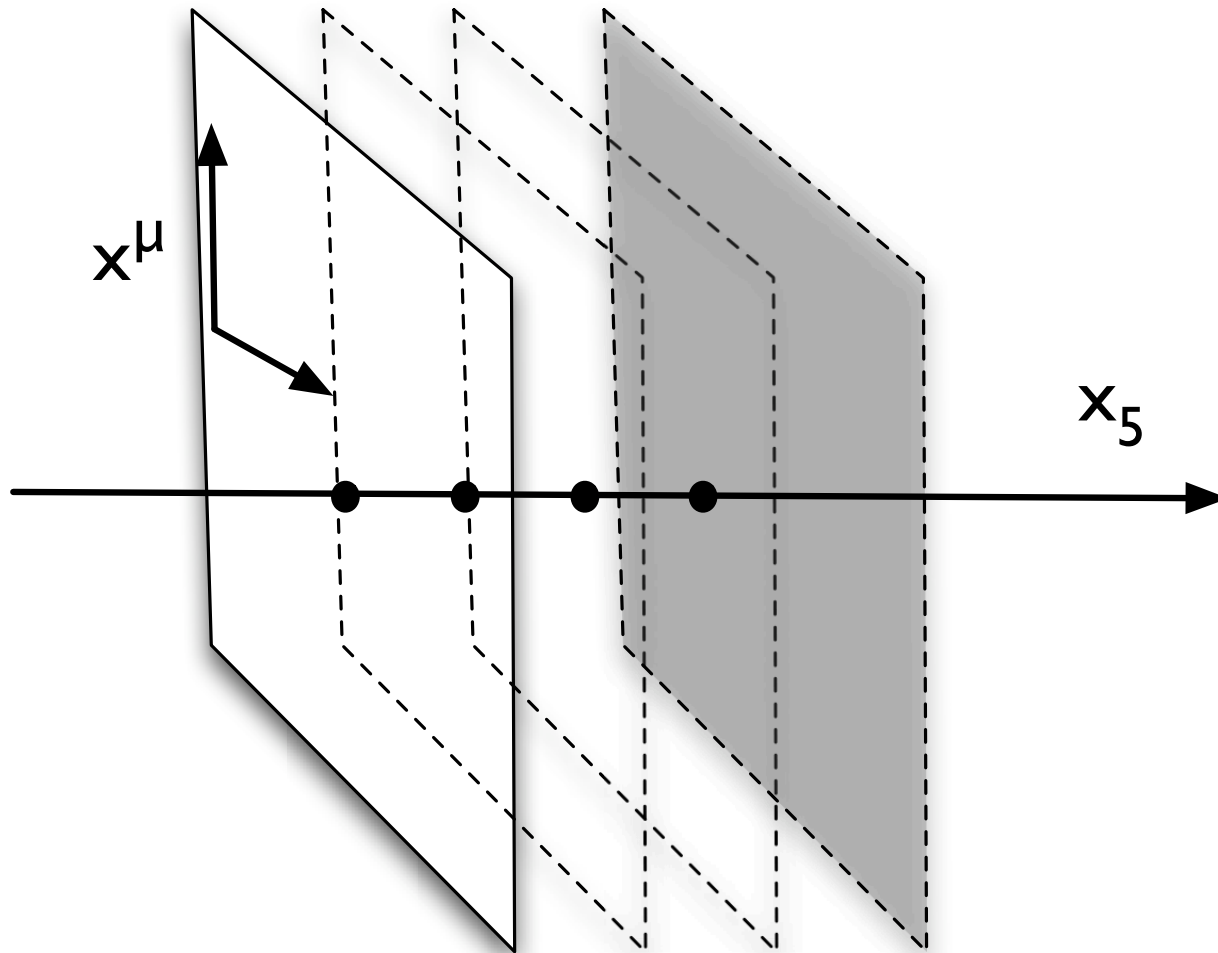


van Gogh

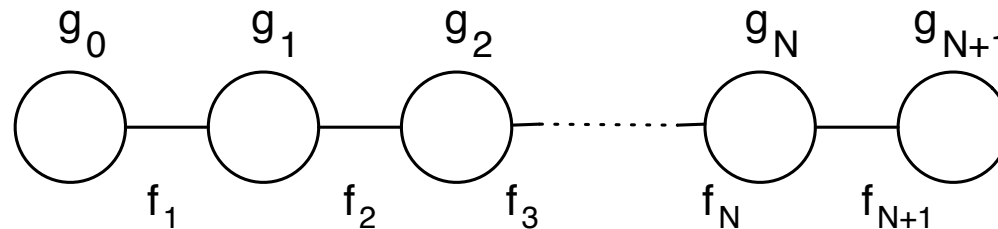


Wolff

Latticize Fifth Dimension



Deconstruction



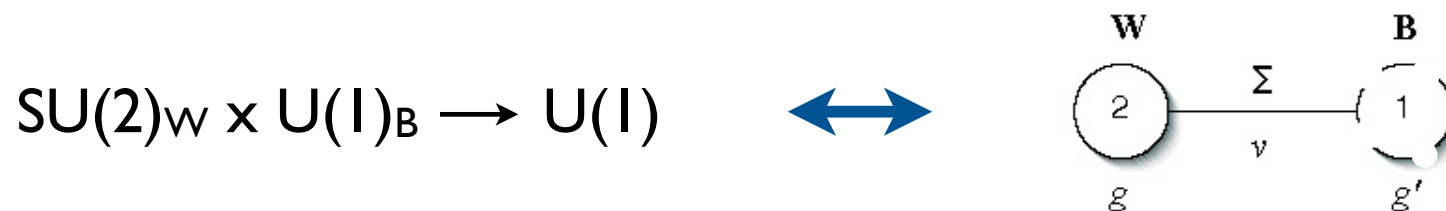
- Discretize fifth dimension ←-----→
- 4D gauge group at each site ○
- Nonlinear sigma model link fields —
- To include warping: vary f_j
- For spatially dependent coupling: vary g_k
- Continuum Limit: take $N \rightarrow$ infinity

Aside: Moose notation



Reveals symmetry (breaking) structure at a glance

A familiar example:



Each circle represents a global SU(2) of which all (solid, left) or a U(1) subgroup (dashed, right) is gauged

Low-energy \mathcal{L}_{eff} description of symmetry-breaking sector

employs non-linear sigma-model fields Σ

A solid line linking two circles is an $[\text{SU}(2) \times \text{SU}(2) / \text{SU}(2)]$ non-linear sigma model field; at the scale v this breaks the gauged or global symmetries of the attached circles

Note: Σ is a 2x2 matrix field transforming as $\Sigma \rightarrow L\Sigma R^\dagger$ under the SU(2) groups which it connects.

An $SU(2) \times SU(2) \times U(1) \times U(1)$ model with the following symmetry-breaking pattern:

$$SU(2)_L \times SU(2)_W \times U(1)_B \times U(1)_R$$

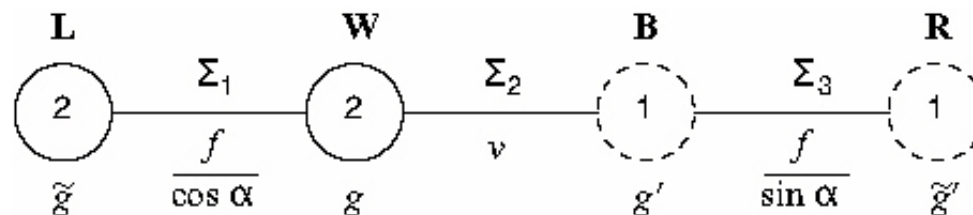


$$SU(2)_{\text{weak}} \times U(1)_Y$$

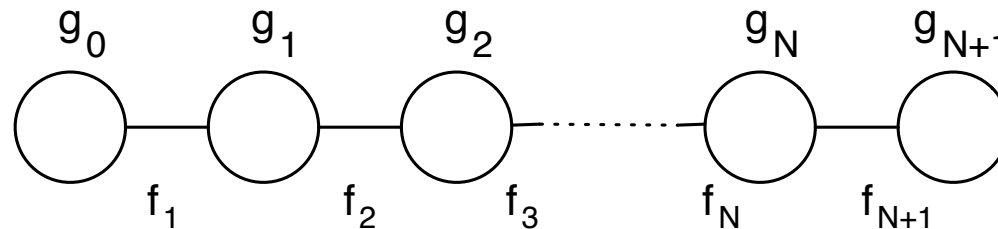


$$U(1)_{\text{EM}}$$

Can be represented compactly in Moose notation

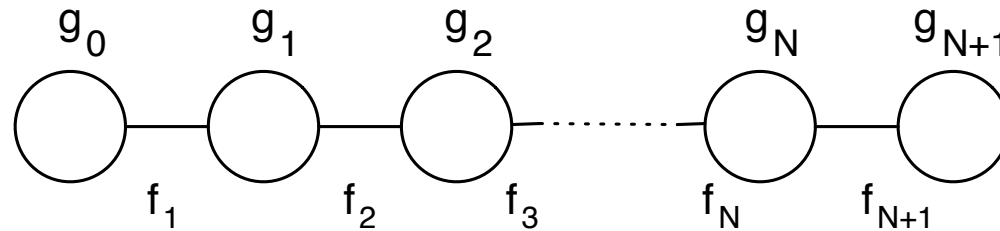


Deconstruction



- Discretize fifth dimension ←-----→
- 4D gauge group at each site ○
- Nonlinear sigma model link fields —
- To include warping: vary f_j
- For spatially dependent coupling: vary g_k
- Continuum Limit: take $N \rightarrow$ infinity

Continuum Limit



$$\mathcal{S} = - \int d^4x \sum_{j=0}^{N+1} \frac{1}{2g_j^2} \text{tr} (F_{\mu\nu}^j F^{j\mu\nu}) + \int d^4x \sum_{j=1}^{N+1} \frac{f_j^2}{4} \text{tr} \left((D_\mu U_j)^\dagger (D^\mu U_j) \right)$$

$$D_\mu U_j = \partial_\mu U_j - iA_\mu^{j-1} U_j + iU_j A_\mu^j$$

Unitary Gauge: $U_j = 1$

$$g_i = g \sqrt{N+2} \kappa_i \quad \frac{1}{g^2} = \sum_{i=0}^{N+1} \frac{1}{g_i^2},$$

$$f_i = f \sqrt{N+1} h_i \quad \frac{1}{f^2} = \sum_{i=1}^{N+1} \frac{1}{f_i^2}.$$

Define coordinate $y_i = \frac{i}{N+1}$

f and g , κ_i and h_i fixed as $N \rightarrow \infty$

As $N \rightarrow \infty$

$$y_i \rightarrow y ,$$

$$\Delta y = \frac{1}{N+1} \rightarrow dy ,$$

$$\frac{1}{N+1} \sum_{i=0}^{N+1} \rightarrow \int_0^1 dy ,$$

$$A_\mu^j(x) \rightarrow A_\mu(x, y) ,$$

$$(N+1) [A_\mu^{j+1}(x) - A_\mu^j(x)] \rightarrow \frac{\partial A_\mu}{\partial y}$$

$$\kappa_i, h_i \rightarrow \kappa(y), h(y) ,$$

$$\frac{1}{N+2} \sum_{i=0}^{N+1} \frac{1}{\kappa_i^2} \rightarrow \int_0^1 dy \frac{1}{\kappa^2(y)} = 1 ,$$

$$\frac{1}{N+1} \sum_{i=1}^{N+1} \frac{1}{h_i^2} \rightarrow \int_0^1 dy \frac{1}{h^2(y)} = 1 .$$

$$\mathcal{S}_5 = \int d^4x dy \left[-\frac{1}{2g^2\kappa^2(y)} \text{tr}(F_{\mu\nu}F^{\mu\nu}) + \frac{f^2 h^2(y)}{4} \text{tr}(F_{\mu y}F^{\mu y}) \right]$$

5D Yang Mills

$$\mathcal{S}_5 = \int d^4x dy \left[-\frac{1}{2g^2\kappa^2(y)} \text{tr}(F_{\mu\nu}F^{\mu\nu}) + \frac{f^2h^2(y)}{4} \text{tr}(F_{\mu y}F^{\mu y}) \right]$$

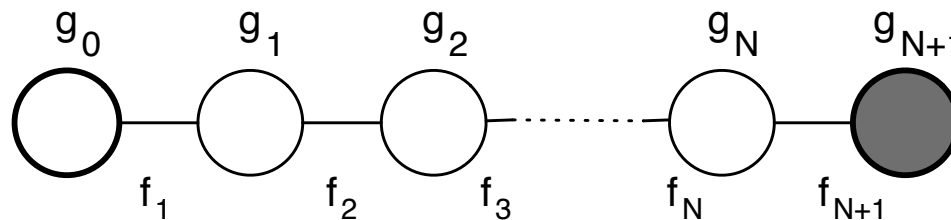
Equivalent to:

$$\mathcal{S}_{5dYM} = -\frac{1}{2} \int d^4x dx^5 \sqrt{G} \text{tr}(F_{MN}F^{MN})$$

With warped metric:

$$ds^2 = \frac{(gf)^2}{4} \kappa^2(y) h^2(y) \eta_{\mu\nu} dx^\mu dx^\nu - dy^2$$

Simplest Higgsless Models



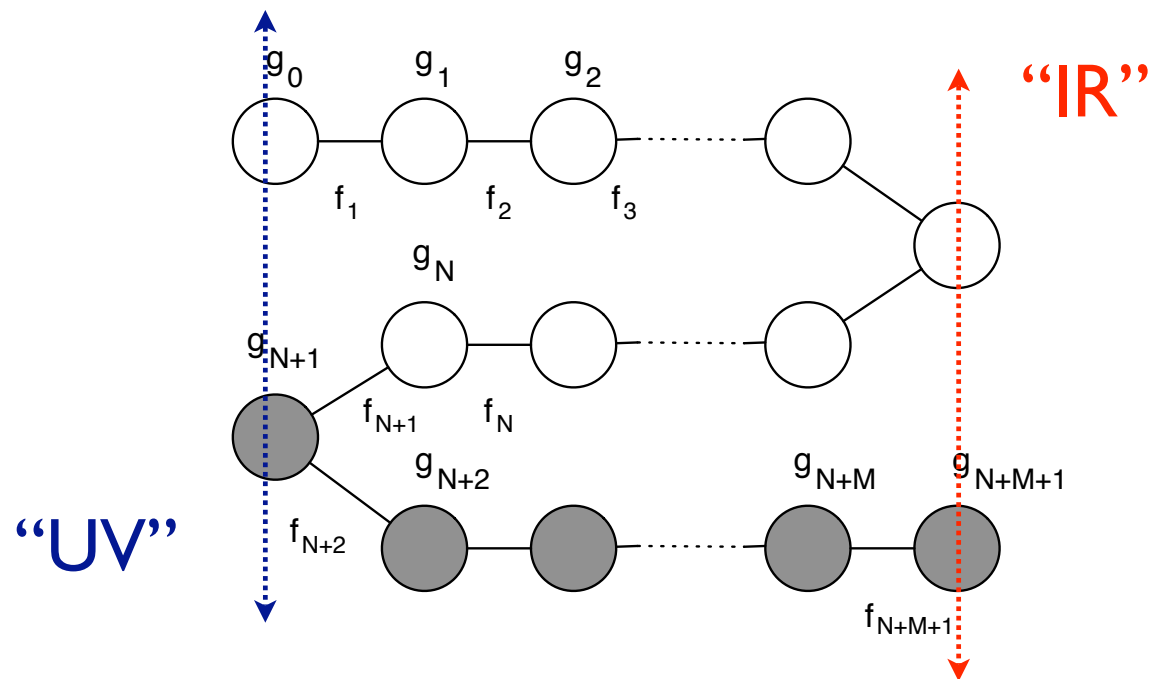
- $SU(2)^N \times U(1)$; general f_j and g_k
- Fermions sit on “branes” [sites 0 and $N+1$]
- Many 4-D/5-D theories are limiting cases...
study them all at once!
- e.g., $N=1$ equivalent to technicolor/one-Higgs

cf. “BESS” and “HLS”

Foadi, *et. al.* & Chivukula *et. al.*

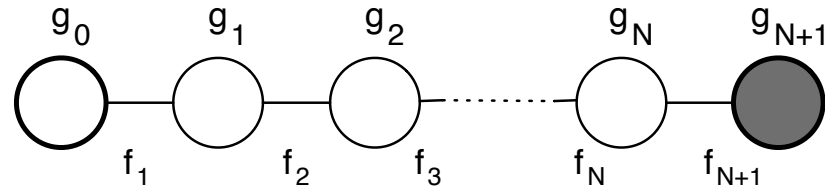
Generalizations

- by folding, represent $SU(2) \times SU(2) \times U(1)$ in “bulk”
- modify fermions’ location (brane? bulk?)



This talk starts with the simplest moose, then looks at generalizations

Mass Matrix & Spectrum



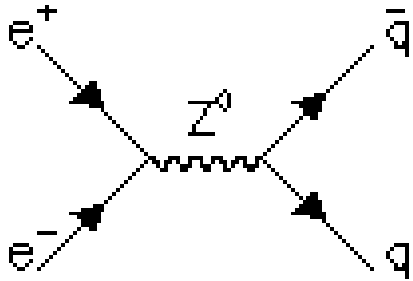
$$M_Z^2 = \frac{1}{4} \begin{pmatrix} g_0^2 f_1^2 & -g_0 g_1 f_1^2 & & & & \\ -g_0 g_1 f_1^2 & g_1^2 (f_1^2 + f_2^2) & -g_1 g_2 f_2^2 & & & \\ & -g_1 g_2 f_2^2 & g_2^2 (f_2^2 + f_3^2) & -g_2 g_3 f_3^2 & & \\ & & \ddots & \ddots & \ddots & \\ & & & -g_{N-1} g_N f_N^2 & g_N^2 (f_N^2 + f_{N+1}^2) & -g_N g_{N+1} f_{N+1}^2 \\ & & & & -g_N g_{N+1} f_{N+1}^2 & g_{N+1}^2 f_{N+1}^2 \end{pmatrix}.$$

- \mathbf{M}_Z^2 as above; Spectrum: Photon, Z, heavy Z's

- \mathbf{M}_W^2 has $g_{N+1} = 0$; Spectrum: W, heavy W's

- EM coupling as expected: $\frac{1}{e^2} = \frac{1}{g_0^2} + \frac{1}{g_1^2} + \dots + \frac{1}{g_{N+1}^2}$

Correlation Functions I



e.g., weak-hypercharge correlation function for Z exchange is within

$$[G(Q^2)]_{0,N+1} \equiv g_0 g_{N+1} \langle 0 | \frac{1}{Q^2 + M_Z^2} | N + 1 \rangle$$

$$Q^2 + M_Z^2 = \begin{pmatrix} Q^2 + g_0^2 f_1^2 / 4 & -g_0 g_1 f_1^2 / 4 & & & & & \\ -g_0 g_1 f_1^2 / 4 & Q^2 + g_1^2 (f_1^2 + f_2^2) / 4 & -g_1 g_2 f_2^2 / 4 & & & & \\ & -g_1 g_2 f_2^2 / 4 & Q^2 + g_2^2 (f_2^2 + f_3^2) / 4 & -g_2 g_3 f_3^2 / 4 & & & \\ & & \ddots & \ddots & \ddots & & \\ & & & -g_{N-1} g_N f_N^2 / 4 & Q^2 + g_N^2 (f_N^2 + f_{N+1}^2) / 4 & -g_N g_{N+1} f_{N+1}^2 / 4 & \\ & & & & -g_N g_{N+1} f_{N+1}^2 / 4 & Q^2 + g_{N+1}^2 f_{N+1}^2 / 4 & \end{pmatrix}$$

By considering the (0,N+1) co-factor, we deduce the form of the correlation function

Correlation Functions II

$$[G(Q^2)]_{0,N+1} = \frac{C}{Q^2(Q^2 + M_Z^2) \prod_{n=1}^N (Q^2 + m_{Z_n}^2)}$$

We know the residue of $Q^2=0$ pole must be $e^2 \dots$

$$[G(Q^2)]_{0,N+1} = \frac{e^2 M_Z^2 \prod_{n=1}^N m_{Z_n}^2}{Q^2(Q^2 + M_Z^2) \prod_{n=1}^N (Q^2 + m_{Z_n}^2)}$$

Other residues are also informative.

S parameter

Correlation function residue at $Q^2 = -M_Z^2$
gives “ $J_3 J_Y$ ” coupling of light Z-boson

$$[\xi_Z]_{WY} = -e^2 \prod_{n=1}^N \frac{1}{1 - \frac{M_Z^2}{m_{Z_n}^2}} = -e^2 \left(1 + \frac{\alpha S}{4s_Z^2 c_Z^2} \right)$$

Requiring $M_Z \ll m_{Z_n}$ yields

$$\alpha S \approx 4s_Z^2 c_Z^2 \sum_{n=1}^N \frac{M_Z^2}{m_{Z_n}^2}$$

for the entire class of models!

Electroweak Parameters I

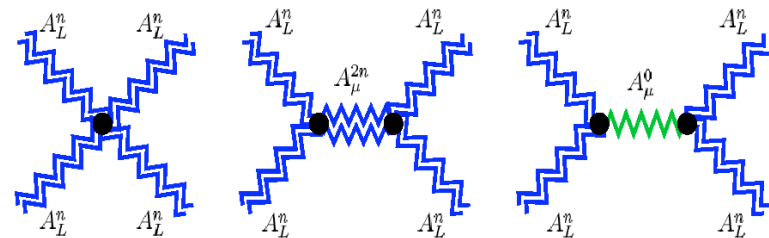
EW corrections (S , T , $\Delta\rho$, δ) defined from amplitudes for “on-shell” 4-fermion processes

$$\begin{aligned}
 -\mathcal{A}_{NC} = & e^2 \frac{QQ'}{Q^2} + \frac{(I_3 - s^2 Q)(I'_3 - s^2 Q')}{\left(\frac{s^2 c^2}{e^2} - \frac{S}{16\pi}\right) Q^2 + \frac{1}{4\sqrt{2}G_F} \left(1 + \frac{\alpha\delta}{4s^2 c^2} - \alpha T\right)} \\
 & + \sqrt{2}G_F \frac{\alpha\delta}{s^2 c^2} I_3 I'_3 + 4\sqrt{2}G_F (\Delta\rho - \alpha T) (Q - I_3)(Q' - I'_3)
 \end{aligned}$$

$$-\mathcal{A}_{CC} = \frac{(I_+ I'_- + I_- I'_+)/2}{\left(\frac{s^2}{e^2} - \frac{S}{16\pi}\right) Q^2 + \frac{1}{4\sqrt{2}G_F} \left(1 + \frac{\alpha\delta}{4s^2 c^2}\right)} + \sqrt{2}G_F \frac{\alpha\delta}{s^2 c^2} \frac{(I_+ I'_- + I_- I'_+)}{2}.$$

Conflict of S & Unitarity

Heavy resonances must unitarize WW scattering
(since there is no Higgs!)



This bounds lightest KK mode mass: $m_{Z_1} < \sqrt{8\pi}v$

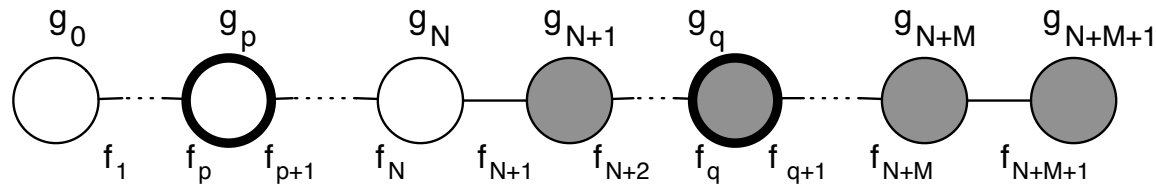
... and yields
$$\alpha S \geq \frac{4s_Z^2 c_Z^2 M_Z^2}{8\pi v^2} = \frac{\alpha}{2}$$

Too large by a factor of a few!

Independent of warping or gauge couplings chosen...

Generalize Localized Fermions?

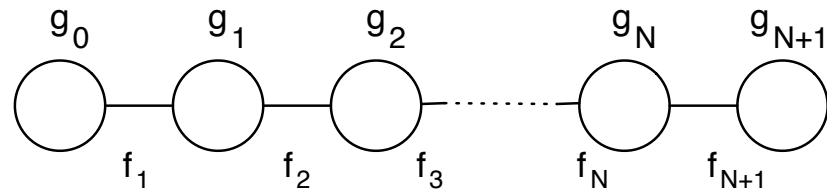
- Since models with fermions localized on the end branes are not viable...
- Consider most **general** model with fermions localized on branes:



How can we tell if its spectrum is acceptable?
(or if **S** is in conflict with unitarity?)

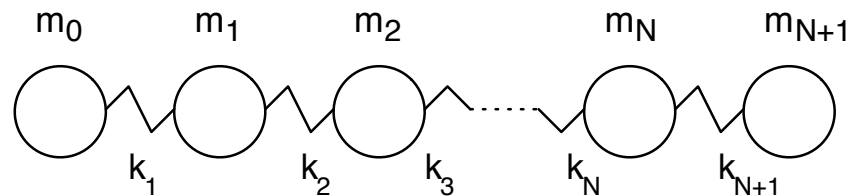
Mass matrix has oscillator analog!

Springs and Mooses



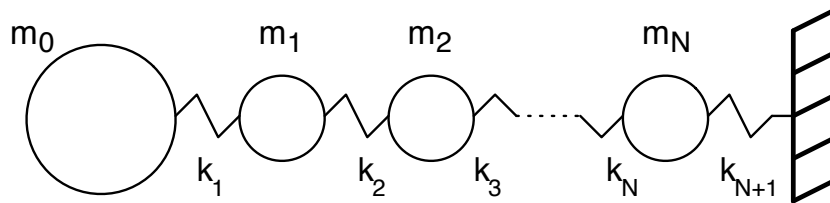
Mass eigenstates
of moose above
are equivalent to
spectrum of
oscillator below

$$k_i \leftrightarrow \frac{f_i^2}{4} \quad m_i \leftrightarrow \frac{1}{g_i^2}$$



W Moose and Oscillator

W moose with small g_0 is equivalent to



Lowest mode



Light W

$$\frac{1}{k_{eff}} = \frac{1}{k_1} + \frac{1}{k_2} + \dots + \frac{1}{k_{N+1}}$$

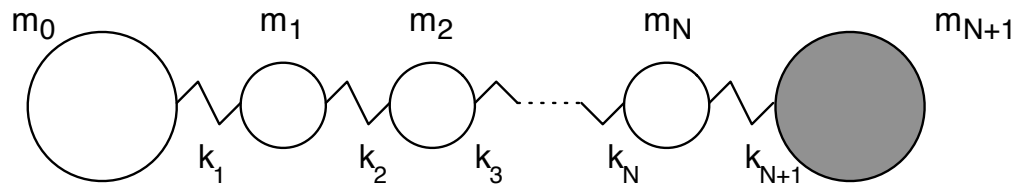
$$\omega_W^2 \approx \frac{k_{eff}}{m_0}$$

$$\frac{4}{v^2} = \frac{4}{f_1^2} + \frac{4}{f_2^2} + \dots + \frac{4}{f_{N+1}^2}$$

$$M_W^2 \approx \frac{g_0^2 v^2}{4}$$

Z Moose and Oscillator

Spectrum of Z moose with g_0 and g_{N+1} small?



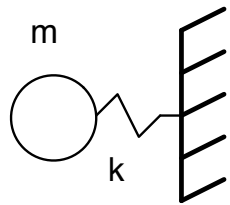
Zero mode (translation) \longleftrightarrow Photon

Next-lowest mode (breathing) \longleftrightarrow Light Z

$$\omega_Z^2 \approx k_{eff} \left(\frac{1}{m_0} + \frac{1}{m_{N+1}} \right)$$

$$M_Z^2 \approx \frac{(g_0^2 + g_{N+1}^2)v^2}{4}$$

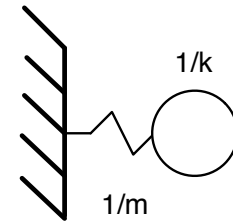
Spring Duality



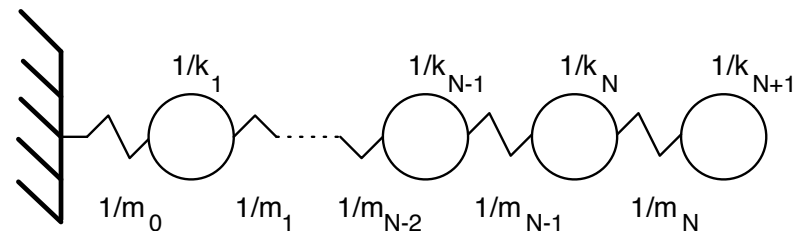
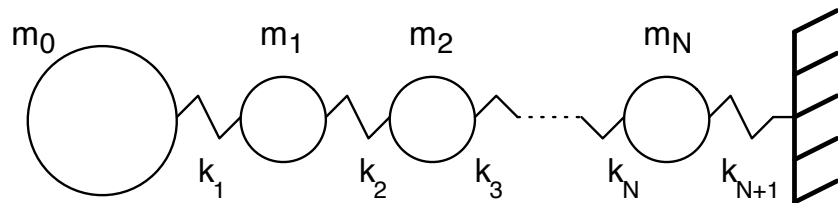
$$\omega^2 = \frac{k}{m}$$

invariant
under

$$k \leftrightarrow m^{-1}$$

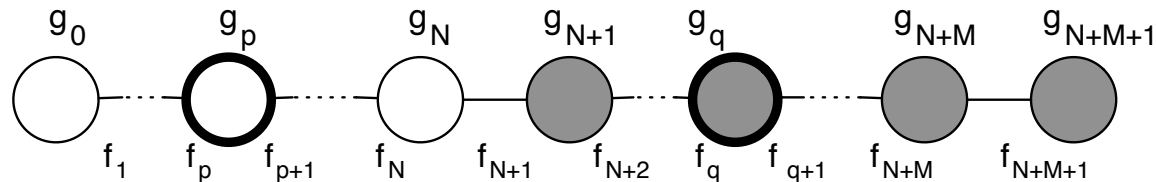


This generalizes to coupled springs!



All non-zero modes same for two systems.
Moose duality: related to “eaten” Goldstone Bosons

Spring Analysis of General Moose

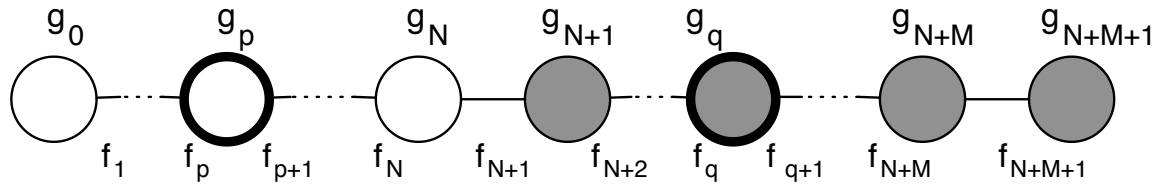


W spectrum (oscillator with wall at site $N+1$):

- identical f_j , one small $g_k \Rightarrow$ 1 light W
- identical f_j , 2 small $g_k \Rightarrow$ 2 light W 's
- by **duality**, same g_k , 2 small $f_j \Rightarrow$ 2 light W 's

Small \hat{S}, W ? with wall at site p , must have no heavy masses in either sub-oscillator; can g_k be small for $k \neq p$? think of stiff springs...

Preliminary Conclusion

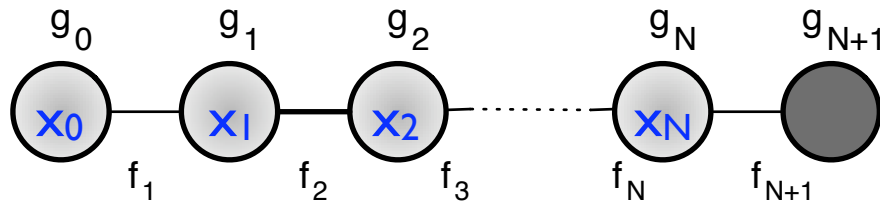


Any model of this kind that

- has localized fermions (at any sites p, q),
- & has only a light photon, W , and Z ,
- & is unitary ...

also has $\hat{S} > 5 \times 10^{-3}$... which is **too large!**

A New Hope?



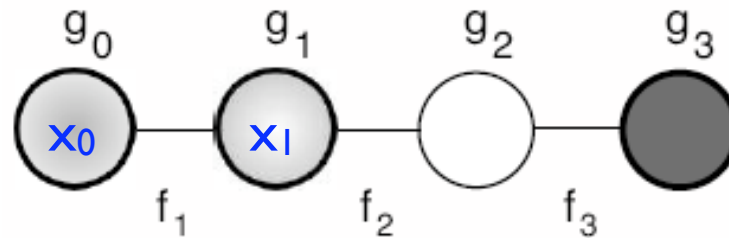
Since Higgsless models with localized fermions are not viable, look at:

Delocalized Fermions, .i.e., mixing of “brane” and “bulk” modes

$$\mathcal{L}_f = \vec{J}_L^\mu \cdot \left(\sum_{i=0}^N x_i \vec{A}_\mu^i \right) + J_Y^\mu A_\mu^{N+1}$$

How will this affect precision EW observables?

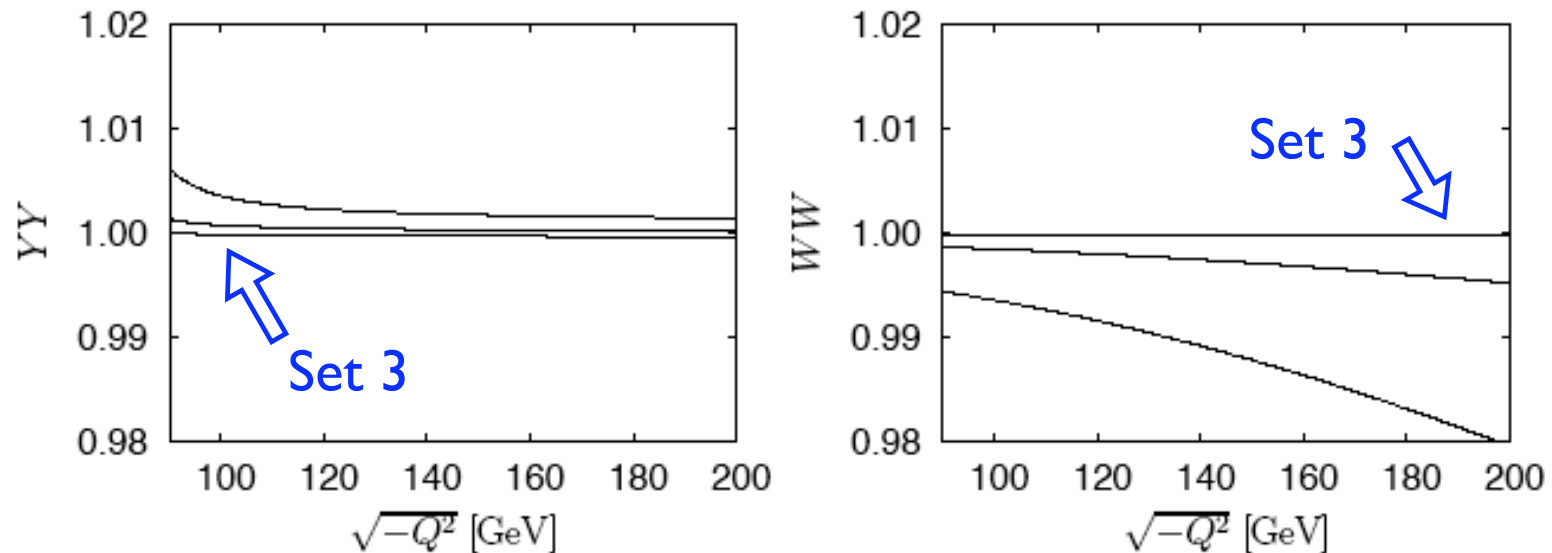
A Toy Model of Delocalization



Massless
Fermions!

	Set 1	Set 2	Set 3
Inputs			
f_1	1000 GeV	2000 GeV	300 GeV
$f_2 = f_3$	356.303 GeV	348.922 GeV	591.850 GeV
g_0	0.664421	0.663478	0.657164
$g_1 = g_2$	4.0	4.0	4.0
g_3	0.356505	0.356651	0.357650
x_1 (fraction on site 1)	0.139231	0.480892	0.014771
Calculated Physical Masses			
M_W	79.9486 GeV	79.9080 GeV	79.9599 GeV
m_{Z1}	976.990 GeV	983.725 GeV	892.459 GeV
m_{W1}	975.913 GeV	982.737 GeV	888.827 GeV
m_{Z2}	2162.17 GeV	4114.49 GeV	1944.08 GeV
m_{W2}	2162.17 GeV	4144.49 GeV	1943.39 GeV

Correlation Functions at LEP-I and II

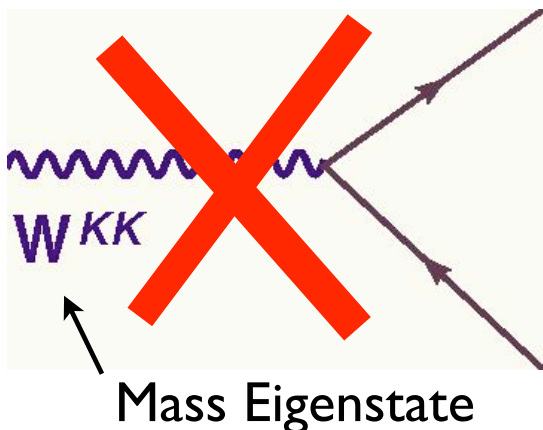


Ratio to Standard Model is shown
... Set 3 comes extremely close!

How to improve on toy model?

Ideal Delocalization

- Choose delocalization related to W wavefunction: $g_i x_i \propto v_i^W$
- NB: $x_i = |\psi_f(i)|^2 > 0$
- W -wavefunction orthogonal to KK wavefunctions.
- No (tree-level) couplings to heavy modes!

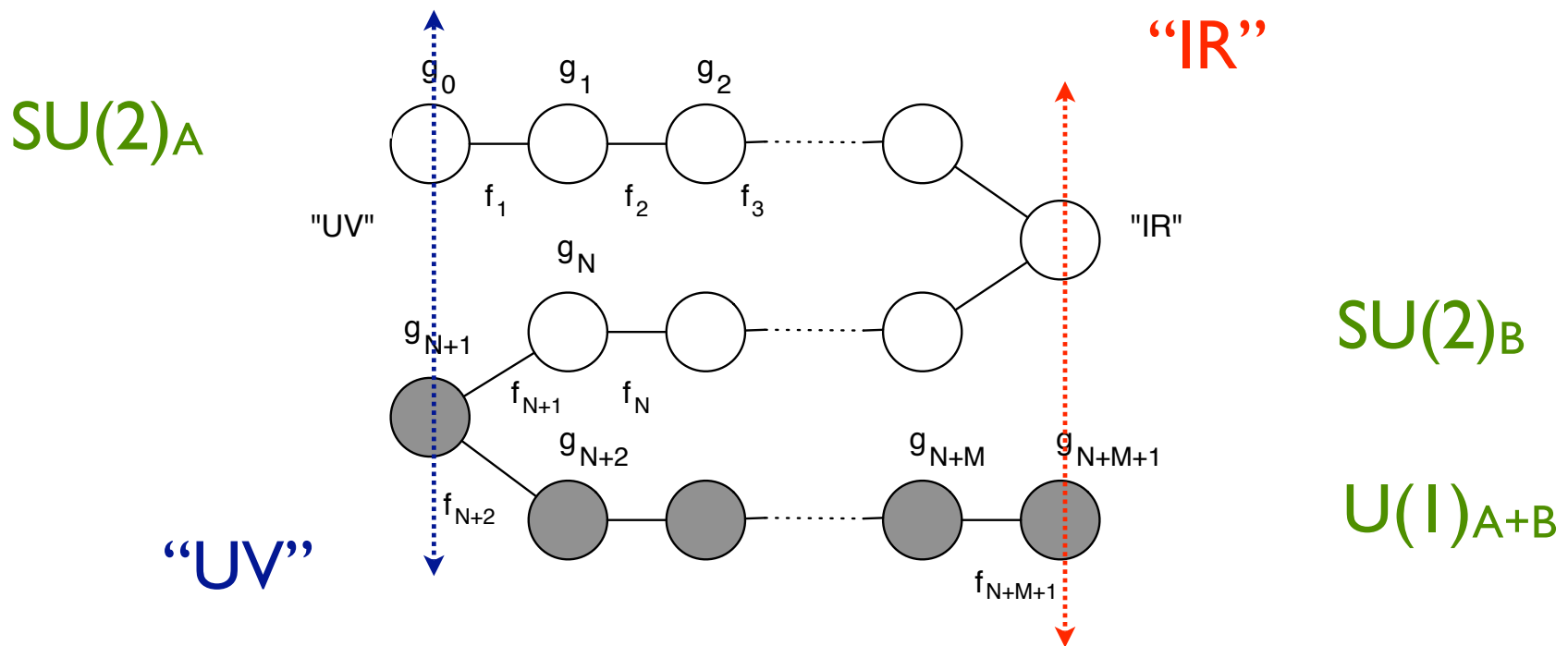


$$\hat{S} = \hat{T} = W = 0$$

$$Y = M_W^2 (\Sigma_W - \Sigma_Z)$$

Signals of ideal delocalization in a sample class of models

- by folding, represent $SU(2) \times SU(2) \times U(1)$ in “bulk”
- ideally delocalize fermions



Triple Gauge Vertices

$$g_{VWW} = \int_0^{\pi R} dz \left\{ \frac{1}{g_{5WA}^2} \chi_V^A(z) |\chi_W^A(z)|^2 + \frac{1}{g_{5WB}^2} \chi_V^B(z) |\chi_W^B(z)|^2 \right\} + \frac{1}{g_0^2} \chi_V^A(0) |\chi_W^A(0)|^2$$

Hagiwara, *et. al.* define:

$$\begin{aligned} \mathcal{L}_{TGV} &= -ie \frac{c_Z}{s_Z} [1 + \Delta\kappa_Z] W_\mu^+ W_\nu^- Z^{\mu\nu} - ie [1 + \Delta\kappa_\gamma] W_\mu^+ W_\nu^- A^{\mu\nu} \\ &- ie \frac{c_Z}{s_Z} [1 + \Delta g_1^Z] (W^{+\mu\nu} W_\mu^- - W^{-\mu\nu} W_\mu^+) Z_\nu \\ &- ie (W^{+\mu\nu} W_\mu^- - W^{-\mu\nu} W_\mu^+) A_\nu , \end{aligned}$$

TGVs in Flat Space with Ideal Delocalization

$$\Delta g_1^Z = \Delta \kappa_Z = \frac{\lambda}{12c^2} \frac{7 + \kappa}{1 + \kappa} > 0$$

$$\lambda = \frac{g_0^2 \pi R}{g_{5WA}^2 + g_{5WB}^2} \quad \kappa = \frac{g_{5WB}^2}{g_{5WA}^2}$$

NB: $\Delta \kappa_\gamma \equiv 0$ $\lambda \rightarrow 0$ as $M_W^2 \rightarrow 0$

As consistent with EM Gauge Invariance

LEP II Constraints

LEP II measurements of WWZ vertex yield

$$\Delta g_1^Z \leq 0.028 \quad @ \quad 95\% \text{CL}$$

In our flat space SU(2) x SU(2) x U(1) model

$$\Delta g_1^Z = \frac{\pi^2}{12c^2} \left(\frac{M_W}{M_{W_1}} \right)^2 \left[\frac{1}{4} \cdot \frac{7 + \kappa}{1 + \kappa} \right]$$

$$M_{W_1} \geq 500 \text{ GeV} \cdot \sqrt{\frac{0.028}{\Delta g_{max}} \left[\frac{1}{4} \cdot \frac{7 + \kappa}{1 + \kappa} \right]}$$

Chiral Lagrangian Parameters

Appelquist-Longhitano Operators:

$$\begin{aligned}\mathcal{L}_1 &\equiv \frac{1}{2}\alpha_1 g_W g_Y B_{\mu\nu} \text{Tr}(TW^{\mu\nu}) \\ \mathcal{L}_2 &\equiv \frac{1}{2}i\alpha_2 g_Y B_{\mu\nu} \text{Tr}(T[V^\mu, V^\nu]) \\ \mathcal{L}_3 &\equiv i\alpha_3 g_W \text{Tr}(W_{\mu\nu}[V^\mu, V^\nu]) \\ \mathcal{L}_4 &\equiv \alpha_4 [\text{Tr}(V^\mu V^\nu)]^2 \\ \mathcal{L}_5 &\equiv \alpha_5 [\text{Tr}(V_\mu V^\mu)]^2\end{aligned}$$

$$T \equiv U\tau_3 U^\dagger \qquad V_\mu \equiv (D_\mu U)U^\dagger$$

U arises from $SU(2)_L \times SU(2)_R \rightarrow SU(2)_V$

Flat Space Results

flat $SU(2) \times SU(2)$ Longhitano parameters	brane localized	ideally delocalized
$e^2 \alpha_1$	$-\frac{2}{3} \lambda s^2$	0
$e^2 \alpha_2$	$-\frac{1}{12} \left(\frac{7+\kappa}{1+\kappa} \right) \lambda s^2$	$-\frac{1}{12} \left(\frac{7+\kappa}{1+\kappa} \right) \lambda s^2$
$e^2 \alpha_3$	$-\frac{1}{12} \left(\frac{1+7\kappa}{1+\kappa} \right) \lambda s^2$	$\frac{1}{12} \left(\frac{7+\kappa}{1+\kappa} \right) \lambda s^2$
$e^2 \alpha_4$	$\frac{1}{30} \frac{(1+14\kappa+\kappa^2)}{(1+\kappa)^2} \lambda s^2$	$\frac{1}{30} \frac{(1+14\kappa+\kappa^2)}{(1+\kappa)^2} \lambda s^2$
$e^2 \alpha_5$	$-\frac{1}{30} \frac{(1+14\kappa+\kappa^2)}{(1+\kappa)^2} \lambda s^2$	$-\frac{1}{30} \frac{(1+14\kappa+\kappa^2)}{(1+\kappa)^2} \lambda s^2$

g_{WWW}

$\alpha S, L_{10}$

(Final!) Conclusions

- **Higgsless** models are intriguing candidate solutions to the puzzle of EWSB.
- **Deconstruction** gives framework for studying 5-d gauge theories as consistent effective field theories.
- Higgsless models with localized fermions are **not** phenomenologically viable (S too large).
- **Delocalized fermions** can yield viable models
- **Ideal Delocalization**: \hat{S} , \hat{T} , W vanish; Y is small ...
 - * best limits from TGV ($M_{KK} > 500\text{-}700$ GeV)
 - * chiral Lagrangian parameters calculable / visible