Gauge-Invariant Gravitational Corrections from Vilkovisky-DeWitt Effective Action and Asymptotic Freedom of All Gauge Theories

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Motivations for our Study

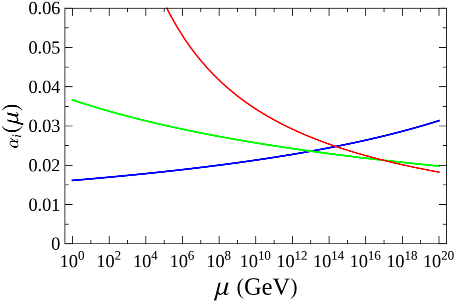
Background 1:

- Four fundamental interactions in nature.
- Strong, Electromagnetic, Weak $\Rightarrow$ Standard Model.
- Gravitation as an exception: No complete quantum gravity.
- Gravitation as an **effective field theory**.
- Seeking important gravitational corrections in standard model.

Motivations for our Study

Background 2:

- 2004 Nobel Prize: Asymptotic freedom of QCD.

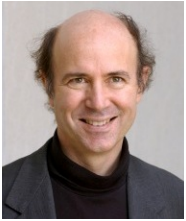


Question: **What will happen to the the three fundamental interactions when gravitational corrections enter?**

Motivations for our Study

Background 2 (Continued):

- Robinson & Wilczek: Phys. Rev. Lett. **96**, 231601(2006), MIT Ph.D Thesis
  - "We calculate the contribution of graviton exchange to the running of gauge couplings at lowest nontrivial order in perturbation theory."



- But it was then found by a few authors to be gauge-dependent and the net result vanishes by using the conventional BFM.

Out of the Shadow of Gauge-Dependence

- The problem of gauge-dependence in traditional Effective Action was pointed out by Jackiw in his study of Coleman-Weinberg model in 1974.
- Later in 1980s, Vilkovisky and DeWitt systematically developed the so-called **geometrical effective action** (Vilkovisky-DeWitt method), in order to get out of the shadow of gauge-dependence.

Out of the Shadow of Gauge-Dependence

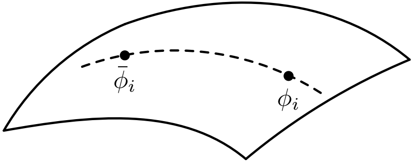
Why is traditional effective action **gauge dependent**?

Key Point: **Gauge transformation** $\Rightarrow$ **Field reparametrization**

- The definition of the traditional effective action $\Gamma[\bar{\phi}]$:

$$e^{i\Gamma[\bar{\phi}]} = \int \mathcal{D}\phi \, e^{iS[\phi]+i\int \mathrm{d}^d x \, \frac{\delta\Gamma[\phi]}{\delta\bar{\phi}^i(x)}[\phi^i(x)-\bar{\phi}^i(x)]}$$

- The definition of $\Gamma[\bar{\phi}]$ introduces the difference of two point $(\phi^i - \bar{\phi}^i)$ in field space.
- This difference of two points in field space is generally NOT a well-defined vector in field-space!

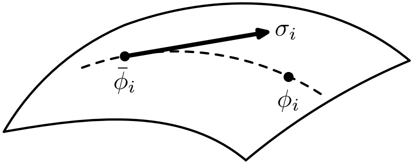


Geometrical Effective Action

- A natural way to solve the problem is to define a **metric** G_{ij} in field space
- Then a set of **connection coefficients** Γ^k_{ij} can be induced from G_{ij} by condition:

$$\nabla_i G^{ij} = 0.$$

- Then, replace the coordinate difference $\phi^i - \bar{\phi}^i$ by a vector $\sigma^i[\phi, \bar{\phi}]$, defined as a directional derivative attach to the geodesic linking ϕ^i and $\bar{\phi}^i$



Geometrical Effective Action

$$e^{i\Gamma_{\mathsf{G}}[\bar{\phi}]} = \int \mathcal{D}\phi\, e^{iS[\phi]+i\int \mathrm{d}^d x\, \frac{\delta\Gamma_{\mathsf{G}}[\phi]}{\delta\bar{\phi}^j(x)}(C^{-1})^j{}_i\sigma^i[\bar{\phi},\phi]}$$

- It can be shown that the geometrical effective action Γ_{G} defined in this way is indeed **reparametrization invariant**, and thus is **gauge independent**.
- To evaluate this effective action, we also need to find a perturbative expansion for it. The result up to 1-loop level is:

$$\Gamma_{\mathsf{G}}[\bar{\phi}] = S[\bar{\phi}] + S_{\mathsf{ghost}}[\bar{\phi}] + \frac{i}{2}\,\mathrm{Tr}\,\log\Big(\tilde{S}_{,i,j}[\bar{\phi}] - \Gamma^k_{ij}S_{,k}[\bar{\phi}]\Big),$$

with

$$\tilde{S}[\phi] = S[\phi] + S_{\mathsf{Gauge\text{-}Fixing}}[\phi].$$

Gravitational Corrections to QED β Functions

- Choosing a metric:

$$G_{g_{\mu\nu}(x)g_{\rho\sigma}(y)} = \frac{1}{\kappa^2}\sqrt{-g(x)}\big[g^{\mu(\rho}(x)g^{\sigma)\nu}(x) - \tfrac{1}{2}g^{\mu\nu}(x)g^{\rho\sigma}(x)\big]\delta(x-y),$$
$$G_{A_\mu(x)A_\nu(y)} = \sqrt{-g(x)}g^{\mu\nu}(x)\delta(x-y).$$

- Evaluating the connection coefficients Γ^k_{ij} from G_{ij};
- Choosing a gauge (Landau-DeWitt gauge):

$$\chi = \partial^\mu a_\mu, \qquad \chi_\mu = \partial^\lambda h_{\mu\lambda} - \tfrac{1}{2}\partial_\mu h + \tfrac{\kappa}{2}a^\lambda \bar{F}_{\mu\lambda}.$$

- Expanding all fields around their background values, up to quadratic order on quantum fluctuations.

Gravitational Corrections to QED β Functions

- The β function can be deduced from perturbative evaluation of the photon self-energy. Generally, it can be parameterized as

$$\beta(g,\mu) = -\frac{b_0}{(4\pi)^2}g^3 + \frac{a_0}{(4\pi)^2}(\kappa^2\mu^2)g.$$

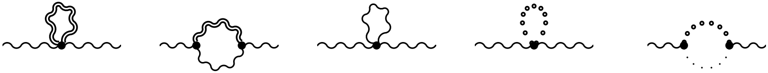
- To perform the calculation in Landau-DeWitt gauge, we need the gauge fixing term in Lagrangian:

$$\mathcal{L}_{\text{gauge-fixing}} = \tfrac{1}{2\zeta}\chi_\mu\chi^\mu - \tfrac{1}{2\xi}\chi^2,$$

  in which the gauge parameters ζ and ξ should be sent to zero.

- We focus on the gravitational corrections to the coefficient a_0. Technically, this amounts to calculating loop diagrams. All the relevant photon self-energy 1-loop diagrams are shown in the next slide.

Gravitational Corrections to QED β Functions



$$
\begin{aligned}
&\text{(a)} = (1+2\zeta)\kappa^2(p^2\eta_{\mu\nu} - p_\mu p_\nu)\mathcal{I}_2\,,\\
&\text{(b)} = \big(-\tfrac{1+\xi}{4\zeta} - 2 - \zeta\big)\kappa^2(p^2\eta_{\mu\nu} - p_\mu p_\nu)\mathcal{I}_2\,,\\
&\text{(c)} = \big(\tfrac{1+\xi}{4\zeta} + \tfrac{1-\xi}{8}\big)\kappa^2(p^2\eta_{\mu\nu} - p_\mu p_\nu)\mathcal{I}_2\,,\\
&\text{(d)} = -\kappa^2(p^2\eta_{\mu\nu} - p_\mu p_\nu)\mathcal{I}_2\,,\\
&\text{(e)} = 0\,,
\end{aligned}
$$

Summing up all this, and sending $\zeta, \xi \to 0$, we get:

$$\text{(a)} + \text{(b)} + \text{(c)} + \text{(d)} + \text{(e)} = -\frac{15}{8}\kappa^2(p^2\eta_{\mu\nu} - p_\mu p_\nu)\mathcal{I}_2.$$

Note that all ζ-pole cancels exactly, which is a consistency check for our calculation.

Gravitational Corrections to QED β Functions

- We define the integral:

$$\mathcal{I}_2 \equiv \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{1}{k^2},$$

  which gets regularized at $d < 2$ via DRED with a pole at $d = 2$.

- After summing up all diagrams and extracting the coefficients of this integral, we are free to re-regularize it at $d = 4$ by placing a common physical momentum cutoff Λ,

$$\mathcal{I}_2 = \int^{\Lambda} \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{1}{k^2} = -\frac{\mathrm{i}}{16\pi^2}\Lambda^2.$$

- Then we can deduce the QED gauge-coupling renormalization

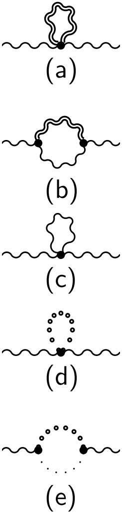
$$\frac{1}{g^2(\mu)} = \frac{1}{g^2(\Lambda)} - \frac{15\kappa^2(\Lambda^2 - \mu^2)}{128\pi^2 g^2},$$

- Finally, we deduce the gauge-invariant gravitational power-law corrections to the beta function,

$$\boxed{\beta(g) = -\frac{15\kappa^2 g\mu^2}{128\pi^2},}$$

which predicts $a_0 = -15/8$ and is Asymptotically Free.

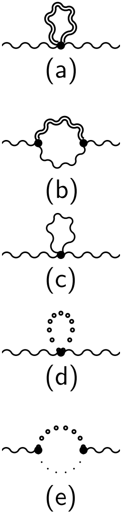
Generalization to Non-Abel Gauge Theory



In order to generalize the result above to non-Abelian case, we note that a non-Abelian gauge theory adds no new graviton-induced self-energy 1-loop diagram, except the couplings in these diagrams may differ from QED.

- Fig.(a) and (d) remain the same, since they contain pure gravitational interactions only.
- Fig.(b) and (e) do not change too, since both graviton and graviton-ghost carry no gauge-charge.

Generalization to Non-Abel Gauge Theory



- Fig.(c) could receive a change due to possible "color" summation over the gauge-loop. There are two kinds of contributions:

$$\bar{F}^a_{\mu\nu}\bar{F}^b_{\rho\sigma}a^a_\alpha a^b_\beta \quad \text{and} \quad \bar{F}^a_{\mu\nu}\bar{F}^a_{\rho\sigma}a^b_\lambda a^b_\kappa$$

  The former causes no summation on gauge indices, while the latter does. However, explicit calculation shows that the contribution to the latter form actually vanishes.

Therefore we conclude that our graviton induced power-law correction is universal for both Abelian and non-Abelian gauge theories.

Gravity-Assisted Gauge Unification

With the gauge-invariant gravitational power-law corrections, we can resolve running gauge coupling $\alpha_i(\mu) = g_i^2(\mu)/4\pi$ from the RGE * :

$$\frac{e^{-c\mu^2}}{\alpha_i(\mu)} - \frac{e^{-c\mu_0^2}}{\alpha_i(\mu_0)} = \frac{b_{0i}}{4\pi}\int_{\mu_0^2}^{\mu^2}\frac{\mathrm{d}x}{x}e^{-cx}.$$

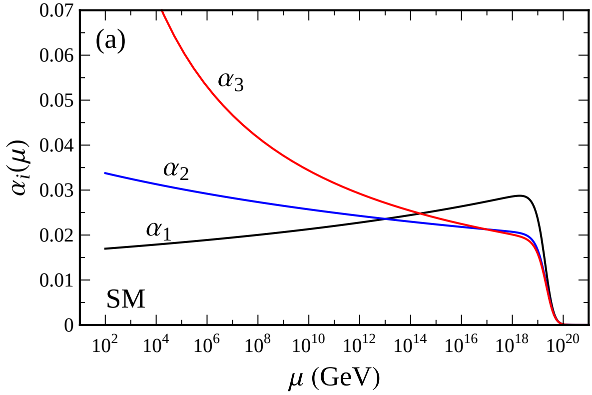
with $c \equiv 15/(8\pi M_P^2)$. So we further deduce,

$$\alpha_i(\mu) = \frac{\alpha_i(\mu_0)e^{-c\mu^2}}{e^{-c\mu_0^2} + \frac{b_{0i}\alpha_i(\mu_0)}{4\pi}\int_{\mu_0^2}^{\mu^2}\frac{\mathrm{d}x}{x}e^{-cx}}.$$

*Here $i = 1, 2, 3$ stand for $U(1)$, $SU(2)$ and $SU(3)$ gauge groups, respectively.

Gravity-Assisted Gauge Unification

For SM with gravitational corrections, we have:



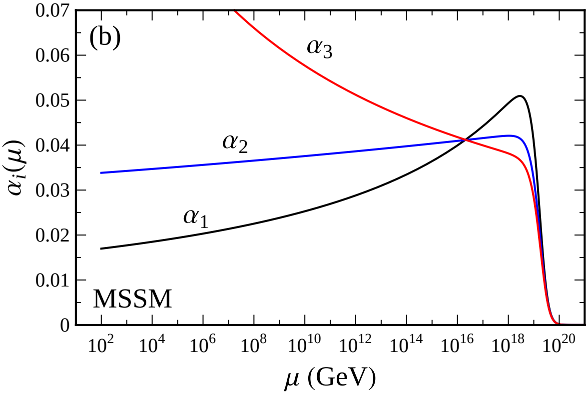
Input:

$$\alpha_{1,M_Z}^{-1} = 59.00 \pm 0.02,$$
$$\alpha_{2,M_Z}^{-1} = 29.57 \pm 0.02,$$
$$\alpha_{3,M_Z}^{-1} = 8.50 \pm 0.14,$$

$$b_{01} = -41/10,$$
$$b_{02} = 19/6,$$
$$b_{03} = 7.$$

Gravity-Assisted Gauge Unification

For MSSM with gravitational corrections, we have:



Input:

$$\alpha_{1,M_Z}^{-1} = 59.00 \pm 0.02,$$
$$\alpha_{2,M_Z}^{-1} = 29.57 \pm 0.02,$$
$$\alpha_{3,M_Z}^{-1} = 8.50 \pm 0.14,$$

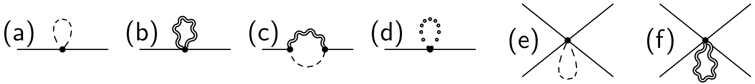
$$b_{01} = -33/5,$$
$$b_{02} = -1,$$
$$b_{03} = 3.$$

$$S_\phi = \int \mathrm{d}^4 x \sqrt{-g}\Big[g^{\mu\nu}(\mathrm{D}_\mu\Phi)^\dagger(\mathrm{D}_\nu\Phi) + \mu^2\Phi^\dagger\Phi - \lambda(\Phi^\dagger\Phi)^2\Big],$$

with Higgs doublet:

$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix}\pi^1(x)+i\pi^2(x)\\ \sigma(x)+\nu+i\pi^3(x)\end{pmatrix}.$$

Graviton-induced self-energy and vertex diagrams with quadratical divergences:



Graviton-induced power-law corrections to scalar self-energy and quartic vertex. (In each entry of the contribution, a common factor $\kappa^2\mathcal{I}_2$ is factorized out.)

	Conventional Approach	Connection-Induced only (from VDA)	Sum ($\zeta \to 0$)
Fig.(a)	0	$(\frac{1}{2}-\frac{1}{4\zeta})p^2 - m^2$	$(\frac{1}{2}-\frac{1}{4\zeta})p^2 - m^2$
Fig.(b)	$-(3+2\zeta)m^2$	0	$-3m^2$
Fig.(c)	ζp^2	$(-1+\frac{1}{4\zeta})p^2$	$(-1+\frac{1}{4\zeta})p^2$
Fig.(d)	0	p^2	p^2
Γ_2	$\zeta p^2 - (3+2\zeta)m^2$	$\frac{1}{2}p^2 - m^2$	$\frac{1}{2}p^2 - 4m^2$
Fig.(e)	0	-6λ	-6λ
Fig.(f)	$-6(3+2\zeta)\lambda$	0	-18λ
Γ_4	$-6(3+2\zeta)\lambda$	-6λ	-24λ

Then we can deduce:

$$\Delta\beta(\lambda,\mu) = +\frac{3\lambda}{8\pi^2}(\kappa^2\mu^2).$$

- Parameterizing the β-function as before:

$$\beta(\lambda,\mu) = \frac{\tilde{b}_0}{(4\pi)^2}\lambda^2 + \frac{\tilde{a}_0}{(4\pi)^2}\lambda\kappa^2\mu^2,$$

  then we see that $\tilde{a}_0 = 6$.

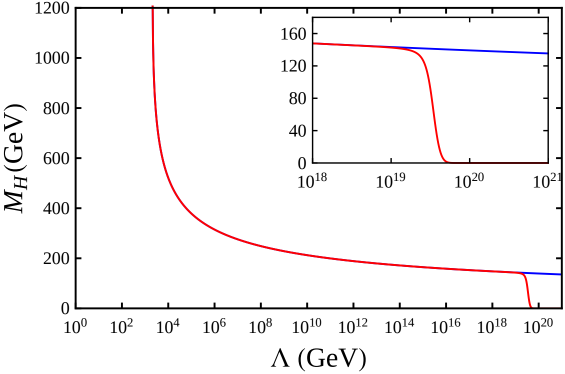
- From this we can deduce the triviality bound:

$$m_H^2 \ln\frac{\Lambda^2}{m_H^2} < \frac{8\pi^2 v^2}{3(1+X)},$$

$$X \equiv \int_{m_H}^{\Lambda}\left[e^{\tilde{a}_0\kappa^2 x^2/32\pi^2} - 1\right]\frac{\mathrm{d}x}{x}\Bigg/\ln\frac{\Lambda}{m_H} > 0.$$

- For the SM case, $\tilde{a}_0 = 0 \;\Rightarrow\; X = 0$. Thus the triviality bound for SM Higgs boson is further tightened by gravitational corrections.

- The modified **Triviality Bound** of the SM Higgs boson with gravitational power-law corrections:



Conclusion

- Gravitational corrections induce a new UV fixed point, and make ALL Abel + Non-Abel gauge theories asymptotically free!

- It is possible that the real GUT will be naturally realized around the Planck scale, and thus will allow the attractive possibility of simultaneous unification of all 4 fundamental forces in nature.

- Graviton further induces power-law corrections to the scalar β-function which is not asymptotically free, thus the triviality bound for SM Higgs boson is further tightened.

Acknowledgments

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- My great thanks also go to my collaborator Xu-Feng Wang, who makes an essential contribution to this work.

THANK YOU FOR YOUR ATTENTION!