2005 International Workshop-Summer School

on physics, detector and accelerator at the linear collider

July 15-20, 2005

Center for High Energy Physics

Tsinghua University, Beijing 100084, China

Lecture 1

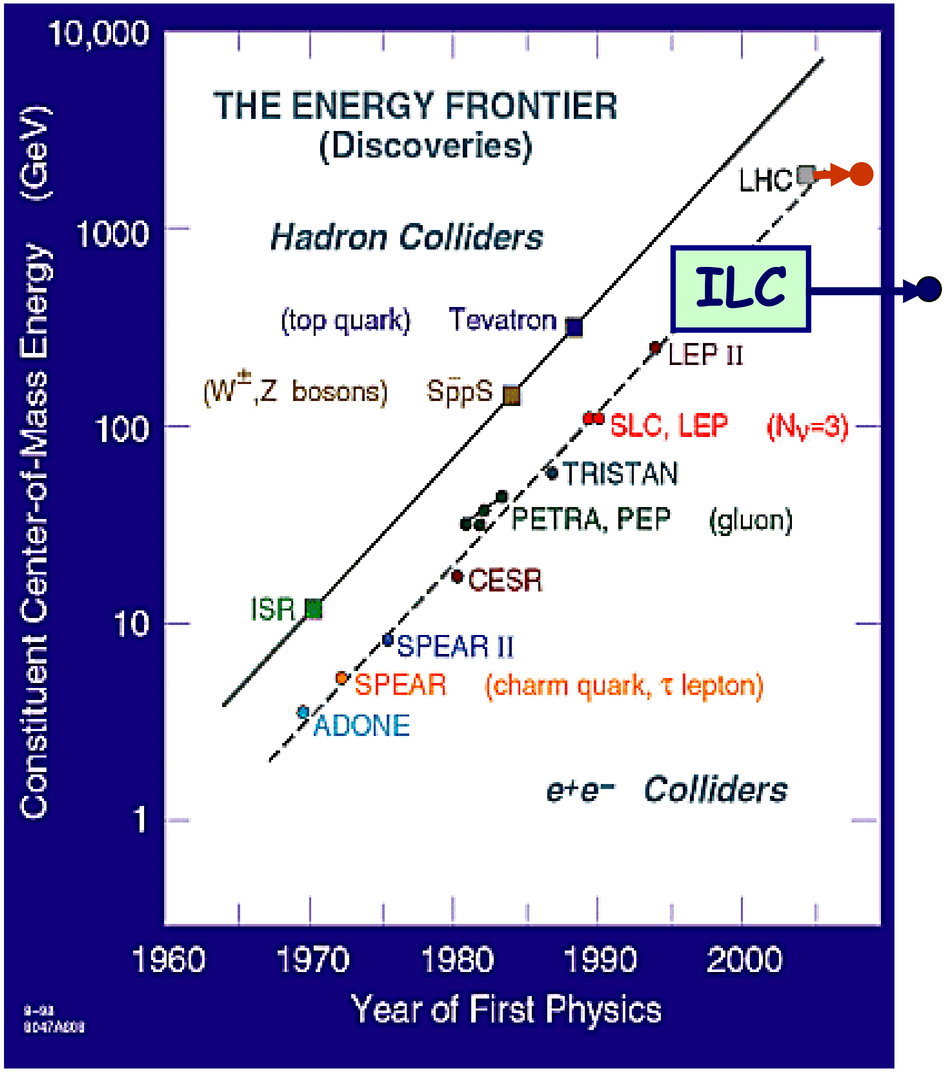
From Electrostatic Accelerators to Linear Colliders

Carlo Pagani

INFN Milano and DESY

On leave from University of Milano

Energy Frontier and e^+e^- Colliders



Relation of LHC and Linear Collider

- Since the ILC will start after the start of LHC, it **must add significant amount of information**. **This is the case!**

- **Neither LC nor HC's can draw the whole picture alone**. ILC will add new discoveries and precision of ILC will be essential for a better understanding of the underlying physics.

- There are probably pieces which can only be explored by the LHC due to the higher mass reach. **Joint interpretation of the results** will improve the overall picture

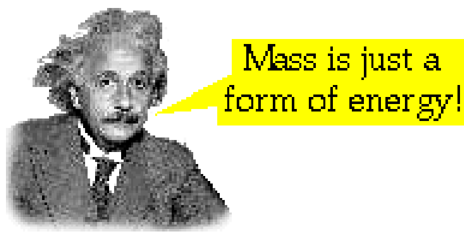
- **Overlapping running of both machines** will further increase the potential of both machines and might be mandatory, depending on the physics scenario realized

Basic Concepts: Energy

- **Energy of a relativistic particle**

$$E = m\,c^2$$

E = total energy
m = relativistic mass
c = speed of light



$m = \gamma\, m_0 \qquad E_0 = m_0\, c^2$

$\gamma = (1-\beta^2)^{-1/2} \qquad \beta = v/c$

m_0 = rest mass
γ = relativistic γ-factor
v = particle speed

- Kinetic energy, T, and momentum, **p**, of a relativistic particle

$$T = m\,c^2\,(\gamma - 1) = E - E_0$$

$$\boldsymbol{p} = m\,\boldsymbol{v} = m_0\,\gamma\,\boldsymbol{v}$$

$$E^2/c^2 = p^2 + m_0^{\,2}\,c^2$$

$$p \approx m\,c \quad if \quad v \approx c$$

- Non relativistic approximation: $v << c$

$$E \approx m_0\,c^2 + \tfrac{1}{2}\,m_0\,v^2$$

$m_0\,c^2$ = rest energy
$\tfrac{1}{2}\,m_0\,v^2$ = classical kinetic energy

- Useful numbers:

Speed of light: c = 2.9979 · 10^8 ms^{-1}
Energy unit: 1eV = 1.6021 · 10^{-19} joule

Electron rest energy: E_0 = 0.511 MeV
Proton rest energy: E_0 = 938 MeV

Basic Concepts: Fields

- **Equation of motion and Lorentz force**

$$\vec{F}_{em} = \vec{F}_{Lorentz} = \frac{d\vec{p}}{dt} = q\ (\vec{E} + \vec{v} \times \vec{B}) = \vec{F}_{el} + \vec{F}_{mag}$$

  - Electric field can transfer energy to the particles

$$\Delta E = \Delta T = \int \vec{F}_{em} \cdot d\vec{s} = q \int \vec{E} \cdot \vec{v} \cdot dt$$

  - Magnetic field can guide the beam in a stable path

- ***All Particle Accelerators are based on these rules***
  - The beam moves inside a vacuum chamber
  - Electromagnetic objects placed on the beam path perform the tasks
    - Magnets guide the beam on the chosen trajectory and produce focusing
    - Resonant RF cavities are used to apply the electric accelerating field
    - The few exceptions are: Betatron, RFQ and Electrostatic Accelerators

Focusing Forces from Magnetic Field

Expanding the magnetic component of the Lorentz force we have

$$\vec{F}_{mag} = q\left(\vec{v} \times \vec{B}\right) = q \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ v_x & v_y & v_z \\ B_x & B_y & B_z \end{vmatrix} = \vec{i}\,q(v_y B_z - v_z B_y) + \vec{j}\,q(v_z B_x - v_x B_z) + \vec{k}\,q(v_x B_y - v_y B_x)$$

The 3 magnetic field components, properly combined with the 3 beam velocity components are used to produce the forces required to guide the beam in a stable orbit

$$F_x^{mag} = q\left(v_y B_z - v_z B_y\right)$$

$$F_y^{mag} = q\left(v_z B_x - v_x B_z\right)$$

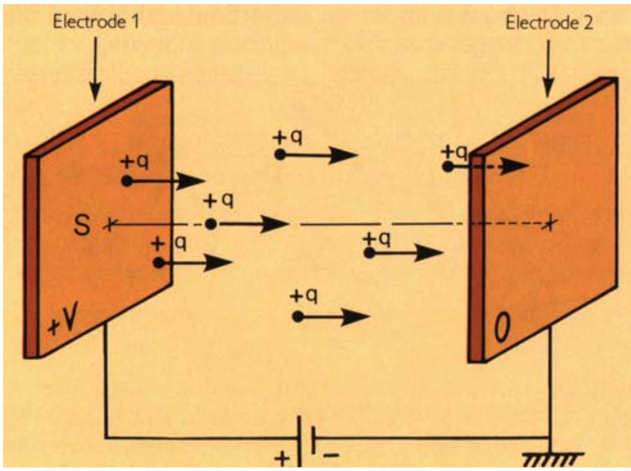
$$F_z^{mag} = q\left(v_x B_y - v_y B_x\right)$$

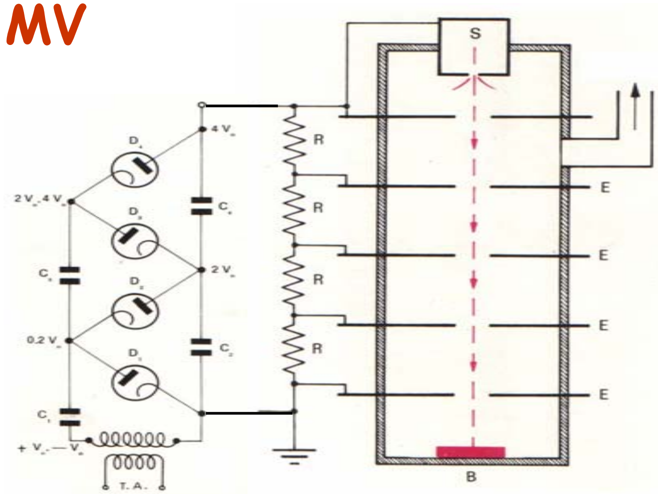
Particle Accelerators

- The name Particle Accelerator is a historical one connected to the concept of an energy increase related to a velocity change, that is an acceleration.
- For protons and ions that has been the case for a while
  - Electrostatic accelerators
  - Linacs
  - Cyclotrons
- Synchrotron concept and strong focusing scheme pushed energies to e level where the Energy increase is dominated by the particle mass increase and the velocity is very close to the speed of light
- The Touschek idea of colliding beams of the same mass but opposite charge in the same synchrotron ring opened the way of the modern colliders

Electrostatic: Cockroft&Walton

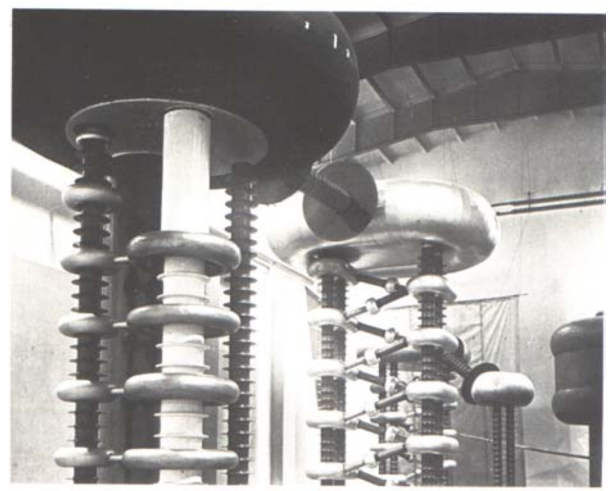
Up to 1-2 MV





the energy gain W of a particle of charge q moving through a potential V is:

W = q V



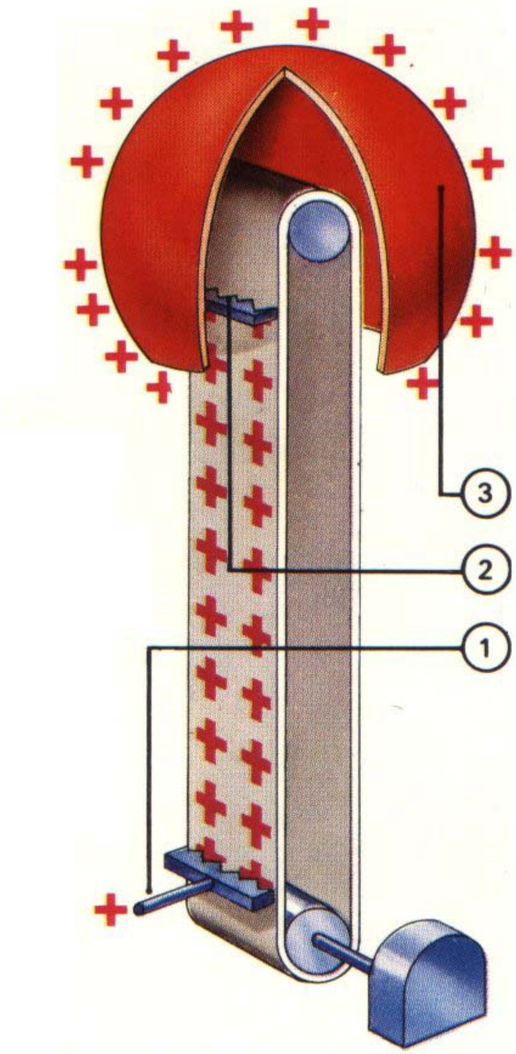
Problems:

- Voltage can be used once
- Source at high voltage
- Voltage limited by sparks

$$\nabla \times \vec{E} = 0$$

Electrostatic: van de Graaff & Tandem

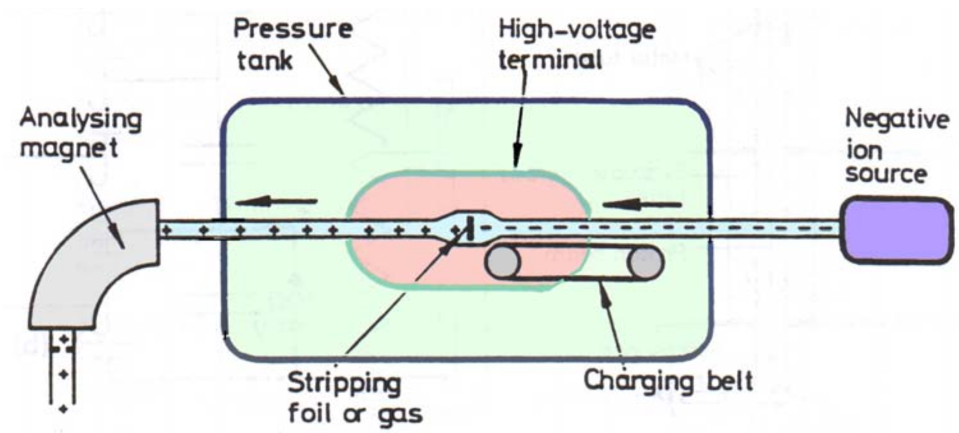
van de Graaff



Up to ~ 20 MV

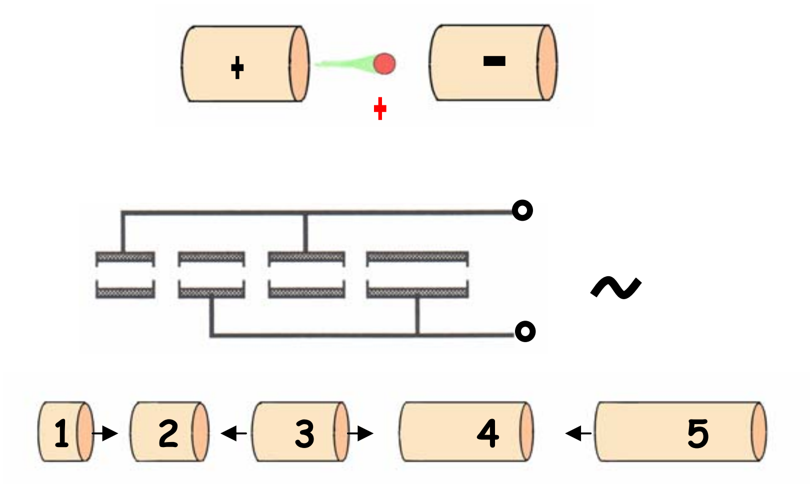
- Charge transported to the High Voltage terminal by a charging belt
- Use of SF_6 for higher voltage

Tandem: Voltage used twice changing the charge sign with a stripper

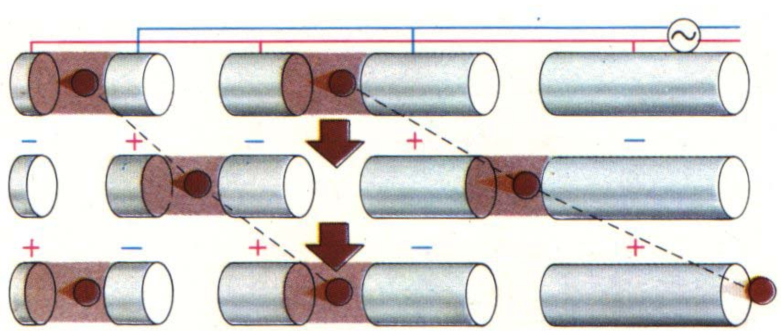


RF acceleration: Wideroe Linac

Linac concept from Wideroe

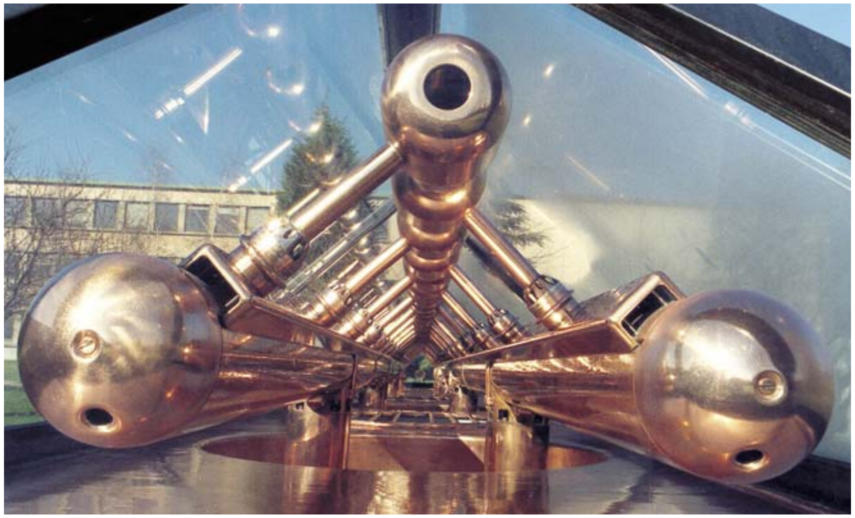


π *mode*



$$\nabla \times \vec{E} \neq 0$$

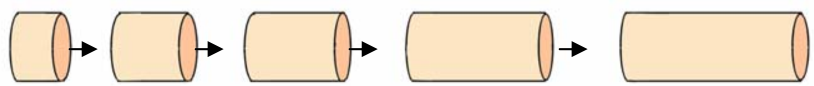
The particle beam can be accelerated many times by the same field but must be bunched to be synchronous with the RF field

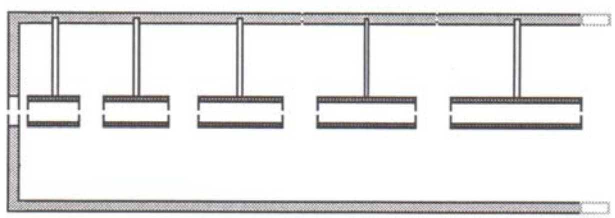


Wideroe type structure: ALICE, Orsay

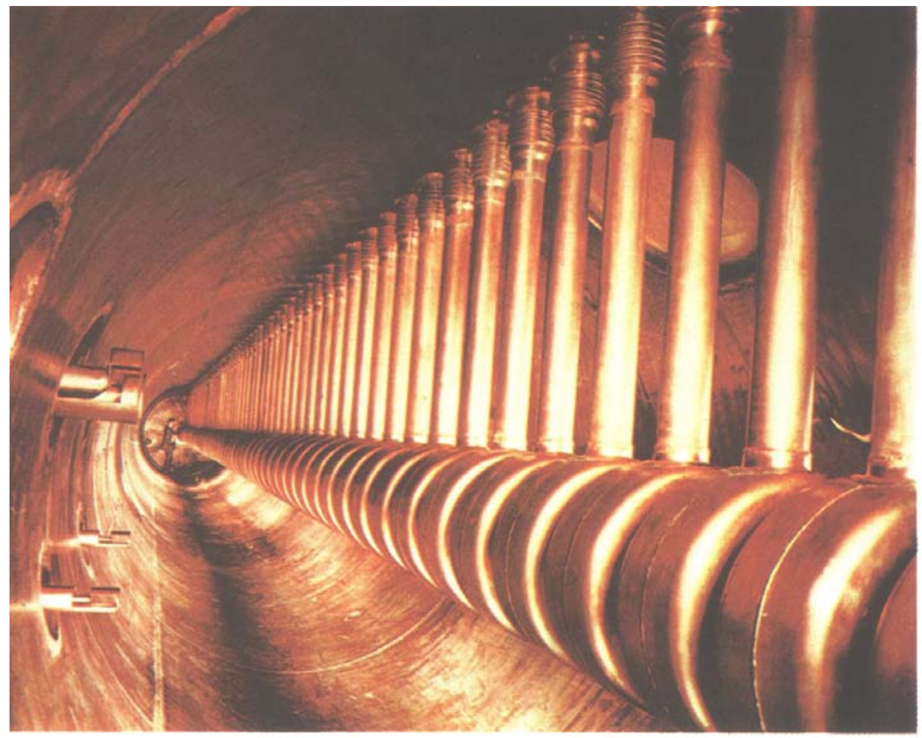
RF acceleration: Alvarez Linac

The Alvarez 2π mode scheme





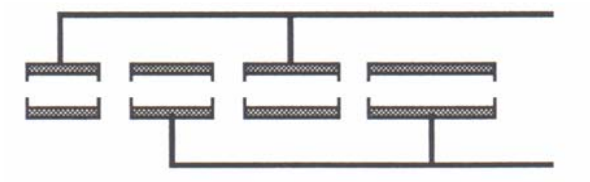
- More efficient structure
- Less RF power dissipation
- Focalization inside the electrodes

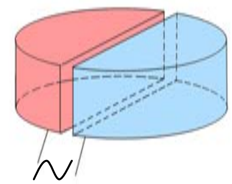


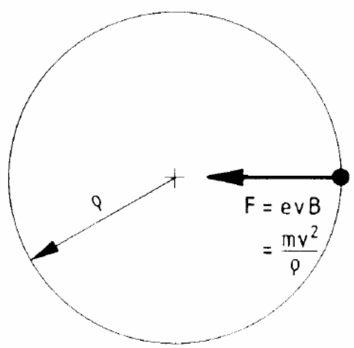
Alvarez, Drift Tube Linac (DTL), Saturne, Saclay

RF acceleration: Cyclotron

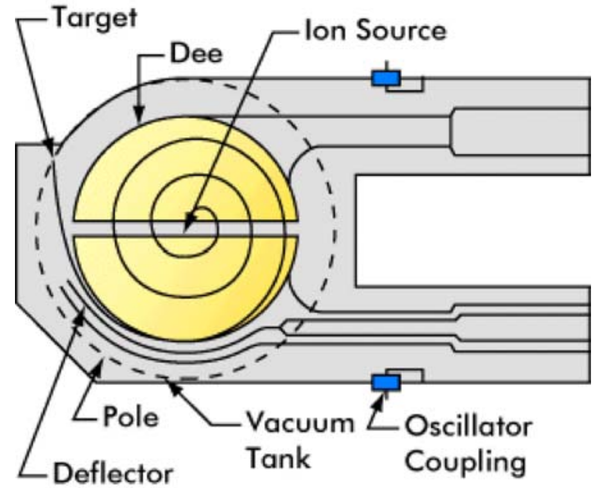
Folded Wideroe Linac in a constant magnetic field







$$B\rho = \frac{mv}{q} = \frac{p}{q}$$



$$f_{rev} = \frac{\omega_{rev}}{2\pi} = \frac{1}{2\pi}\frac{v}{\rho} = \frac{1}{2\pi}\frac{qB}{m}$$

For m and B = constant

also f_{rev} = constant

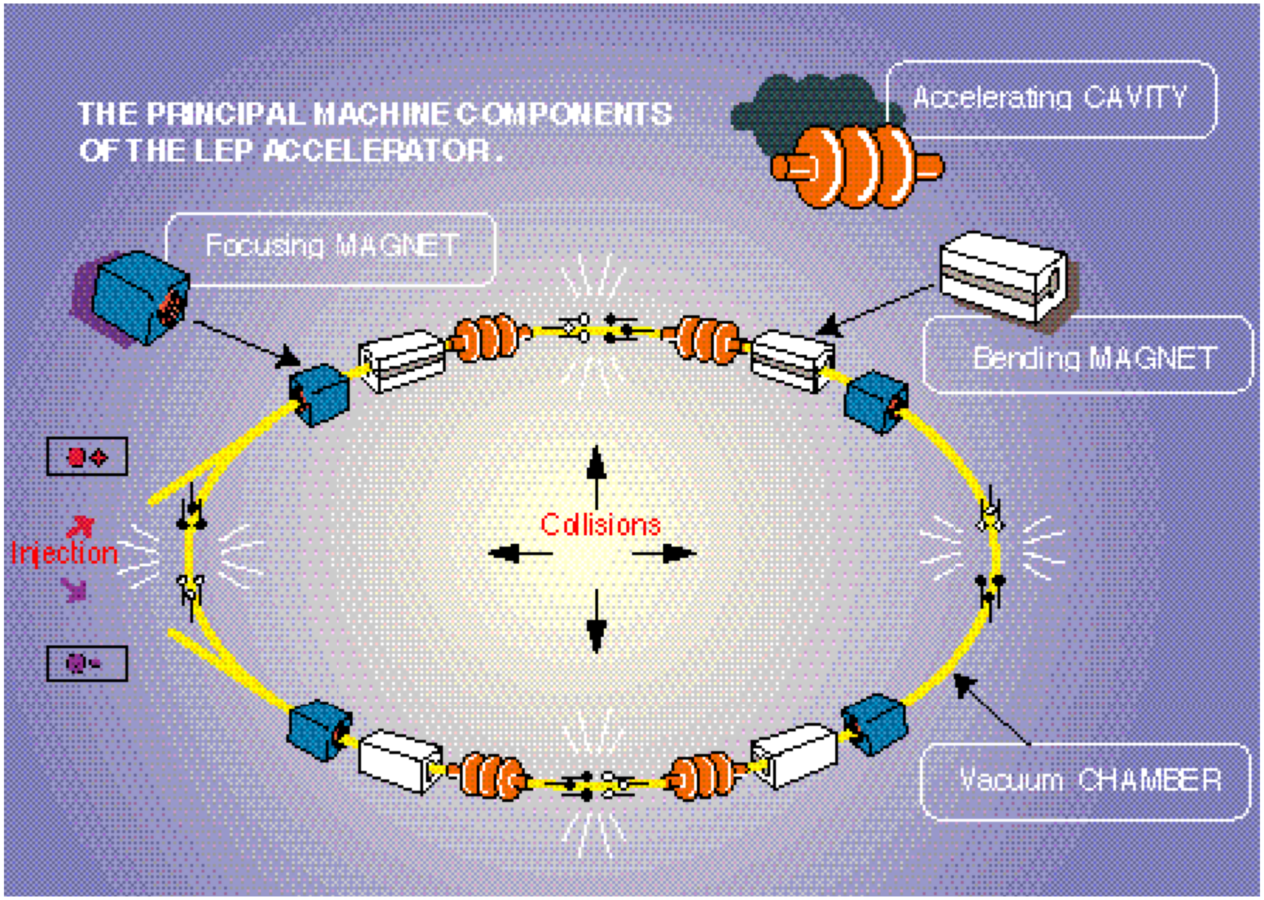
The PSI Cyclotron, Switzerland

RF acceleration: Synchrotron

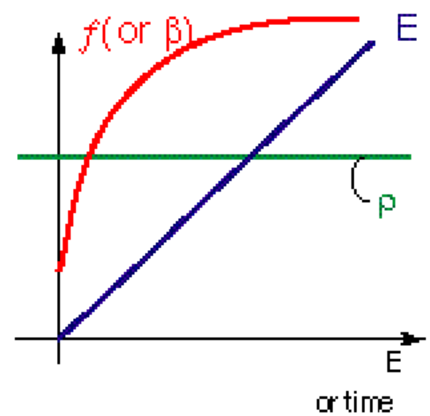
The LEP Example

Cyclotron: constant B Synchrotron: constant ρ

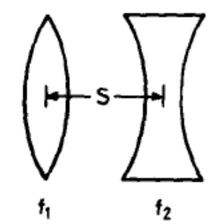
$$B\rho = \frac{mv}{q} = \frac{p}{q}$$



Accelerating cycle



Strong focusing concept

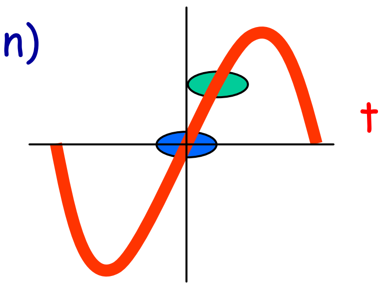


$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{s}{f_1 \cdot f_2}$$

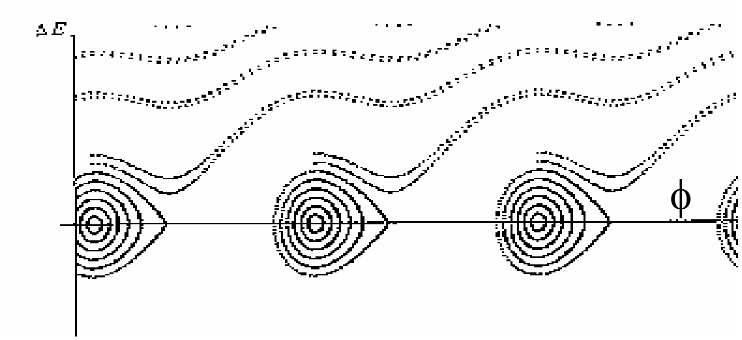
For $\beta \approx c$ E [GeV] ≈ 0.3 B [T] $\cdot \rho$ [m]

Longitudinal stability

- Bunch passing cavity:
  centre of bunch called the **"synchronous particle"**
- Particles see voltage $V_0 \sin 2\pi\omega_{rf} t = V_0 \sin \phi(t)$
  - For synchronous particle $\phi_s = 0$ (no acceleration)
  - Particles arriving early see $\phi < 0$
  - Particles arriving late see $\phi > 0$



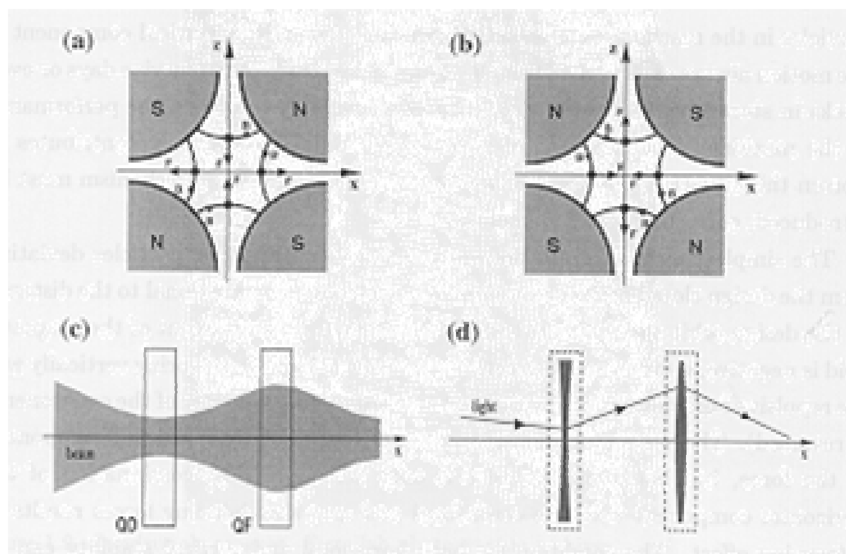
- energy of those in advance is decreased and vice versa: **"Bunching"**
- To accelerate, make $0 < \phi_s < \pi$ $\Delta E = qV_0 \sin \phi_s$
- For longitudinal (phase) stability, make $-\pi/2 < \phi_s < +\pi/2$



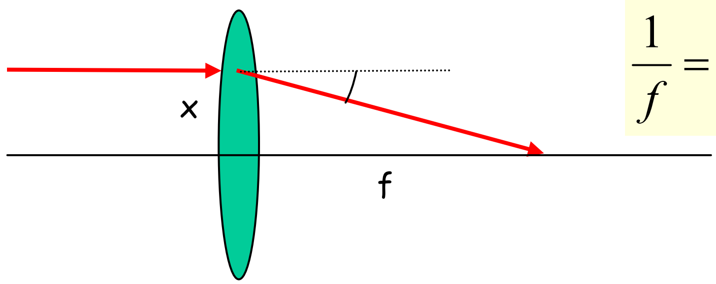
Not all particles are stable. There is a limit to the stable region (the separatrix or "bucket") and, at high intensity, it is important to design the machine so that all particles are confined within this region and are "trapped".

Transverse "Strong Focusing"

- Alternating gradient (AG) principle (1950's)
- A sequence of focusing-defocusing fields provides a stronger net focusing force.
- Quadrupoles focus horizontally, defocus vertically or vice versa. Forces are proportional to displacement from axis.
- A succession of opposed elements enable particles to follow stable trajectories, making small oscillations about the design orbit.
- Technological limits on magnets are high: iron saturation and dissipated power for high current
- Superconducting magnets are required for high field
- Solenoids are preferred at low energy, with high space charge forces: continuous focusing



Thin lens analogy of AG focusing



$$\frac{1}{f} = \left(\frac{B'}{B\rho}\right) L$$

Focusing: $F = \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}$

Defocusing: $D = \begin{pmatrix} 1 & 0 \\ \frac{1}{f} & 1 \end{pmatrix}$

Drift: $O = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}$

Drift space effect

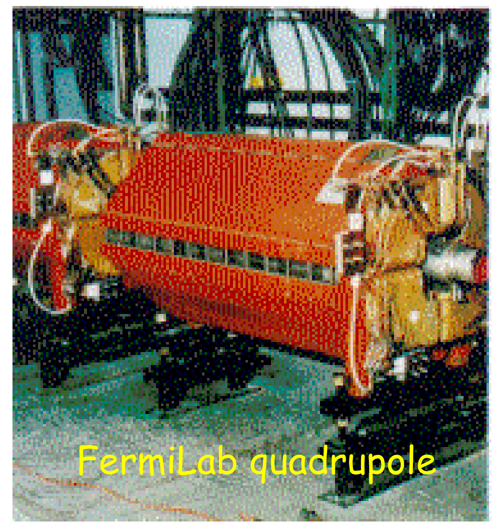
- $x_o = x_i + x_i' L$
- $x_o' = x_i'$

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{out} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_{in}$$

F-O-D

Transfer Matrix

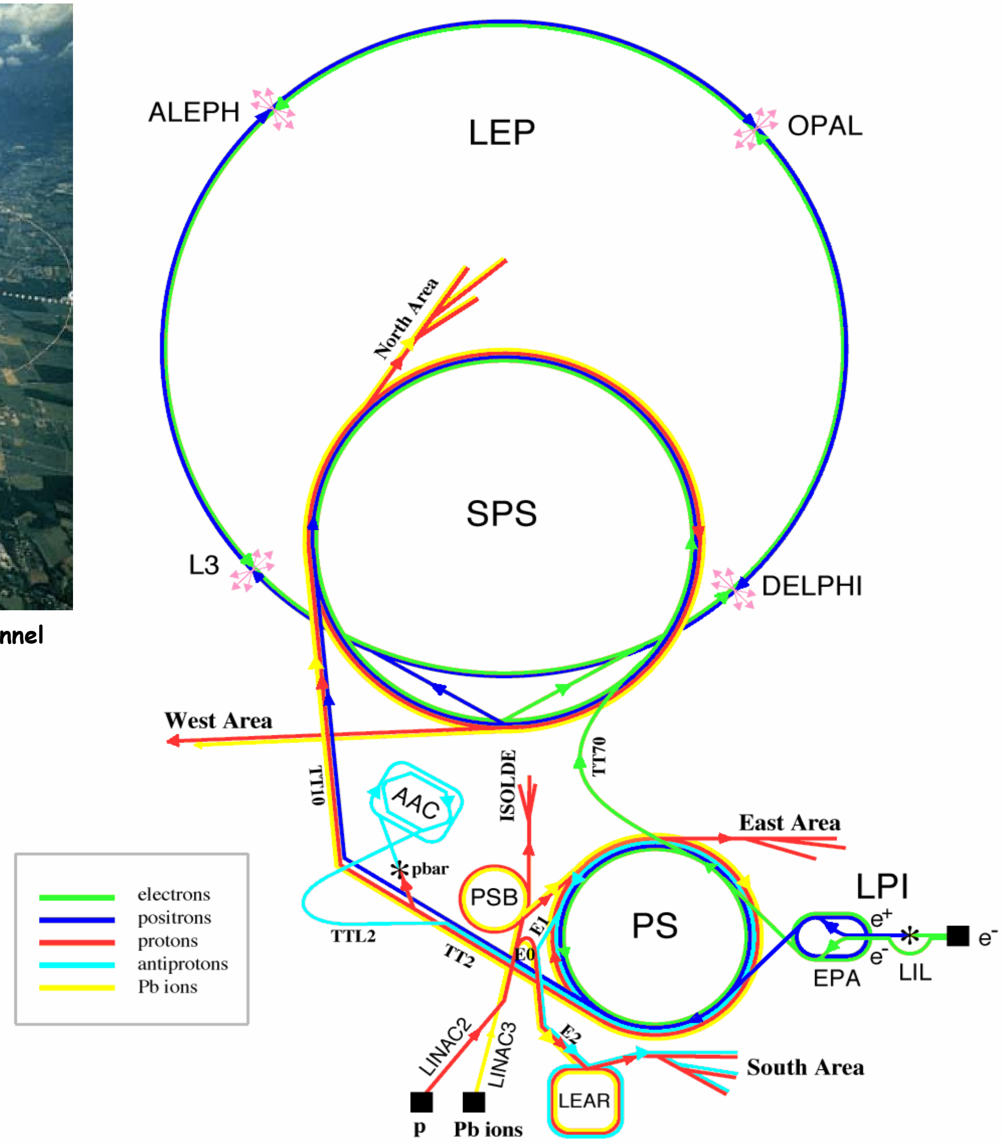
$$\begin{pmatrix} 1 & 0 \\ \frac{1}{f} & 1 \end{pmatrix} \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix} = \begin{pmatrix} 1+\frac{l}{f} & L \\ -\frac{L}{f^2} & 1-\frac{L}{f} \end{pmatrix}$$



- Thin lens of focal length f^2/L, focusing if $L \ll f$
- Same for D-O-F ($f \rightarrow -f$)
- A system of AG lenses can focus in both planes

The LEP CERN Accelerator Complex

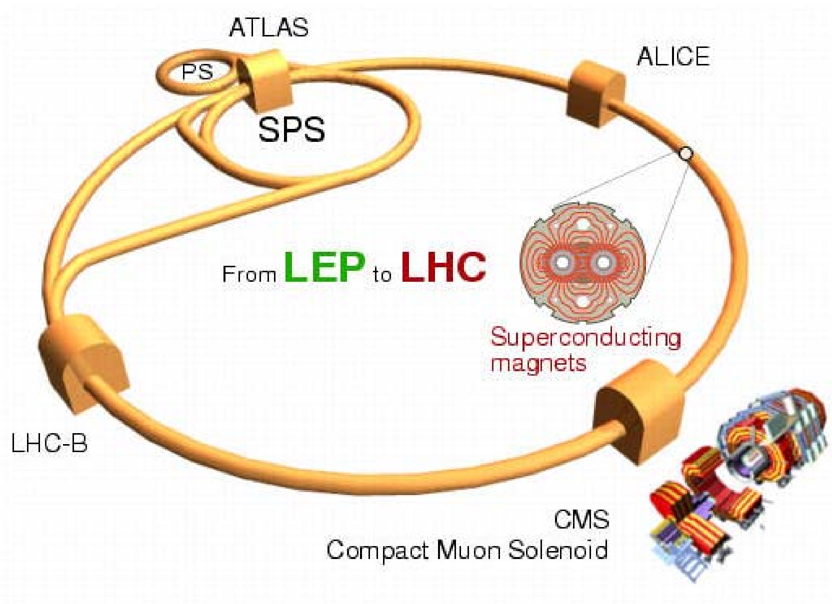
Aerial view of the CERN site with an indication of the circular LEP tunnel



- Linacs and synchrotrons were used to inject in the 28 km synchrotron where both electron and positrons were accelerated up to 100 GeV to collide with a centre of mass energy of 200 GeV
- LHC now under construction is making use of most of the LEP injection accelerator complex

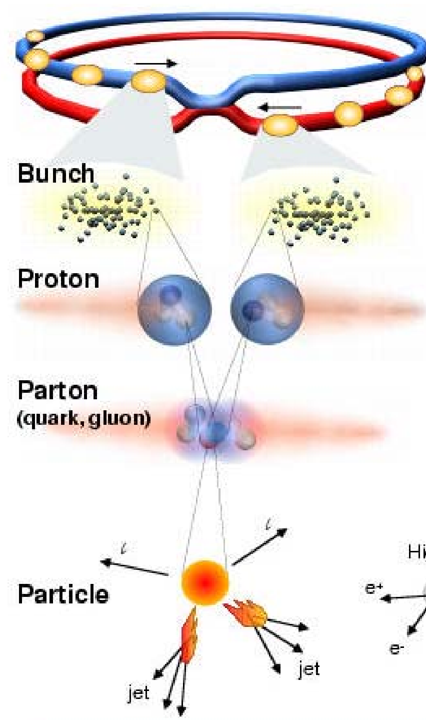
From LEP to LHC

The Large Hadron Collider (LHC)



	Beams	Energy		Luminosity
LEP	e+ e-	200	GeV	10^{32} cm^{-2}s^{-1}
LHC	p p	14	TeV	10^{34}
	Pb Pb	1312	TeV	10^{27}

Collisions at LHC



Proton-Proton	(2835 x 2835 bunches)
Protons/bunch	10^{11}
Beam energy	7 TeV (7×10^{12} eV)
Luminosity	10^{34} cm^{-2} s^{-1}
Crossing rate	40 MHz
Collisions ≈	10^7 - 10^9 Hz

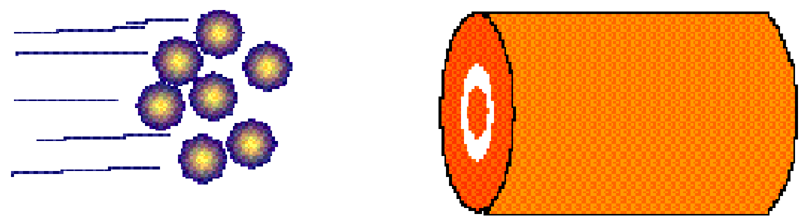
Selection of 1 in 10,000,000,000,000

Colliding Beams for High Energy

W = Energy available in center-of-mass for making new particles

For fixed target :

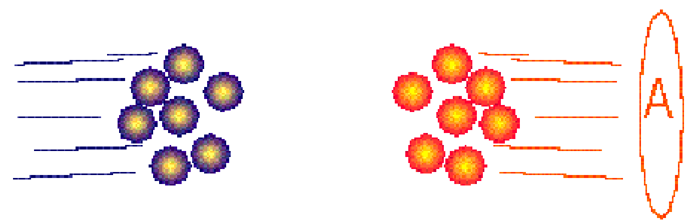
$$E_{c.m} \cong \sqrt{2 m_T E_B}$$



... and we rapidly run out of money trying to gain a factor 10 in c.m. energy

But a storage ring , colliding two beams, gives:

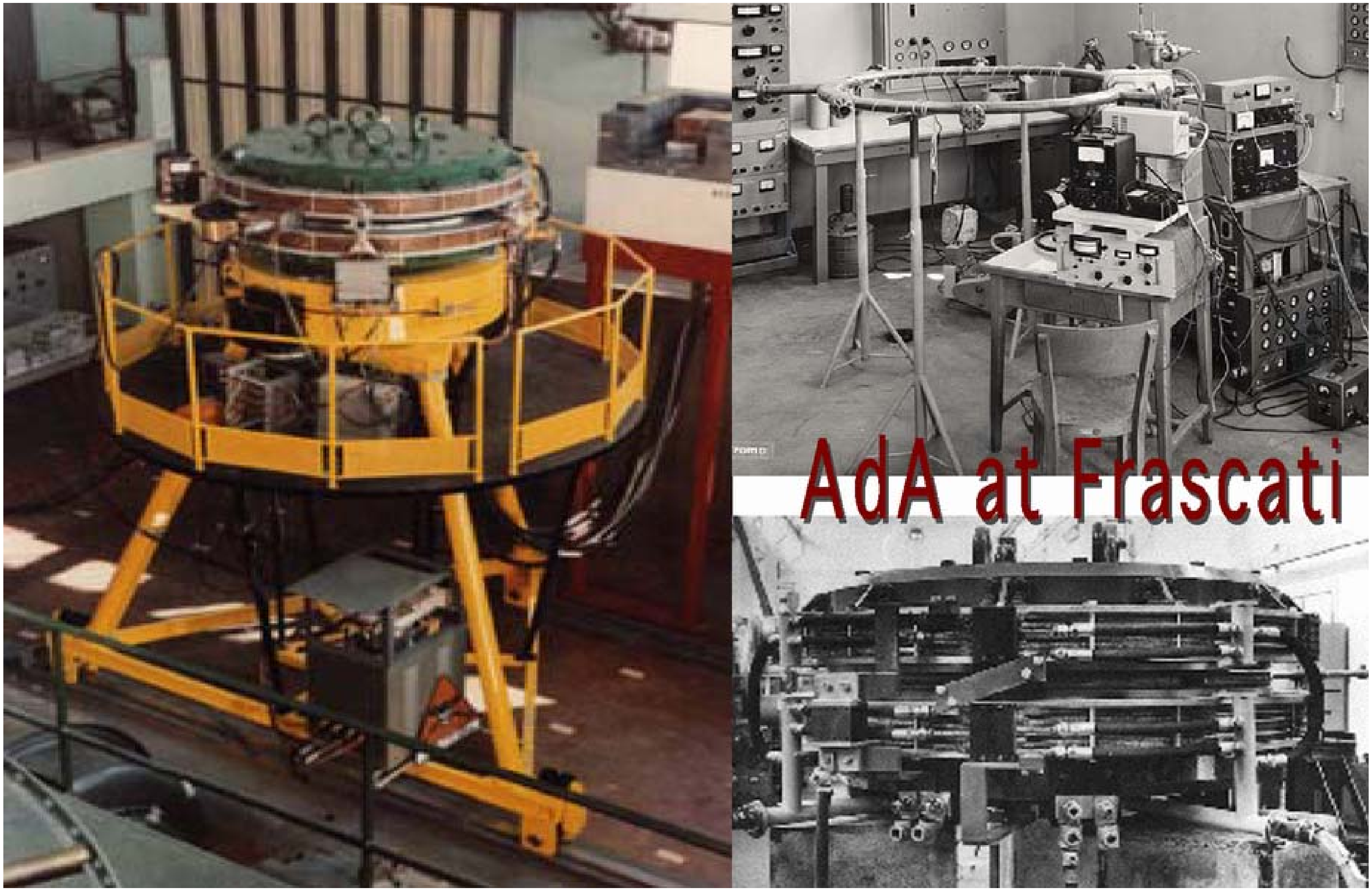
$$E_{c.m.} \cong 2\ E_B$$



Problem: Smaller probability that accelerated particles collide .... "Luminosity" of a collider

$$L = N_1 N_2 \frac{1}{A} \frac{\beta c}{2\pi R} \approx 10^{29} \ldots 10^{34}\, cm^{-2} s^{-1}$$

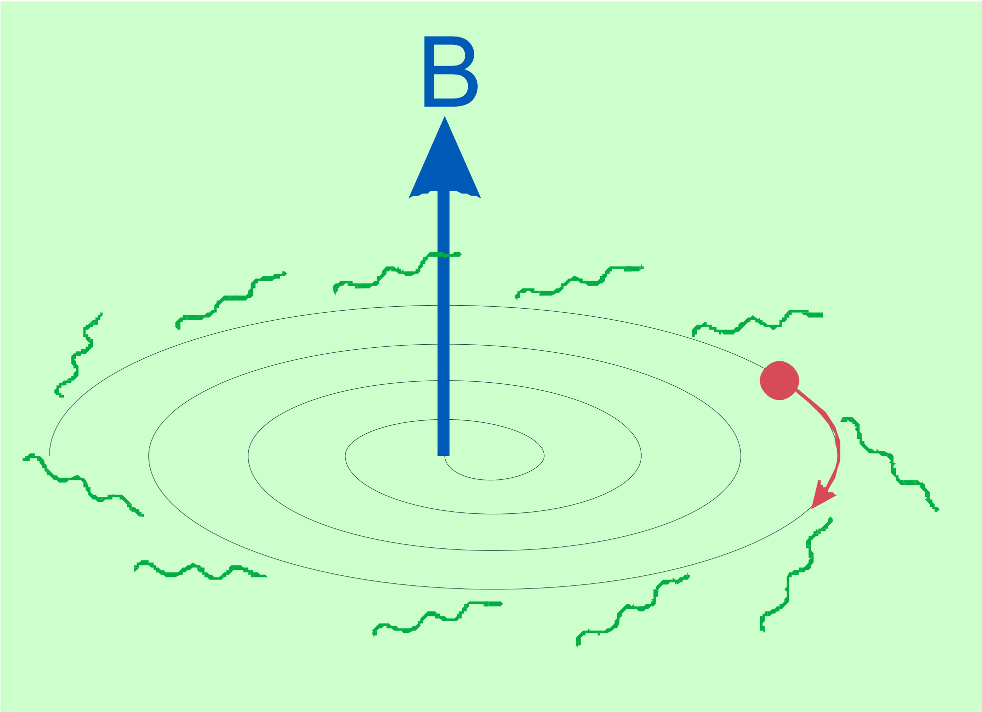
The First e^+e^- Collider



No Circular e^+e^- Collider after LEP

Synchrotron Radiation

From an electron in a magnetic field:



Energy loss must be replaced by RF system

cost scaling $ $\propto E_{cm}^2$

A Simple Exercise

- Synchrotron Radiation (SR) becomes prohibitive for electrons in a circular machine above LEP energies:

$$U_{SR}\,[\mathrm{GeV}] = 6 \cdot 10^{-21} \cdot \gamma^4 \cdot 1/r[km]$$

U_{SR} = energy loss per turn
γ = relativistic factor
r = machine radius

- RF system must replace this loss, and r scale as E^2
- LEP @ 100 GeV/beam: 27 km around, 2 GeV/turn lost
- Possible scale to 250 GeV/beam i.e. E_{cm} = **500 GeV**:
  - **170 km** around
  - **13 GeV/turn** lost

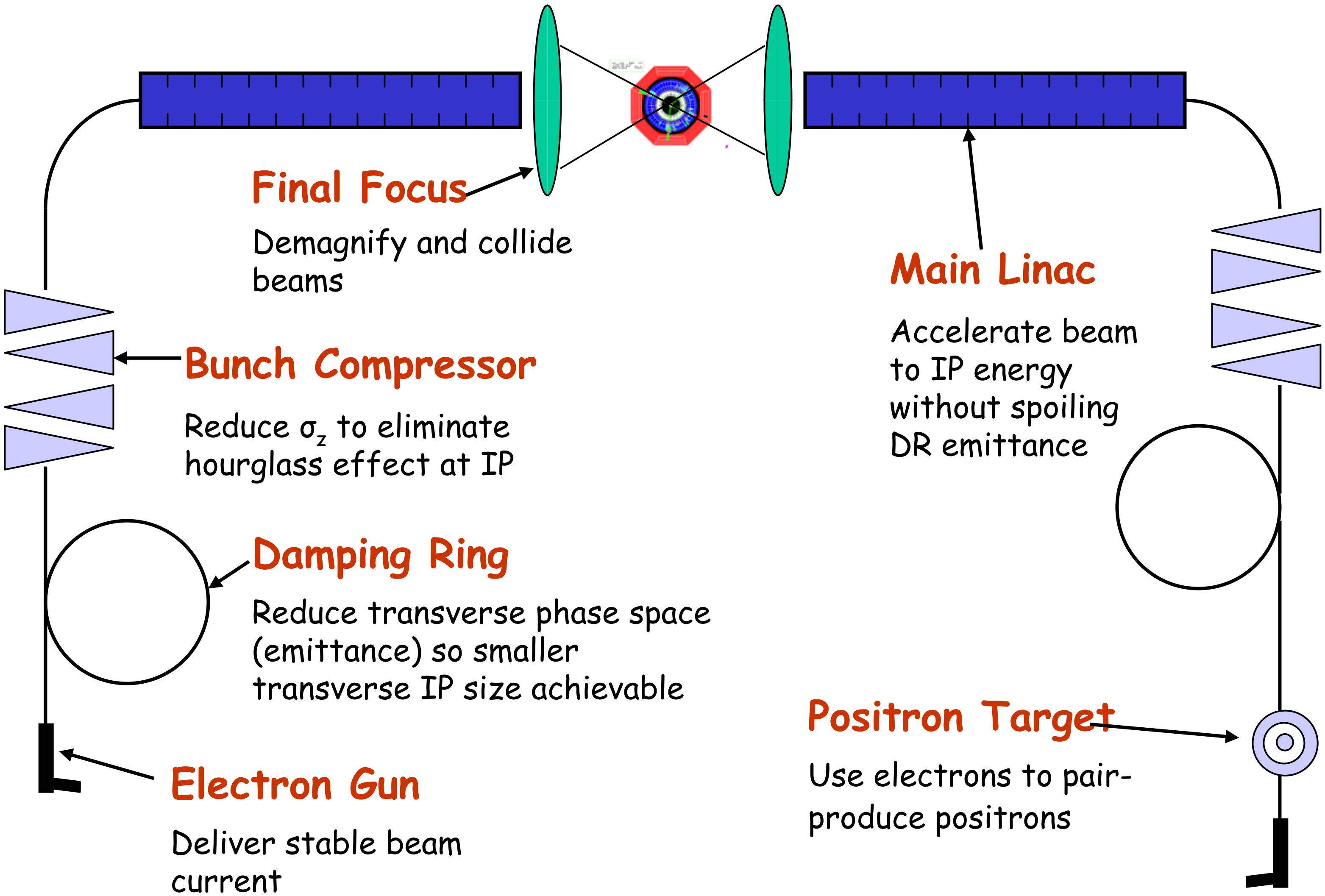
$\gamma_{250GeV} = 4.9 \,.\, 10^5$

- Consider also the luminosity
  - For a **luminosity of ~ $10^{34}/cm^2/second$**, scaling from b-factories gives ~ 1 Ampere of beam current
  - 13 GeV/turn x 2 amperes = **26 GW RF power**
  - Because of conversion efficiency, this collider would consume more power than the state of **California in summer: ~ 45 GW**

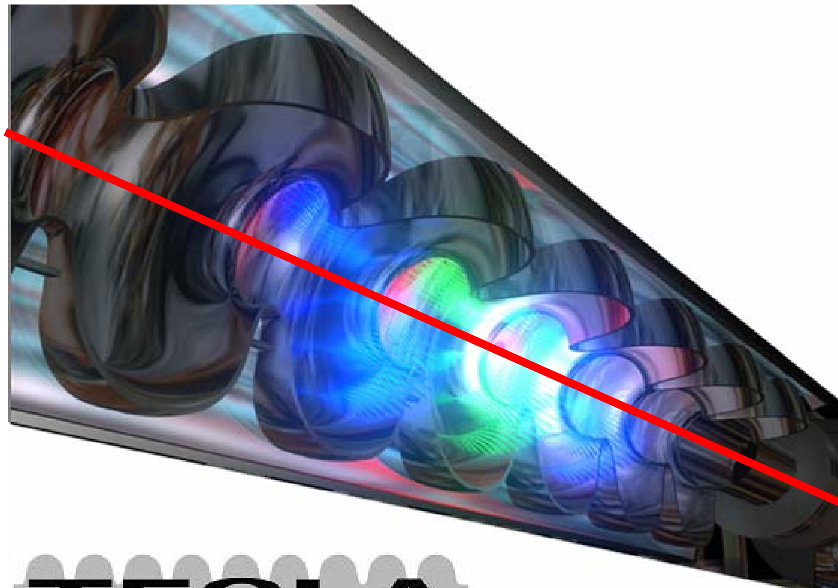
Circulating beam power = 500 GW

- Both size and power seem excessive

Linear Collider Conceptual Scheme

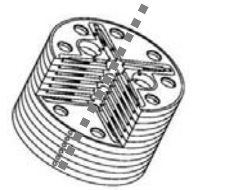


Competing technologies



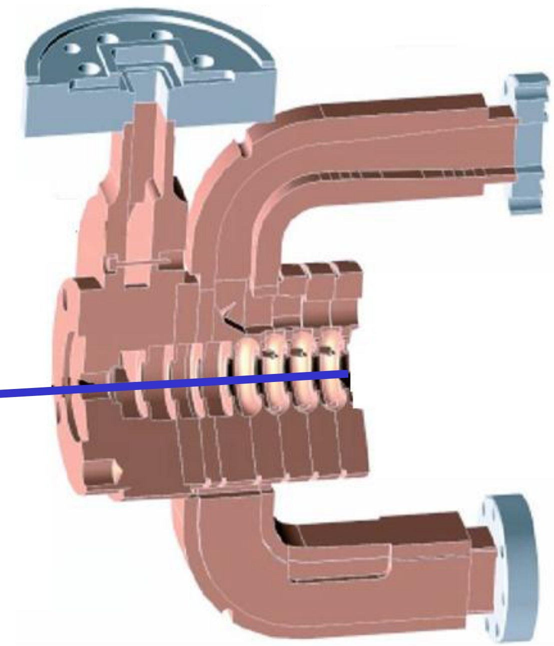
TESLA

1.3 GHz - Cold



CLIC

30 GHz-Warm



NLC JLC

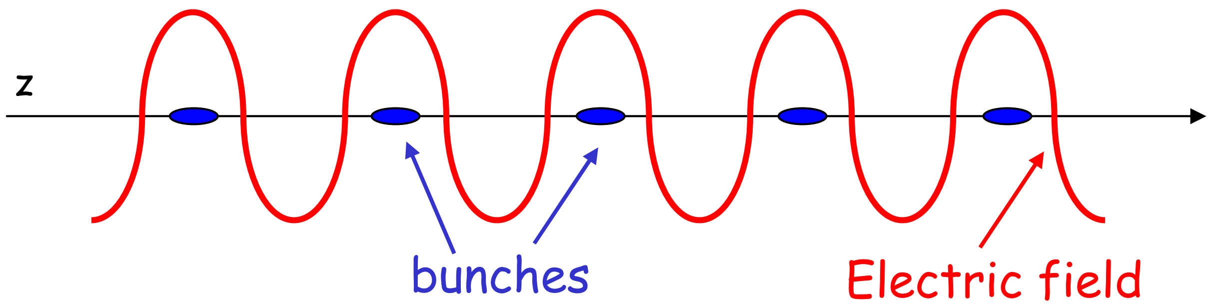
11.4 GHz - Warm

The Linac Accelerator Concept

- An RF source is used to generate an electric field in a region of a resonant metallic structure
- The particles of the beam need to be localized in *bunches* and properly phased with respect to the field so that the beam is "accelerated"

$$\frac{d\left(\gamma m_0 c^2\right)}{ds} = qE_z\left(s,t\right)$$

- In order to keep acceleration along the linac this synchronism condition needs to be maintained.



Maxwell Equations and Waves

Electromagnetic fields are described by Maxwell Equations that in empty space are:

$$\begin{cases} \nabla \cdot \vec{E} = 0 \\ \nabla \cdot \vec{B} = 0 \\ \nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t} \\ \nabla \times \vec{B} = \mu_0 \varepsilon_0 \dfrac{\partial \vec{E}}{\partial t} \end{cases}$$

From Maxwell Equation we obtain the Wave Equations for Electric and Magnetic Fields

$$\nabla^2 \begin{Bmatrix} \vec{E} \\ \vec{B} \end{Bmatrix} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \begin{Bmatrix} \vec{E} \\ \vec{B} \end{Bmatrix} = 0$$

where: $\boxed{c = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}}}$

Plane Wave: Equations

Defining the z-axis parallel to the direction of propagation, we can solve the wave equation as a superposition of travelling plane waves:

$$\vec{E} = \vec{E}_0 e^{i(\omega t - kz)} \qquad \vec{H} = \vec{H}_0 e^{i(\omega t - kz)}$$

Absence of boundaries (isotropic, homogeneous vacuum) requires that vectors $\mathbf{E}_0$ and $\mathbf{H}_0$ be constant for all time and space

Applying the wave equation to the electric and magnetic fields yields:

$$\nabla^2 \vec{E} = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \vec{E}_0 e^{i(\omega t - kz)} \qquad \nabla^2 \vec{H} = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \vec{H}_0 e^{i(\omega t - kz)}$$

Since E_0 and H_0 are constant, both the time derivative and Laplacian operate only on the complex exponential. After cancellation of constant factors, we find:

$$\boxed{k^2 = \frac{\omega^2}{c^2} \rightarrow \frac{\omega}{k} = \frac{\partial \omega}{\partial k} = c} \qquad \boxed{c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}}$$

i.e., a plane wave with a phase velocity and a group velocity = c

Free-Space Solution

Go back and apply Maxwell's equations to this solution:

$$\vec{\nabla} \cdot \vec{E} = \frac{\partial \vec{E}}{\partial z} = -ikE_{0,z} e^{i(\omega t - kz)} = 0$$

That is: k or $E_{0,z}$ must be zero!

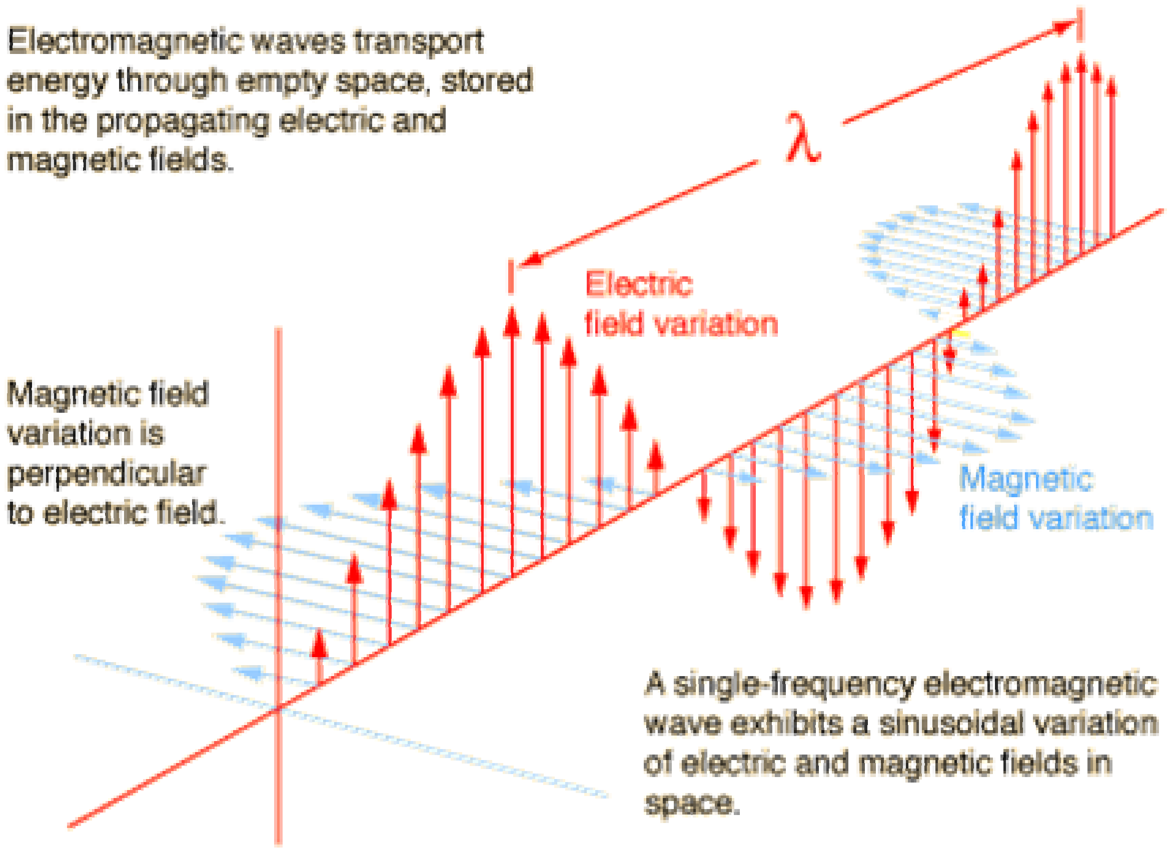
k=0: trivial solution, no wave!

$E_{0,z}$ = 0: electric field accelerates beam transverse to direction of wave propagation! If beam is accelerated in x while wave moves in z, then wave will first accelerate, then decelerate, the beam!

No good for acceleration!

Planar Wave: Pictorial View

Electromagnetic wave in empty space



The energy transport per unit area is described by the Poynting vector ***S***

$$\boxed{\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}}$$

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

ε_0 = electric permittivity
μ_0 = magnetic permeability

The Phase velocity v_{ph} is the velocity of an observer sitting at constant phase

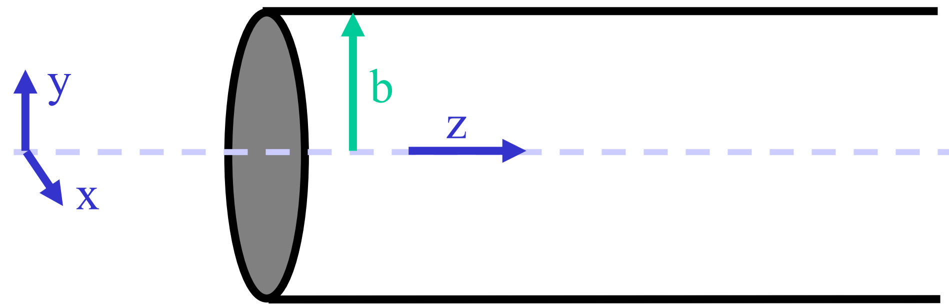
Group velocity v_{gr} is the velocity of the energy propagation

$$v_{ph} = v_{gr} = c$$

Bounded Solution to the Wave Equation

Apply some kind of boundaries in *x* and *y*, so that non-zero *x* and *y* derivatives of the electric field can cancel z derivative (i.e. permits non-zero $E_{0,z}$ while still obeying Maxwell).

Try a conducting pipe of radius b, oriented along z axis:



$$\vec{E} = \vec{E}_0 e^{i(\omega t - kz)}$$

$$\vec{H} = \vec{H}_0 e^{i(\omega t - kz)}$$

This time vectors E_0 and H_0 are functions of transverse coordinates *x* and *y* (or *r* and *θ*) but not *z* or *t*. Thus we can simplify some derivatives:

$$\frac{\partial}{\partial z} = -ik, \frac{\partial^2}{\partial z^2} = -k^2$$

$$\frac{\partial}{\partial t} = +i\omega, \frac{\partial^2}{t^2} = -\omega^2$$

Perfect Conductor Solution - 2

Using cylindrical coordinates we have,

- at the boundary, i.e. at $r=b$, the *normal* component of **B** and the *tangential* component of **E** are continuous.
- if the conductor is perfect, then *within* the conductor the electric and magnetic field are identically zero. Thus at $r=b$, H_r, E_z, and E_θ ➔ 0.
- since $E_\theta=0$, the θ component of the magnetic curl equation must go to zero.

$$\begin{cases} \vec{n}\cdot\vec{B}=0 \\ \vec{n}\times\vec{E}=0 \end{cases}$$

In total

$$E_\theta = E_z = H_r = \frac{\partial H_z}{\partial r} = 0$$

With some algebra and canceling the common complex component (time dependence) we get from the wave equation the longitudinal electric field:

$$\nabla^2 \vec{E} = \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\vec{E}_0 e^{i(\omega t - kz)}$$

$$E_{0,z} = \sum_{n=0}^{\infty} a_n J_n(k_c r)\cos(n\theta + \theta_n)$$

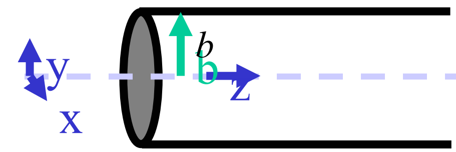
Where:

J_n are the Bessel functions

$$k_c^2 \equiv \left(\frac{\omega^2}{c^2} - k^2\right)$$

n must be is an integer

Perfect Conductor Solution - 3



Because: $E_z = 0$ @ $r = b$,

We can set: $k_c b = z_{np}$, where z_{np} is the p^{th} zero of J_n.

As a result: $k_c > 0$

$$E_{0,z} = \sum_{p=1}^{\infty}\sum_{n=0}^{\infty} a_{np} J_n(k_{c,np} r)\cos(n\theta + \theta_{np})$$

$$k_{c,np} = \frac{z_{np}}{b} = \sqrt{\frac{\omega^2}{c^2} - k^2}$$

$k^2 \geq 0$ because k must be real for propagation and for $k = 0$ we have the cutoff frequency:

$$\nabla^2 \vec{E} = \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\vec{E}_0 e^{i(\omega t - kz)}$$

$$\omega_{c,np} = c\frac{z_{np}}{b} = \omega_c$$

Cutoff frequency

$\omega > \omega_c$ Traveling wave: propagation

$\omega < \omega_c$ Evanescent wave: can't propagate

But also:

$$v_{gr} = \frac{\partial \omega}{\partial k} = c \cdot \frac{\sqrt{\omega^2 - \omega_c^2}}{\omega} < c$$

$$v_{ph} = \frac{\omega}{k} = \sqrt{c^2 + \frac{\omega_c^2}{k^2}} > c$$

TM and TE Modes

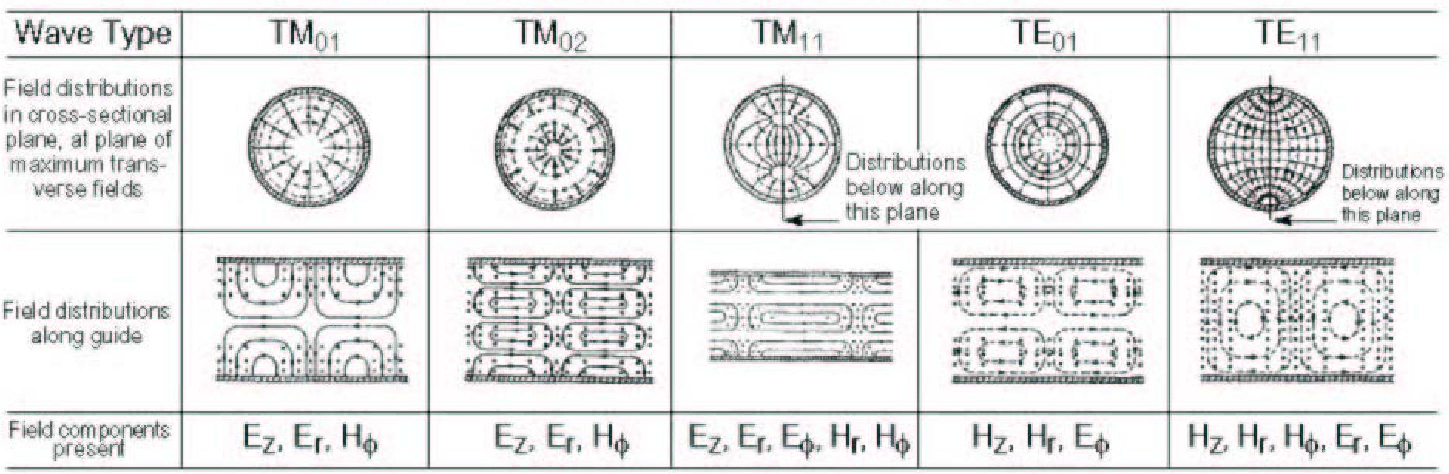
A similar solution is available for the magnetic field vector

In general a wave with a given phase and group velocity cannot have both a longitudinal electric field and a longitudinal magnetic field!

Waves with $H_{0,z} \equiv 0$ are called TM (transverse magnetic) modes; waves with $E_{0,z} \equiv 0$ are called TE (transverse electric) modes. Usually the modes are referred to with their index numbers, TE_{uv} or TM_{np}

TM01 mode has nonzero E_z, E_r, H_θ components only

Table 3 Mode Patterns in Circular Waveguide.



Wave Type	TM_{01}	TM_{02}	TM_{11}	TE_{01}	TE_{11}
Field distributions in cross-sectional plane, at plane of maximum transverse fields			Distributions below along this plane		Distributions below along this plane
Field distributions along guide					
Field components present	E_z, E_r, H_ϕ	E_z, E_r, H_ϕ	$E_z, E_r, E_\phi, H_r, H_\phi$	H_z, H_r, E_ϕ	$H_z, H_r, H_\phi, E_r, E_\phi$

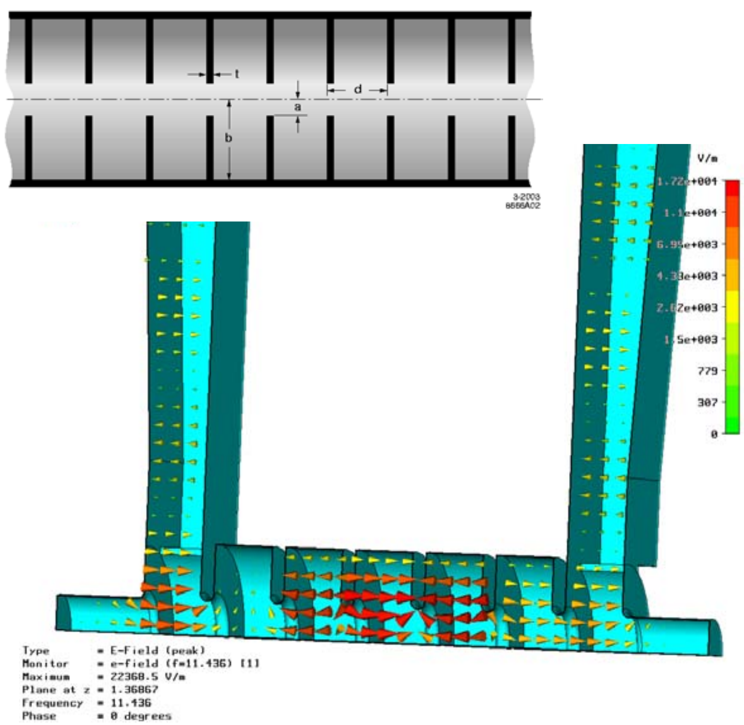
7–98
8365A213

Real Accelerating Structures: Cavities

Imposing boundary condition in the longitudinal direction, z, we have for each mode (for example the TM_{01}) two waves: rightward (+z)-propagating wave and a leftward-propagating wave The combination can give a wave with phase velocity $v_{ph} \le c$

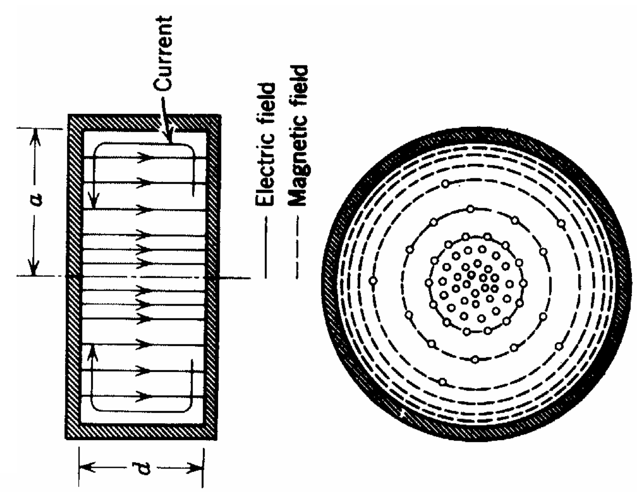
Traveling wave structure

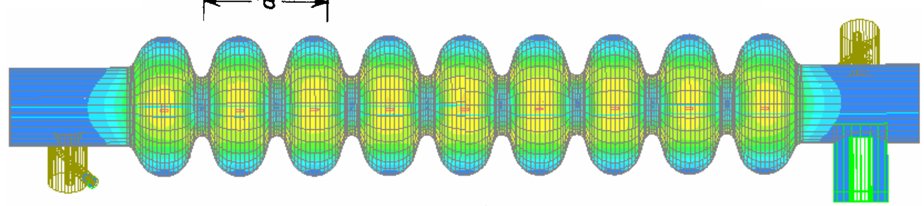
$V_{ph} \approx c$ and $Vg < c$



Standing wave structure

$V_{ph} = 0$ and $Vg = c$



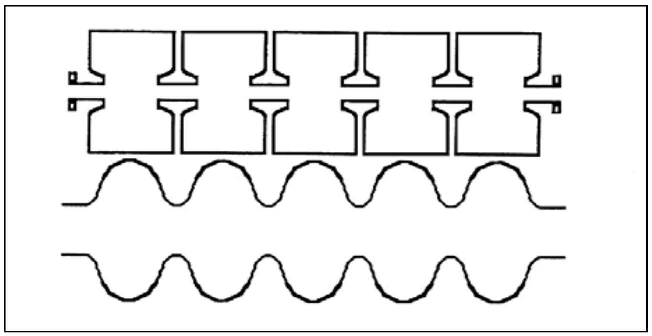


π mode

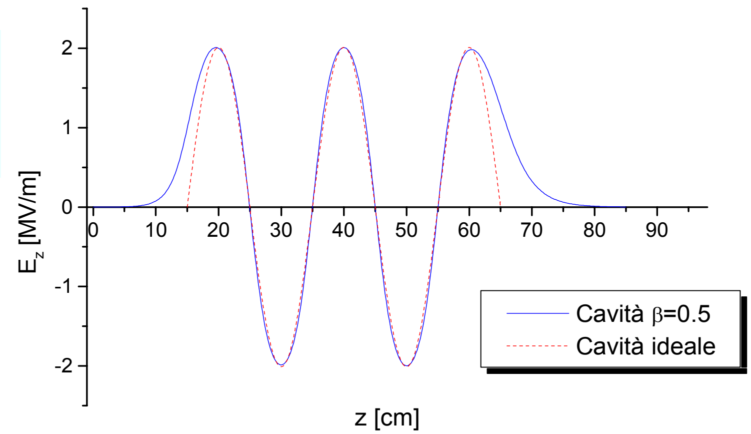
Realistic multicell cavities

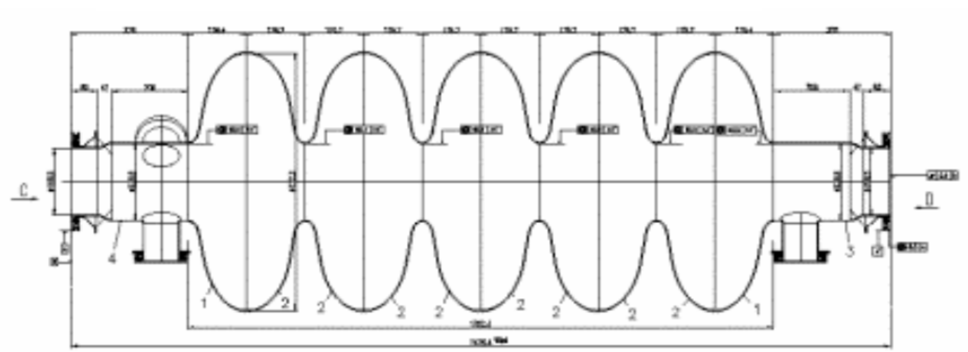
standing wave: resonant structure

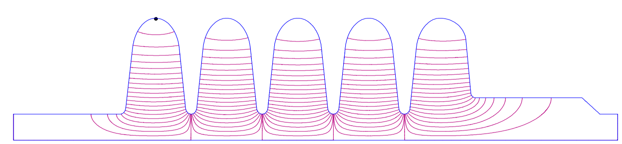
- In order to efficiently accelerate the beam, *multicell resonators are used*, by periodically repeating the resonant structure and providing coupling between the different cells. Any geometry can be computed with existing numerical codes
  - The simplest coupling is represented by the **E** field through the beam hole (capacitive coupling)
- The beam needs to keep the relative phase with the field



$$L = \frac{\lambda_{RF}\beta}{2}$$







Electrons and Protons

- electron and proton masses

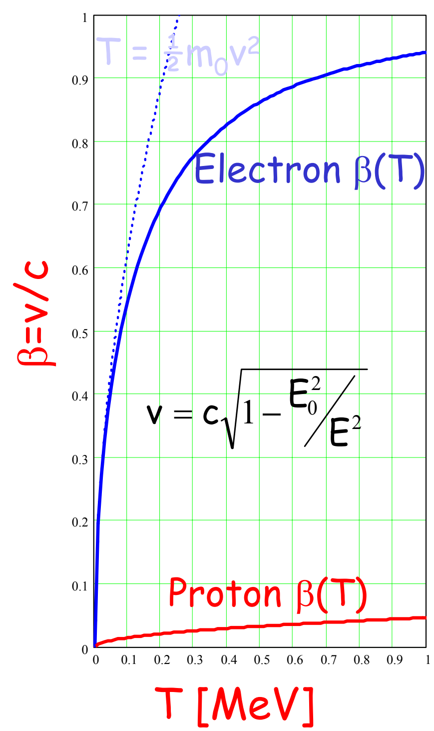
$$E_{0,proton} \approx 2000\ E_{0,electron}$$

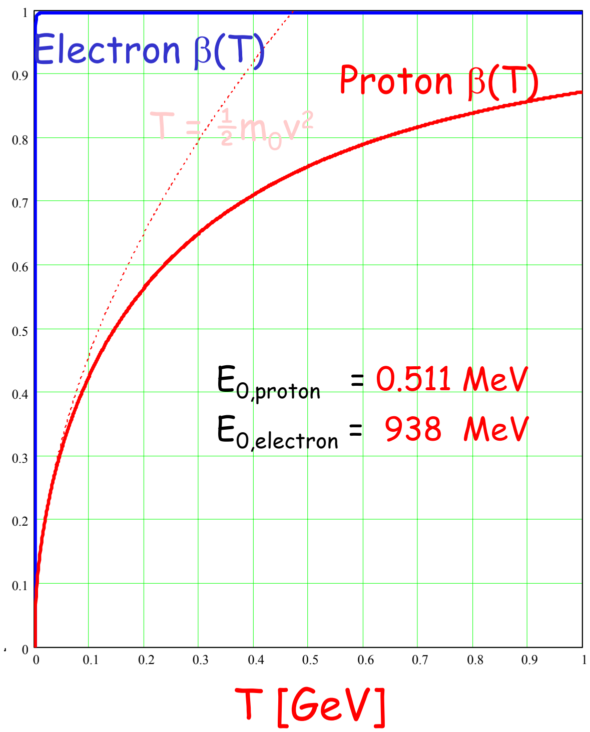
- A proton varies its velocity on a much higher kinetic energy range
- "Synchronous" condition for a multicell cavity:

$$L = \frac{\lambda_{RF}\beta}{2}$$

- The cell length depends on the particle velocity.
- Synchronism is exact only for a given velocity value.

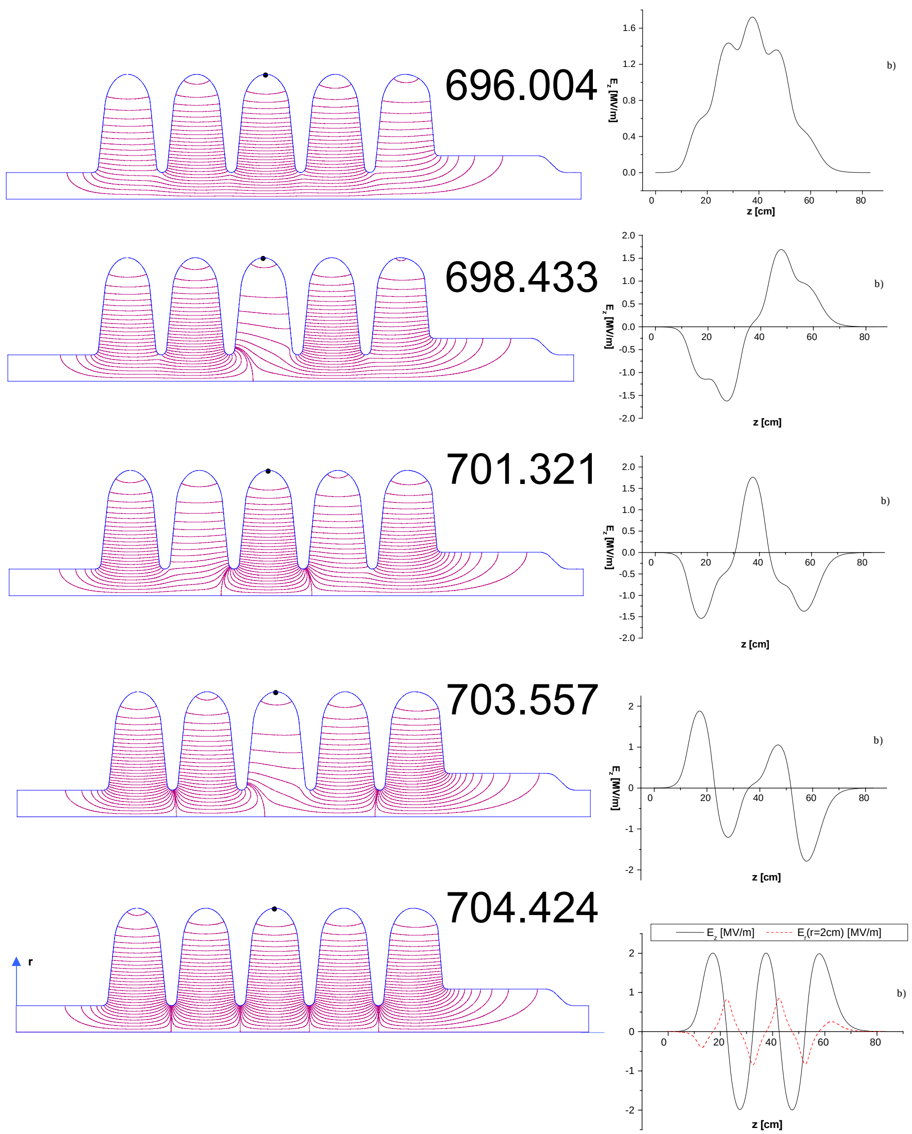
→ Cavities operated in a velocity range.





- For electrons all RF cavities are identical
- For protons, cavity geometries follow the particle velocity, that is the particle β.
- **Below β ≈ 0.5, special structures are required**

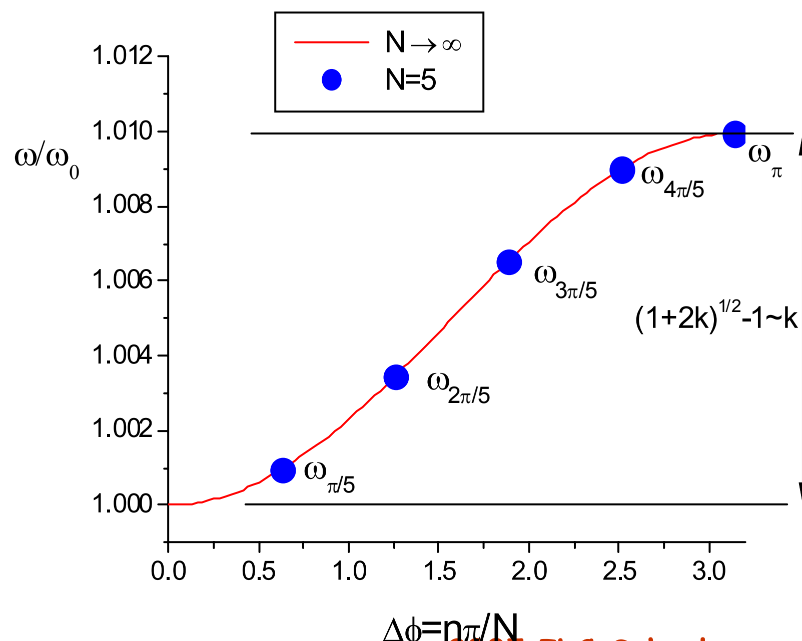
Modes of the multicell cavity



A coupled system of N resonant oscillators at ω_0 can oscillate in N normal modes, "fist band" with eigenfrequencies close to ω_0

$$\omega_n = \omega_0 \sqrt{1 + k\left[1 - \cos\left(\frac{n\pi}{N}\right)\right]}$$

Where k is the coupling coefficient



Energy gain and dissipated power

- To accelerate particles efficiently, very high electric field is required

$$\Delta E = \Delta T = \int \vec{F}_{Lor} \cdot d\vec{s} = q \int \vec{E} \cdot \vec{v}\, dt$$

- In any structure (cavity) holding an electromagnetic field, both dissipated power and stored energy scale quadratically with the fields
- The efficiency of a cavity depends from:
  - Its quality factor, Q

    driven by the surface resistance, R_s

$$Q = \frac{\omega U}{P_{diss}}$$

- U is the energy stored in the cavity
- P_{diss} is the power dissipated on its surface

  - Its shunt impedance, r

    function of the cavity geometry and of the surface resistance, R_s

$$r = \frac{(\Delta V)^2}{P_{diss}}$$

- ΔV is the voltage seen by the beam

$$\frac{r}{Q} = \frac{(\Delta V)^2}{\omega U}$$

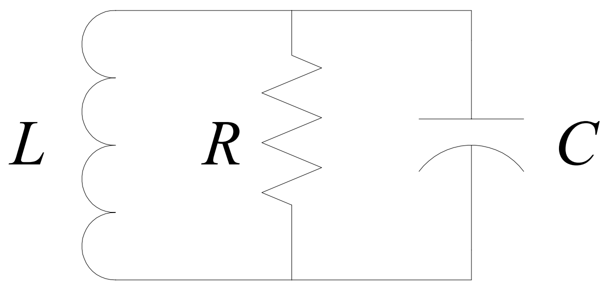
"r over Q" is purely a geometrical factor

- For efficient acceleration Q, r and r/Q must all be as high as possible

→
- Good material for maximum Q and r (that is minimum P_{diss})
- Good design for maximum r/Q

Cavity lumped circuit model and R_S

- A cavity at the fundamental mode has an equivalent resonant lumped circuit



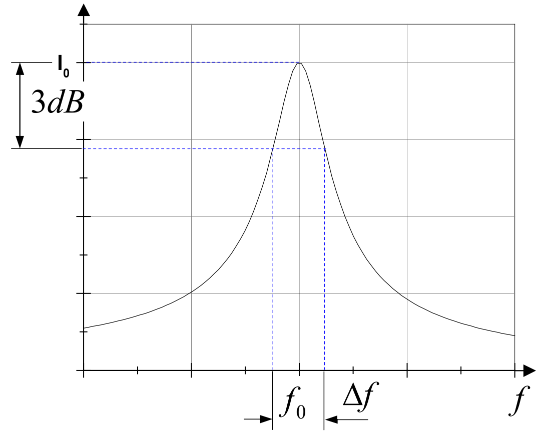
$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad Q = \omega_0 RC$$

$$\omega_0 = 2\pi f_0$$

$$P_{diss} = \frac{V^2}{2R}$$

- Q determines the frequency band Δf

$$\Delta f = \frac{f_0}{Q}$$



- R proportional to Q determines P_{diss}

$$R \propto Q$$

- R depends inversely from the cavity R_s through a geometrical factor

$$R \propto \frac{1}{R_s}$$

- In practice, for a given geometry and a given accelerating field the surface resistance R_s plays the crucial role of determining the dissipated power, that is the power required to sustain the field

$$\boxed{R_s}$$

Superconductivity whenever possible

- For a good but not perfect conductor ($\rho \neq 0$), the fields and currents penetrate into the conductor in a small layer at the cavity surface (the skin depth, δ)

$$R_s = \frac{\rho}{\delta} \neq 0$$

- With RF fields, a SC cavity dissipate power, not all electrons are in Cooper pairs.

$$P_{diss} = \frac{R_s}{2} \int_S H^2 dS$$

Nb

$$R_s[\mathrm{n}\Omega] = 9 \times 10^4 \frac{f^2[\mathrm{GHz}]}{T[\mathrm{K}]} \exp\left(-\frac{17.664}{T[\mathrm{K}]}\right)$$

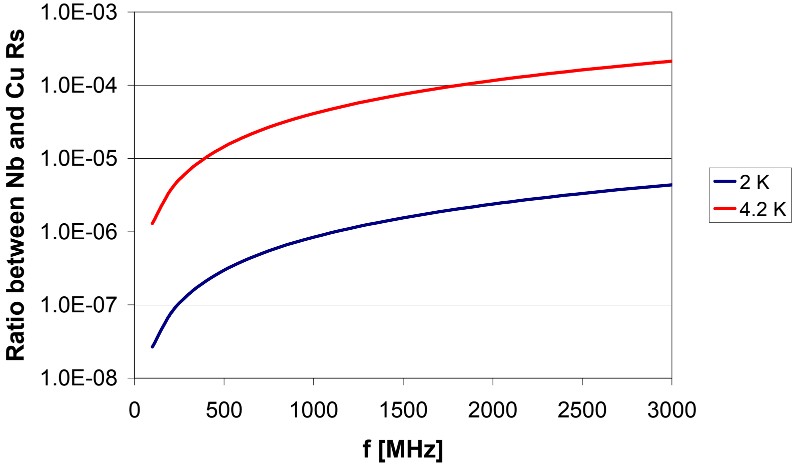
Cu

$$R_s[\mathrm{m}\Omega] = 7.8 f^{\frac{1}{2}}[\mathrm{GHz}]$$

SC SuperConducting

NC or RT NormalConducting

- In NC linac a huge amount of power is deposited in the copper structure: MW to have MV
  - Pulsed operation and Low Duty Cycle



- Superconductivity, drastically reduces the dissipated power. But some drawbacks
  - Higher complexity: refrigeration and cryomodules
  - Higher technology: cavity treatments
  - Carnot and refrigeration plant efficiencies
  - Simpler geometries: lower shunt impedance

$$\eta_C = \frac{T_2}{T_1 - T_2} = \begin{cases} 1/70 \text{ for } T_1 = 300\mathrm{K}, T_2 = 4.2\mathrm{K} \\ 1/150 \text{ for } T_1 = 300\mathrm{K}, T_2 = 2\mathrm{K} \end{cases} \qquad \eta_{th} = \begin{cases} 25-30\% \text{ at } T = 4.2\mathrm{K} \\ 15-20\% \text{ at } T = 2\mathrm{K} \end{cases}$$

And two big advantages:

  - Large bore radius: less beam losses
  - CW or high duty cicle preferred

$$\eta_{tot} = \eta_C \eta_{th} \approx \begin{cases} 250\mathrm{W} \text{ at } 300\mathrm{K} \text{ for } 1\mathrm{W} \text{ at } T = 4.2\mathrm{K} \\ 800\mathrm{W} \text{ at } 300\mathrm{K} \text{ for } 1\mathrm{W} \text{ at } T = 2\mathrm{K} \end{cases}$$