Lepton Flavour Violation in SUSY GUTs

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November 7, 2006

‘TeV scale Physics at the LHC’, CCAST Beijing,
Sept 15th 2006
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\[ U_{e3} \text{ and BR}(\mu \rightarrow e\gamma) \text{ in SUSY SO}(10) \]

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$U_{e3}$ and BR($\mu \rightarrow e\gamma$) in SUSY SO(10)

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Outline

- Introduction to $SO(10)$
- Two Simple examples
- Invariance of the Casas-Ibara $R$ matrix
- $U_{e3}$ versus $\mu \rightarrow e\gamma$
- Summary
General SO(10)

Each SM family, plus a right-handed neutrino in a single 16-dim rep.

\[ W_{SO(10)}^Y = h_{ij}^{10} 16_i 16_j 10_H + h_{ij}^{126} 16_i 16_j 126_H + h_{ij}^{120} 16_i 16_j 120_H \]  

Here the matrices \( h^{10,126} \) are symmetric and \( h^{120} \) is anti-symmetric.

\[
\begin{align*}
16 \ 16 \ 10 & \supset 5(uu^c + \nu\nu^c) + \bar{5}(dd^c + ee^c) \\
16 \ 16 \ 126 & \supset 1\nu^c\nu^c + 15\nu\nu + 5(uu^c - 3\nu\nu^c) + 4\bar{5}(dd^c - 3ee^c) \\
16 \ 16 \ 120 & \supset 5\nu\nu^c + 45uu^c + \bar{5}(dd^c + ee^c) + 4\bar{5}(dd^c - 3ee^c)
\end{align*}
\]

The resulting tree level mass matrices are as follows,

\[
\begin{align*}
M^u &= M_5^{10} + M_5^{126} + M_5^{120} \quad (3) \\
M^\nu &= M_5^{10} - 3M_5^{126} + M_5^{120} \quad (4) \\
M^d &= M_5^{10} + M_5^{126} + M_5^{120} + M_5^{120} \quad (5) \\
M^e &= M_5^{10} - 3M_5^{126} + M_5^{120} - 3M_5^{120} \quad (6) \\
M^\nu_{LL} &= M^{15}_{126} \quad (7) \\
M^\nu_R &= M^1_{126} \quad (8)
\end{align*}
\]
**General SO(10)**

Each SM family, plus a right-handed neutrino in a single 16-dim rep.

\[
W^Y_{\text{SO}(10)} = h_{ij}^{10} 16_i 16_j 10_H + h_{ij}^{126} 16_i 16_j 126_H + h_{ij}^{120} 16_i 16_j 120_H
\]  \hspace{1cm} (9)

Here the matrices \( h^{10,126} \) are symmetric and \( h^{120} \) is anti-symmetric.

\[
\begin{align*}
16 & \ 16 & 10 \ni 5(uu^c + \nu\nu^c) + 5(dd^c + ee^c) \\
16 & \ 16 & 126 \ni 1\nu^c\nu^c + 15\nu\nu + 5(uu^c - 3\nu\nu^c) + 45(dd^c - 3ee^c) \\
16 & \ 16 & 120 \ni 5\nu\nu^c + 45uu^c + 5(dd^c + ee^c) + 45(dd^c - 3ee^c)
\end{align*}
\]  \hspace{1cm} (10)

The resulting tree level mass matrices are as follows,

\[
\begin{align*}
M^u &= M^5_{10} + M^5_{126} + M^{45}_{120} \hspace{1cm} (11) \\
M^\nu &= M^5_{10} - 3M^5_{126} + M^5_{120} \hspace{1cm} (12) \\
M^d &= M^5_{10} + M^{45}_{126} + M^5_{120} + M^{45}_{120} \hspace{1cm} (13) \\
M^e &= M^5_{10} - 3M^{45}_{126} + M^5_{120} - 3M^{45}_{120} \hspace{1cm} (14) \\
M^\nu_{LL} &= M^1_{126} \hspace{1cm} (15) \\
M^\nu_R &= M^1_{126} \hspace{1cm} (16)
\end{align*}
\]
General SO(10)

Each SM family, plus a right-handed neutrino in a single 16-dim rep.

$$W_{\text{SO}(10)}^Y = h_{ij}^{10}16_i16_j10_H + h_{ij}^{126}16_i16_j126_H + h_{ij}^{120}16_i16_j120_H$$ \tag{17}

Here the matrices $h^{10,126}$ are symmetric and $h^{120}$ is anti-symmetric.

$$16\ 16\ 10 \supset 5(uu^c + \nu^c) + \bar{5}(dd^c + ee^c)$$
$$16\ 16\ 126 \supset 1\nu^c\nu^c + 15\nu\nu + 5(uu^c - 3\nu^c) + 45(dd^c - 3ee^c)$$ \tag{18}
$$16\ 16\ 120 \supset 5\nu\nu + 45uu^c + \bar{5}(dd^c + ee^c) + 4\bar{5}(dd^c - 3ee^c)$$

The resulting tree level mass matrices are as follows,

$$M^u = M_{10}^5 + M_{126}^5 + M_{120}^{45}$$ \tag{19}
$$M^\nu = M_{10}^5 - 3M_{126}^5 + M_{120}^5$$ \tag{20}
$$M^d = M_{10}^5 + M_{126}^{45} + M_{120}^5 + M_{120}^{45}$$ \tag{21}
$$M^e = M_{10}^5 - 3M_{126}^{45} + M_{120}^5 - 3M_{120}^{45}$$ \tag{22}
$$M_{LL}^\nu = M_{126}^{15}$$ \tag{23}
$$M_{R}^\nu = M_{126}^1$$ \tag{24}
**General SO(10)**

Each SM family, plus a right-handed neutrino in a single 16-dim rep.

\[ W_{SO(10)}^Y = h_{ij}^{10} 16_i 16_j 10_H + h_{ij}^{126} 16_i 16_j 126_H + h_{ij}^{120} 16_i 16_j 120_H \]  \hspace{1cm} (25)

Here the matrices \( h^{10,126} \) are symmetric and \( h^{120} \) is anti-symmetric.

\[
16 \hspace{0.2cm} 16 \hspace{0.2cm} 10 \hspace{0.2cm} \supset \hspace{0.2cm} 5(uu^c + \nu \nu^c) + 5(dd^c + ee^c) \\
16 \hspace{0.2cm} 16 \hspace{0.2cm} 126 \hspace{0.2cm} \supset \hspace{0.2cm} 1\nu^c \nu^c + 15\nu \nu + 5(uu^c - 3\nu \nu^c) + 45(dd^c - 3ee^c) \\
16 \hspace{0.2cm} 16 \hspace{0.2cm} 120 \hspace{0.2cm} \supset \hspace{0.2cm} 5\nu \nu^c + 45uu^c + 5(dd^c + ee^c) + 45(dd^c - 3ee^c) \]  \hspace{1cm} (26)

The resulting tree level mass matrices are as follows,

\[
M^u = M^5_{10} + M^5_{126} + M^{45}_{120} \\
M^\nu = M^5_{10} - 3M^5_{126} + M^5_{120} \\
M^d = M^5_{10} + M^{45}_{126} + M^5_{120} + M^{45}_{120} \\
M^e = M^5_{10} - 3M^{45}_{126} + M^5_{120} - 3M^{45}_{120} \\
M^\nu_{LL} = M^{15}_{126} \\
M^\nu_R = M^1_{126} \]  \hspace{1cm} (27-32)
Two simple models of $Y_\nu$ mixing

The size of mixing in $Y_\nu$ determines LFV rates

Masiero et.al. hep-ph/0209303

- **Case I:** CKM type mixing in $Y_\nu \implies$ small LFV rates
  - $\tan\beta = 5$ and $\tan\beta = 50$

- **Case II:** MNS type mixing in $Y_\nu \implies$ large LFV rates
  - $\tan\beta = 5$ and $\tan\beta = 50$
I: CKM type mixings in $Y_\nu$

Introduce separate Higgs couplings for the up and down type fermions.

$$W_{\text{SO}(10)} = \frac{1}{2} h_{ij}^{u}16i16j10_u + \frac{1}{2} h_{ij}^{d,e}16i16j10_d + \frac{1}{2} h_{ij}^{R}16i16j126$$  \hspace{1cm} (33)

Leading to the $MSSM + \nu_R$ superpotential,

$$W = h_{ij}^{u}Q_iu_jH_u + h_{ij}^{\nu}L_i\nu_jH_u + h_{ij}^{d}Q_i d_jH_d + h_{ij}^{e}L_i e_jH_d + \frac{1}{2} M_{ij}^{R} \nu_i \nu_j$$  \hspace{1cm} (34)

Here, $h^u = h^\nu$ and $h^d = h^e$. Here the symmetric $h^u$ is diagonalised by,

$$h_{\text{diag}}^u = V_{\text{CKM}} h^u V_{\text{CKM}}^T$$  \hspace{1cm} (35)

so that,

$$h^\nu = V_{\text{CKM}}^T h_{\text{diag}}^u V_{\text{CKM}}$$  \hspace{1cm} (36)

The type I seesaw mechanism then gives,

$$m_\nu = h^\nu M_R^{-1} h^\nu T v_u^2$$  \hspace{1cm} (37)

So that the Majorana Mass matrix is of the form,

$$M_R = v_u^2 V_{\text{CKM}}^T h_{\text{diag}}^u V_{\text{CKM}} U_{\text{MNS}} (m_{\nu}^{\text{diag}})^{-1} U_{\text{MNS}}^T V_{\text{CKM}}^T h_{\text{diag}}^u V_{\text{CKM}}$$  \hspace{1cm} (38)
II: MNS type mixings in $Y_\nu$

Introduce separate Higgs couplings for the up and down type fermions. To get MNS like mixing we must introduce an asymmetrical Higgs coupling, $\Phi$.

$$W_{SO(10)} = \frac{1}{2} h_{ij}^{u,\nu} 16i16j10u + \frac{1}{2} h_{ij}^{d,\nu} 16i16j126 $$

(39)

If we write it in $SU(5)$ language then we have,

$$W_{SU(5)} = \frac{1}{2} h_{ii}^u 10i10j5u + h_{ii}^{\nu,\bar{5}} 1i5u + h_{ij}^d 10i\bar{5}j\bar{5}d + \frac{1}{2} M_{ij} 1i1j $$

(40)

Here, $h_{ii}^u = h_{ii}^{\nu}$. Rotating to the diagonal down quark basis,

$$h_{\text{diag}^d} = V_{\text{CKM}}^T h_{\text{diag}^d} U_{\text{MNS}}^T $$

(41)

And,

$$h_{\nu}^u = V_{\text{CKM}}^T h_{\text{diag}^u} V_{\text{CKM}} $$

(42)

$$h_{\nu}^\nu = U_{\text{MNS}} h_{\text{diag}^u} $$

(43)

The type I seesaw mechanism then gives,

$$m_{\nu} = h_{\nu}^\nu M_{\nu}^{-1} h_{\nu}^u T \nu^2 $$

(44)

Inverting this relation we have the right-handed neutrino mass,

$$M_{\nu} = h_{\nu}^\nu T m_{\nu}^{-1} h_{\nu}^u \nu^2 = h_{\text{diag}^u} (m_{\nu}^{-1})^{-1} h_{\text{diag}^u} \nu^2 = \text{diag} \left\{ \frac{m_u^2}{m_{\nu_1}}, \frac{m_c^2}{m_{\nu_2}}, \frac{m_t^2}{m_{\nu_3}} \right\} $$

(45)
The Casas-Ibara R matrix

Usually defined in the ‘Flavour Basis’; diagonal $M_{RR}$, $Y_{LR}^E$

$$\mathcal{L} = -Y_{LR}^E \text{diag} H_d \bar{L} E_R - Y_{LR}^\nu H_u \bar{L} \nu_R + \frac{1}{2} \nu_R^T M_{RR}^\text{diag} \nu_R$$  \hspace{1cm} (46)

The seesaw mass is,

$$m_\nu = v_u^2 Y_{LR}^\nu M_{RR}^\text{diag}^{-1} Y_{LR}^\nu$$ \hspace{1cm} (47)

Which is diagonalised by the MNS matrix,

$$m_\nu^\text{diag} = U_{MNS}^\dagger m_\nu U_{MNS}^*$$ \hspace{1cm} (48)

Inverting this equation gives you,

$$1 = v_u^2 (m_\nu^\text{diag})^{-1/2} U_{MNS}^\dagger Y_{LR}^\nu M_{RR}^\text{diag}^{-1/2} Y_{LR}^\nu U_{MNS}^* (m_\nu^\text{diag})^{-1/2}$$ \hspace{1cm} (49)

$$= RR^T$$ \hspace{1cm} (50)

So that,

$$R = v_u (m_\nu^\text{diag})^{-1/2} U_{MNS}^\dagger Y_{LR}^\nu M_{RR}^\text{diag}^{-1/2}$$ \hspace{1cm} (51)

Is the R matrix of Casas-Ibara, hep-ph/0103065, defined in the flavour basis.

$$Y_{LR}^\nu = U_{MNS} (m_\nu^\text{diag})^{1/2} R M_{RR}^\text{diag}^{1/2} 1/v_u$$ \hspace{1cm} (52)
Invariance of the Casas-Ibara R matrix

Recently S.F. King hep-ph/0610239, showed that R is basis invariant.

All models with the same, R matrix, are related by a simple basis rotation. i.e. $Y_{LR}^E$, $Y_{LR}^\nu$, $M_{RR}$ and $R$ defines a ‘class’ of models:

In a general basis we have the seesaw mass,

$$m_\nu = v_u^2 Y_{LR}^\nu M_{RR}^{-1} Y_{LR}^{\nu T}$$  \hspace{1cm} (53)

Inverting again gives,

$$1 = v_u^2 m_\nu^{-1} Y_{LR}^\nu \sqrt{M_{RR}^{-1}} \sqrt{M_{RR}^{-1}}^{T} Y_{LR}^{\nu T} (m^{-1})^T = R R^T$$  \hspace{1cm} (54)

Where, $M_{RR}^{-1} = \sqrt{M_{RR}^{-1}} \sqrt{M_{RR}^{-1}}^{T}$ and $m_\nu = m_{\sqrt{\nu}} m_{\sqrt{\nu}}^{T}$

So that the R matrix in a general basis is,

$$R = v_u^2 m_{\sqrt{\nu}}^{-1} Y_{LR}^\nu \sqrt{M_{RR}^{-1}}$$  \hspace{1cm} (55)

And R is clearly basis invariant under a basis rotation,

$$m_{\sqrt{\nu}}^{-1} \rightarrow m_{\sqrt{\nu}}^{-1} V_{EL}^\dagger$$  \hspace{1cm} (56)

$$\sqrt{M_{RR}^{-1}} \rightarrow V_{\nu R} \sqrt{M_{RR}^{-1}}$$  \hspace{1cm} (57)

$$Y_{LR}^\nu \rightarrow V_{EL} Y_{LR}^\nu V_{\nu R}^\dagger$$  \hspace{1cm} (58)
R for our two simple models

• CKM mixing Case:

\[ R = 1 \]

• MNS mixing Case:

\[ R = 1 \]

The two models are related by a non-unitary transformation of the right handed neutrino field,

\[ S^{-1} = V_{\text{CKM}}^T h_{\text{diag}}^{-1} V_{\text{CKM}} U_{\text{MNS}} h_{\text{diag}}^u \]

(59)

With R also invariant under,

\[ Y_{\text{LR}}^{\nu} \rightarrow Y_{\text{LR}}^{\nu} S^{-1} \]  
(60)

\[ M_R \rightarrow S^{T-1} M_R S^{-1} \]  
(61)
\( \mu \to e\gamma \) in model II

Approximate off-diagonal slepton mass terms induced by RG running as,

\[
[m_1^2]_{21} \approx -\frac{1}{8\pi^2}(3m_0^2 + A_0^2) [h^\nu h^{\nu T}]_{21} \ln \left( \frac{M_{\text{GUT}}}{M_R} \right)
\]  \hspace{1cm} (62)

Then the branching ratio for \( l_i \to l_j \gamma \) is approximately,

\[
\text{Br}(l_i \to l_j \gamma) \approx \frac{\alpha^2 ([m_1^2]_{21})^2}{G_F^2 m_{\text{SUSY}}^8 \tan^2 \beta} \leq B
\]  \hspace{1cm} (63)

Here \( B \) is the experimental bound set on this LFV decay. The present value is \( B = 1.2 \times 10^{-11} \) at 90\% C.L., but in the near future it could become \( B = 5 \times 10^{-14} \) at 90\% C.L.

For the MNS mixing model we have,

\[
[h^\nu h^{\nu T}]_{21} = y_u^2 U_{\mu 1} U_{e 1} + y_c^2 U_{\mu 2} U_{e 2} + y_t^2 U_{\mu 3} U_{e 3}
\]  \hspace{1cm} (64)

Set a limit on the neutrino mixing angle \( \sin \theta_{13} \) from the above muon decay bound.
Limits set on $\theta_{13}$ from the present and future bounds on the LFV decay $\mu \rightarrow e\gamma$. (a) $\tan \beta = 50$, $B = 5 \times 10^{-14}$, (b) $\tan \beta = 50$, $B = 1.2 \times 10^{-11}$, (c) $\tan \beta = 5$, $B = 1.2 \times 10^{-11}$, (d) $\tan \beta = 5$, $B = 5 \times 10^{-14}$, The area above(below) the curves is allowed(excluded).
Summary

• Casas-Ibara $R$ matrix is basis invariant and a general form of $R$ has been given.

• Two limiting $SO(10)$ models were presented and the interplay of $\sin \theta_{13}$ and $\mu \to e\gamma$ was studied.