Physics with Extra Dimensions:

Aren't four enough?

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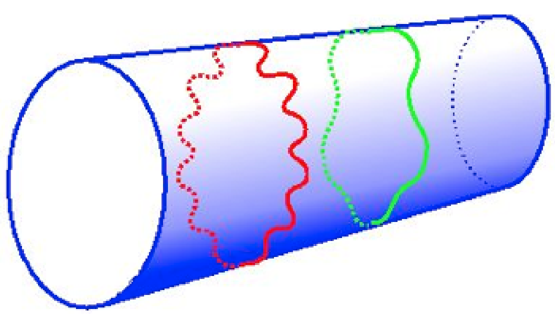
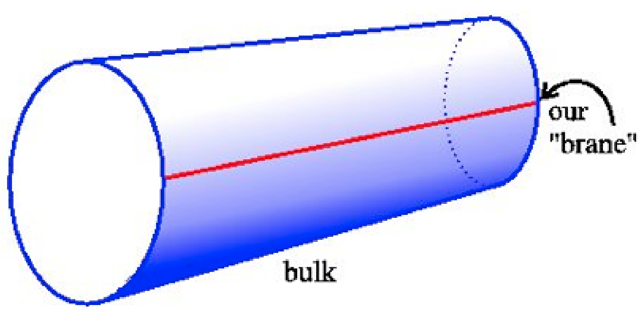
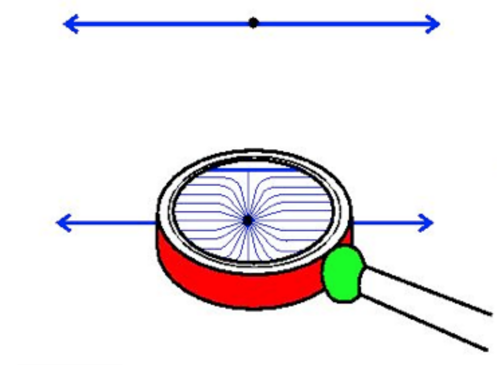






Topical Seminar on Frontier of Particle Physics 2006:

Beyond the Standard Model

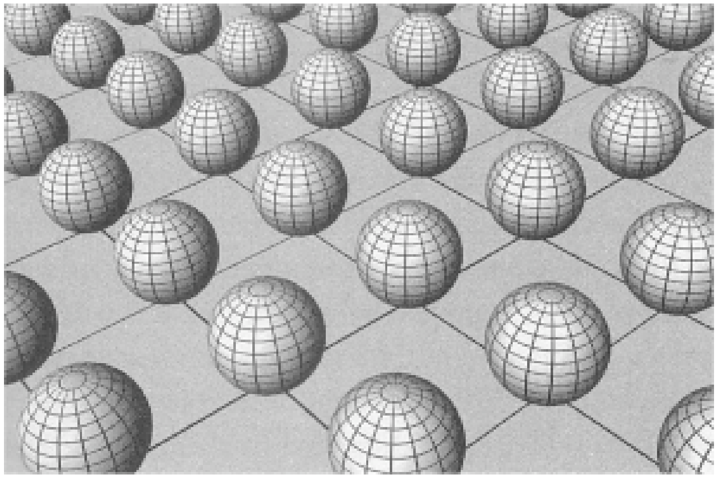
Beijing, China, August 7-11, 2006

They could be there,...

Gunnar Nordstrom

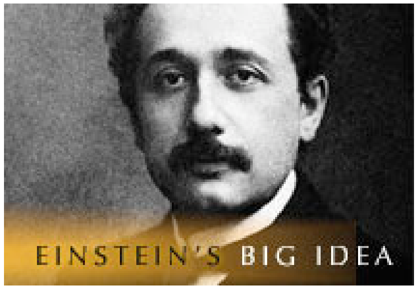
Théodore Kaluza

Oscar Klein

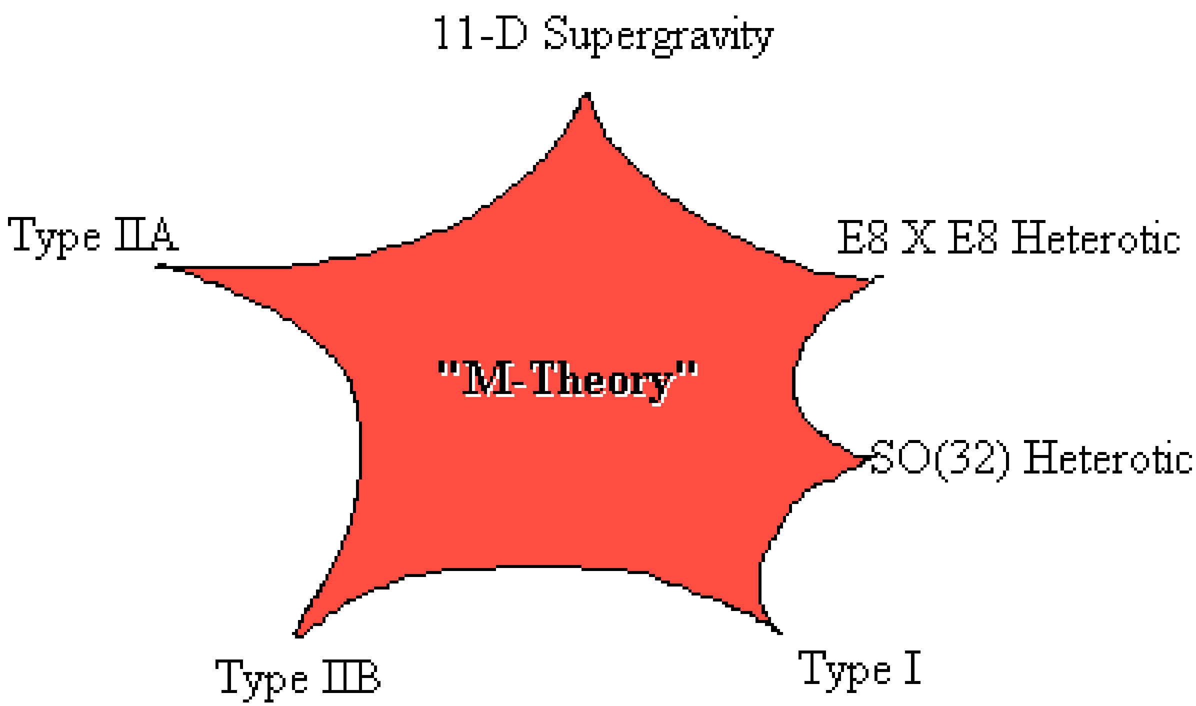


In GR Symmetries of Extra Dimensions yield Gauge Fields

Quantum Version?
Sources?

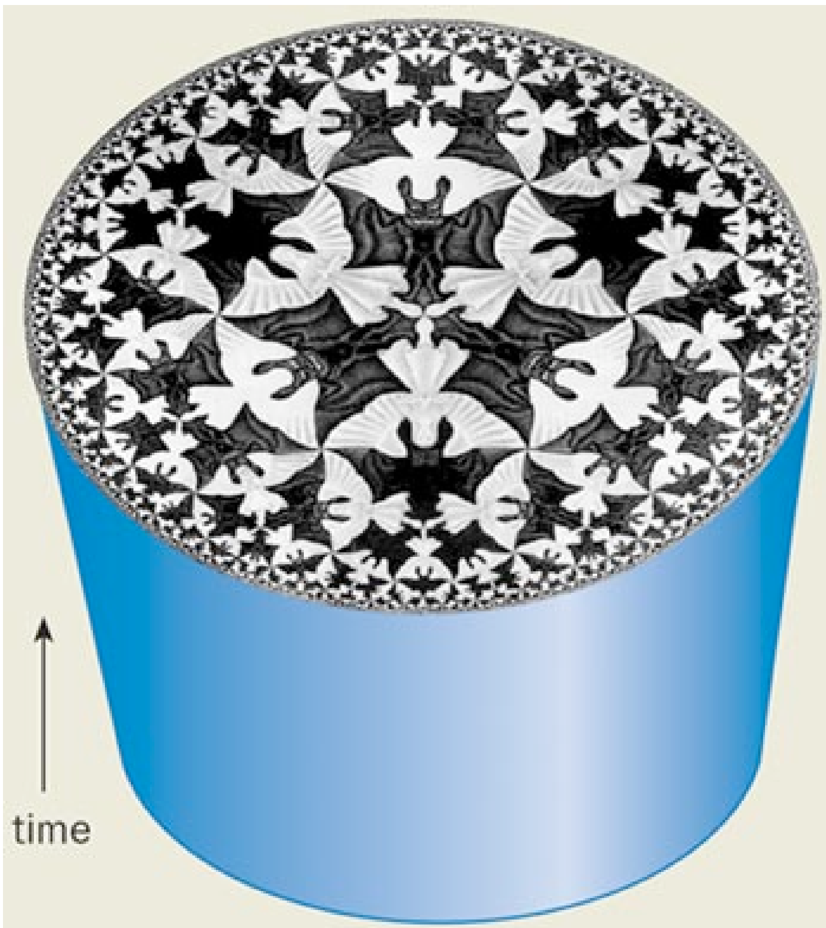


... and are required by string theory,...



Consistency requires 10 or 11 dimensions

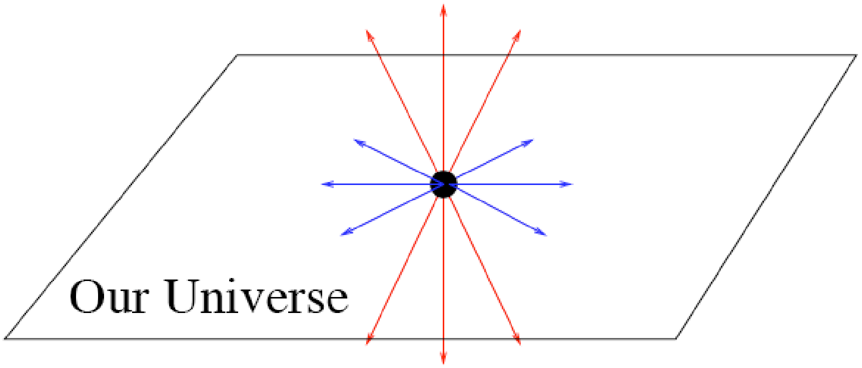
.. which inspires holography/duality,...



Equivalence of theory on boundary of space-time to that on the interior...

(More Later)

... and, with branes, their existence could address the hierarchy.



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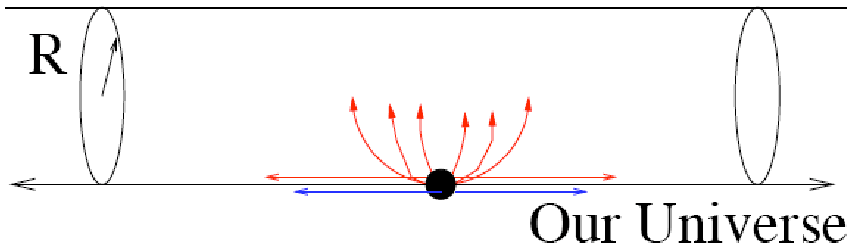
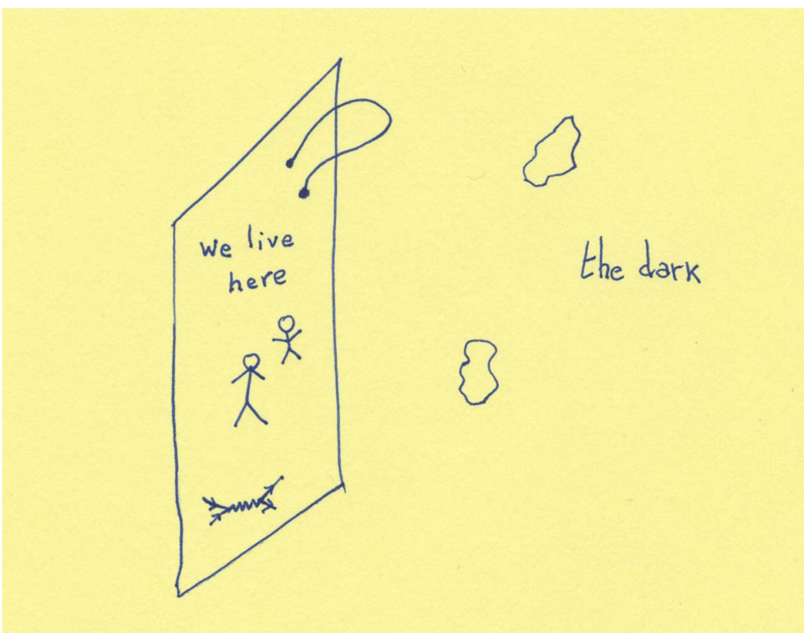
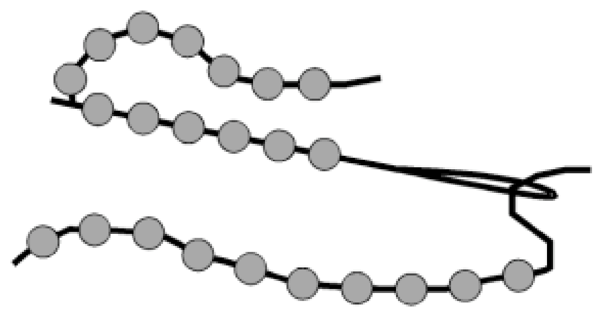
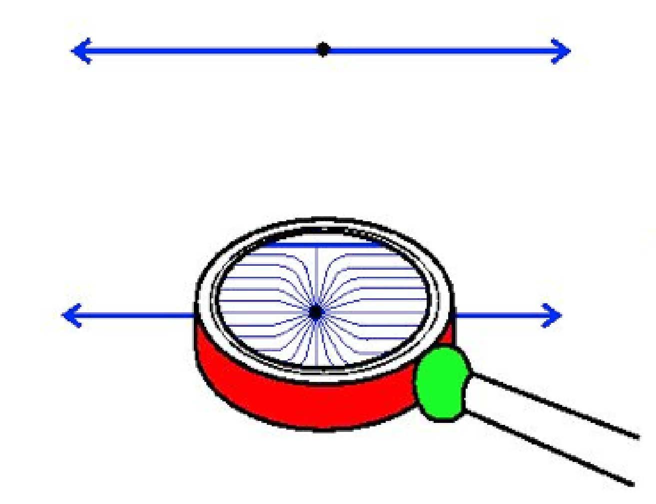






We're could be like beads on a string!

Large Extra Dimensions



$$V_{Grav}(r) \propto \begin{cases} \frac{m_1 m_2}{M_{Pl}^2\, r} & r \gg R \\ \frac{m_1 m_2}{M^{2+n}\, r^{n+1}} & r \ll R \end{cases}$$

$$M_{Pl}^2 \propto M^{2+n}\, R^n$$

For M = 1 TeV:

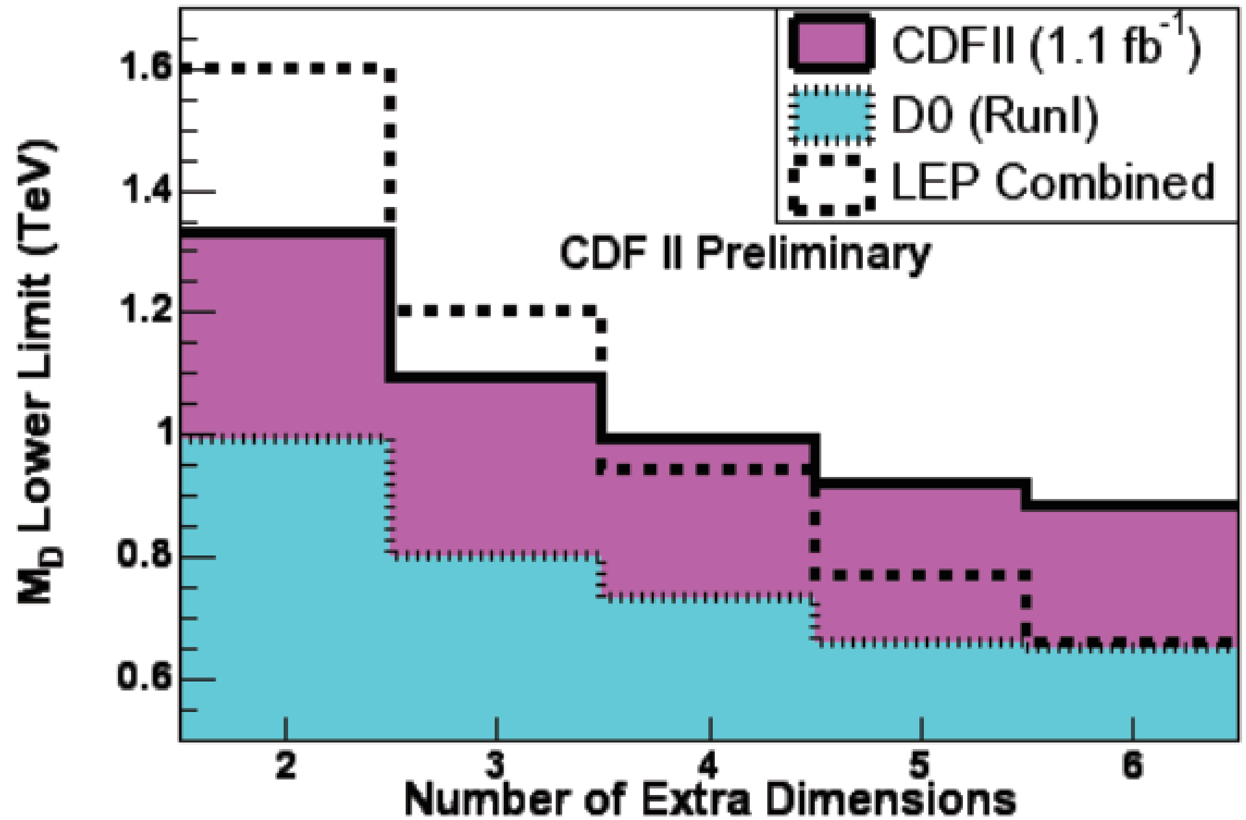
- $n = 2 \Rightarrow R = \mathcal{O}(1\,\mathrm{mm})$ & $R^{-1} \simeq 10^{-4}\,\mathrm{eV}$

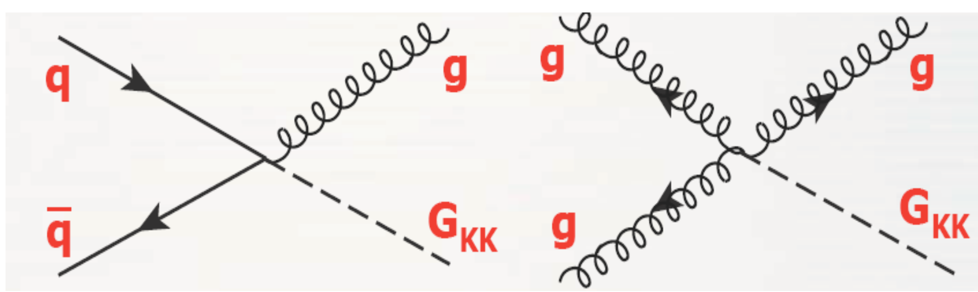
  $SuperNovae\ require\ M \geq 50\ TeV\ for\ n = 2$

- $n = 6 \Rightarrow R = \mathcal{O}(10^{-12}\,\mathrm{cm})$ & $R^{-1} \simeq 10\,\mathrm{MeV}$

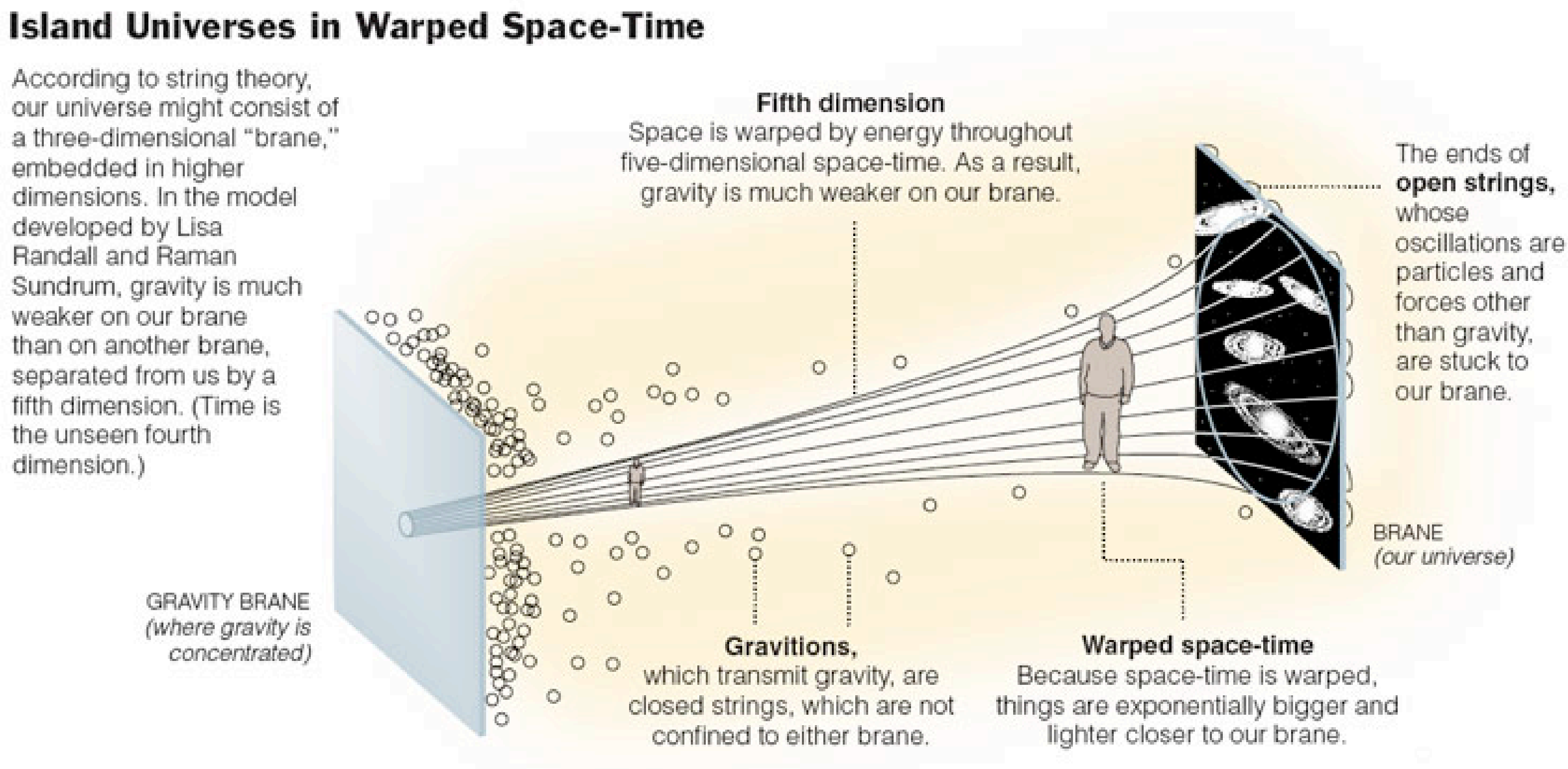
Arkani-Hamed, Dimopoulos, Dvali, & Antoniadis, Lykken, Dudas, Gerghetta, ...

Collider Limits on LED





Warped Extra Dimensions

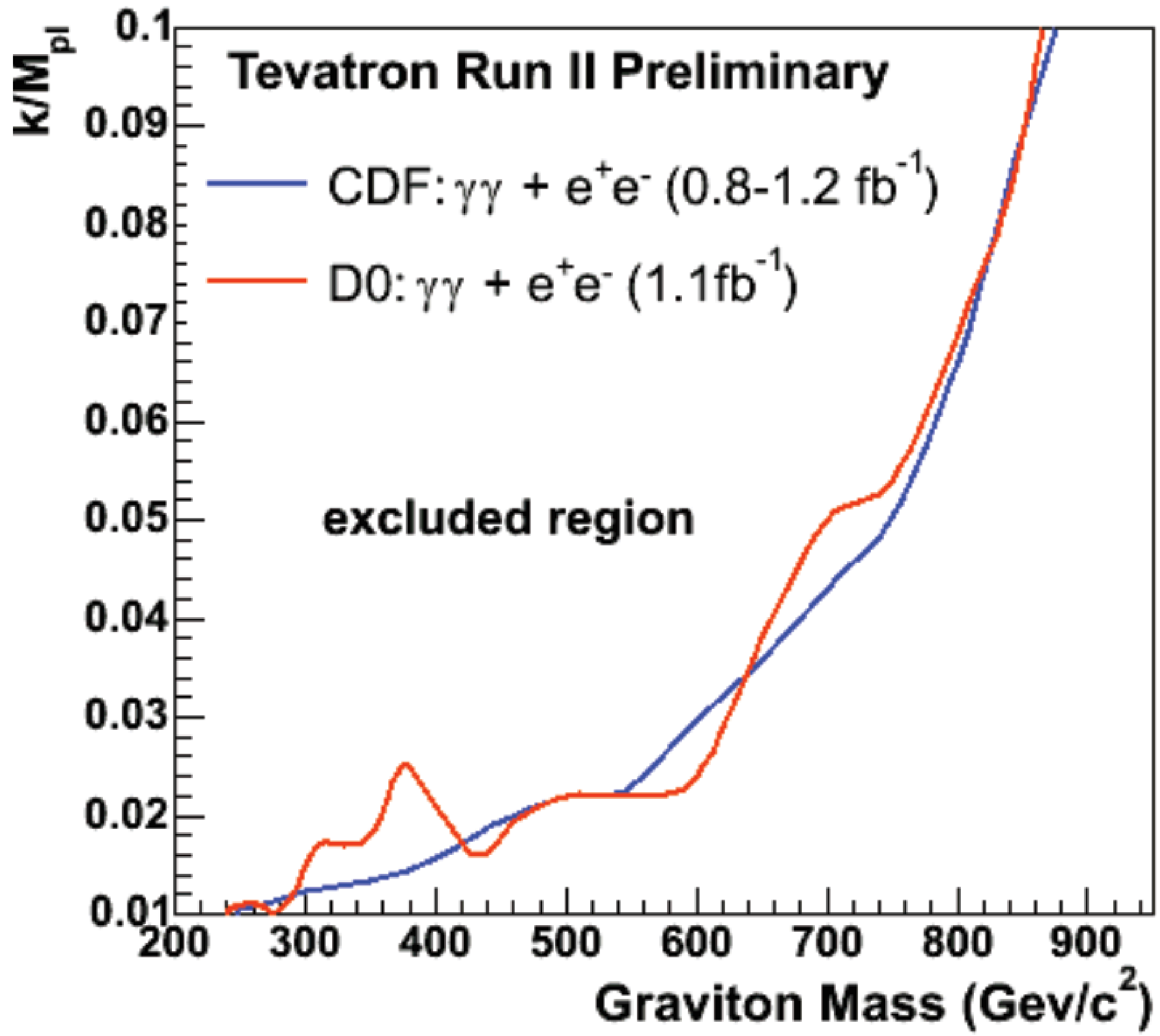


$$ds^2 = e^{-2kr_c|\varphi|}d^2x_4 + r_c^2 d\varphi^2$$

$$M_{Pl}^2 \propto \frac{M^3}{k}\left(1 - e^{-2k\pi r_c}\right)$$

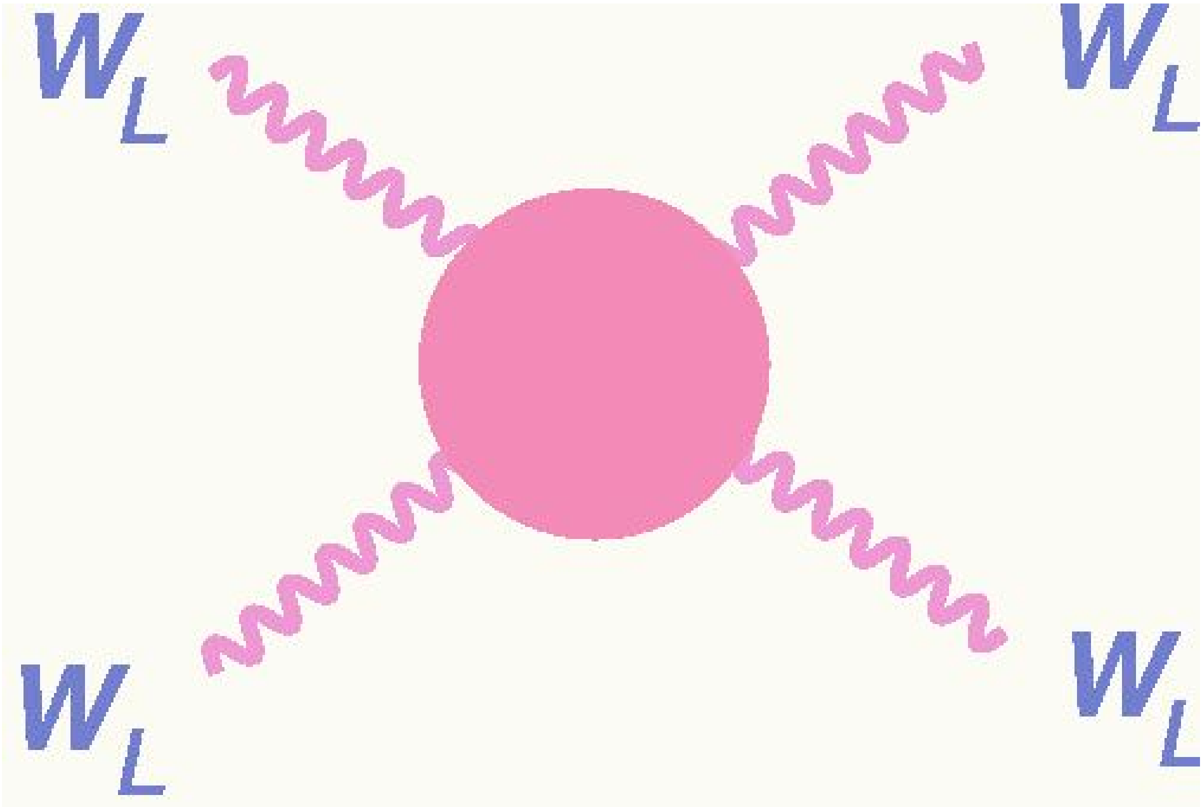
Randall and Sundrum, ...

RS Experimental Limits

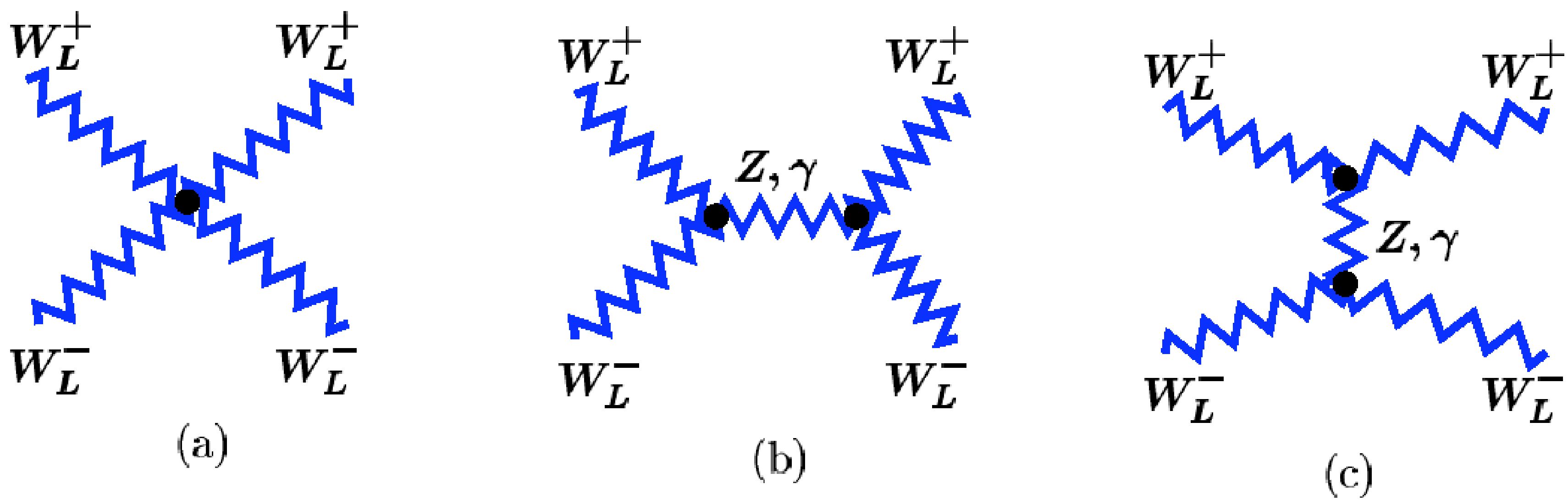


Can Extra-D be related to EWSB?

Consider Loss of Unitarity in



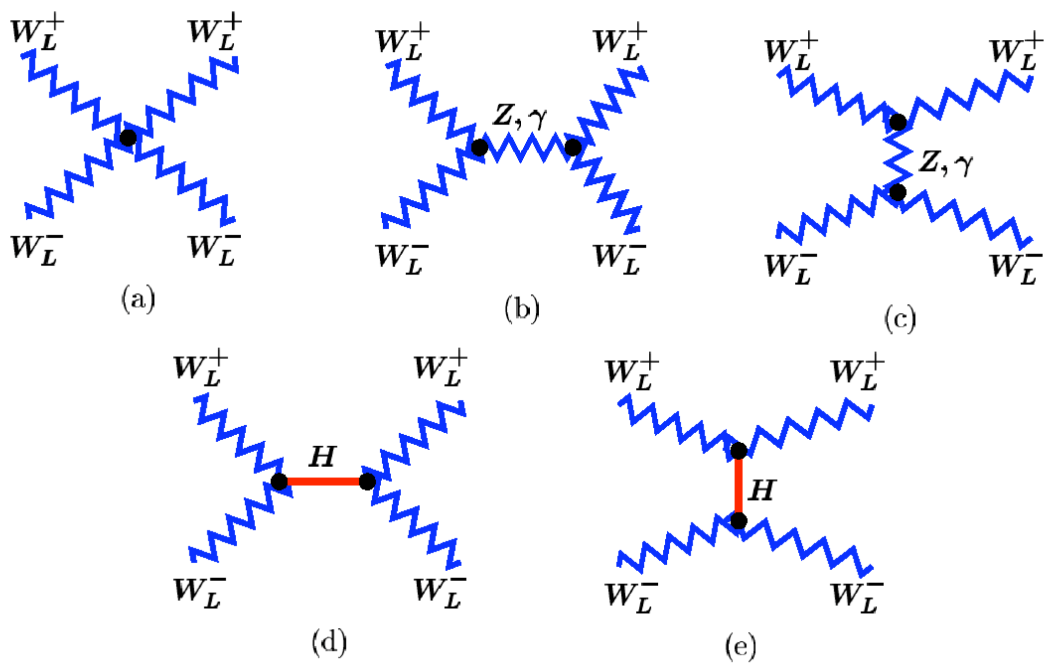
SU(2) x U(1) @ E^4



Graphs	$g^2 \frac{E^4}{m_w^4}$
(a)	$-3 + 6\cos\theta + \cos^2\theta$
(b)	$-4\cos\theta$
(c)	$+3 - 2\cos\theta - \cos^2\theta$
Sum	0

$$\epsilon_L^\mu(k) = \frac{k^\mu}{m_w} + \mathcal{O}\left(\frac{m_w}{E}\right)$$

SU(2) x U(1) @ E^2

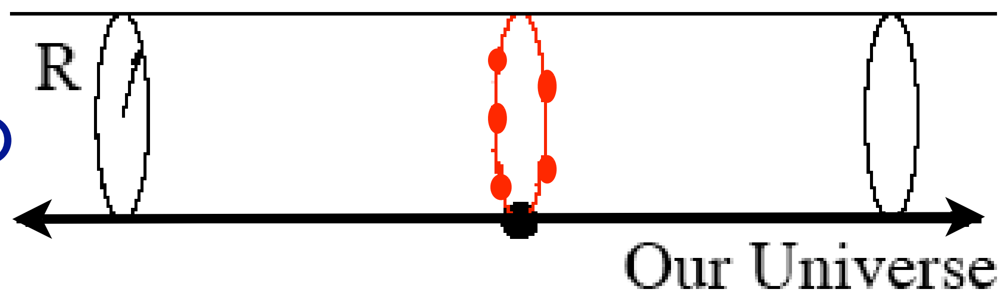


Graphs	$g^2 \frac{E^2}{m_w^2}$
(a)	$+2 - 6\cos\theta$
(b)	$-\cos\theta$
(c)	$-\frac{3}{2} + \frac{15}{2}\cos\theta$
(d + e)	$-\frac{1}{2} - \frac{1}{2}\cos\theta$
Sum including (d+e)	**0**

- $\mathcal{O}(E^0) \Rightarrow$ 4d m_H bound: $m_H < \sqrt{16\pi/3}\, v \simeq 1.0$ TeV
- If no Higgs $\Rightarrow \mathcal{O}(E^2) \Rightarrow \; E < \sqrt{4\pi}\, v \simeq 0.9$ TeV

Massive Gauge Bosons arise from Extra-D YM Theories!

Flat Space:



Expand 5-D gauge bosons in eigenmodes:

e.g. for S^1/Z_2:

$$\widehat{A}^a_\mu = \frac{1}{\sqrt{\pi R}}\left[A^{a0}_\mu(x_\nu) + \sqrt{2}\sum_{n=1}^{\infty} A^{an}_\mu(x_\nu)\cos\left(\frac{n x_5}{R}\right)\right]$$

$$\widehat{A}^a_5 = \sqrt{\frac{2}{\pi R}}\sum_{n=1}^{\infty} A^{an}_5(x_\nu)\sin\left(\frac{n x_5}{R}\right)$$

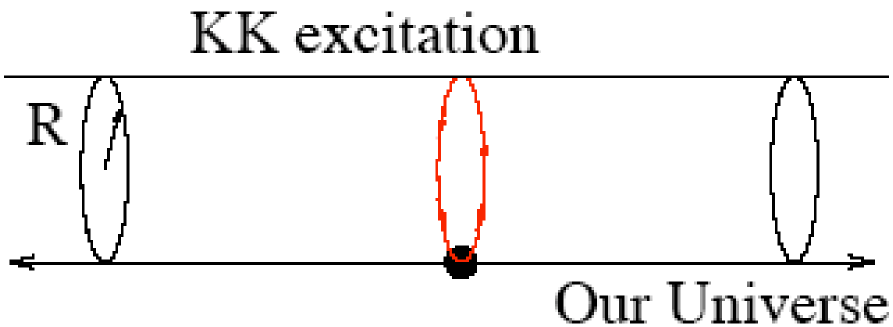
4-D gauge kinetic term contains

$$M_n = \frac{n}{R}$$

$$\frac{1}{2}\sum_{n=1}^{\infty}\left[M_n^2 (A^{an}_\mu)^2 - 2 M_n A^{an}_\mu \partial^\mu A^{an}_5 + (\partial_\mu A^{an}_5)^2\right]$$

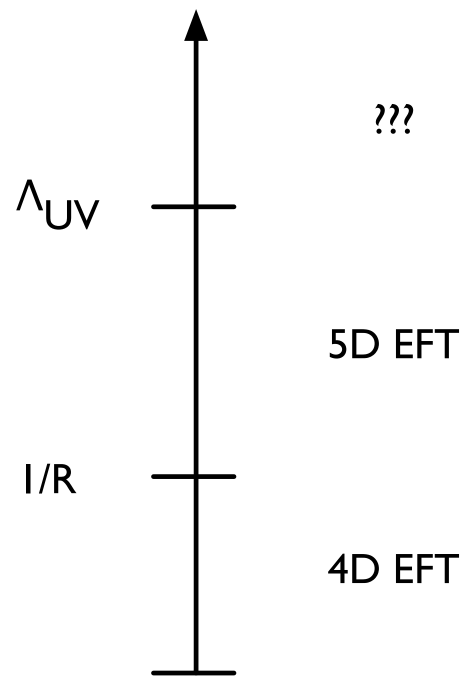
i.e., $A^{an}_L \leftrightarrow A^{an}_5$

Energy Scales and Couplings



$$\hat{A}^a_\mu = \frac{1}{\sqrt{\pi R}}\left[A^{a0}_\mu(x_\nu) + \sqrt{2}\sum_{n=1}^{\infty} A^{an}_\mu(x_\nu)\cos\left(\frac{n x_5}{R}\right)\right]$$

$$\hat{A}^a_5 = \sqrt{\frac{2}{\pi R}}\sum_{n=1}^{\infty} A^{an}_5(x_\nu)\sin\left(\frac{n x_5}{R}\right)$$



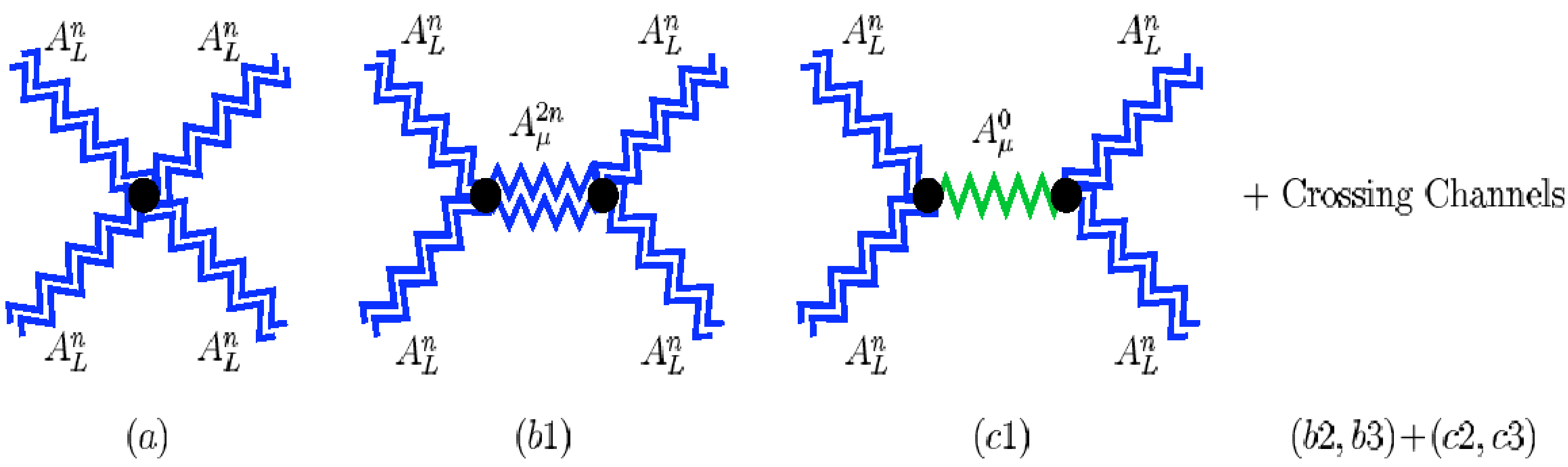
$$g_4 = \frac{g_5}{\sqrt{\pi R}} \qquad M_n = \frac{n}{R}$$

$$\boxed{\Lambda_{UV} \propto \frac{1}{g_5^2}}$$

Generalizations

- Applies to any 5-D manifold with Lorentz Invariance for fixed x^5: $A^\mu = \sum_n A_n^\mu(x^\nu) f_n(x^5)$
- KK eigenvalues and wavefunctions depend on metric and boundary conditions
- KK couplings related to overlap of KK wavefunctions: $g_{nmp} \propto \int dx^5 \, f_n(x^5)\, f_m(x^5)\, f_p(x^5)$
- Yields Lorentz-invariant effective field theory for infinite tower of KK modes.

4-D KK Mode Scattering



Cancellation of bad high-energy behavior through exchange of massive vector particles

graph	$g^2C^{eab}C^{ecd}$	$g^2C^{eac}C^{edb}$	$g^2C^{ead}C^{ebc}$
(a)	$6c(x^4-x^2)$	$\frac{3}{2}(3-2c-c^2)x^4$ $-3(1-c)x^2$	$\frac{-3}{2}(3+2c-c^2)x^4$ $+3(1+c)x^2$
$(b1)$	$-2c(x^4+x^2)$		
$(c1)$	$-4cx^4$		
$(b2,3)$		$\frac{-1}{2}(3-2c+c^2)x^4$ $+3(1-c)x^2$	$\frac{1}{2}(3+2c-c^2)x^4$ $-3(1+c)x^2$
$(c2,3)$		$(-3+2c+c^2)x^4$ $-8cx^2$	$(3+2c-c^2)x^4$ $-8cx^2$
Sum	$-8cx^2$	$-8cx^2$	$-8cx^2 \Rightarrow$ **0**

RSC, H.J. He, D. Dicus

Elastic Unitarity

$$S^\dagger S = \mathcal{I} \quad \Rightarrow \quad |s_l|^2 = 1$$

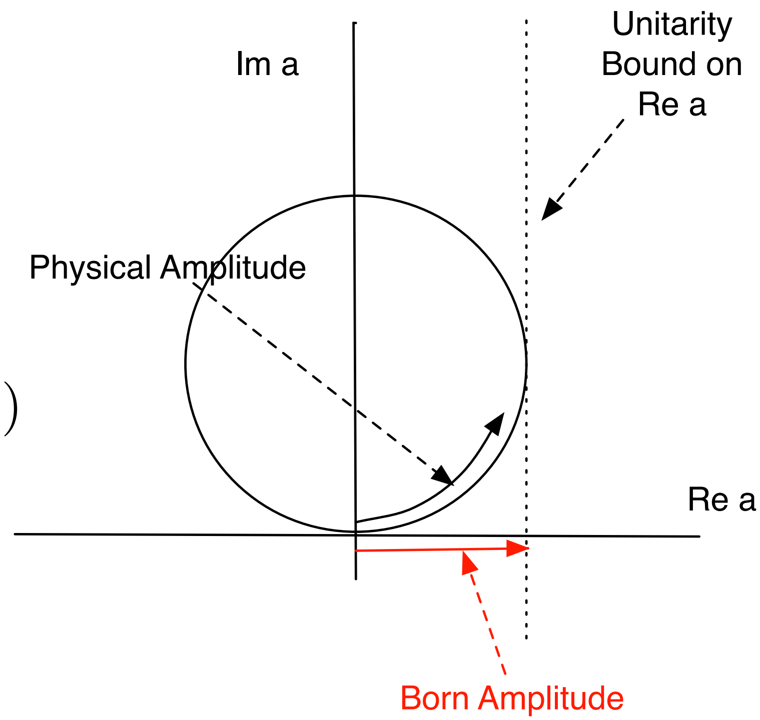
$$s_l = 1 + 2ia_l \quad \Rightarrow \quad \mathrm{Im}(a_l) = |a_l|^2$$

$$a_l = e^{i\delta_l}\sin\delta_l$$

$$a_l = \frac{1}{32(2)\pi}\left[\frac{4k^2}{s}\right]^{1/2}\int_{-1}^{1} d\cos\theta\,\mathcal{M}(s,\theta)$$

Identical Particle Factor

Feynman Amplitude



These formulae apply to the elastic scattering of pairs of particles of fixed helicity

Jacob and Wick, 1959

MultiChannel Unitarity

Spin-j: $$S_j = \mathcal{I} + 2i\mathcal{T}_j \quad , \quad \mathcal{T}_j = \left[a_j^{\alpha\to\beta}\right]$$

Rotation Coefficient

$$a_j^{\alpha\to\beta} = \frac{1}{32(\sqrt{2}_i)(\sqrt{2}_f)\pi}\left[\frac{4k_i k_f}{s}\right]^{1/2}\int_{-1}^{1} d\cos\theta\, \mathcal{M}^{\alpha\to\beta}(s,\cos\theta)\, d^j_{\Delta\lambda_i \Delta\lambda_f}(\theta)$$

Identical Particle Factors

Feynman Amplitude

$$S_j^\dagger S_j = \mathcal{I} \quad \Rightarrow \quad \mathcal{T}_j = U^\dagger\, e^{i\Delta_j} \sin\Delta_j\, U$$

Eigenvalues of $\mathcal{T}_j$ bounded : $\max\left[\left|\mathrm{Re}\left(e^{i\Delta_j}\sin\Delta_j\right)\right|\right] < \frac{1}{2}$

These formulae apply to the scattering of pairs of particles of fixed helicity

see, for example Durand and Lopez, PRD 40, 207 (1989)

No Free Lunch

Non-renormalizability of 5-D YM implies lingering unitarity issues ... how is this manifest in KK scattering?

Consider a state composed of KK pairs with $n \leq N_0$

$$|\psi^{ab}\rangle = \frac{1}{\sqrt{N_0}} \sum_{\ell=1}^{N_0} |A_L^{a\ell} A_L^{b\ell}\rangle$$

Find 4-D s-wave, gauge-singlet amplitude of $|\psi^{aa}\rangle \to |\psi^{cc}\rangle$

$$a_\psi^{00} = \frac{N_0}{R} \frac{k g_5^2}{8\pi^2} \mathcal{O}(1)$$

Grows with N_0!

Moral: Unitarity can be delayed, but not avoided!

- unitary bound on a_ψ^{00} implies highest KK mode number is bounded from above:

$$\frac{N_0}{R} < \frac{\sqrt{32}\pi^2}{k} \frac{\mathcal{O}(1)}{g_5^2}$$

(consistent with 5-d intuition)

Higgsless Models

- Can we use this idea in EWSB?
- Unitarize TeV-scale $W_L W_L$ scattering using vector bosons?
- If KK modes exist, $M_W << M_{KK}$!!
- Luckily, unitarization generalizes to a large class of 5-d manifolds and boundary conditions!

Csaki, Grojean, Murayama, Pilo, Terning

Simplest TC-inspired Model

$$\mathcal{L} = -\frac{1}{4g_5^2} W^{a\,MN} W^a_{MN}$$

Brane localized fermions

SU(2) Breakling

$$-\delta(y - \pi R)\frac{1}{4g^2} W^{a\,\mu\nu} W^a_{\mu\nu} \qquad -\delta(y)\frac{1}{4g'^2} W^{3\,\mu\nu} W^3_{\mu\nu}$$

$$\delta(x_5, x_5 - \pi R) \equiv \lim_{\varepsilon \to 0^+} \delta(x_5 - \varepsilon, x_5 - \pi R + \varepsilon)$$

$$\frac{dW^a_\mu}{dx_5}|_{x_5=\pi R} = 0\,, \quad \frac{dW^3_\mu}{dx_5}|_{x_5=0} = 0 \;\; \& \;\; W^{1,2}_\mu(0) = 0$$

Foadi, Gopalkrishna, Schmidt hep-ph/0312324

Carena, Tait, Wagner hep-ph/0207056

Power Counting and Scales

Standard Model Recovered in limit: $g,\, g' \ll \frac{g_5}{\sqrt{\pi R}}$

$$M_W \approx \frac{g}{g_5\sqrt{\pi R}}, \quad M_Z \approx \frac{\sqrt{g^2 + g'^2}}{g_5\sqrt{\pi R}}$$

$$M_{W^*} \simeq \frac{1}{R}$$

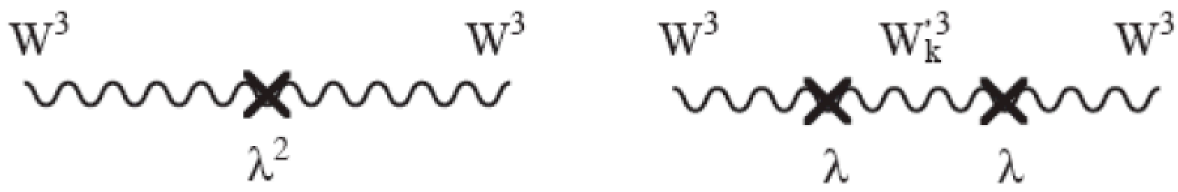
Need moderately strong g_5 in order to gain anything!

$$\Lambda_5 \simeq \frac{2\sqrt{2}\pi^2}{g_5^2} \gg \sqrt{4\pi} v$$

$$\frac{\pi^{3/2}}{\sqrt{2}} \gg \frac{g_5}{\sqrt{\pi R}} \gg g \simeq \mathcal{O}(1)$$

Just barely OK?

S, again!



$$\alpha S = \frac{2e^2}{3}\frac{\pi R}{g_5^2} \simeq \frac{e^2}{3\pi\sqrt{2}}\frac{\Lambda_5}{M_{W^*}}$$

$$\alpha S < \mathcal{O}(10^{-3}) \Rightarrow \frac{M_{W^*}}{\Lambda_5} > 7$$

INCONSISTENT

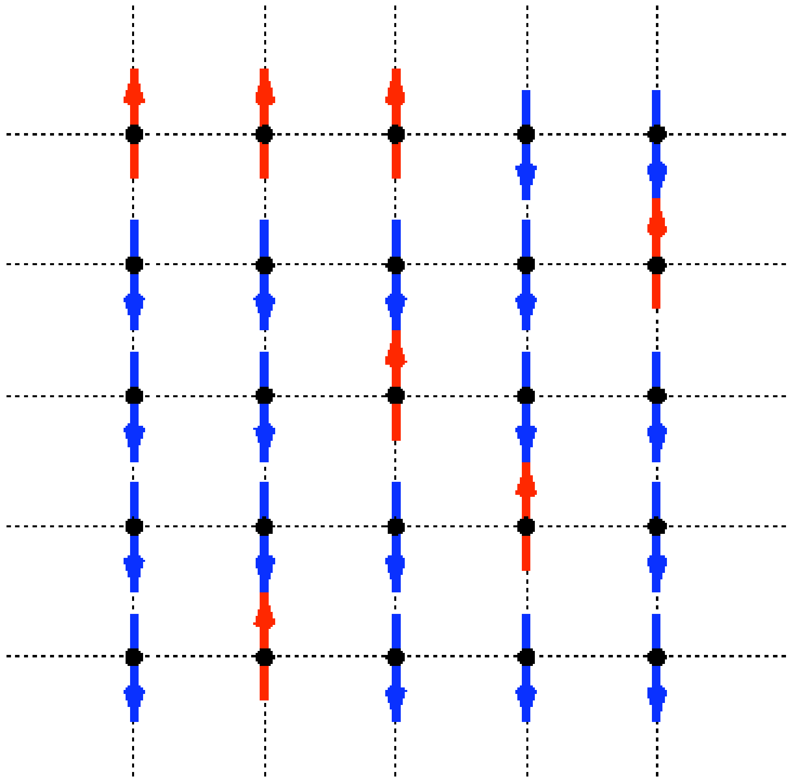
Recipe for a Higgsless Model:

- Choose "bulk" gauge group, location of fermions, and boundary conditions
- Choose $g(x_5)$
- Choose metric/manifold: $g_{MN}(x_5)$
- Calculate spectrum & eigenfunctions
- Calculate fermion couplings
- Compare to Standard Model: S, T, U, ...
- Can we find a viable model?
- How can we analyze many at once? Deconstruction!

More Next Lecture!

AdS/CFT...

Duality: 2-D Ising Model



$$Z[K \equiv \beta J] = \int [d\sigma_i] \exp\left(-\beta J \sum_{p,q} \sigma_p \sigma_q\right)$$

K = "coupling"

$$\sinh(2K)\sinh(2K^*) = 1$$

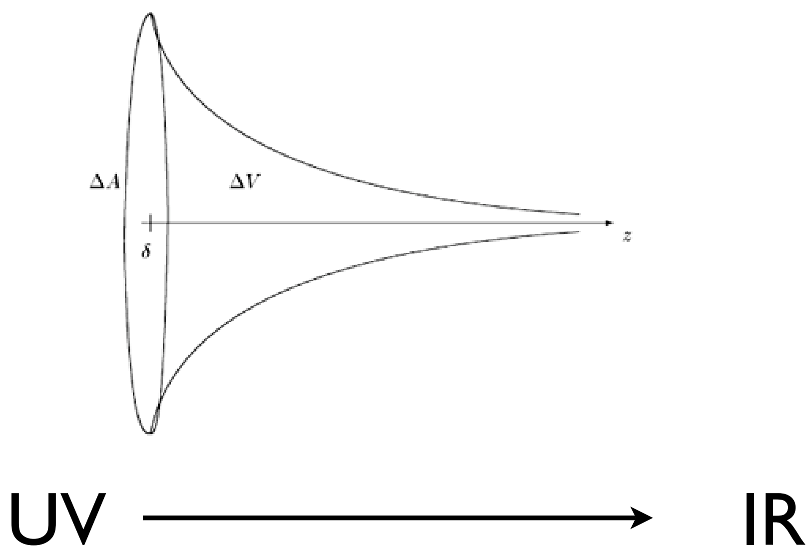
$$\frac{Z[K]}{\sinh^{N/2}(2K)} \equiv \frac{Z[K^*]}{\sinh^{N/2}(2K^*)}$$

"strong-coupling/weak-coupling duality"

Kramers and Wannier, 1941

AdS/CFT Duality

Conjecture: Equivalence of 5D theory in AdS and 4D CFT



$$ds^2 = \left(\frac{R}{z}\right)^2 \left[\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right]$$

$$R < z < R'$$

NB: Rescaling Invariance!

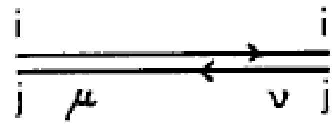
Strong evidence for N=4 SUSY YM string theory on AdS

Strongly-coupled CFT ⇔ Weakly-coupled 5D Theory!

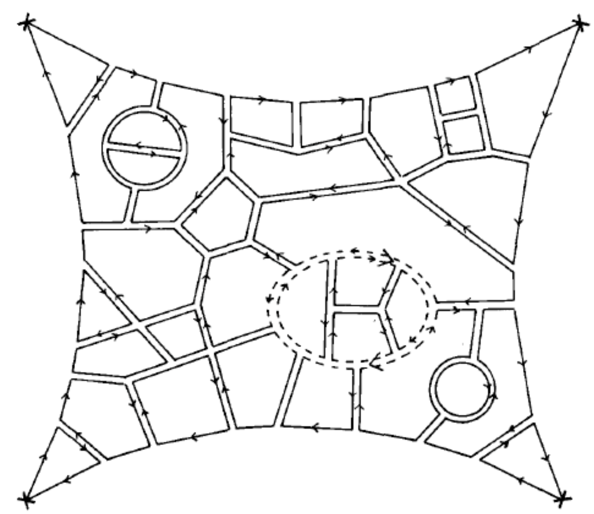
AdS/CFT and Large-N

Tree-level in dual theory = "Large-N" limit of CFT

$g^2_{CFT} N_{CFT} = \lambda$ fixed

Gluon: 

Leading Term arises from planar diagrams :



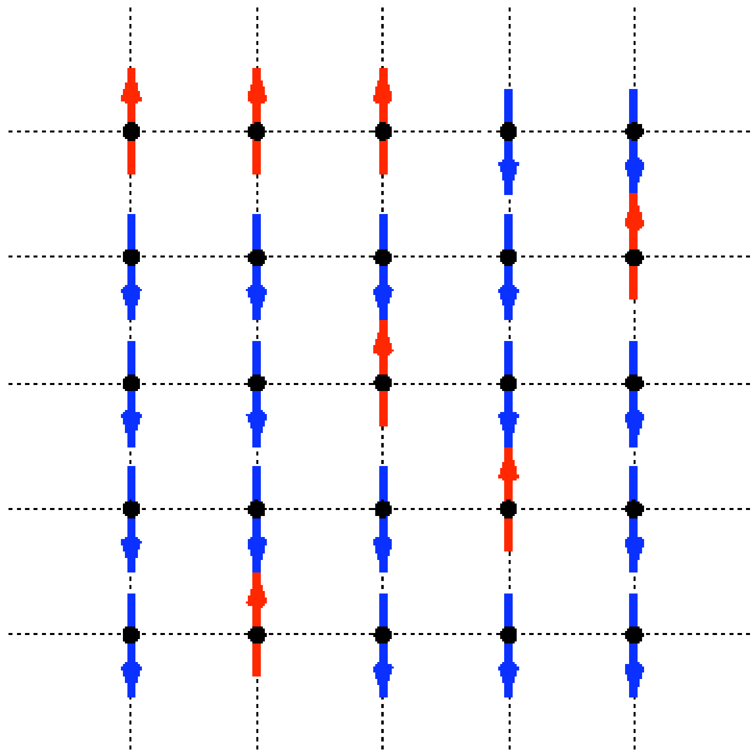
't Hooft, NPB 72 (1974) 461

AdS/CFT Dictionary

Bulk of AdS	$\leftrightarrow$	CFT
Coordinate (z) along AdS	$\leftrightarrow$	Energy scale in CFT
Appearance of UV brane	$\leftrightarrow$	CFT has a cutoff
Appearance of IR brane	$\leftrightarrow$	conformal symmetry broken spontaneously by CFT
KK modes localized on IR brane	$\leftrightarrow$	composites of CFT
Modes on the UV brane	$\leftrightarrow$	Elementary fields coupled to CFT
Gauge fields in bulk	$\leftrightarrow$	CFT has a global symmetry
Bulk gauge symmetry broken on UV brane	$\leftrightarrow$	Global symmetry not gauged
Bulk gauge symmetry unbroken on UV brane	$\leftrightarrow$	Global symmetry weakly gauged
Higgs on IR brane	$\leftrightarrow$	CFT becoming strong produces composite Higgs
Bulk gauge symmetry broken on IR brane by BC's	$\leftrightarrow$	Strong dynamics that breaks CFT also breaks gauge symmetry

Csaki hep-ph/0510275

Duality and Higgsless Models



A Higgsless Theory on AdS could be “dual” to a conformally-invariant model of dynamical EWSB : like walking TC!

Energy Scales and Couplings with AdS/CFT

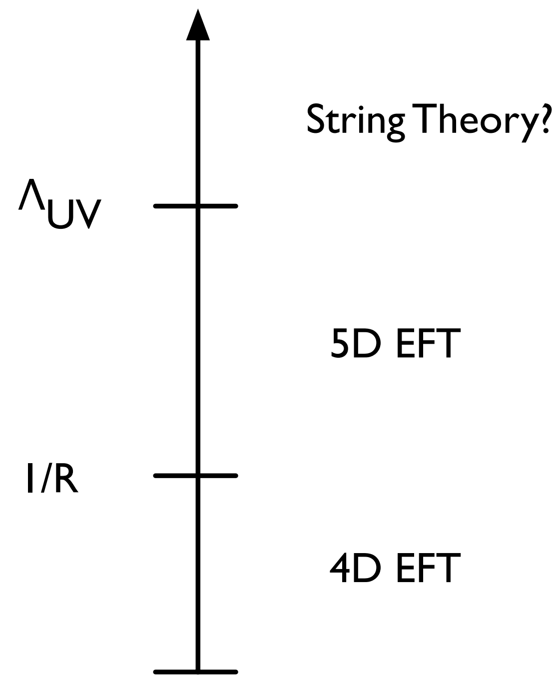
Quantum Corections in 5D KK Theory:

$$\mathcal{O}\left(\frac{g_4^2}{16\pi^2}\right) = \mathcal{O}\left(\frac{1}{N_{CFT}}\right)$$

$$g_4^2 = \frac{g_5^2}{R\log\frac{R'}{R}} \qquad M_{KK} \approx \frac{\pi}{4R'}$$

Naive Dimensional Analysis:

$$\Lambda_{NDA} = \frac{24\pi^3}{g_5^2} \simeq \frac{6\ N_{CFT}\ M_{KK}}{\log\frac{R'}{R}}$$



How close are we to large-N?

think way different

Summary

- Extra dimensions could be present, and are motivated by various theoretical ideas.
- Higgsless models utilize 5-D gauge theories with BC's to realize EWSB.
- AdS/CFT interpretation of Higgsless models $\Leftrightarrow$ "Walking Technicolor"
- Can we analyze theories in general and construct viable ones?

Next Time: Deconstruction

References

- Sundrum, TASI04, hep-th/0508134
- Csaki, TASI04 hep-ph/0510275
- Gerghetta, LesHouches05, hep-0601213
- Maldacena, TASI03, hep-ph/0309246