Lepton Flavour Violation in SUSY GUTs

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November 7, 2006

'TeV scale Physics at the LHC', CCAST Beijing,
Sept 15th 2006

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Outline

- **Introduction to** $SO(10)$
- **Two Simple examples**
- **Invariance of the Casas-Ibara** R **matrix**
- U_{e3} **versus** $\mu \to e\gamma$
- **Summary**

General SO(10)

Each SM family, plus a right-handed neutrino in a single 16-dim rep.

$$\mathbf{W^{Y}_{SO(10)} = h^{10}_{ij} 16_i 16_j 10_H + h^{126}_{ij} 16_i 16_j 126_H + h^{120}_{ij} 16_i 16_j 120_H} \qquad (1)$$

Here the matrices $h^{10,126}$ are symmetric and h^{120} is anti-symmetric.

$$\begin{aligned} 16\,16\,10 &\supset 5(uu^c + \nu\nu^c) + \bar{5}(dd^c + ee^c) \\ 16\,16\,126 &\supset 1\nu^c\nu^c + 15\nu\nu + 5(uu^c - 3\nu\nu^c) + \bar{45}(dd^c - 3ee^c) \\ 16\,16\,120 &\supset 5\nu\nu^c + 45uu^c + \bar{5}(dd^c + ee^c) + \bar{45}(dd^c - 3ee^c) \end{aligned} \qquad (2)$$

The resulting tree level mass matrices are as follows,

$$M^u = M^5_{10} + M^5_{126} + M^{45}_{120} \qquad (3)$$
$$M^\nu = M^5_{10} - 3M^5_{126} + M^5_{120} \qquad (4)$$
$$M^d = M^{\bar{5}}_{10} + M^{\bar{45}}_{126} + M^{\bar{5}}_{120} + M^{\bar{45}}_{120} \qquad (5)$$
$$M^e = M^{\bar{5}}_{10} - 3M^{\bar{45}}_{126} + M^{\bar{5}}_{120} - 3M^{\bar{45}}_{120} \qquad (6)$$
$$M^\nu_{LL} = M^{15}_{126} \qquad (7)$$
$$M^\nu_R = M^1_{126} \qquad (8)$$

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Here the matrices $h^{10,126}$ are symmetric and h^{120} is anti-symmetric.

$$\begin{aligned} \mathbf{16\,16\,10} &\supset \mathbf{5(uu^c + \nu\nu^c) + \bar{5}(dd^c + ee^c)} \\ \mathbf{16\,16\,126} &\supset \mathbf{1\nu^c\nu^c + 15\nu\nu + 5(uu^c - 3\nu\nu^c) + \bar{45}(dd^c - 3ee^c)} \\ \mathbf{16\,16\,120} &\supset \mathbf{5\nu\nu^c + 45uu^c + \bar{5}(dd^c + ee^c) + \bar{45}(dd^c - 3ee^c)} \end{aligned} \tag{10}$$

The resulting tree level mass matrices are as follows,

$$M^u = M^5_{10} + M^5_{126} + M^{45}_{120} \tag{11}$$

$$M^\nu = M^5_{10} - 3M^5_{126} + M^5_{120} \tag{12}$$

$$M^d = M^{\bar{5}}_{10} + M^{\bar{45}}_{126} + M^{\bar{5}}_{120} + M^{\bar{45}}_{120} \tag{13}$$

$$M^e = M^{\bar{5}}_{10} - 3M^{\bar{45}}_{126} + M^{\bar{5}}_{120} - 3M^{\bar{45}}_{120} \tag{14}$$

$$M^\nu_{LL} = M^{15}_{126} \tag{15}$$

$$M^\nu_R = M^1_{126} \tag{16}$$

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Here the matrices $h^{10,126}$ are symmetric and h^{120} is anti-symmetric.

$$\begin{aligned} \mathbf{16\,16\,10} &\supset \mathbf{5(uu^c + \nu\nu^c) + \bar{5}(dd^c + ee^c)} \\ \mathbf{16\,16\,126} &\supset \mathbf{1\nu^c\nu^c + 15\nu\nu + 5(uu^c - 3\nu\nu^c) + \bar{45}(dd^c - 3ee^c)} \\ \mathbf{16\,16\,120} &\supset \mathbf{5\nu\nu^c + 45uu^c + \bar{5}(dd^c + ee^c) + \bar{45}(dd^c - 3ee^c)} \end{aligned} \tag{18}$$

The resulting tree level mass matrices are as follows,

$$M^u = M^5_{10} + M^5_{126} + M^{45}_{120} \tag{19}$$
$$M^\nu = M^5_{10} - 3M^5_{126} + M^5_{120} \tag{20}$$
$$M^d = M^{\bar{5}}_{10} + M^{\bar{45}}_{126} + M^{\bar{5}}_{120} + M^{\bar{45}}_{120} \tag{21}$$
$$M^e = M^{\bar{5}}_{10} - 3M^{\bar{45}}_{126} + M^{\bar{5}}_{120} - 3M^{\bar{45}}_{120} \tag{22}$$
$$M^\nu_{LL} = M^{15}_{126} \tag{23}$$
$$M^\nu_R = M^1_{126} \tag{24}$$

General SO(10)

Each SM family, plus a right-handed neutrino in a single 16-dim rep.

$$\mathbf{W^{Y}_{SO(10)} = h^{10}_{ij} 16_i 16_j 10_H + h^{126}_{ij} 16_i 16_j 126_H + h^{120}_{ij} 16_i 16_j 120_H} \tag{25}$$

Here the matrices $h^{10,126}$ are symmetric and h^{120} is anti-symmetric.

$$\begin{aligned}
\mathbf{16\,16\,10} &\supset \mathbf{5(uu^c + \nu\nu^c) + \bar{5}(dd^c + ee^c)} \\
\mathbf{16\,16\,126} &\supset \mathbf{1\nu^c\nu^c + 15\nu\nu + 5(uu^c - 3\nu\nu^c) + \bar{45}(dd^c - 3ee^c)} \\
\mathbf{16\,16\,120} &\supset \mathbf{5\nu\nu^c + 45uu^c + \bar{5}(dd^c + ee^c) + \bar{45}(dd^c - 3ee^c)}
\end{aligned} \tag{26}$$

The resulting tree level mass matrices are as follows,

$$M^u = M^5_{10} + M^5_{126} + M^{45}_{120} \tag{27}$$

$$M^\nu = M^5_{10} - 3M^5_{126} + M^5_{120} \tag{28}$$

$$M^d = M^{\bar{5}}_{10} + M^{\bar{45}}_{126} + M^{\bar{5}}_{120} + M^{\bar{45}}_{120} \tag{29}$$

$$M^e = M^{\bar{5}}_{10} - 3M^{\bar{45}}_{126} + M^{\bar{5}}_{120} - 3M^{\bar{45}}_{120} \tag{30}$$

$$M^\nu_{LL} = M^{15}_{126} \tag{31}$$

$$M^\nu_R = M^1_{126} \tag{32}$$

Two simple models of Y_ν mixing

The size of mixing in Y_ν determines LFV rates

Masiero et.al. hep-ph/0209303

- **Case I: CKM type mixing in $Y_\nu \Longrightarrow$ small LFV rates**
  - $tan\beta = 5$ **and** $tan\beta = 50$

- **Case II: MNS type mixing in $Y_\nu \Longrightarrow$ large LFV rates**
  - $tan\beta = 5$ **and** $tan\beta = 50$

I: CKM type mixings in Y_ν

Introduce separate Higgs couplings for the up and down type fermions.

$$\mathbf{W_{SO(10)} = \frac{1}{2}h^{u,\nu}_{ij}16_i16_j10_u + \frac{1}{2}h^{d,e}_{ij}16_i16_j10_d + \frac{1}{2}h^{R}_{ij}16_i16_j126} \tag{33}$$

Leading to the $MSSM + \nu_R$ superpotential,

$$\mathbf{W = h^u_{ij}Q_iu_jH_u + h^\nu_{ij}L_i\nu_iH_u + h^d_{ii}Q_id_jH_d + h^e_{ii}L_ie_jH_d + \frac{1}{2}M^R_{ij}\nu_i\nu_j} \tag{34}$$

Here, $h^u = h^\nu$ and $h^d = h^e$. Here the symmetric h^u is diagonalised by,

$$\mathbf{h^u_{diag} = V_{CKM}h^uV^T_{CKM}} \tag{35}$$

so that,

$$\mathbf{h^\nu = V^T_{CKM}h^u_{diag}V_{CKM}} \tag{36}$$

The type I seesaw mechanism then gives,

$$\mathbf{m_\nu = h^\nu M_R^{-1}h^{\nu\,T}v_u^2} \tag{37}$$

So that the Majorana Mass matrix is of the form,

$$\mathbf{M_R = v_u^2V^T_{CKM}h^u_{diag}V_{CKM}U_{MNS}(m_\nu^{diag})^{-1}U^T_{MNS}V^T_{CKM}h^u_{diag}V_{CKM}} \tag{38}$$

II: MNS type mixings in Y_ν

Introduce separate Higgs couplings for the up and down type fermions. To get MNS like mixing we must introduce an asymmetrical Higgs coupling, Φ.

$$\mathbf{W_{SO(10)} = \frac{1}{2}h^{u,\nu}_{ii}16_i16_i10_u + \frac{1}{2}h^{d,e}_{ij}16_i16_j\Phi + \frac{1}{2}h^{R}_{ij}16_i16_j126} \tag{39}$$

If we write it in SU(5) language then we have,

$$\mathbf{W_{SU(5)} = \frac{1}{2}h^{u}_{ii}10_i10_i5_u + h^{\nu}_{ii}\bar{5}_i1_i5_u + h^{d}_{ij}10_i\bar{5}_j\bar{5}_d + \frac{1}{2}M^{R}_{ij}1_i1_j} \tag{40}$$

Here, $h^u_{ii} = h^\nu_{ii}$. Rotating to the diagonal down quark basis,

$$\mathbf{h^d_{diag} = V^T_{CKM}h^dU^T_{MNS}} \tag{41}$$

And,

$$h^u = V^T_{CKM}h^u_{diag}V^T_{CKM} \tag{42}$$

$$\mathbf{h^\nu = U_{MNS}h^u_{diag}} \tag{43}$$

The type I seesaw mechanism then gives,

$$\mathbf{m_\nu = h^\nu M_R^{-1}h^{\nu\,T}v_u^2} \tag{44}$$

Inverting this relation we have the right-handed neutrino mass,

$$\mathbf{M_R = h^{\nu\,T}m_\nu^{-1}h^\nu v_u^2 = h^u_{diag}\left(m_\nu^{diag}\right)^{-1}h^u_{diag}v_u^2} = \mathrm{diag}\left\{\mathbf{\frac{m_u^2}{m_{\nu_1}}, \frac{m_c^2}{m_{\nu_2}}, \frac{m_t^2}{m_{\nu_3}}}\right\} \tag{45}$$

The Casas-Ibara R matrix

Usually defined in the 'Flavour Basis'; diagonal M_{RR}, Y^E_{LR}

$$\mathcal{L} = -\mathbf{Y^{E\,diag}_{LR} H_d \bar{L} E_R - Y^{\nu}_{LR} H_u \bar{L} \nu_R + \frac{1}{2} \nu^T_R M^{diag}_{RR} \nu_R} \tag{46}$$

The seesaw mass is,

$$\mathbf{m_\nu = v_u^2 Y^{\nu}_{LR} M^{diag\,-1}_{RR} Y^{\nu\,T}_{LR}} \tag{47}$$

Which is diagonalised by the MNS matrix,

$$\mathbf{m^{diag}_\nu = U^\dagger_{MNS}\, m_\nu\, U^*_{MNS}} \tag{48}$$

Inverting this equation gives you,

$$\mathbf{1} = \mathbf{v_u^2 (m^{diag}_\nu)^{-1/2}\, U^\dagger_{MNS}\, Y^\nu_{LR} M^{diag\,-1/2}_{RR} M^{diag\,-1/2}_{RR} Y^{\nu\,T}_{LR}\, U^*_{MNS}\, (m^{diag}_\nu)^{-1/2}} \tag{49}$$

$$= \mathbf{RR^T} \tag{50}$$

So that,

$$\mathbf{R = v_u (m^{diag}_\nu)^{-1/2}\, U^\dagger_{MNS}\, Y^\nu_{LR} M^{diag\,-1/2}_{RR}} \tag{51}$$

Is the R matrix of Casas-Ibara, hep-ph/0103065, defined in the flavour basis.

$$\mathbf{Y^\nu_{LR} = U_{MNS} (m^{diag}_\nu)^{1/2}\, R\, M^{diag\,1/2}_{RR}\, 1/v_u} \tag{52}$$

Invariance of the Casas-Ibara R matrix

Recently S.F.King hep-ph/0610239, showed that R is basis invariant.

All models with the same, R matrix, are related by a simple basis rotation.
i.e. Y^E_{LR}, Y^ν_{LR}, M_{RR} *and* R *defines a 'class' of models:*

In a general basis we have the seesaw mass,

$$\mathbf{m}_\nu = \mathbf{v_u^2 Y^\nu_{LR} M^{diag\,-1}_{RR} Y^{\nu\,T}_{LR}} \tag{53}$$

Inverting again gives,

$$\mathbf{1 = v_u^2 m^{-1}_{\sqrt{\nu}} Y^\nu_{LR} \sqrt{M^{-1}_{RR}} \sqrt{M^{-1}_{RR}}^T Y^{\nu\,T}_{LR} (m^{-1}_{\sqrt{\nu}})^T = R\,R^T} \tag{54}$$

Where, $\mathbf{M^{-1}_{RR} = \sqrt{M^{-1}_{RR}}\sqrt{M^{-1}_{RR}}^T}$ **and** $\mathbf{m_\nu = m_{\sqrt{\nu}}\, m^T_{\sqrt{\nu}}}$

So that the R matrix in a general basis is,

$$\mathbf{R = v_u^2 m^{-1}_{\sqrt{\nu}} Y^\nu_{LR} \sqrt{M^{-1}_{RR}}} \tag{55}$$

And R is clearly basis invariant under a basis rotation,

$$\mathbf{m^{-1}_{\sqrt{\nu}}} \rightarrow \mathbf{m^{-1}_{\sqrt{\nu}} V^\dagger_{E_L}} \tag{56}$$

$$\mathbf{\sqrt{M^{-1}_{RR}}} \rightarrow \mathbf{V_{\nu_R} \sqrt{M^{-1}_{RR}}} \tag{57}$$

$$\mathbf{Y^\nu_{LR}} \rightarrow \mathbf{V_{E_L} Y^\nu_{LR} V^\dagger_{\nu_R}} \tag{58}$$

R for our two simple models

- CKM mixing Case:

$$\mathbf{R = 1}$$

- MNS mixing Case:

$$\mathbf{R = 1}$$

The two models are related by a non-unitary transformation of the right handed neutrino field,

$$\mathbf{S^{-1} = V^T_{CKM} h^{u\,-1}_{diag} V_{CKM} U_{MNS} h^u_{diag}} \tag{59}$$

With R also invariant under,

$$\mathbf{Y^{\nu}_{LR} \;\rightarrow\; Y^{\nu}_{LR} S^{-1}} \tag{60}$$

$$\mathbf{M_R \;\rightarrow\; S^{T\,-1} M_R S^{-1}} \tag{61}$$

$\mu \to e\gamma$ in model II

Approximate off-diagonal slepton mass terms induced by RG running as,

$$\left[\mathbf{m}^2_{\tilde{\mathbf{l}}}\right]_{\mathbf{21}} \approx -\frac{\mathbf{1}}{\mathbf{8}\pi^{\mathbf{2}}}(\mathbf{3m_0^2} + \mathbf{A_0^2})\,[\mathbf{h}^\nu \mathbf{h}^{\nu \mathrm{T}}]_{\mathbf{21}} \ln\left(\frac{\mathbf{M}_{\mathrm{GUT}}}{\mathbf{M_R}}\right) \tag{62}$$

Then the branching ratio for $l_i \to l_j\gamma$ is approximately,

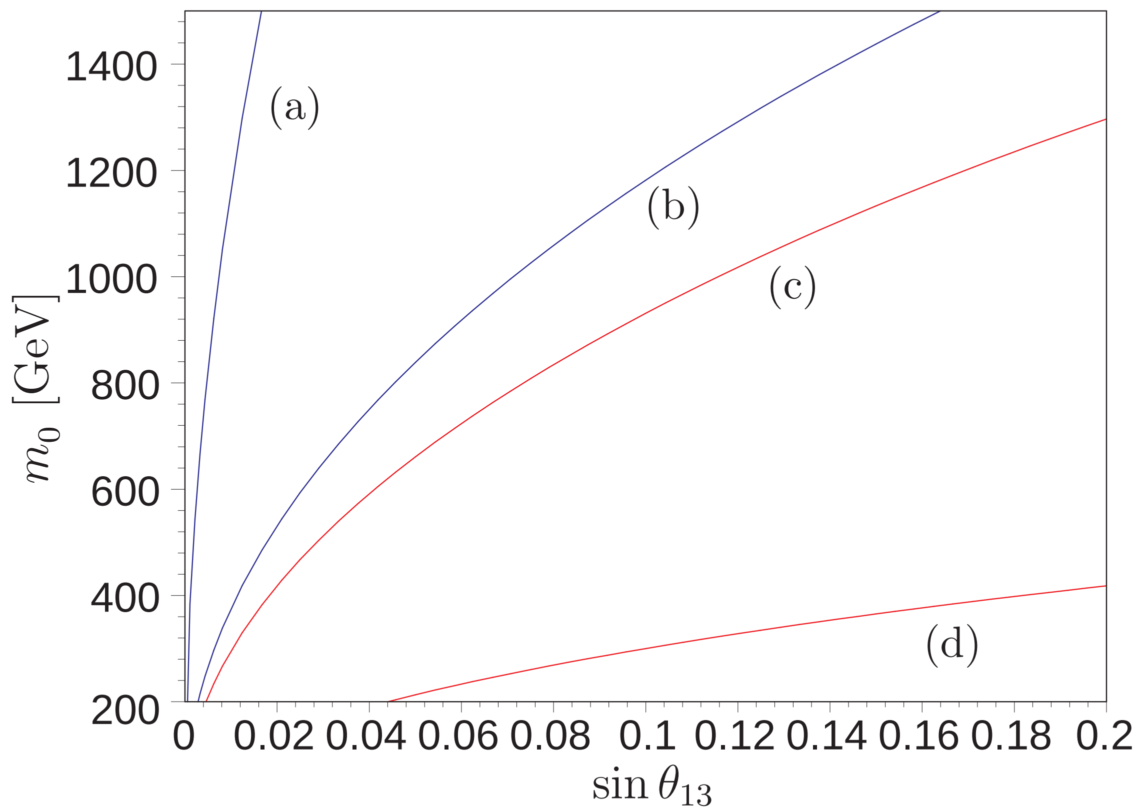
$$\mathbf{Br}(\mathbf{l_i} \to \mathbf{l_j}\gamma) \approx \frac{\alpha^{\mathbf{2}}}{\mathbf{G_F^2}} \frac{([\mathbf{m}^2_{\tilde{\mathbf{l}}}]_{\mathbf{21}})^{\mathbf{2}}}{\mathbf{m}^{\mathbf{8}}_{\mathrm{SUSY}}} \tan^{\mathbf{2}}\beta \leq \mathbf{B} \tag{63}$$

Here B is the experimental bound set on this LFV decay. The present value is $B = 1.2 \times 10^{-11}$ at 90% C.L., but in the near future it could become $B = 5 \times 10^{-14}$ at 90% C.L.

For the MNS mixing model we have,

$$[\mathbf{h}^\nu \mathbf{h}^{\nu \mathrm{T}}]_{\mathbf{21}} = \mathbf{y_u^2}\,\mathbf{U}_{\mu\mathbf{1}}\mathbf{U_{e1}} + \mathbf{y_c^2}\,\mathbf{U}_{\mu\mathbf{2}}\mathbf{U_{e2}} + \mathbf{y_t^2}\,\mathbf{U}_{\mu\mathbf{3}}\mathbf{U_{e3}} \tag{64}$$

Set a limit on the neutrino mixing angle $\sin theta_{13}$ from the above muon decay bound.



Limits set on θ_{13} from the present and future bounds on the LFV decay $\mu \to e\gamma$. (a) $\tan\beta = 50$, $B = 5 \times 10^{-14}$**, (b)** $\tan\beta = 50$, $B = 1.2 \times 10^{-11}$**, (c)** $\tan\beta = 5$, $B = 1.2 \times 10^{-11}$**, (d)** $\tan\beta = 5$, $B = 5 \times 10^{-14}$**, The area above(below) the curves is allowed(excluded).**

Summary

- **Casas-Ibara R matrix is basis invariant and a general form of R has been given.**

- **Two limiting SO(10) models were presented and the interplay of** $\sin\theta_{13}$ **and** $\mu \to e\gamma$ **was studied.**