Supersymmetric Grand Unified Theories and Their Phenomenological Consequences

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Tsinghua University, July 18, 2005

I. INTRODUCTION

Standard Model:

- Gauge hierarchy problem
- Charge quantization
- Gauge coupling unification
- Fermion masses and mixings
- Dark matter
- Baryon asymmetry
- ......

SUSY GUTs:

- Unification of all the known gauge interactions
- Gauge coupling unification in MSSM
- Yukawa coupling unification for the third family
- Radiative electroweak symmetry breaking
- Weak mixing angle at weak scale M_Z
- Neutrino masses by see-saw mechanism
- ......

SUSY GUTs are very interesting.

II. SUSY GUTs

(A) SU(5) Model

- 10_i (U_i^c, E_i^c and Q_i) and $\overline{5}_i$ (D_i^c and L_i)
- Higgs fields: Φ (24), H (5), $\overline{H}$ ($\overline{5}$)

$$W = a\mathrm{Tr}\Phi + b\mathrm{Tr}\Phi^2 + c\mathrm{Tr}\Phi^3 \ .$$

(1) Vacuum degeneracy

- $\langle\Phi\rangle = 0$
- $\langle\Phi\rangle = v_G \, \mathrm{diag}(1, 1, 1, -\frac{3}{2}, -\frac{3}{2})$
- $\langle\Phi\rangle = v_G \, \mathrm{diag}(1, 1, 1, 1, -4)$

Question: Why is SU(5) broken down to the SM gauge symmetry?

- $W = \lambda S \left(\mathrm{Tr}\Phi^2 - \mu^2\right)$
- Supergravitational Interaction

(2) Gauge coupling unification

We define M_G as the value of μ where

$$\alpha_1(M_G) = \alpha_2(M_G) \equiv \alpha_G.$$

$$M_G \approx 3 \times 10^{16} \ \text{GeV} \ , \quad \alpha_G^{-1} \approx 24.$$

Good fits to the low energy data require

$$\epsilon_3 \equiv \frac{\alpha_3(M_G) - \alpha_G}{\alpha_G} \sim -3\% \ \text{ to } \ -4\%.$$

(3) Proton Decay

(i) The dimension 6 operators are derived from gauge boson exchange. For example,

$$\frac{g_G^2}{2M_X^2}\,\bar{u}^\dagger\, Q\, \bar{d}^\dagger\, L.$$

The proton lifetime is given by

$$\tau_p \sim 5 \times 10^{36}\, \left(\frac{M_X}{3 \times 10^{16}\ \mathrm{GeV}}\right)^4 \left(\frac{0.015\ \mathrm{GeV}^3}{\beta_{lattice}}\right)^2 \ \mathrm{Years}.$$

The dominant decay mode is

$$p \longrightarrow \pi^0 + e^+.$$

Bound: $M_X > 5 \times 10^{15}$ GeV.

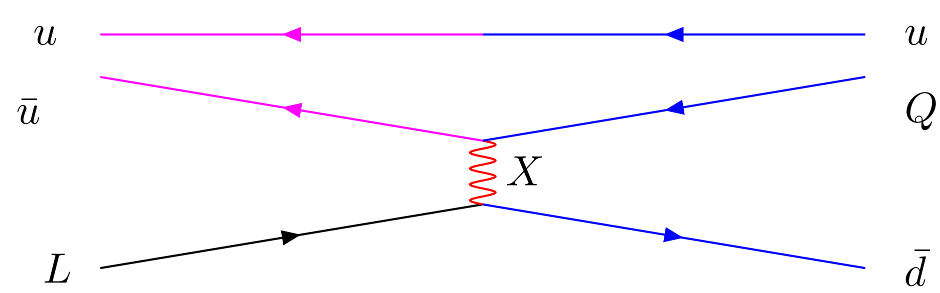


Figure 1: X boson exchange diagram giving the dimension 6 four fermion operator for proton and neutron decay.

(ii) Dimension 4 operators are dangerous

$$(U^c \; D^c \; D^c) + (Q \; L \; D^c) + (E^c \; L \; L).$$

All the dimension 3 and 4 baryon and lepton number violating operators are forbidden by R parity.

(iii) Dimension 5 operators

$$\frac{1}{M_T^{eff}} \left[Q \frac{1}{2} c_{qq} Q \; Q c_{ql} L + \overline{U} c_{ud} \overline{D} \; \overline{U} c_{ue} \overline{E} \right].$$

are obtained when integrating out heavy color triplet Higgs fields

$$W = y^u_{ij} 10_i 10_j H + y^d_{ij} 10_i \bar{5}_j \overline{H}.$$

Unlike the non-SUSY GUTs, the proton decay modes in SUSY GUTs are

$$p \rightarrow K^+ + \bar{\nu} \; , \quad n \rightarrow K^0 + \bar{\nu}.$$

The proton decay amplitude is then given generically by the expression

$$T(p \to K^+ + \bar{\nu}) \sim \frac{c^2}{M_T^{eff}} (\text{Loop Factor}) \, \frac{\beta_{lattice}}{f_\pi} \, m_p.$$

Point: Yukawa coupling suppression for proton decay.

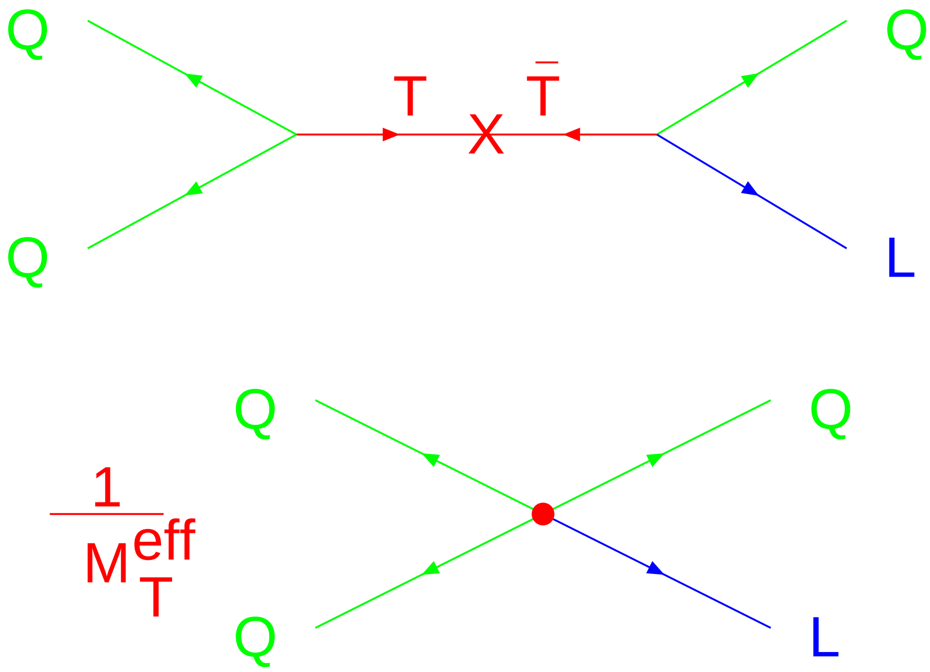


Figure 2: Color triplet Higgs exchange diagram giving the dimension 5 superpotential operator for proton and neutron decay.

(4) Doublet-Triplet Splittings

$$W = \lambda \overline{H} \Phi H + M \overline{H} H \ .$$

D-T splitting:

$$\frac{3}{2} \lambda v_G = M \ .$$

(i) Sliding Singlet

$$W = 2\overline{H}\Phi H + S\overline{H}H\ .$$

$$F_{H_d} = H_u\left(-3v_X + \langle S\rangle\right)\ .$$

Problem: Unstable against radiative corrections.

(ii) Missing Partner Mechanism

Higgs fields: Σ **(**75**)**, Θ **(50)**, $\overline{\Theta}$ **(**$\overline{50}$**)**

$$W = \lambda_1 \Theta \Sigma \overline{H} + \lambda_2 \overline{\Theta} \Sigma H + M \Theta \overline{\Theta} + f(\Sigma) \; .$$

- Σ has a SM singlet–Breaking SU(5).
- Θ has a color triplet but no doublet.
- Triplets are very massive while doublets are very light.

(5) Fermion masses and Mixings: Froggatt-Nielsen Mechanism

The ratios of the masses at the GUT scale are

$$m_u : m_c : m_t \sim \lambda^7 : \lambda^4 : 1 \ ,$$

$$m_d : m_s : m_b \sim \lambda^4 : \lambda^2 : 1 \ ,$$

$$m_e : m_\mu : m_\tau \sim \lambda^5 : \lambda^2 : 1 \ ,$$

where $\lambda = 0.22$. And the CKM matrix is given by

$$V_{\rm CKM} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ -\lambda & 1 & \lambda^2 \\ -\lambda^3 & -\lambda^2 & 1 \end{pmatrix} \ .$$

In the hierarchy case, we can estimate the ratio of the neutrino masses

$$m_{\nu_2} : m_{\nu_3} \sim \sqrt{\frac{\Delta m^2_{\rm solar}}{\Delta m^2_{\rm atm.}}} \sim \lambda^2 : 1 \ ,$$

and neutrino mixings are bilarge.

Anomalous $U(1)_A$ gauge symmetry in string theory

- ϕ with $U(1)_A$ charge -1 obtains a VEV $\langle\phi\rangle = v_S$, and $v_S/M_S \sim \lambda$.

- $U(1)_A$ charge assignment:

$$Q_{10_1} = 3\ ,\ Q_{10_2} = 2\ ,\ Q_{10_3} = 0\ ,\ Q_{\bar{5}_1} = 1, Q_{\bar{5}_2} = Q_{\bar{5}_3} = 0\ .$$

- Yukawa couplings

$$W = y^u_{ij}\left(\frac{\phi}{M_S}\right)^{Q_{10_i}+Q_{10_j}} 10_i 10_j H + y^d_{ij}\left(\frac{\phi}{M_S}\right)^{Q_{10_i}+Q_{\bar{5}_j}} 10_i \bar{5}_j \overline{H}\ .$$

$$Y_U \sim y_t \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \ , \ \ m_\nu \sim m_{\nu_\tau} \begin{pmatrix} \lambda^2 & \lambda & \lambda \\ \lambda & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix} \ ,$$

$$Y_D = (Y_E)^T \sim y_b \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda & 1 & 1 \end{pmatrix} \ .$$

(6) Comments:

- Minimal SUSY SU(5) is ruled out by gauge coupling unification and proton decay

  Solution: introducing non-renomalizable operators or large threshold corrections

- Planck scale can not suppress the dimensional 5 proton decay operators

$$W = \lambda \frac{1}{M_{\rm Pl}} 10_i 10_j 10_k \overline{5}_l \ .$$

  Solution: Need flavour symmetry.

(B) SO(10) Model

Among several candidates of the GUT gauge groups SU(5), SO(10), and E_6, SO(10) has several particularly attractive features:

- SO(10) is the smallest real simple GUT group.
- Fermion unification.
- Yukawa unification.
- The Dimopoulos-Wilczek mechanism: the doublet-triplet splitting.

(1) Symmetry Breaking Chains:

- $SO(10) \longrightarrow SU(5)_{GG} \times U(1)_X \longrightarrow G_{\rm SM}$
- Higgs fields: (i) 16 and $\overline{16}$; (ii) 45 (24)
- $SO(10) \longrightarrow SU(4)_C \times SU(2)_L \times SU(2)_R \longrightarrow G_{\rm SM}$
- Higgs fields: (i) 54 or 210; (ii) 16 and $\overline{16}$ or 126 and $\overline{126}$
- $SO(10) \longrightarrow SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \longrightarrow G_{\rm SM}$
- Higgs fields: (i) 45 and 54; (ii) 16 and $\overline{16}$ or 126 and $\overline{126}$
- $SO(10) \longrightarrow$ Flipped $SU(5) \times U(1)' \longrightarrow G_{\rm SM}$

(2) Dimopoulos-Wilczek Mechanism

- Higgs fields: A (45); H_1 (10); H_2 (10)

$$W = \lambda H_1 A H_2 \, .$$

- $\langle A \rangle = i\sigma_2 \times \mathrm{diag}\,(v_o, v_o, v_o, 0, 0)$
- Triplets are heavy while doublets are light
- Problem: 4 Higgs doublets (2 must be heavy)

(3) Fermion Masses and Mixings

It is difficult to explain the fermion masses and mixings in the $SO(10)$ model due to the fermion unification.

- $U(1)$ and extra discrete flavour symmetry.
- $U(2)$ flavour symmetry.
- $SU(2) \times U(1)$ flavour symmetry.
- $SU(2) \times Z_2 \times Z_2 \times Z_2$ flavour symmetry.
- $SU(3) \times U(1)$ flavour symmetry.
- $SU(3)$ flavour symmetry.
- ......

(C) $SU(4)_C \times SU(2)_L \times SU(2)_R$**–Pati-Salam Model**

Higgs fields: X $(4, 1, 2)$, $\overline{X}$ $(\bar{4}, 1, 2)$; H $(1, 2, 2)$

Additional fields: Y$(6, 1, 1)$

$$W = \lambda_1 XYX + \lambda_2 \overline{X} Y \overline{X} + \lambda_3 S(\mu_0^2 - X\overline{X}) \, .$$

- No doublet-triplet splitting problem.
- The representations of Higgs fields are smaller than the adjoint representation.

(D) Flipped $SU(5) \times U(1)_X$ Model

$$T_{\mathrm{U(1)_{Y'}}} = \mathrm{diag}\left(-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, \frac{1}{2}, \frac{1}{2}\right).$$

$$Q_Y = \frac{1}{5}\left(Q_X - Q_{Y'}\right).$$

SM fermions:

$$F_i = (10, 1),\ \bar{f}_i = (\bar{5}, -3),\ l_i^c = (1, 5),$$

In flipped $SU(5)$, the up-type and down-typ quarks are flipped in the 10 and $\overline{5}$ representations with respect to the usual $SU(5)$.

Higgs fields:

$$H = (10, 1),\ \bar{H} = (\bar{10}, -1),\ h = (5, -2),\ \bar{h} = (\bar{5}, +2).$$

$$H = (Q_H, D^c_H, N_H),\ \bar{H} = (\bar{Q}_{\bar{H}}, \bar{D}^c_{\bar{H}}, \bar{N}_{\bar{H}}).$$

$$h = (D_h, D_h, D_h, H_1),\ \bar{h} = (\bar{D}_{\bar{h}}, \bar{D}_{\bar{h}}, \bar{D}_{\bar{h}}, H_2).$$

Doublet-Triplet Splittings

$$W_{\rm GUT} = \lambda_1 H H h + \lambda_2 \bar{H} \bar{H} \bar{h} + S(\bar{H} H - M_{\rm V}^2).$$

- $< N_H >=< \bar{N}_{\bar{H}} >= M_{\rm V}$.
- The superfields in H and $\bar{H}$ are eaten or acquire large masses via the supersymmetric Higgs mechanism, except for D^c_H and $\bar{D}^c_{\bar{H}}$.
- The superpotential $\lambda_1 H H h$ and $\lambda_2 \bar{H} \bar{H} \bar{h}$ combine the D^c_H and $\bar{D}^c_{\bar{H}}$ with the D_h and $\bar{D}_{\bar{h}}$, respectively, to form the massive eigenstates with masses $\lambda_1 < N_H >$ and $\lambda_2 < \bar{N}_{\bar{H}} >$.
- The triplets in h and $\bar{h}$ only have small mixing through the μ term, the higgsino-exchange mediated proton decay are negligible.

(E) Comments:

- Neutrino mixings are bilarge.
- In the framework of the supersymmetric see-saw mechanism, the flavor mixings in slepton mass matrix can be induced, which then give implications on the $\mu \to e\gamma$ and $\tau \to \mu, e + \cdots$ processes.
- In $SU(5)$ or $SO(10)$, the flavor mixings in down-type squark mass matrix can also be induced, and give the interesting phenomenological implications for the Kaon and B meson physics.
- In flipped $SU(5)$ model, the flavor mixings in up-type squark mass matrix can be generated, and give the interesting phenomenological implications for the top and charm physics, for example, D meson physics and top decay.

III. ORBIFOLD GUTS

(A) N=1 SUSY 5D Orbifold SU(5) on $M^4 \times S^1/(Z_2 \times Z_2')$

Set-up:

- Space-time $M^4 \times S^1$, coordinates x^μ, ($\mu = 0, 1, 2, 3$), $y \equiv x^5$.
- The orbifold $S^1/(Z_2 \times Z_2')$ is obtained by S^1 moduloing two equivalent classes:

$$P : \; y \sim -y \; , \; P' : \; y' \sim -y' \; ,$$

where $y' \equiv y + \pi R/2$.

- 5D N=1 SUSY corrresponds to 4D N=2 SUSY.
- 5D vector multiplet can be decomposed as one gauge multiplet (V) and one chiral multiplet (Σ) in 4D N=1 SUSY.
- 5D hypermultiplet corresponds to 4D chiral multiplet (X) and its Hermitian conjugate X^c.

  Higgs fields: H and H^c; $\overline{H}$ and $\overline{H}^c$

Transformation properties:

$$V_\mu(x^\mu, y) \to V_\mu(x^\mu, -y) = PV_\mu(x^\mu, y)P^{-1} \ ,$$

$$\Sigma(x^\mu, y) \to \Sigma(x^\mu, -y) = -P\Sigma(x^\mu, y)P^{-1} \ ,$$

$$H(x^\mu, y) \to H(x^\mu, -y) = PH(x^\mu, y) \ .$$

$$H^c(x^\mu, y) \to H^c(x^\mu, -y) = -PH^c(x^\mu, y) \ .$$

$$P = \mathrm{diag}\,(+1, +1, +1, +1, +1) \;\; ,$$

$$P' = \mathrm{diag}\,(-1, -1, -1, +1, +1) \;\; .$$

Fields	(P, P')
V^a, H_u, H_d	(+, +)
$V^{\hat{a}}$, T, $\overline{T}$	(+, -)
$\Sigma^{\hat{a}}$, T^c, $\overline{T}^c$	(-, +)
Σ^a, H_u^c, H_d^c	(-, -)

Points:

- Only the fields with parity (+, +) have zero modes.
- The SU(5) gauge symmetry is broken down to the SM gauge symmetry for the zero modes and on the 3-brane at $y = \pi R/2$.
- 4D N=2 SUSY is broken down to the 4D N=1 SUSY for the zero modes and on the 3-branes at $y = 0$ and $\pi R/2$.
- Natural doublet-triplet splittings.
- $U(1)_R$ symmetry to forbid dimension-5 proton decay operators.

In the orbifold gauge symmetry breaking, only the broken gauge symmetry is preserved on at least one 3-brane at the fixed point.

(B) $SU(3)_C \times SU(3)$ Model on $M^4 \times S^1/(Z_2 \times Z_2')$

About thirty-two years ago, Weinberg tried to unify the $SU(2)_L$ weak and $U(1)_Y$ hypercharge interactions into the $SU(3)$ gauge interaction where the tree-level weak mixing angle ($\sin^2\theta_W$) arising from the breaking of a $SU(3)$ group is 0.25 which is within 10% of the present experimental value 0.2312.

$$T_{U(1)_Y} = \text{diag}\,(\frac{1}{2}, \frac{1}{2}, -1) \equiv \sqrt{3}\lambda^8/2 \,,$$
$$g_Y = g_3/\sqrt{3} \,, \; g_2 = g_3 \,.$$

$$\sin^2\theta_W = \frac{g_Y^2}{g_Y^2 + g_2^2} = \frac{1}{4} \,.$$

However, the quark doublets can not be accommodated in the theory due to their small hypercharge quantum numbers.

We can construct the 5-D $SU(3)_C \times SU(3)$ model on $M^4 \times S^1/(Z_2 \times Z_2')$ where only the SM gauge symmetry is preserved on the 3-brane at $y = 0$. And we put the SM quarks on that 3-brane.

The successful features of the supersymmetric model:

- The desirable weak mixing angle $\sin^2\theta_W = 0.2312$ at m_Z scale can be generated naturally.
- The $U(1)_Y$ charge quantization can be obtained due to the gauge invariant of the Yukawa couplings and anomaly free conditions.
- A complete gauge coupling unification might occur at 10^6 GeV if the compactification scale of the fifth dimension ($1/R$) is 10^4 GeV for the supersymmetric model without embedding it into a GUT model.
- The model can be tested at the future colliders, for example, the LHC.

The problems in the model:

- Why the gauge interactions are not completely unified?
- μ problem for supersymmetric model.
- Neutrino mass problem. The left-handed neutrinos can obtain masses via the dimension-5 operators $y_{\nu ij} L_i L_j H_u H_u / M_*$, where M_* is about 10^6 GeV. So, $m_\nu \sim y_\nu \times 10^7$ eV.

These problems can be solved when we consider the 6-dimensional $\mathcal{N} = 2$ supersymmetric $SU(6)$ models on T^2 orbifolds.

(C) Low Energy Gauge Unification Theory (SU(5))

Grand Unified Theory (GUT):

- Gauge Interactions Unification.
- Fermion and Higgs Unification.
- Problems from fermion and Higgs unification: proton decay, D-T splitting, and fermion mass problem.

We might conjecture that:

The realistic fundamental theory, which describes the nature, might be the theory with the gauge unification and without the fermion and Higgs unification. And the unification scale could be around $O(100\text{TeV})$.

6-Dimensional N=2 Supersymmetric $SU(5)$ Model on $M^4 \times S^1/(Z_2 \times Z_2') \times S^1/(Z_2 \times Z_2')$

- Proton Decay. We forbid the dimension-5 proton decay operators, and the dimension-6 proton decay operators via X and Y exchanges. We also separate the quarks and letptons.

- Anomaly Cancellation. Introducing the Chern-Simons term on the 4-brane at $z = \pi R_2/2$ or 6-dimensional topological term.

- Charge Quantization. Gauge invariance of the localized superpotential and anomaly cancellation.

- Accelerated Gauge Coupling Unification. For example, assuming $1/R_2 = 20/R_1$, $1/R_1 = 10$ TeV and $M_{SUSY} \sim$ 200-1000 GeV, we can achieve the gauge coupling unification at around 530 TeV.

(D) Gauge-Fermion Unification and Flavour Symmetry (E_6)

Among several candidates of the GUT gauge groups SU(5), SO(10), and E_6, SO(10) has several particularly attractive features:

- SO(10) is the smallest real simple GUT group.
- Fermion unification.
- Yukawa unification.
- The Dimopoulos-Wilczek mechanism: the doublet-triplet splitting.

However, it is difficult to explain the fermion masses and mixings in the $SO(10)$ model due to the fermion unification.

Three interesting questions

(1) The maximal supersymmetry in the supersymmetric gauge theory is the 4-dimensional $\mathcal{N} = 4$ supersymmetry, why is there no 4-dimensional $\mathcal{N} = 4$ supersymmetry in the nature?

(2) Why are there three families of fermions in the Standard Model (SM)?

(3) To explain the fermion masses and mixings in $SO(10)$ model by Froggatt-Nielsen mechanism, one introduced non-trivial flavour symmetries, for example, the $SU(2) \times U(1)$ flavour symmetry. So, what is the origin of the flavour symmetry?

$\mathcal{N}=(1,1)$ Supersymmetric E_6 Models on $M^4 \times T^2/Z_3$

- 6D $\mathcal{N}=(1,1)$ supersymmetry corresponds to the 4D $\mathcal{N}=4$ supersymmetry.

- The vector multiplet in the 6D $\mathcal{N}=(1,1)$ SUSY gauge theory, contains one gauge supermultiplet and three chiral multiplets in the 4D $\mathcal{N}=1$ SUSY language.

- The SM fermions may come from the zero modes of three chiral multiplets due to the orbifold gauge symmetry breaking, which can explain why there are three families of fermions in the SM.

$$\mathbf{78} = (\mathbf{45}, \mathbf{0}) \oplus (\mathbf{16}, -\mathbf{3}) \oplus (\overline{\mathbf{16}}, \mathbf{3}) \oplus (\mathbf{1}, \mathbf{0}) \ .$$

- The R symmetry ($SO(2)_{56} \times U(1)_{\mathbf{4}_+} \times SU(2)_{\mathbf{4}_-}$) can give us the flavour symmetry to generate the fermion masses and mixings.

(E) Fermion Masses and Mixings in 5-D Orbifold $SO(10)$ Model

During last several years, there is a mechanism to generate the flavor hierarchy based on the wave-function profiles in the extra dimension [a]. This new mechanism can generate the flavour hierarchy elegantly in 5-dimensional orbifold SU(5) model [b].

$SO(10) \supset SU(5)_{GG} \times U(1)_X$

New Idea: *The $U(1)_X$ breaking effect can directly contribute to the wave-function profiles of the SM fermions, i. e., the source of the fermion mass hierarchy via the bulk mass m of the particles.*

The fermion masses and mixings in 5-D orbifold $SO(10)$ models can be explained neatly via the 5-D wave-function localization.

[a] Arkani-Hamed and Schmaltz; Mirabelli and Schmaltz; Kaplan and Tait; Kakizaki and Yamaguchi; Haba and Maru.

[b] Hebecker and March-Russell.

(F) Coupling Unifications in Gauge-Higgs-Fermions Unified Orbifold Models

- 6D N=2 and 7D N=1 SUSY correspond to 4D N=4 SUSY
- The SM fermions and Higgs fields can arise from the zero modes of the three chiral multiplets
- We can construct the models with gauge-fermion-Higgs unification.
- The gague couplings, Yukawa couplings for the third family, and/or Higgs trilinear couplings can be unified.

6D $SU(9)$ Model on $M^4 \times T^2/Z_6$ and 7D $SU(9)$ Model on $M^4 \times T^2/Z_6 \times S^1/Z_2$.

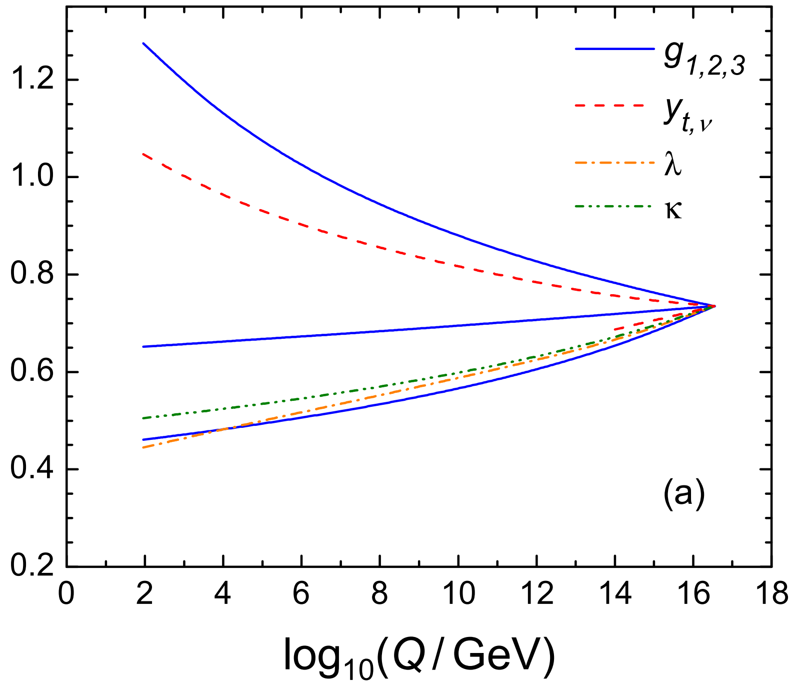
- One pair of Higgs doublets H_u and H_d, and three SM singlets S and S_i arise from zero modes of three chiral multiplets

$$W = \lambda S H_u H_d + \kappa S S_1 S_2 \,,$$

- The third family up-type SM fermions or down-type SM fermions come from the zero modes of three chiral multiplets.

- Up-type or down-type partial coupling unification

$$g_1 = g_2 = g_3 = \lambda = \kappa = y_t = y_{\nu_\tau} \,, \qquad g_1 = g_2 = g_3 = \lambda = \kappa = y_b = y_\tau \,.$$



1.2
1.0
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0.4
0.2
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2
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8
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$g_{1,2,3}$
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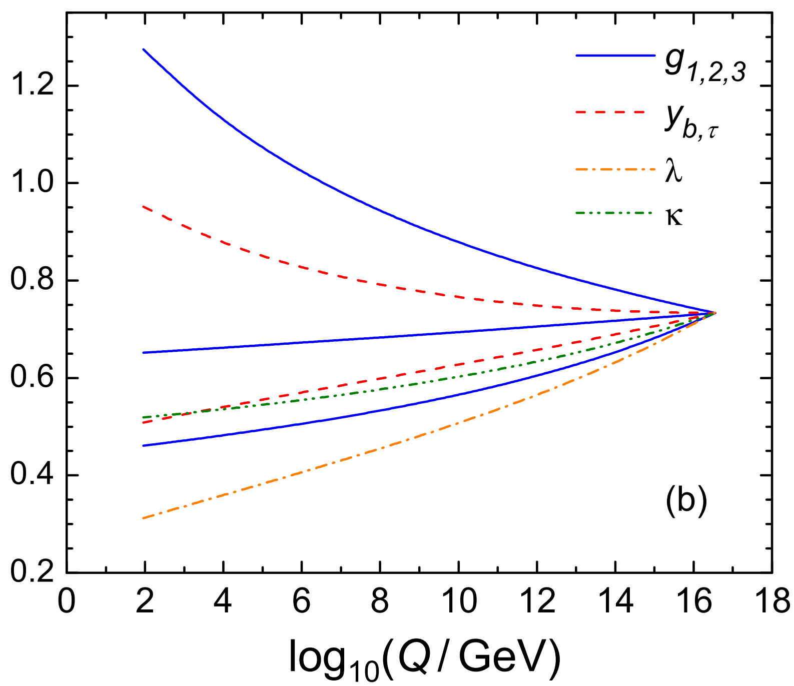
(a)
$\log_{10}(Q\,/\,\mathrm{GeV})$



1.2
1.0
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$y_{b,\tau}$
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(b)
$\log_{10}(Q\,/\,\mathrm{GeV})$

Summary

SUSY GUTs:

- Gauge coupling unification: SUSY or GUT scale threshold corrections, or non-renormalizable operators.
- Doublet-triplet splittings: SU(5): complicated missing partner mechanism; SO(10): Dimopoulos-Wilczek mechanism; PS model: no problem; Flipped SU(5): elegant missing partner mechanism.
- Dimensional 5 proton decay: Need flavour symmetry.
- Fermion masses and Mixings: Froggatt-Nielsen Mechanism.
- Neutrino bilarge mixings can induce interesting flavour physics.

Orbifold GUTs:

- Elegant solution to the Doublet-triplet splitting problem.
- No problem for dimensional 5 proton decay.
- Fermion masses and mixings can be explained via the wave-function profiles in the extra dimension.
- A lot of new models with very good features.