Physics at a Linear Collider

— Basic Knowledge and Techniques

Tao Han
Univ. of Wisconsin - Madison

ILC Workshop/Summer School,
TsingHua University, Beijing, China
(July 15 – 20, 2005)

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I. Basics for e^+e^- Physics: SM expectations
II. Beyond the SM: SUSY and CP Violation
III. Other Operational Modes in a Linear Collider
IV. Techniques and Tools
V. High Energy Physics: Where We Are

I. Basics for e^+e^- Physics: SM expectations

About an e^+e^- Collider

The collisions between e^- and e^+ have major advantages:

- The system of an electron and a positron has zero charge, zero lepton number etc., so that it is suitable to create new particles after e^+e^- annihilation.

- With symmetric beams between the electrons and positrons, the laboratory frame is the same as the c.m. frame, so that the total c.m. energy is fully exploited to reach the highest possible physics threshold.

- With well-understood beam properties, the scattering kinematics is well-constrained.

- Backgrounds low and well-undercontrol.

- It is possible to achieve high degrees of beam polarizations, so that chiral couplings and other asymmetries can be effectively explored.

Disadvantages

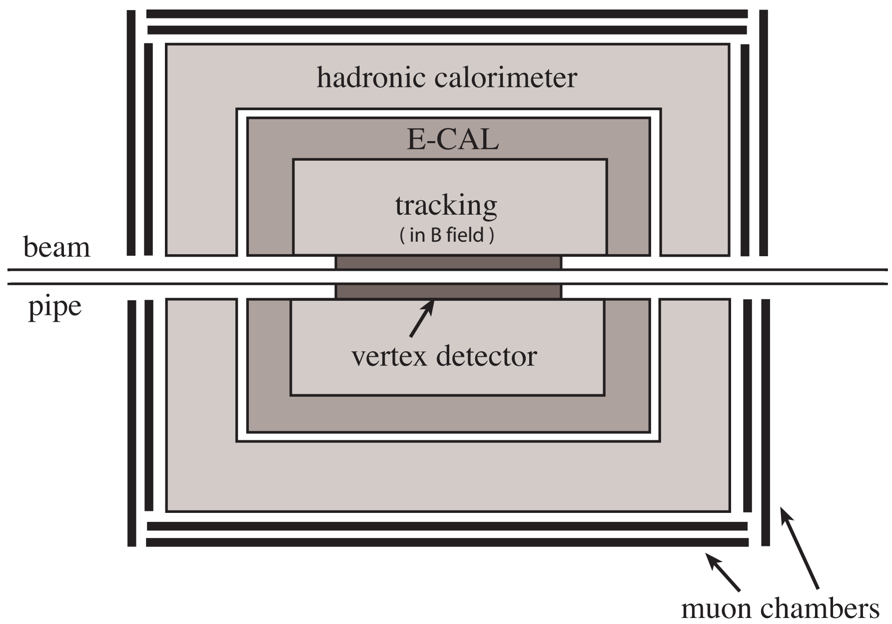
- Large synchrotron radiation due to acceleration,

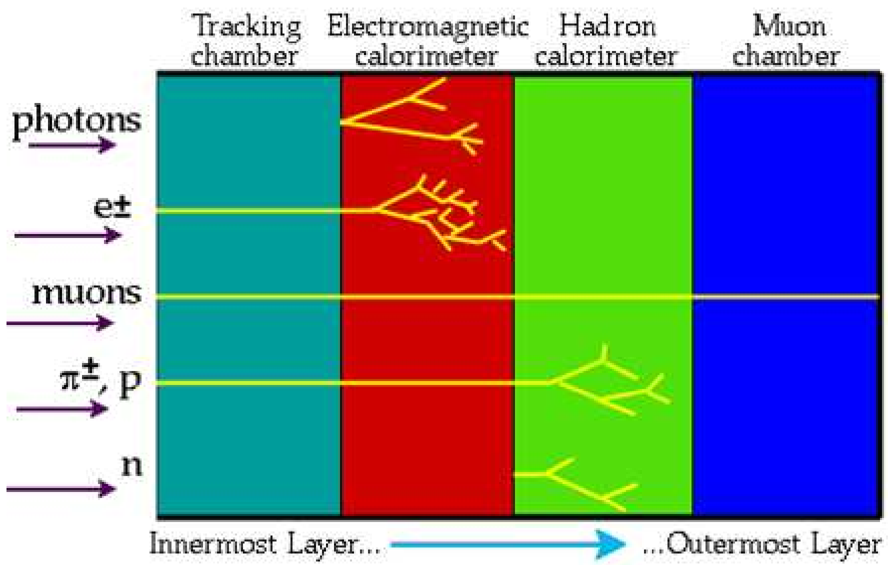
$$\Delta E \sim \frac{1}{R} \left(\frac{E}{m_e}\right)^4 .$$

  Thus, a multi-hundred GeV e^+e^- collider will have to be made a linear accelerator.

- This becomes a major challenge for achieving a high luminosity when a storage ring is not utilized;
  beamsstrahlung severe.

Our magnifying glass: Detector complex





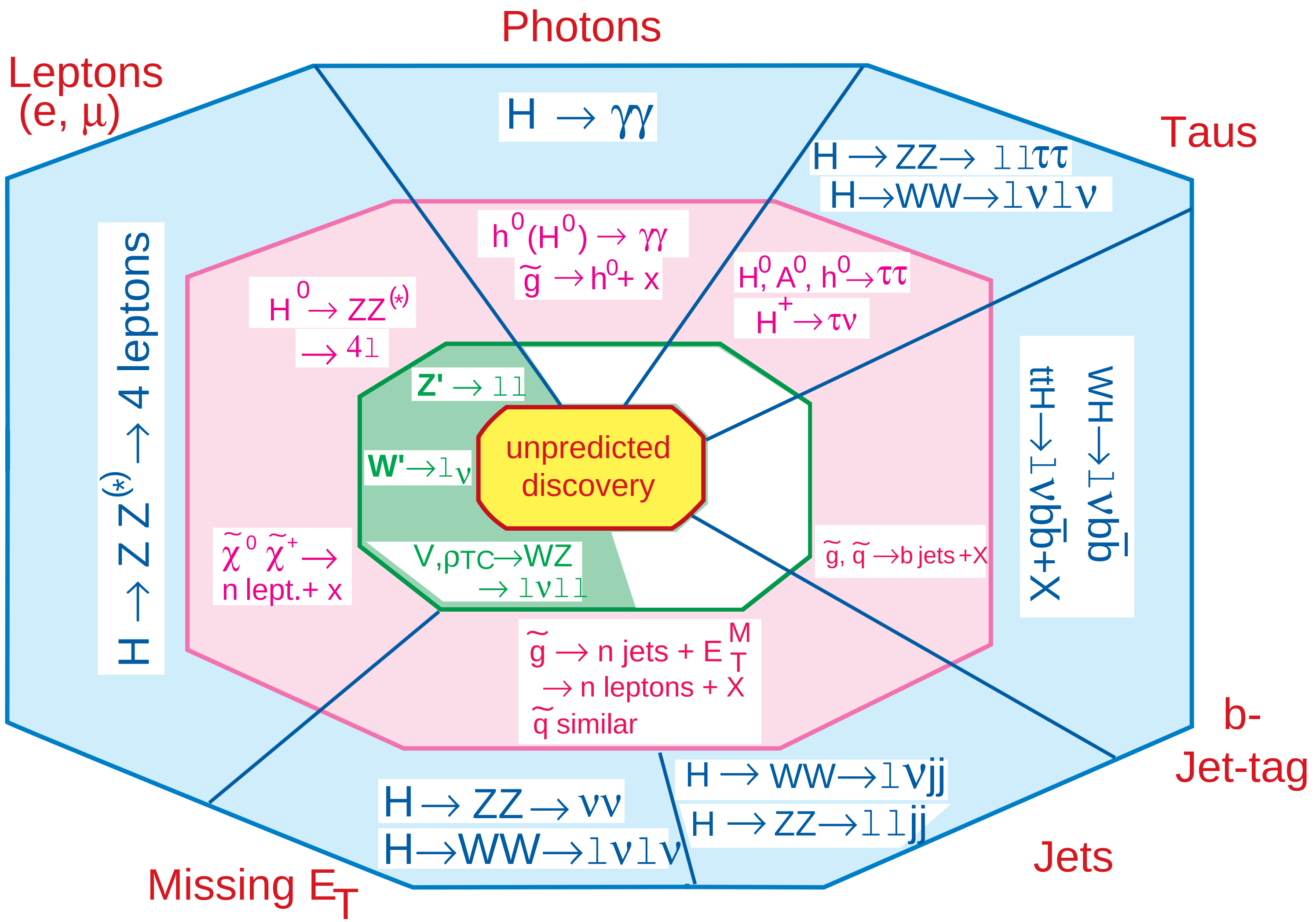
Something a theorist should know:

— What do the SM particles look like in a detector?

Although gauge interactions present remarkable universality among the SM fermions, we theorists should always remember the significant differences how the SM particles actually look like in a detector...

Leptons	Vetex	Tracking	ECAL	HCAL	Muon Cham.
$e^\pm$	$\times$	$\vec{p}$	E	$\times$	$\times$
$\mu^\pm$	$\times$	$\vec{p}$	$\times$	$\times$	$\vec{p}$
$\tau^\pm$	$\surd\times$	$\surd$	$e^\pm$	$h^\pm$; $3h^\pm$	$\mu^\pm$
ν_e, ν_μ, ν_τ	$\times$	$\times$	$\times$	$\times$	$\times$
Quarks					
u, d, s	$\times$	$\surd$	$\times$	E	$\times$
$c \to D$	$\surd$	$\surd$	$e^\pm$	h's	$\mu^\pm$
$b \to B$	$\surd$	$\surd$	$e^\pm$	h's	$\mu^\pm$
$t \to bW^\pm$	b	$\surd$	$e^\pm$	$b +$ 2 jets	$\mu^\pm$
Gauge bosons					
γ	$\times$	$\times$	E	$\times$	$\times$
g	$\times$	$\surd$	$\times$	E	$\times$
$W^\pm \to \ell^\pm \nu$	$\times$	$\vec{p}$	$e^\pm$	$\times$	$\mu^\pm$
$\to q\bar{q}'$	$\times$	$\surd$	$\times$	2 jets	$\times$
$Z^0 \to \ell^+\ell^-$	$\times$	$\vec{p}$	$e^\pm$	$\times$	$\mu^\pm$
$\to q\bar{q}$	$(b\bar{b})$	$\surd$	$\times$	2 jets	$\times$

— How to search for new particles?



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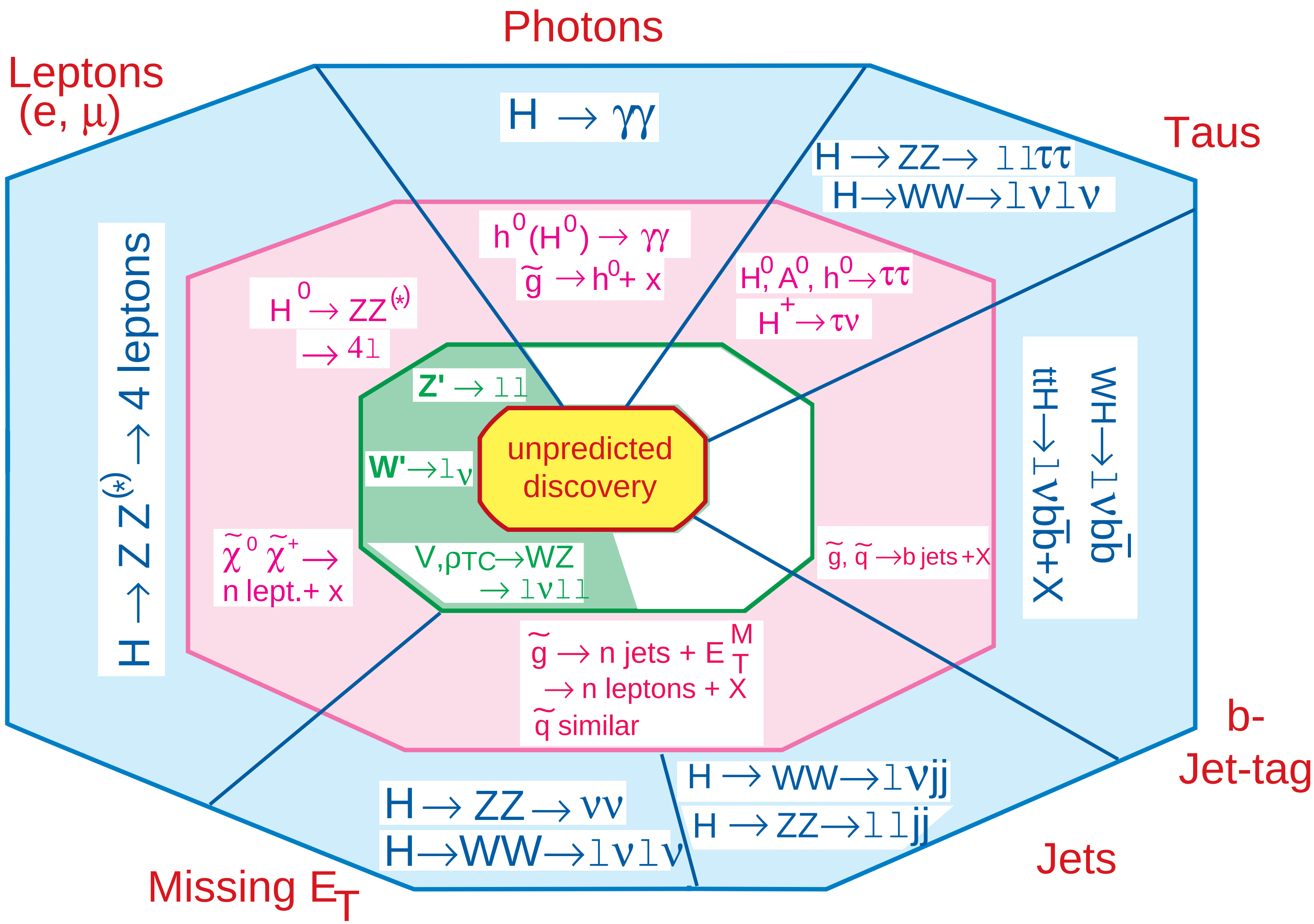

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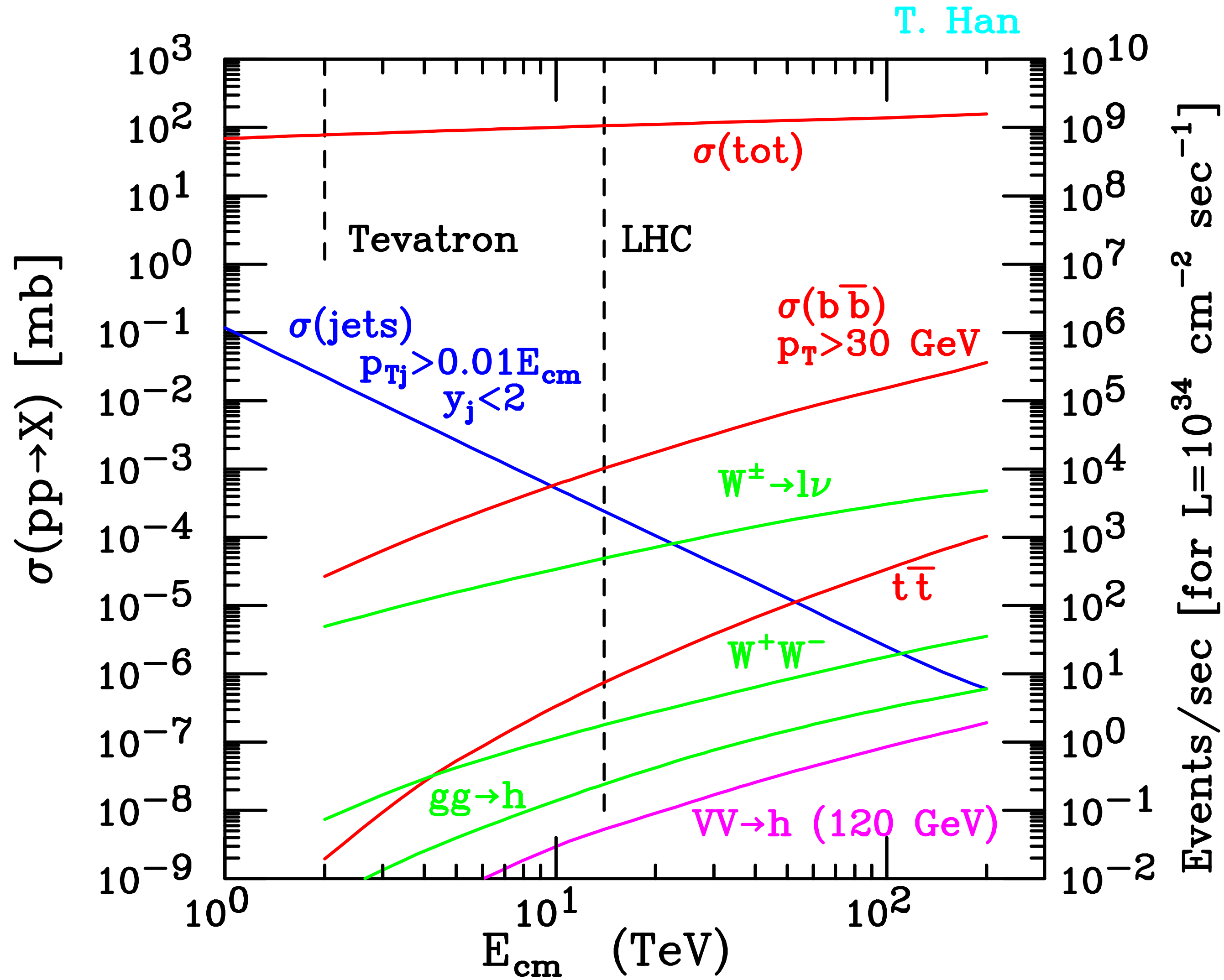
Homework I-1: For Z' particle that couples to SM fermions and W^+W^-.

List the experimental final states.

We know that at the LHC ...

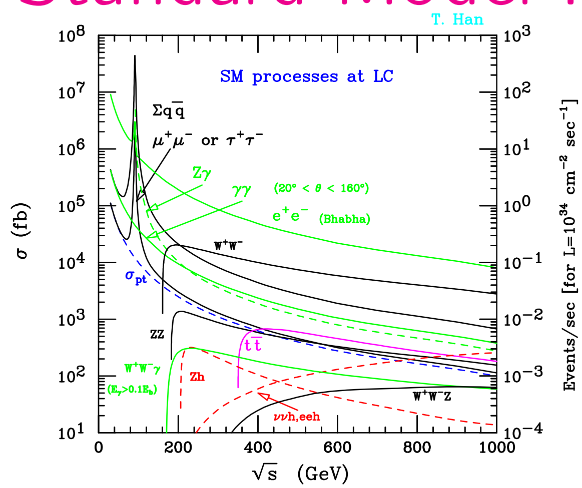
Huge reach for physics goals:



Event rate very high,

but background suppression $10^{-6} - 10^{-10}$!

Baisc Standard Model Processes at an e^+e^- Collider



— more details to come ...

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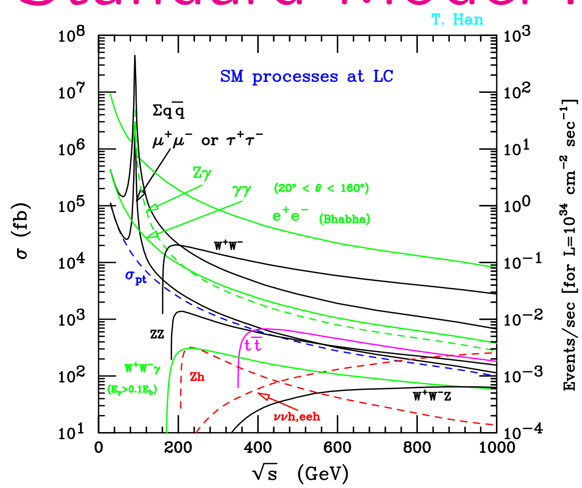

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Dynamics and Kinematics go hand-by-hand

The Mandelstam variables are defined for $a + b \to 1 + 2$:

$$\begin{aligned} s &= (p_a + p_b)^2 = (p_1 + p_2)^2 = E_{cm}^2, \\ t &= (p_a - p_1)^2 = (p_b - p_2)^2 = m_a^2 + m_1^2 - 2(E_a E_1 - p_a p_1 \cos\theta_{a1}), \\ u &= (p_a - p_2)^2 = (p_b - p_1)^2 = m_a^2 + m_2^2 - 2(E_a E_2 - p_a p_2 \cos\theta_{a2}). \end{aligned}$$

Assume that $m_a = m_1$ and $m_b = m_2$, (Homework I-2: work out the kinematics.)

$$\begin{aligned} t &= -2p_{cm}^2(1 - \cos\theta_{a1}^*), \\ u &= -2p_{cm}^2(1 + \cos\theta_{a1}^*) + \frac{(m_1^2 - m_2^2)^2}{s}, \end{aligned}$$

where $p_{cm} = \lambda^{1/2}(s, m_1^2, m_2^2)/2\sqrt{s}$ is the momentum magnitude in the c.m. frame.

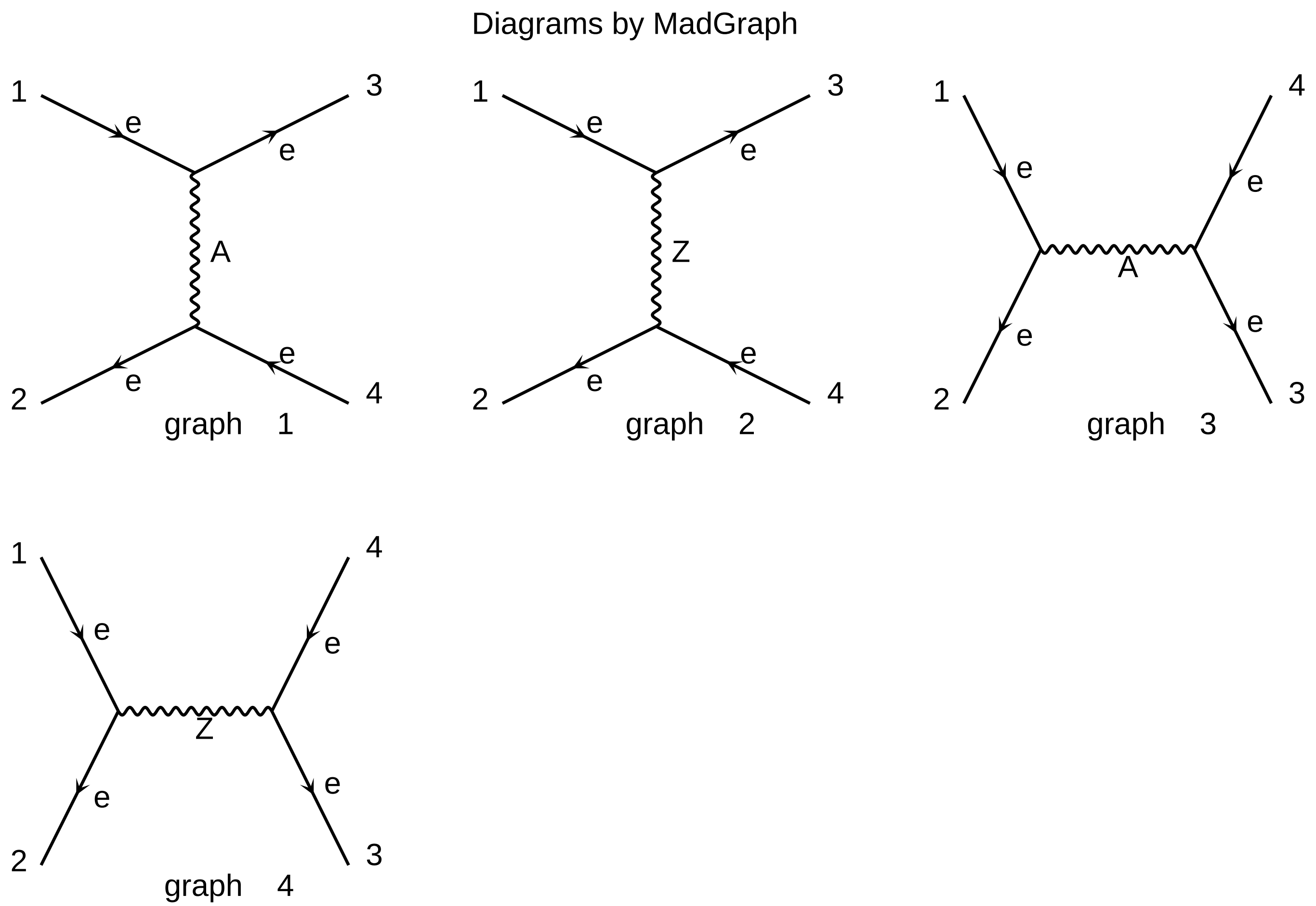
s- and t-channel singularities characteristic.

• $e^+e^- \to e^+e^-$

The Bhabha scattering: The leading order QED process;

It presents s, t-channel "singularities" (see last page).

`(Think 1: which diagrams? are they physical? observable?)`



- $e^+e^- \to f\bar{f}$ $(\mu\mu,\ \tau\tau,\ b\bar{b},\ t\bar{t}, \ldots)$

The simplest reaction would be the QED process $e^+e^- \to \gamma^* \to \mu^+\mu^-$:

$$\sigma(e^+e^- \to \gamma^* \to \mu^+\mu^-) \equiv \sigma_{pt} = \frac{4\pi\alpha^2}{3s} \approx \frac{100\ \text{fb}}{(\text{E}_{\text{cm}}/1\ \text{TeV})^2}.$$

• $e^+e^- \to f\bar{f}\ (\mu\mu,\ \tau\tau,\ b\bar{b},\ t\bar{t}, ...)$

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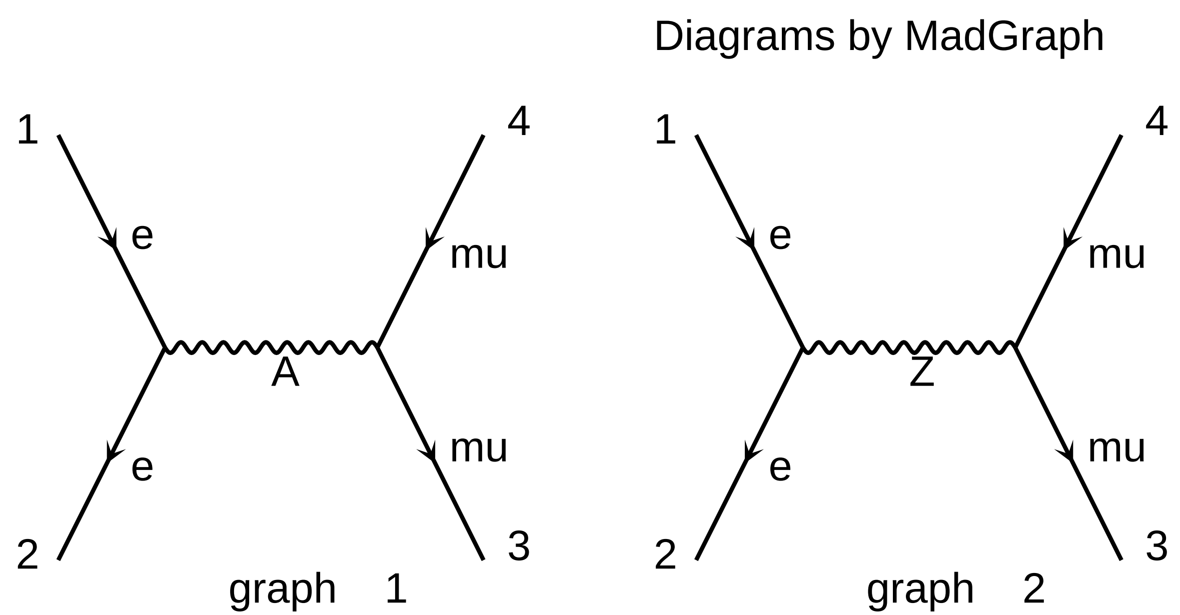
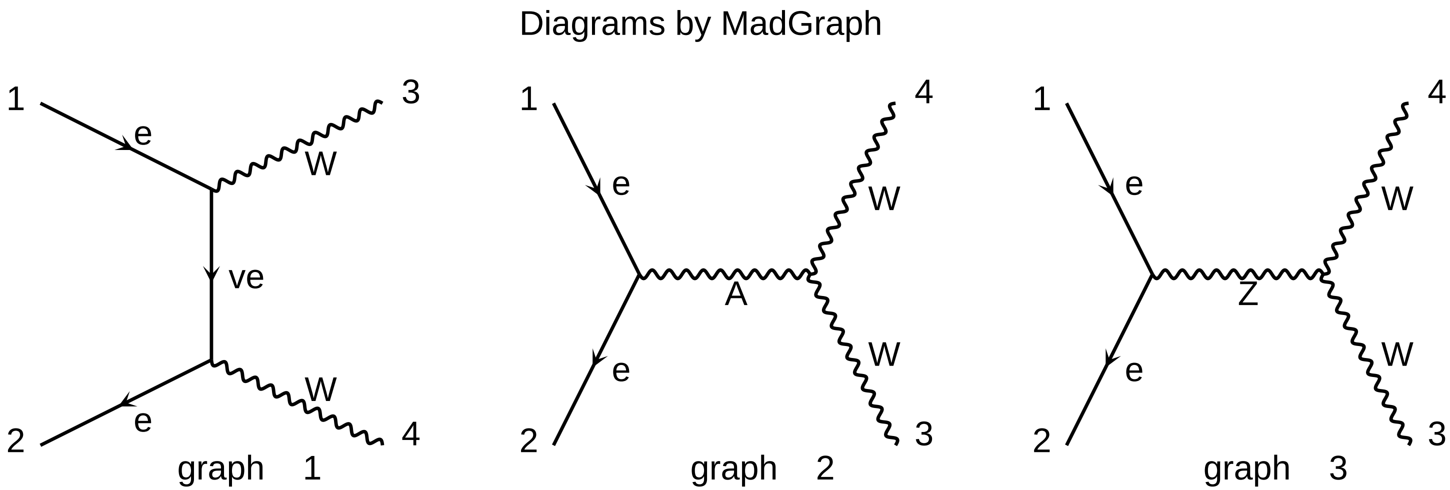
σ_{pt} has become standard units.

Z contribution becomes important at $\sqrt{s} \sim M_Z$.



Very important to produce new particle states ($J = 1$ channel).

- $e^+e^- \to W^+W^-,\ ZZ,\ \gamma\gamma$



Important to test non-Abelian gauge field self interactions.

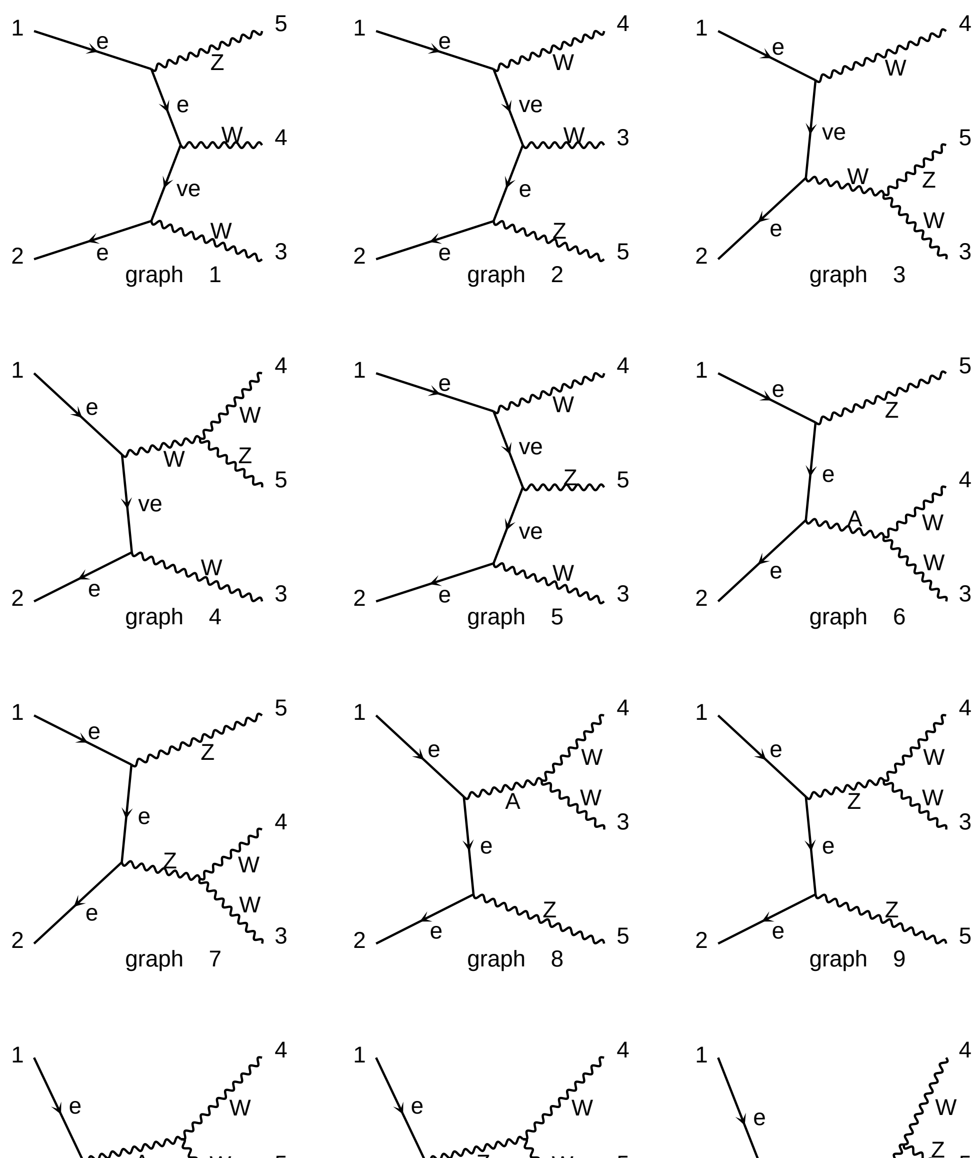
Important to produce new states (via $J = 1$ channel).

- $e^+e^- \to W^+W^-Z,\ W^+W^-\gamma ...$

Important to check 4-point couplings;

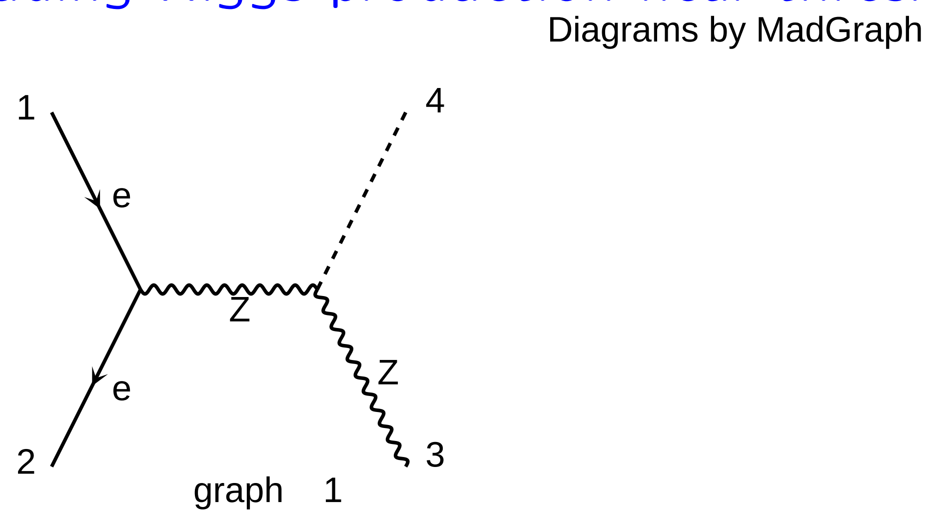
First time for $H \to WW, ZZ$ to contribute.

Diagrams by MadGraph



- $e^+e^- \to Zh$

Leading Higgs production near threshold.



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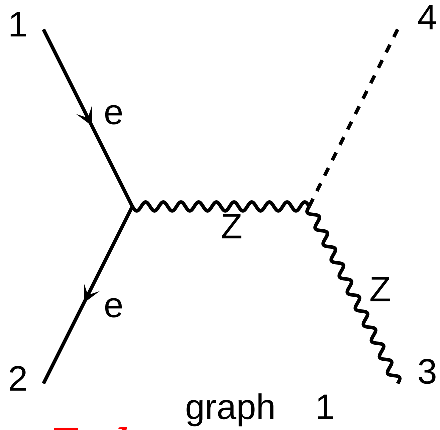
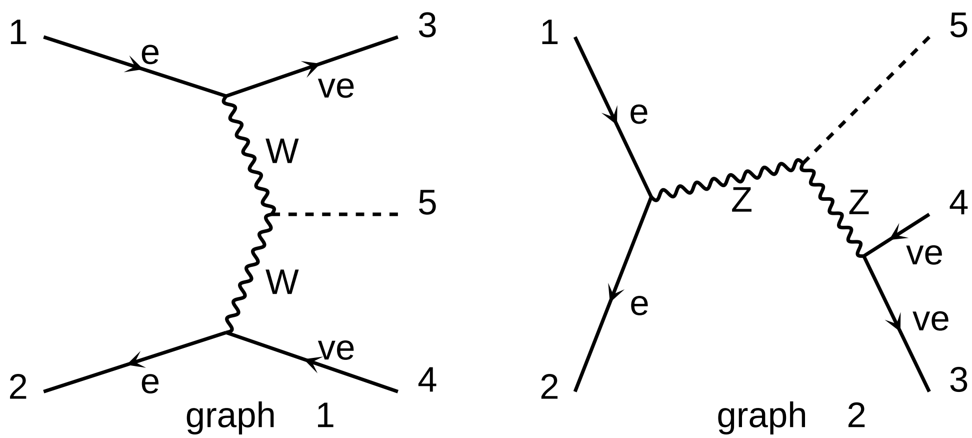


- $e^+e^- \to W^*W^*h \to \bar{\nu}\nu h$

Leading Higgs production for heavier H.

Diagrams by MadGraph



Simple Formalism

For the production of two-particle a, b, the differential cross section is given by

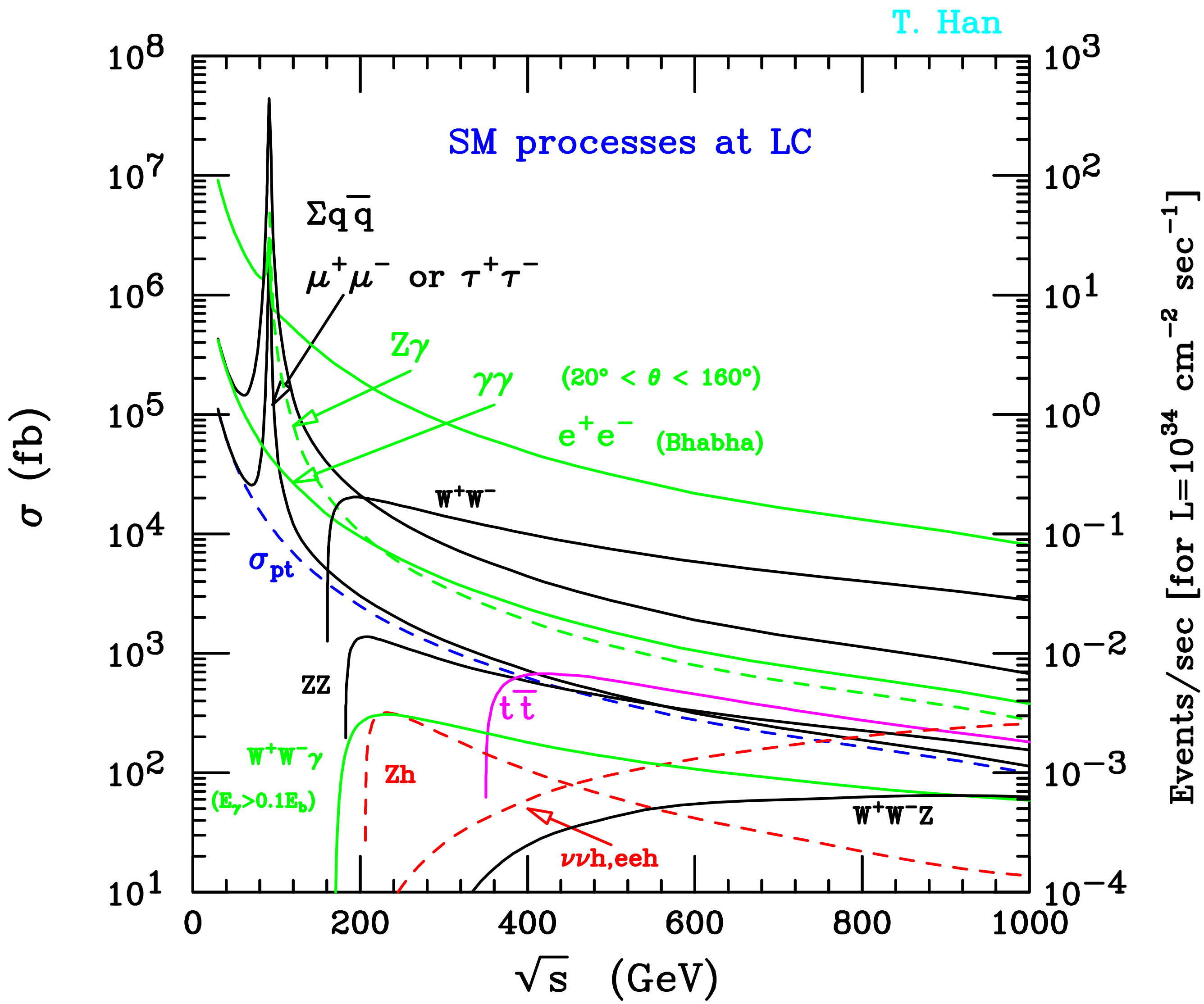
$$\frac{d\sigma(e^+e^- \to ab)}{d\cos\theta} = \frac{\beta}{32\pi s}\overline{\sum}|\mathcal{M}|^2$$

where

• $\beta = \lambda^{1/2}(1, m_a^2/s, m_b^2/s)$, is the speed factor for the out-going particles in the c.m. frame, and $p_{cm} = \beta\sqrt{s}/2$,

• $\overline{\sum|\mathcal{M}|^2}$ the squared matrix element, summed and averaged over quantum numbers
(like color and spins etc.)

• unpolarized beams so that the azimuthal angle trivially integrated out,



- The Z resonance prominent: Breit-Wigner resonance

$$\frac{1}{(s-M_V^2)^2+\Gamma_V^2 M_V^2}$$

Resonant production:

For an arbitrary resonance V,

If the energy spread $\delta\sqrt{s} \ll \Gamma_V$, the line-shape mapped out:

$$\sigma(e^+e^- \to V \to X) = \frac{4\pi(2j+1)\Gamma(V \to e^+e^-)\Gamma(V \to X)}{(s - M_V^2)^2 + \Gamma_V^2 M_V^2}\ \frac{s}{M_V^2},$$

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If $\delta\sqrt{s} \gg \Gamma_V$, the narrow-width approximation:

$$\frac{1}{(s-M_V^2)^2 + \Gamma_V^2 M_V^2} \quad \to \quad \frac{\pi}{M_V \Gamma_V}\ \delta(s = M_V^2),$$

$$\sigma(e^+e^- \to V \to X) \quad = \quad \frac{4\pi^2(2j+1)\Gamma(V \to e^+e^-)BF(V \to X)}{M_V^3} \frac{dL(\hat{s} = M_V^2)}{d\tau}$$

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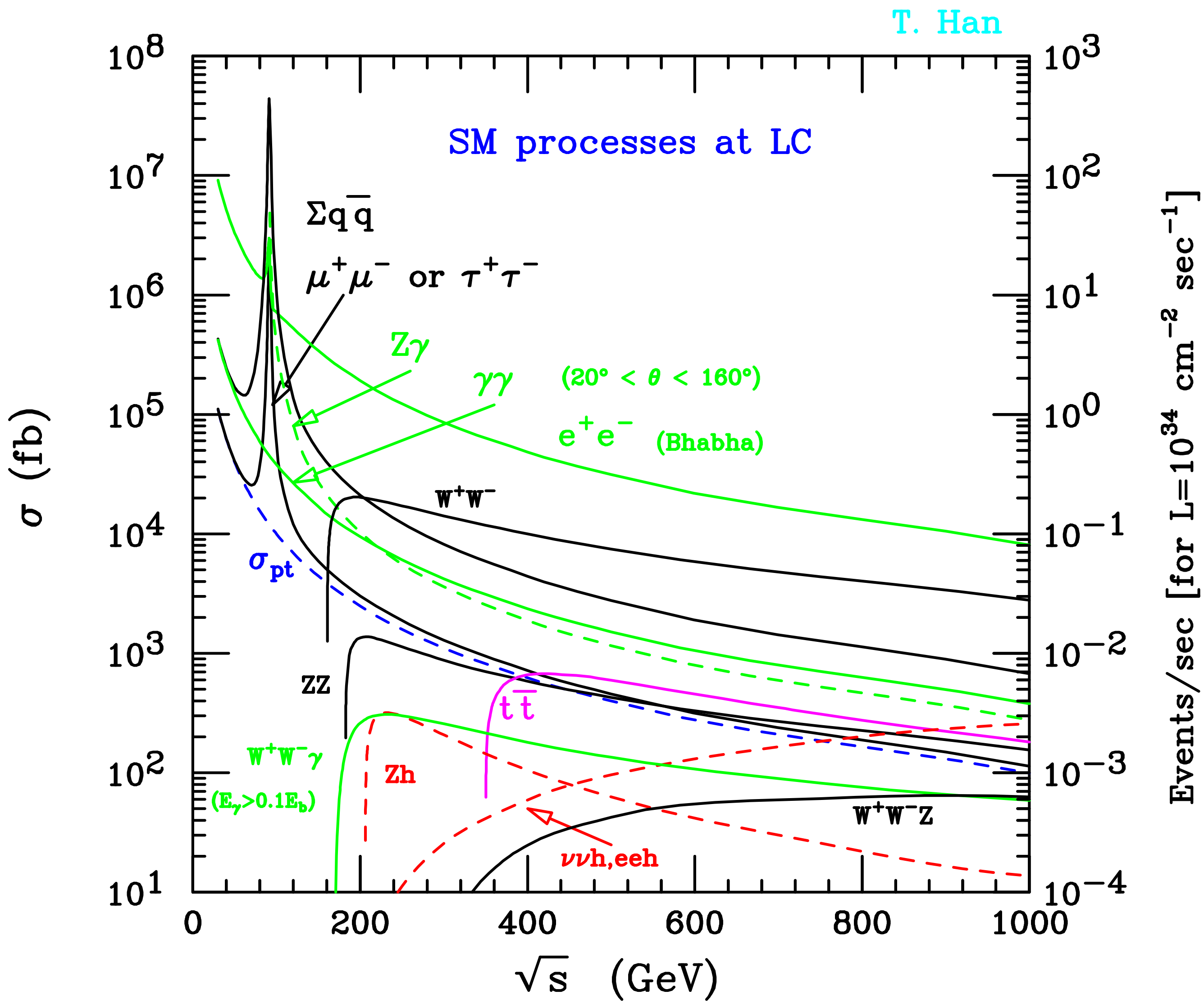
`Homework I-3: sketch the derivation of these two formulas.`

For a more general relation between $\delta\sqrt{s} - \Gamma_V$,
need more careful convolution $dL/d\tau$.



Away from the resonance and for finite-angle scattering:

$$\sigma \sim \frac{1}{s} \quad \text{or} \quad \sigma \sim \frac{1}{M_V^2} \ln^2 \frac{s}{M_V^2}.$$

Fermion production:

Common processes: $e^-e^+ \to f\bar{f}$.
For most of the situations, the scattering matrix element can be casted into a $V \pm A$ chiral structure of the form (sometimes with the help of Fierz transformations)

$$\mathcal{M} = \frac{e^2}{s} Q_{\alpha\beta}\ [\bar{v}_{e^+}(p_2)\gamma^\mu P_\alpha u_{e^-}(p_1)]\ [\bar{\psi}_f(q_1)\gamma_\mu P_\beta \psi'_{\bar{f}}(q_2)],$$

where $P_\mp = (1 \mp \gamma_5)/2$ are the L, R chirality projection operators, and $Q_{\alpha\beta}$ are the bilinear couplings governed by the underlying physics of the interactions with the intermediate propagating fields.
With this structure, the scattering matrix element squared:

$$\begin{aligned}\overline{\sum}|\mathcal{M}|^2 &= \frac{e^4}{s^2}\left[(|Q_{LL}|^2 + |Q_{RR}|^2)\ u_i u_j + (|Q_{LL}|^2 + |Q_{RL}|^2)\ t_i t_j\right.\\ &+ \left. 2Re(Q^*_{LL}Q_{LR} + Q^*_{RR}Q_{RL}) m_f m_{\bar{f}} s\right],\end{aligned}$$

where $t_i = t - m_i^2 = (p_1 - q_1)^2 - m_i^2$ and $u_i = u - m_i^2 = (p_1 - q_2)^2 - m_i^2$.

`Homework I-4: Verify this formula.`

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- The Z resonance prominent (or other M_V),

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- At the ILC $\sqrt{s} = 500$ GeV,

$$\sigma(e^+e^- \to e^+e^-) \sim 100\sigma_{pt} \sim 40 \text{ pb.}$$

(anglular cut dependent.)

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$$\sigma_{pt} \sim \sigma(ZZ) \sim \sigma(t\bar{t}) \sim 400 \text{ fb;}$$
$$\sigma(u,d,s) \sim 9\sigma_{pt} \sim 3.6 \text{ pb;}$$
$$\sigma(WW) \sim 20\sigma_{pt} \sim 8 \text{ pb.}$$

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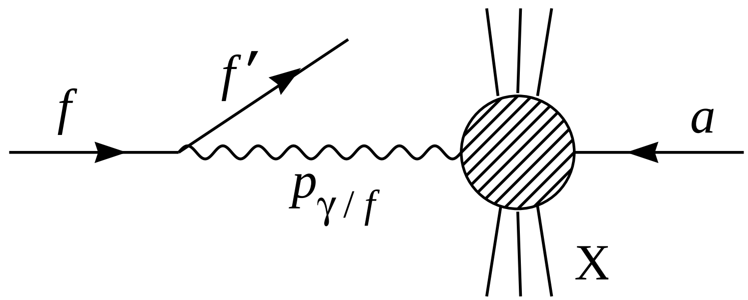
and

$$\sigma(ZH) \sim \sigma(WW \to H) \sim \sigma_{pt}/4 \sim 100 \text{ fb;}$$
$$\sigma(WWZ) \sim 0.1\sigma_{pt} \sim 40 \text{ fb.}$$

Gauge boson radiation:

A qualitatively different process is initiated from gauge boson radiation, typically off fermions:



The simplest case is the photon radiation off an electron, like:

$$e^+e^- \to e^+,\ \gamma^* e^- \to e^+ e^-.$$

The dominant features are due to the result of a t-channel singularity, induced by the collinear photon splitting:

$$\sigma(e^- a \to e^- X) \approx \int dx\ P_{\gamma/e}(x)\ \sigma(\gamma a \to X).$$

The so called the effective photon approximation.

For an electron of energy E, the probability of finding a collinear photon of energy xE is given by

$$P_{\gamma/e}(x) = \frac{\alpha}{2\pi}\frac{1+(1-x)^2}{x}\ln\frac{E^2}{m_e^2},$$

known as the Weizsäcker-Williams spectrum.

`(Think 2:  How to derive this splitting function?)`

We see that:

- m_e enters the log to regularize the collinear singularity;
- $1/x$ leads to the infrared behavior of the photon;
- This picture of the photon probability distribution is also valid for other photon spectrum:

Based on the back-scattering laser technique, it has been proposed to produce much harder photon spectrum, to construct a "photon collider"...

(massive) Gauge boson radiation:

A similar picture may be envisioned for the electroweak massive gauge bosons, $V = W^{\pm}, Z$.

Consider a fermion f of energy E, the probability of finding a (nearly) collinear gauge boson V of energy xE and transverse momentum p_T (with respect to $\vec{p}_f$) is approximated by

$$P^T_{V/f}(x, p_T^2) = \frac{g_V^2 + g_A^2}{8\pi^2} \frac{1 + (1-x)^2}{x} \frac{p_T^2}{(p_T^2 + (1-x)M_V^2)^2},$$

$$P^L_{V/f}(x, p_T^2) = \frac{g_V^2 + g_A^2}{4\pi^2} \frac{1-x}{x} \frac{(1-x)M_V^2}{(p_T^2 + (1-x)M_V^2)^2}.$$

Although the collinear scattering would not be a good approximation until reaching very high energies $\sqrt{s} \gg M_V$, it is instructive to consider the qualitative features.

Think 3: Compare the qualitative features of the p_T spectra for P^T, P^L distributions.

Beam polarization:

One of the merits for an e^+e^- linear collider is the possible high polarization for both beams.
Consider first the longitudinal polarization along the beam line direction. Denote the average $e^\pm$ beam polarization by $P^L_\pm$, with $P^L_\pm = -1$ purely left-handed and $+1$ purely right-handed.

The polarized squared matrix element, based on the helicity amplitudes $\mathcal{M}_{\sigma_{e-}\sigma_{e+}}$:

$$\overline{\sum}|\mathcal{M}|^2 = \frac{1}{4}[(1-P^L_-)(1-P^L_+)|\mathcal{M}_{--}|^2 + (1-P^L_-)(1+P^L_+)|\mathcal{M}_{-+}|^2 + (1+P^L_-(1-P^L_+)|\mathcal{M}_{+-}|^2 + (1+P^L_-)(1+P^L_+)|\mathcal{M}_{++}|^2].$$

Since the electroweak interactions of the SM and beyond are chiral:
Certain helicity amplitudes can be suppressed or enhanced by properly choosing the beam polarizations: *e.g.*, $W^\pm$ exchange ...

Furthermore, it is possible to produce transversely polarized beams with the help of a spin-rotator.

If the beams present average polarizations with respect to a specific direction perpendicular to the beam line direction, $-1 < P^T_\pm < 1$, then there will be one additional term in the limit $m_e \to 0$,

$$\frac{1}{4}\, 2\ P^T_- P^T_+\ \mathrm{Re}(\mathcal{M}_{-+}\mathcal{M}^*_{+-}).$$

The transverse polarization is particularly important when the interactions produce an asymmetry in azimuthal angle, such as the effect of CP violation.