The Electroweak Theory, Higgs Physics, and Beyond

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A Decade of Discovery Past . . .

- EW theory $\rightarrow$ law of nature [Z, e^+e^-, $\bar{p}p$, νN, $(g-2)_\mu$, . . .]
- Higgs-boson influence in the vacuum [EW experiments]
- ν oscillations: $\nu_\mu \rightarrow \nu_\tau$, $\nu_e \rightarrow \nu_\mu/\nu_\tau$ [$\nu_\odot$, ν_{atm}, reactors]
- Understanding QCD [heavy flavor, Z^0, $\bar{p}p$, νN, ep, ions, lattice]
- Discovery of top quark [$\bar{p}p$]
- Direct $\mathcal{CP}$ violation in $K \rightarrow \pi\pi$ [fixed-target]
- B-meson decays violate $\mathcal{CP}$ [$e^+e^- \rightarrow B\bar{B}$]
- Flat universe: dark matter, energy [SN Ia, CMB, LSS]
- Detection of ν_τ interactions [fixed-target]
- Quarks, leptons structureless at 1 TeV scale [mostly colliders]

Tevatron Collider is breaking new ground in sensitivity

Collider Run II Integrated Luminosity

Collider Run II Peak Luminosity
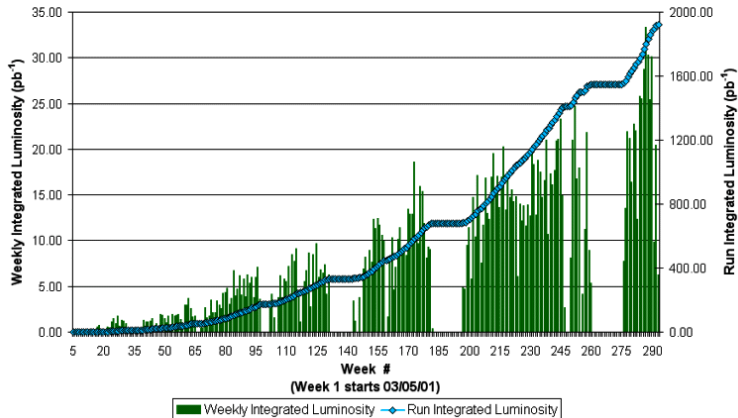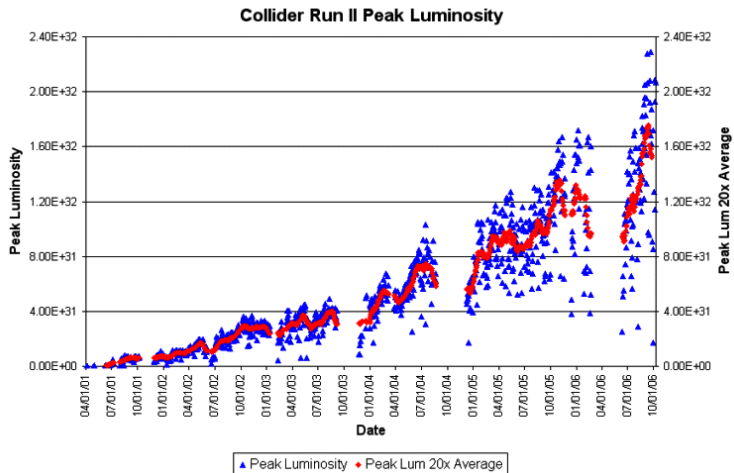
Tevatron Collider in a Nutshell

980-GeV protons, antiprotons (2π km)
frequency of revolution $\approx 45\,000$ s^{-1}
392 ns between crossings
($36 \otimes 36$ bunches)
collision rate $= \mathcal{L} \cdot \sigma_{\text{inelastic}} \approx 10^7$ s^{-1}
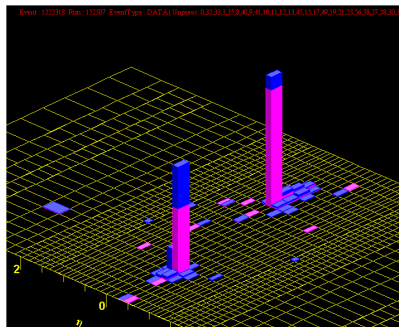
$c \approx 10^9$ km/h; $\quad v_p \approx c - 495$ km/h

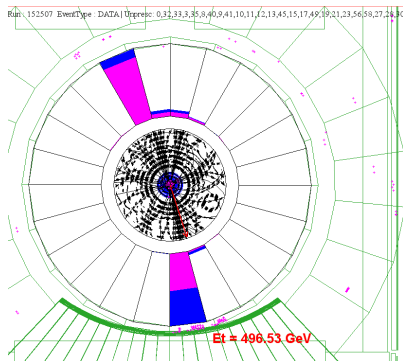Record $\mathcal{L}_{\text{init}} = 2.29 \times 10^{32}$ cm^{-2} s^{-1}
[CERN ISR: pp, 1.4×10^{32} cm^{-2} s^{-1}]

Goal: ≈ 8 fb^{-1} by 10.2009

The World's Most Powerful Microscopes

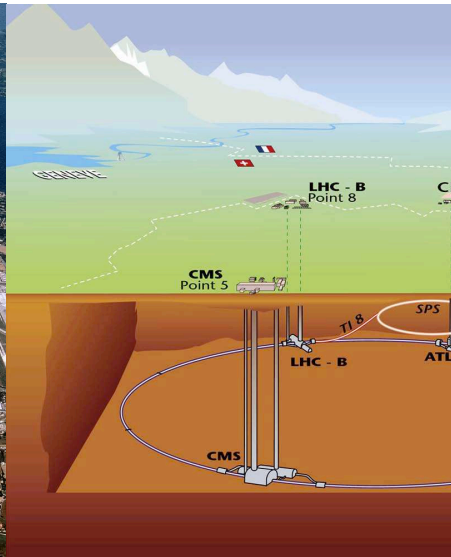## *nanonanophysics*

CDF dijet event ($\sqrt{s} = 1.96$ TeV): $E_T = 1.364$ TeV

$$q\bar{q} \to \text{jet} + \text{jet}$$

LHC will operate *soon*, breaking new ground in E & $\mathcal{L}$

LHC in a nutshell

7-TeV protons on protons (27 km); $v_p \approx c - 10$ km/h
Novel two-in-one dipoles (≈ 9 teslas)

Collisions! ($E_{cm} = 900$ GeV, $\mathcal{L} \approx 10^{29}$ cm^{-2} s^{-1}): 11.07

Commissioning run until end 2007, then shutdown

First collisions at $E_{cm} = 14$ TeV: Spring 2008
First physics run! Goal of few fb^{-1} by end 2008

Eventual: $\mathcal{L} \gtrsim 10^{34}$ cm^{-2} s^{-1}: 100 fb^{-1}/year

Why the LHC is so exciting (I)

- Even low luminosity opens vast new realm:
  10 pb^{-1} *(few days at initial $\mathcal{L}$)* yields
  8000 top quarks, 10^5 *W*-bosons,
  100 QCD dijets beyond Tevatron kinematic limit
  Supersymmetry hints in a few weeks ?

- Essential first step: rediscover the standard model

- The antithesis of a one-experiment machine;
  enormous scope and versatility beyond high-$p_\perp$

- $\mathcal{L}$ upgrade extends $\gtrsim$10-year program ...

You will need a trigger ...
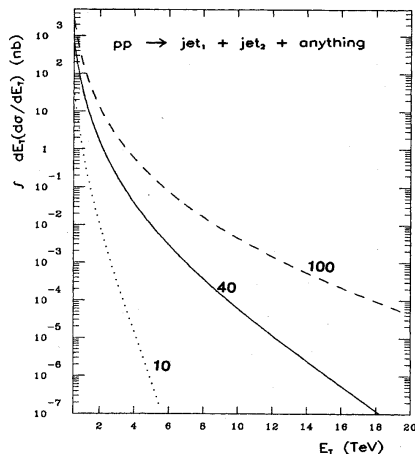
Dijet integral cross section, $|\eta| \leq 2.5$ ...

10 pb^{-1} @ LHC $\rightsquigarrow \gtrsim 10^4$ events with $E_T \gtrsim 1.364$ TeV

The importance of the 1-TeV scale

EW theory does not predict Higgs-boson mass

▷ Conditional *upper bound* from Unitarity
Compute amplitudes $\mathcal{M}$ for gauge boson scattering at high energies, make a partial-wave decomposition

$$\mathcal{M}(s, t) = 16\pi \sum_J (2J + 1)a_J(s)P_J(\cos\theta)$$

Most channels decouple – pw amplitudes are small at all energies (except very near the particle poles, or at exponentially large energies) – $\forall M_H$.

Four interesting channels:

$$W_L^+ W_L^- \quad Z_L^0 Z_L^0/\sqrt{2} \quad HH/\sqrt{2} \quad HZ_L^0$$

L: longitudinal, $1/\sqrt{2}$ for identical particles

In HE limit,[1] s-wave amplitudes $\propto G_F M_H^2$

$$\lim_{s \gg M_H^2} (a_0) \to \frac{-G_F M_H^2}{4\pi\sqrt{2}} \cdot \begin{bmatrix} 1 & 1/\sqrt{8} & 1/\sqrt{8} & 0 \\ 1/\sqrt{8} & 3/4 & 1/4 & 0 \\ 1/\sqrt{8} & 1/4 & 3/4 & 0 \\ 0 & 0 & 0 & 1/2 \end{bmatrix}$$

Require that largest eigenvalue respect pw unitarity condition $|a_0| \leq 1$

$$\implies \quad M_H \leq \left(\frac{8\pi\sqrt{2}}{3 G_F} \right)^{1/2} = 1 \text{ TeV}/c^2$$

condition for perturbative unitarity

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[1]Convenient to calculate using *Goldstone-boson equivalence theorem*, which reduces dynamics of longitudinally polarized gauge bosons to scalar field theory with interaction Lagrangian given by $\mathcal{L}_{int} = -\lambda v h(2w^+ w^- + z^2 + h^2) - (\lambda/4)(2w^+ w^- + z^2 + h^2)^2$, with $1/v^2 = G_F \sqrt{2}$ and $\lambda = G_F M_H^2 / \sqrt{2}$.

- If the bound is respected
  - weak interactions remain weak at all energies
  - perturbation theory is everywhere reliable

- If the bound is violated
  - perturbation theory breaks down
  - weak interactions among $W^{\pm}$, Z, H become strong on 1-TeV scale

  $\Rightarrow$ features of *strong* interactions at GeV energies will characterize *electroweak* gauge boson interactions at TeV energies

  New phenomena are to be found in the EW interactions at energies not much larger than 1 TeV

Threshold behavior of the pw amplitudes a_{IJ} follows from chiral symmetry

$$a_{00} \approx \quad G_F s/8\pi\sqrt{2} \qquad \text{attractive}$$
$$a_{11} \approx \quad G_F s/48\pi\sqrt{2} \qquad \text{attractive}$$
$$a_{20} \approx \quad -G_F s/16\pi\sqrt{2} \qquad \text{repulsive}$$

Lee, Quigg, Thacker, *Phys. Rev. D* **16,** 1519 (1977)

What the LHC *is not really* for . . .

- Find the Higgs boson,
  the Holy Grail of particle physics,
  the source of all mass in the Universe.

- Celebrate.

- Then particle physics will be over.

  We are not ticking off items on a shopping list . . .

We are exploring a vast new terrain
  . . . and reaching the Fermi scale

The Origins of Mass

(masses of nuclei "understood")

$p, [\pi], \rho$ understood: QCD

confinement energy is the source

"Mass without mass" Wilczek, *Phys. Today* (November 1999)

We understand the visible mass of the Universe
. . . without the Higgs mechanism

W, Z electroweak symmetry breaking
$$M_W^2 = \tfrac{1}{2}g^2 v^2 = \pi\alpha/G_F\sqrt{2}\sin^2\theta_W$$
$$M_Z^2 = M_W^2/\cos^2\theta_W$$

$q, \ell^{\mp}$ EWSB + Yukawa couplings

ν_ℓ EWSB + Yukawa couplings; new physics?

All fermion masses $\Leftrightarrow$ physics beyond standard model

H ?? fifth force ??

Challenge: Understanding the Everyday

- Why are there atoms?
- Why chemistry?
- Why stable structures?
- What makes life possible?

What would the world be like, without a (Higgs) mechanism to hide electroweak symmetry and give masses to the quarks and leptons?

Searching for the mechanism of electroweak symmetry breaking, we seek to understand

why the world is the way it is.

This is one of the deepest questions humans have ever pursued, and

it is coming within the reach of particle physics.

Our picture of matter
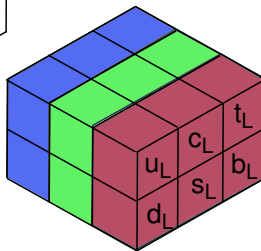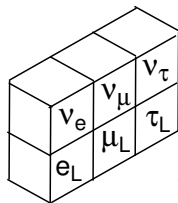
Pointlike constituents ($r < 10^{-18}$ m)

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \qquad \begin{pmatrix} c \\ s \end{pmatrix}_L \qquad \begin{pmatrix} t \\ b \end{pmatrix}_L$$
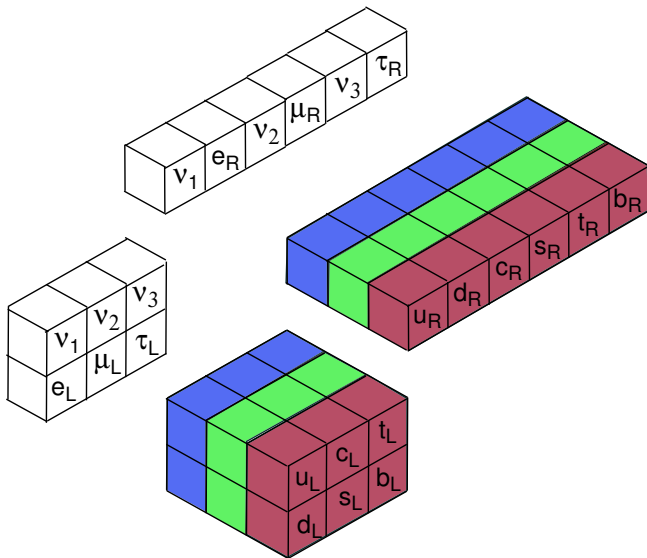
$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \qquad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \qquad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L$$

Few fundamental forces, derived from gauge symmetries

$$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$$

Electroweak symmetry breaking: Higgs mechanism?

Formulate electroweak theory

Three crucial clues from experiment:

- Left-handed weak-isospin doublets,

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \qquad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \qquad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$$

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L \qquad \begin{pmatrix} c \\ s' \end{pmatrix}_L \qquad \begin{pmatrix} t \\ b' \end{pmatrix}_L ;$$

- Universal strength of the (charged-current) weak interactions;
- Idealization that neutrinos are massless.

> First two clues suggest $SU(2)_L$ gauge symmetry

A theory of leptons

$$L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \qquad R \equiv e_R$$

weak hypercharges $Y_L = -1$, $Y_R = -2$
Gell-Mann–Nishijima connection, $Q = I_3 + \frac{1}{2}Y$

$SU(2)_L \otimes U(1)_Y$ gauge group $\Rightarrow$ gauge fields:

- weak isovector $\vec{b}_\mu$, coupling g $\qquad \boxed{b_\mu^\ell = b_\mu^\ell - \varepsilon_{jk\ell}\alpha^j b_\mu^k - (1/g)\partial_\mu\alpha^\ell}$

- weak isoscalar $\mathcal{A}_\mu$, coupling $g'/2$ $\qquad \boxed{\mathcal{A}_\mu \to \mathcal{A}_\mu - \partial_\mu\alpha}$

Field-strength tensors

$$F_{\mu\nu}^\ell = \partial_\nu b_\mu^\ell - \partial_\mu b_\nu^\ell + g\varepsilon_{jk\ell}b_\mu^j b_\nu^k \,, SU(2)_L$$

$$f_{\mu\nu} = \partial_\nu \mathcal{A}_\mu - \partial_\mu \mathcal{A}_\nu \,, U(1)_Y$$

Interaction Lagrangian

$$\mathcal{L} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{leptons}}$$

$$\mathcal{L}_{\text{gauge}} = -\tfrac{1}{4} F^{\ell}_{\mu\nu} F^{\ell\mu\nu} - \tfrac{1}{4} f_{\mu\nu} f^{\mu\nu},$$

$$
\begin{aligned}
\mathcal{L}_{\text{leptons}} &= \overline{\mathsf{R}}\, i\gamma^{\mu}\left(\partial_{\mu} + i\frac{g'}{2}\mathcal{A}_{\mu}Y\right)\mathsf{R} \\
&+ \overline{\mathsf{L}}\, i\gamma^{\mu}\left(\partial_{\mu} + i\frac{g'}{2}\mathcal{A}_{\mu}Y + i\frac{g}{2}\vec{\tau}\cdot\vec{b}_{\mu}\right)\mathsf{L}.
\end{aligned}
$$

Mass term $\mathcal{L}_e = -m_e(\bar{e}_R e_L + \bar{e}_L e_R) = -m_e \bar{e} e$ violates local gauge inv.

Theory: 4 massless gauge bosons ($\mathcal{A}_{\mu}\quad b^1_{\mu}\quad b^2_{\mu}\quad b^3_{\mu}$); Nature: 1 ($\gamma$)
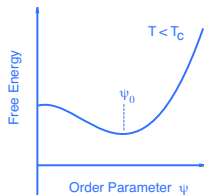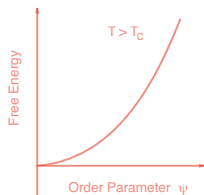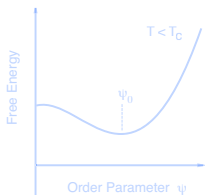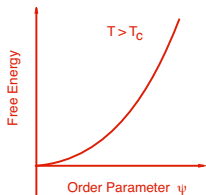
Massive Photon?   *Hiding Symmetry*

Recall **2** miracles of superconductivity:

- No resistance . . .   . . . Meissner effect (exclusion of **B**)

  Ginzburg–Landau Phenomenology (not a theory from first principles)

normal, resistive charge carriers . . .   . . . + superconducting charge carriers

$$\mathbf{B} = 0: \qquad G_{\text{super}}(0) = G_{\text{normal}}(0) + \alpha \, |\psi|^2 + \beta \, |\psi|^4$$

$$T > T_c : \quad \alpha > 0 \quad \langle |\psi|^2 \rangle_0 = 0$$

$$T < T_c : \quad \alpha < 0 \quad \langle |\psi|^2 \rangle_0 \neq 0$$

In a nonzero magnetic field . . .

$$G_{\mathsf{super}}(\mathbf{B}) = G_{\mathsf{super}}(0) + \frac{\mathbf{B}^2}{8\pi} + \frac{1}{2m^*} \left| -i\hbar\nabla\psi - \frac{e^*}{c}\mathbf{A}\psi \right|^2$$

$$\left. \begin{array}{c} e^* = -2 \\ m^* \end{array} \right\} \text{ of superconducting carriers}$$

Weak, slowly varying field: $\quad \psi \approx \psi_0 \neq 0, \ \nabla\psi \approx 0$

Variational analysis $\leadsto$

$$\nabla^2\mathbf{A} - \frac{4\pi e^{*2}}{m^* c^2} \left|\psi_0\right|^2 \mathbf{A} = 0$$

wave equation of a *massive photon*

Photon – *gauge boson* – acquires mass
within superconductor

origin of Meissner effect

Meissner effect levitates Leon Lederman (Snowmass 2001)

Hiding EW Symmetry

Higgs mechanism: relativistic generalization of Ginzburg-Landau superconducting phase transition

- Introduce a complex doublet of scalar fields

$$\phi \equiv \left(\begin{array}{c} \phi^+ \\ \phi^0 \end{array} \right) \quad Y_\phi = +1$$

- Add to $\mathcal{L}$ (gauge-invariant) terms for interaction and propagation of the scalars,

$$\mathcal{L}_{\text{scalar}} = (\mathcal{D}^\mu \phi)^\dagger (\mathcal{D}_\mu \phi) - V(\phi^\dagger \phi),$$

where $\mathcal{D}_\mu = \partial_\mu + i\frac{g'}{2}\mathcal{A}_\mu Y + i\frac{g}{2}\vec{\tau}\cdot\vec{b}_\mu$ and

$$\boxed{V(\phi^\dagger \phi) = \mu^2(\phi^\dagger \phi) + |\lambda|\,(\phi^\dagger \phi)^2}$$

- Add a Yukawa interaction $\mathcal{L}_{\text{Yukawa}} = -\zeta_e \left[\overline{\mathsf{R}}(\phi^\dagger \mathsf{L}) + (\overline{\mathsf{L}}\phi)\mathsf{R} \right]$

- Arrange self-interactions so vacuum corresponds to a broken-symmetry solution: $\mu^2 < 0$

  Choose minimum energy (vacuum) state for vacuum expectation value

$$\langle\phi\rangle_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}, \quad v = \sqrt{-\mu^2/|\lambda|}$$

Hides (breaks) $SU(2)_L$ and $U(1)_Y$

but preserves $U(1)_{em}$ invariance

Invariance under $\mathcal{G}$ means $e^{i\alpha\mathcal{G}}\langle\phi\rangle_0 = \langle\phi\rangle_0$, so $\mathcal{G}\langle\phi\rangle_0 = 0$
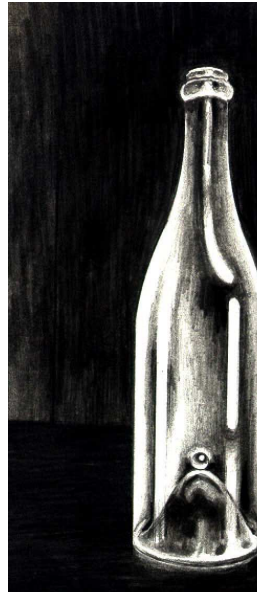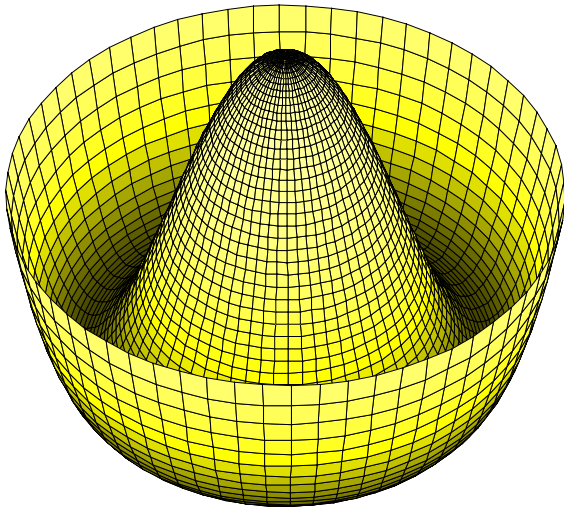
$$\tau_1\langle\phi\rangle_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix} \neq 0 \quad \text{broken!}$$

$$\tau_2\langle\phi\rangle_0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} -iv/\sqrt{2} \\ 0 \end{pmatrix} \neq 0 \quad \text{broken!}$$

$$\tau_3\langle\phi\rangle_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 \\ -v/\sqrt{2} \end{pmatrix} \neq 0 \quad \text{broken!}$$

$$Y\langle\phi\rangle_0 = Y_\phi\langle\phi\rangle_0 = +1\langle\phi\rangle_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \neq 0 \quad \text{broken!}$$

Symmetry of laws $\neq$ symmetry of outcomes

Examine electric charge operator Q on the (neutral) vacuum

$$
\begin{aligned}
Q\langle\phi\rangle_0 &= \tfrac{1}{2}(\tau_3 + Y)\langle\phi\rangle_0 \\
&= \tfrac{1}{2}\begin{pmatrix} Y_\phi + 1 & 0 \\ 0 & Y_\phi - 1 \end{pmatrix}\langle\phi\rangle_0 \\
&= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \\
&= \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \text{\textit{unbroken!}}
\end{aligned}
$$

Four original generators are broken, *electric charge is not*

- $SU(2)_L \otimes U(1)_Y \rightarrow U(1)_{em}$ (will verify)
- Expect massless photon
- Expect gauge bosons corresponding to

$$\tau_1, \ \tau_2, \ \tfrac{1}{2}(\tau_3 - Y) \equiv K \quad \text{to acquire masses}$$

- Electromagnetism is mediated by a massless photon, coupled to the electric charge;
- Mediator of charged-current weak interaction acquires a mass $M_W^2 = \pi\alpha/G_F\sqrt{2}\sin^2\theta_W$,
- Mediator of (new!) neutral-current weak interaction acquires mass $M_Z^2 = M_W^2/\cos^2\theta_W$;
- Massive neutral scalar particle, the Higgs boson, appears, but its mass is not predicted;
- Fermions can acquire mass—values not predicted.

Successful predictions of $SU(2)_L \otimes U(1)_Y$ theory:

- neutral-current interactions
- necessity of charm
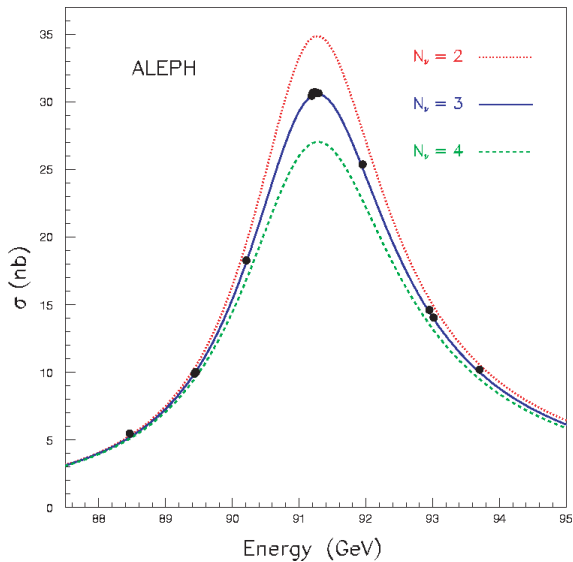- existence and properties of $W^{\pm}$ and Z^0

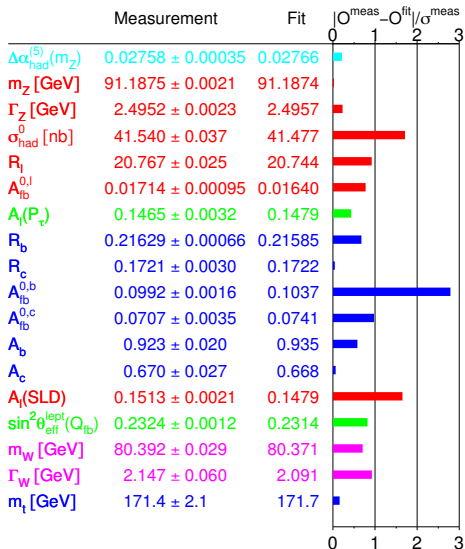$+$ a decade of precision EW tests (one-per-mille)

M_Z	$91\,187.6 \pm 2.1$ MeV/c^2
Γ_Z	2495.2 ± 2.3 MeV
$\sigma^0_{\text{hadronic}}$	41.541 ± 0.037 nb
Γ_{hadronic}	1744.4 ± 2.0 MeV
Γ_{leptonic}	83.984 ± 0.086 MeV
$\Gamma_{\text{invisible}}$	499.0 ± 1.5 MeV

$\Gamma_{\text{invisible}} \equiv \Gamma_Z - \Gamma_{\text{hadronic}} - 3\Gamma_{\text{leptonic}}$

light ν : $N_\nu = \Gamma_{\text{invisible}}/\Gamma^{\text{SM}}(Z \to \nu_i \bar{\nu}_i) = 2.994 \pm 0.012 \quad (\nu_e, \nu_\mu, \nu_\tau)$

Three light neutrinos

|	Measurement	Fit	$\|O^{meas} - O^{fit}\| / \sigma^{meas}$
$\Delta\alpha_{had}^{(5)}(m_Z)$	0.02758 ± 0.00035	0.02766	
m_Z [GeV]	91.1875 ± 0.0021	91.1874	
Γ_Z [GeV]	2.4952 ± 0.0023	2.4957	
σ_{had}^0 [nb]	41.540 ± 0.037	41.477	
R_l	20.767 ± 0.025	20.744	
$A_{fb}^{0,l}$	0.01714 ± 0.00095	0.01640	
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1479	
R_b	0.21629 ± 0.00066	0.21585	
R_c	0.1721 ± 0.0030	0.1722	
$A_{fb}^{0,b}$	0.0992 ± 0.0016	0.1037	
$A_{fb}^{0,c}$	0.0707 ± 0.0035	0.0741	
A_b	0.923 ± 0.020	0.935	
A_c	0.670 ± 0.027	0.668	
A_l(SLD)	0.1513 ± 0.0021	0.1479	
$\sin^2\theta_{eff}^{lept}(Q_{fb})$	0.2324 ± 0.0012	0.2314	
m_W [GeV]	80.392 ± 0.029	80.371	
Γ_W [GeV]	2.147 ± 0.060	2.091	
m_t [GeV]	171.4 ± 2.1	171.7	

LEP Electroweak Working Group, Summer 2006

Why a Higgs boson must exist

▷ Role in canceling high-energy divergences

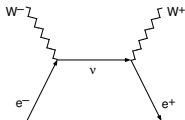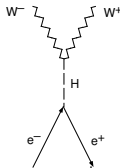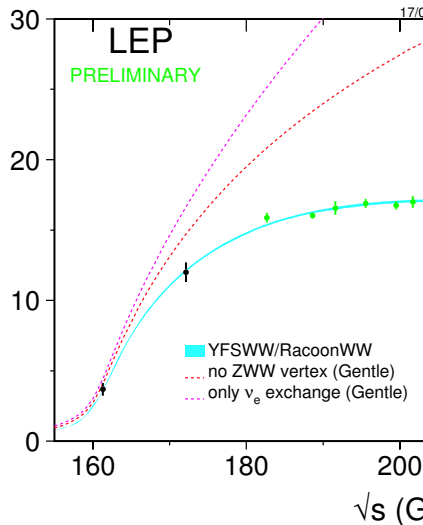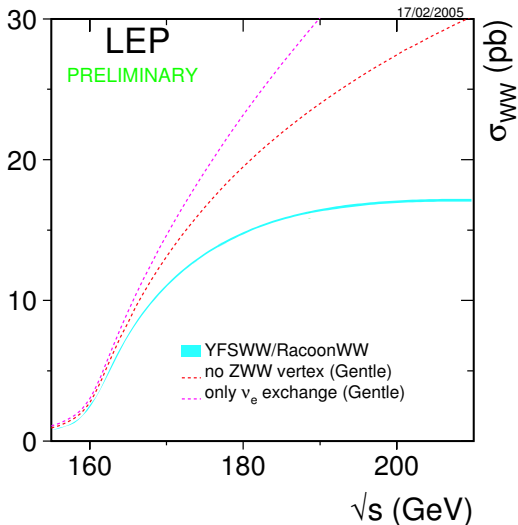S-matrix analysis of $e^+e^- \rightarrow W^+W^-$
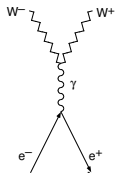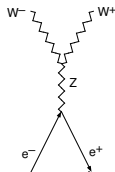
Individual $J = 1$ partial-wave amplitudes $\mathcal{M}_\gamma^{(1)}$, $\mathcal{M}_Z^{(1)}$, $\mathcal{M}_\nu^{(1)}$ have unacceptable high-energy behavior ($\propto s$)

. . . But sum is well-behaved

"Gauge cancellation" observed at LEP2 (Tevatron)

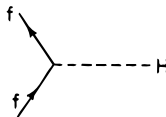$J = 0$ amplitude exists because electrons have mass, and can be found in "wrong" helicity state

$$\mathcal{M}_\nu^{(0)} \propto s^{\frac{1}{2}} \; : \text{unacceptable HE behavior}$$

(no contributions from γ and Z)

This divergence is canceled by the Higgs-boson contribution

$$\boxed{\Rightarrow H e \bar{e} \text{ coupling must be } \propto m_e,}$$

because "wrong-helicity" amplitudes $\propto m_e$

$$\frac{-i m_f}{v} = -i m_f (G_F \sqrt{2})^{1/2}$$

If the Higgs boson did not exist, something else would have to cure divergent behavior

If gauge symmetry were unbroken ...

- no Higgs boson
- no longitudinal gauge bosons
- no extreme divergences
- no wrong-helicity amplitudes

                    ... and no viable low-energy phenomenology

In spontaneously broken theory ...

- gauge structure of couplings eliminates the most severe divergences
- lesser—but potentially fatal—divergence arises because the electron has mass ... due to the Higgs mechanism
- SSB provides its own cure—the Higgs boson

Similar interplay & compensation *must exist* in any acceptable theory

Why hadron colliders?

Rich diversity of elementary processes at high energy

Benchmark: $q\bar{q}$ interactions at 1 TeV ...

$$\langle x \rangle = \tfrac{1}{6} \rightsquigarrow pp \text{ collisions at } \sqrt{s} \approx 6 \text{ TeV}$$

Fixed-target: $p \approx 2 \times 10^4 \text{ TeV} = 2 \times 10^{16} \text{ eV}$
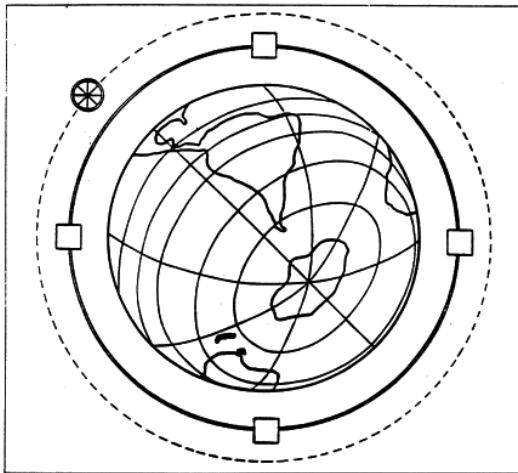
$$r = \frac{10}{3} \cdot \left(\frac{p}{1 \text{ TeV}} \right) / \left(\frac{B}{1 \text{ tesla}} \right) \text{ km.}$$

$$B = 2 \text{ T (iron magnets)} \Rightarrow r = \tfrac{1}{3} \times 10^5 \text{ km.}$$

$$\approx \tfrac{1}{12} \times \text{ lunar orbit!}$$

SC magnets (10 T) $\Rightarrow r \approx R_{\oplus} = 6.4 \times 10^3$ km

Fermi's Dream Machine (1954)

From a 1954 Slide by Enrico Fermi, University of Chicago Special Collections.

5 000-TeV protons, 2-tesla magnets, 8 000-km radius, $\$1.7 \times 10^9$, 40 years

Breakthrough: Colliding beams!

To reach $3 \oplus 3$ TeV, require

$$r_{3\text{ TeV}} = \frac{10\text{ T}}{B}\text{ km}.$$

$\times 2$ (straight sections, quads, correctors) . . .

10-T dipoles: radius of practical machine ≈ 2 km
$\approx 2\times$Tevatron

SC magnets greatly reduce operating cost

Key advances in accelerator technology

- The idea of colliding beams
- Alternating-gradient ("strong") focusing
- Superconducting accelerator magnets
- Vacuum technology: in 20 hours, protons travel $\approx 2 \times 10^{10}$ km, $\approx 150 \times$ Earth – Sun
- Large-scale cryogenic technology
- Active optics
- Intense antiproton sources; beam cooling

Competing technologies?

- None for quark–gluon interactions
- None for highest energies (derate composite protons)
- Lepton–lepton collisions: LEP ($\sqrt{s} \approx 0.2$ TeV) was the last great electron synchrotron?

  Synchrotron radiation $\Rightarrow$ linear colliders for higher $\sqrt{s}$ $\rightsquigarrow$ International Linear Collider

  - Challenge to reach 1 TeV; $\mathcal{L}$ *a great challenge*
  - Can we surpass 1 TeV? CLIC, . . .

Competing technologies?

- Lepton–hadron collisions: HERA ($e^{\pm}p$) as example; energy intermediate between e^+e^-, pp

  $e^{\pm}(u, d)$ leptoquark channel, proton structure, γp
  High $\mathcal{L}$ a challenge: beam profiles don't match

  (Far) future: $\mu^{\pm}p$ collider?

- Heavy-ion collisions: RHIC the prototype; LHC (relatively) modest energy per nucleon; quark-gluon plasma; new phases of matter

Unorthodox projectiles?

- $\gamma\gamma$ Collider: Backscattered laser beams; enhancement of linear collider capabilities
- $\mu^+\mu^-$ collider: Advantage of elementary particle, disadvantage of muon decay (2.2μs).

  Small ring to reach very high effective energies?

  Muon storage ring (neutrino factory) would turn bug into feature!

What is a proton?

(For hard scattering) a broad-band, unselected beam of quarks, antiquarks, gluons, and perhaps other constituents characterized by parton densities

$$f_i^{(a)}(x_a, Q^2),$$

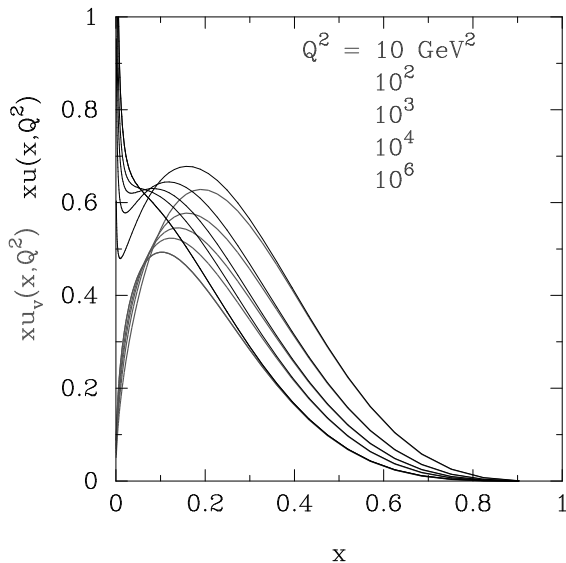. . . number density of species i with momentum fraction x_a of hadron a seen by probe with resolving power Q^2.

Q^2 evolution given by QCD perturbation theory
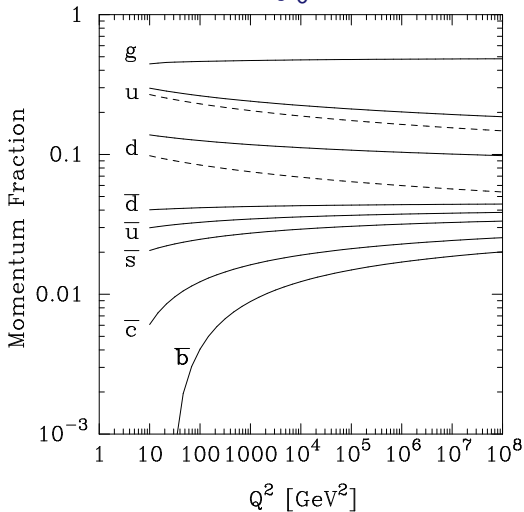
$f_i^{(a)}(x_a, Q_0^2)$: nonperturbative

PDFs determined from deeply inelastic scattering ...

What is a proton?

$Q^2 = 10 \text{ GeV}^2$
10^2
10^3
10^4
10^6
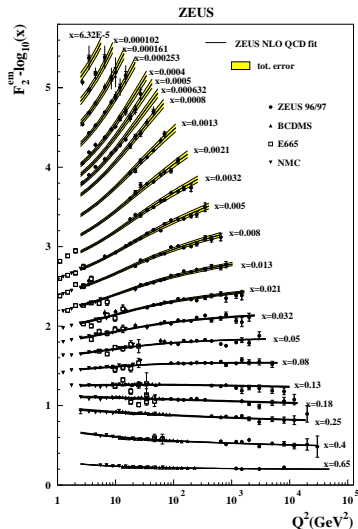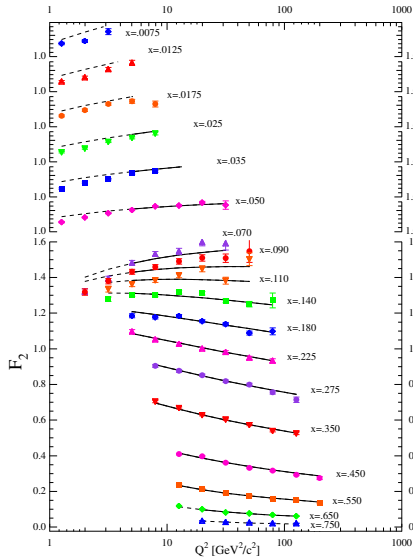
Flavor content of the proton: $\displaystyle\int_0^1 dx\, x\, f_i(x, Q^2)$

Asymptotic limit ($Q^2 \to \infty$): $g : \frac{8}{17}$; $q_s : \frac{3}{68}$; $q_v : 0$

Hard-scattering cross sections

$$d\sigma(a + b \to c + X) = \sum_{ij} \int dx_a dx_b \cdot$$

$$f_i^{(a)}(x_a, Q^2) f_j^{(b)}(x_b, Q^2) d\hat{\sigma}(i + j \to c + X),$$

$d\hat{\sigma}$: elementary cross section at energy $\sqrt{\hat{s}} = \sqrt{x_a x_b s}$

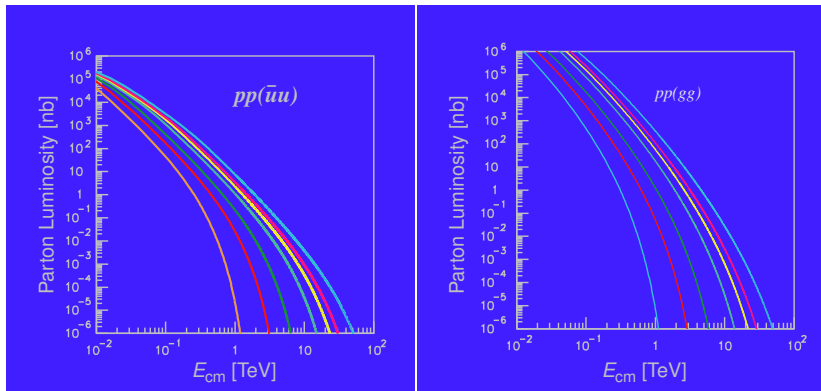Define differential luminosity ($\tau = \hat{s}/s$)

$$\frac{d\mathcal{L}}{d\tau} = \frac{1}{1 + \delta_{ij}} \int_\tau^1 dx \left[f_i^{(a)}(x) f_j^{(b)}(\tau/x) + f_j^{(a)}(x) f_i^{(b)}(\tau/x) \right]$$

parton i–parton j collisions in $(\tau, \tau + d\tau)$ per ab collision

$$d\sigma(a + b \to c + X) = \sum_{ij} \frac{d\mathcal{L}_{ij}}{d\tau} \hat{\sigma}(i + j \to c + X)$$

Hard scattering: $\hat{\sigma} \propto 1/\hat{s}$; Resonance: $\hat{\sigma} \propto \tau$; form $(\tau/\hat{s}) d\mathcal{L}/d\tau$

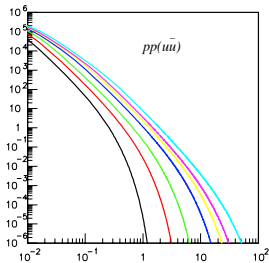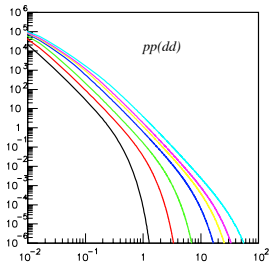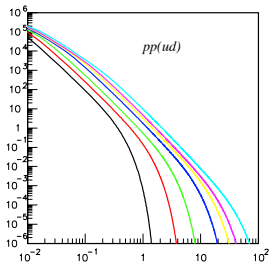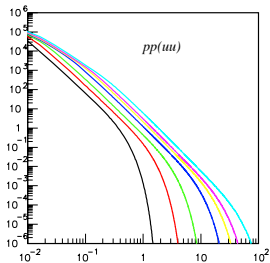# Parton Luminosities $(\tau/\hat{s})d\mathcal{L}/d\tau$

at $\sqrt{s} = 2, 6, 14, 40, 70, 100, 200$ TeV

Background: E. Eichten, I. Hinchliffe, K. Lane, and C. Quigg, *Rev. Mod. Phys.* **56**, 579 (1984). (CTEQ5 parton distributions)
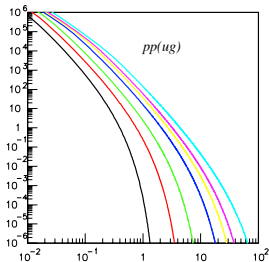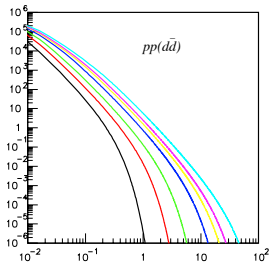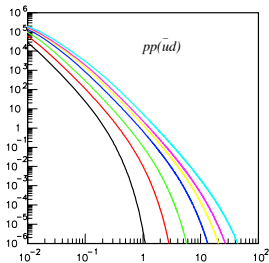
Parton Luminosities . . .
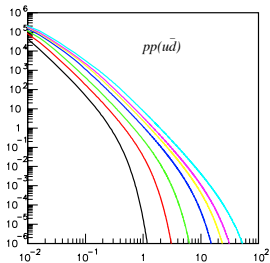
CTEQ5M set

Parton Luminosities . . .

CTEQ5M set
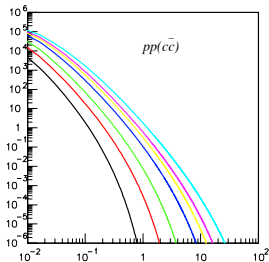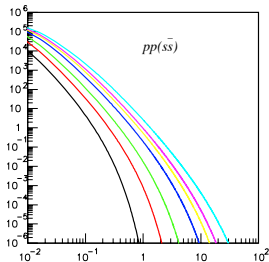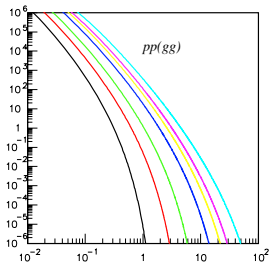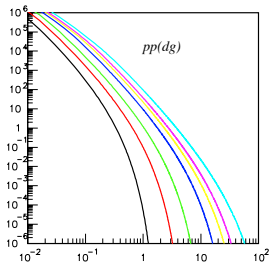
CTEQ5M set

Parton Luminosities . . .
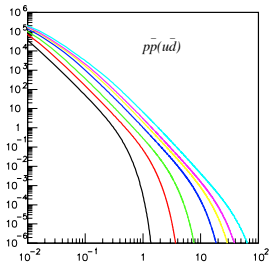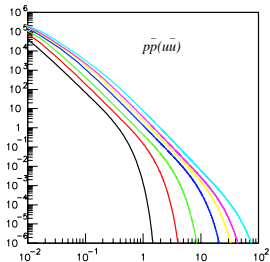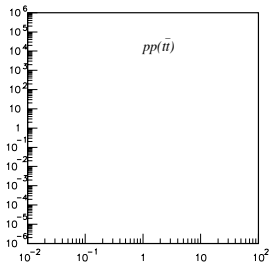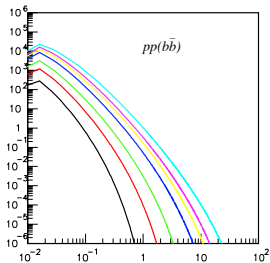
CTEQ5M set

Parton Luminosities . . .

CTEQ5M set
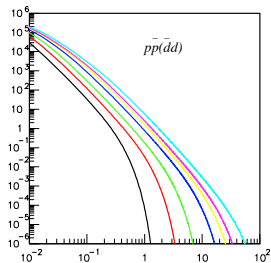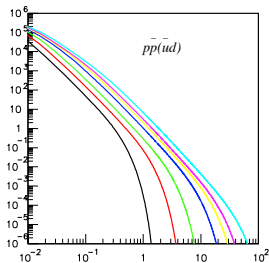
Higgs boson: the missing element of electroweak theory

Bounding M_H from above . . .

Triviality of scalar field theory

- Only *noninteracting* scalar field theories make sense on all energy scales
- Quantum field theory vacuum is a dielectric medium that screens charge
- $\Rightarrow$ *effective charge* is a function of the distance or, equivalently, of the energy scale

$$\boxed{\text{running coupling constant}}$$

In $\lambda\phi^4$ theory, calculate variation of coupling constant λ in perturbation theory by summing bubble graphs
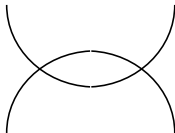
$\lambda(\mu)$ is related to a higher scale Λ by

$$\frac{1}{\lambda(\mu)} = \frac{1}{\lambda(\Lambda)} + \frac{3}{2\pi^2}\log\left(\Lambda/\mu\right)$$

(Perturbation theory reliable only when λ is small, lattice field theory treats strong-coupling regime)

For stable Higgs potential (*i.e.*, for vacuum energy not to race off to $-\infty$), *require* $\lambda(\Lambda) \geq 0$

Rewrite RGE as an inequality

$$\frac{1}{\lambda(\mu)} \geq \frac{3}{2\pi^2} \log\left(\Lambda/\mu\right)$$

. . . implies an *upper bound*

$$\lambda(\mu) \leq 2\pi^2/3 \log\left(\Lambda/\mu\right)$$

If we require the theory to make sense to arbitrarily high energies—or short distances—then we must take the limit $\Lambda \to \infty$ while holding μ fixed at some reasonable physical scale. In this limit, the bound forces $\lambda(\mu)$ to zero.

$\longrightarrow$ free field theory "trivial"

Rewrite as bound on M_H:

$$\Lambda \leq \mu \exp\left(\frac{2\pi^2}{3\lambda(\mu)}\right)$$

Choose $\mu = M_H$, and recall $M_H^2 = 2\lambda(M_H)v^2$

$$\boxed{\Lambda \leq M_H \exp\left(4\pi^2 v^2 / 3 M_H^2\right)}$$

Higgs-boson Mass (GeV) — vertical axis, 0 to 600

Higgs interactions vanish

quantum corrections disfavor

excluded by direct searches

electroweak symmetry not hidden

10^3 10^5 10^7 10^9 10^{11} 10^{13} 10^{15} 10^{17} 10^{19}
energy to which electroweak theory holds (GeV)
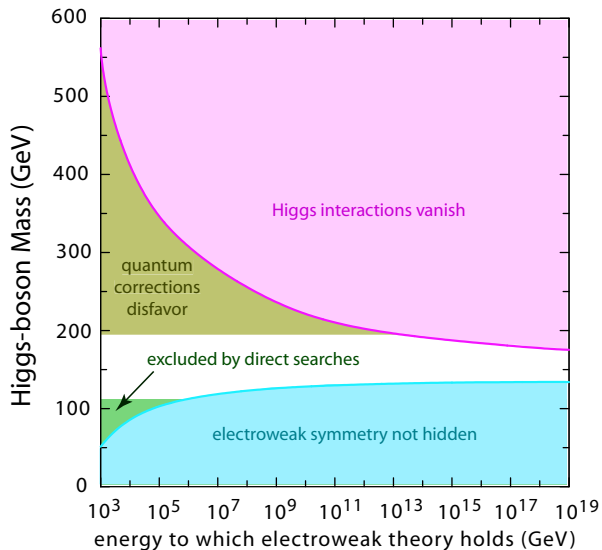
Moral: For any M_H, there is a *maximum energy scale* $\Lambda^\star$ at which the theory ceases to make sense.

The description of the Higgs boson as an elementary scalar is at best an effective theory, valid over a finite range of energies

Perturbative analysis breaks down when $M_H \to 1$ TeV/c^2 and interactions become strong

Lattice analyses $\implies M_H \lesssim 710 \pm 60$ GeV/c^2 if theory describes physics to a few percent up to a few TeV

If $M_H \to 1$ TeV EW theory lives on brink of instability

Lower bound by requiring EWSB vacuum $V(v) < V(0)$

Requiring that $\langle\phi\rangle_0 \neq 0$ be an absolute minimum of the one-loop potential up to a scale Λ yields the vacuum-stability condition ... (for $m_t \lesssim M_W$)

$$M_H^2 > \frac{3 G_F \sqrt{2}}{8\pi^2}(2M_W^4 + M_Z^4 - 4m_t^4) \log(\Lambda^2/v^2)$$

(No illuminating analytic form for heavy m_t)

If the Higgs boson is relatively light (which would require explanation) then the theory can be self-consistent up to very high energies
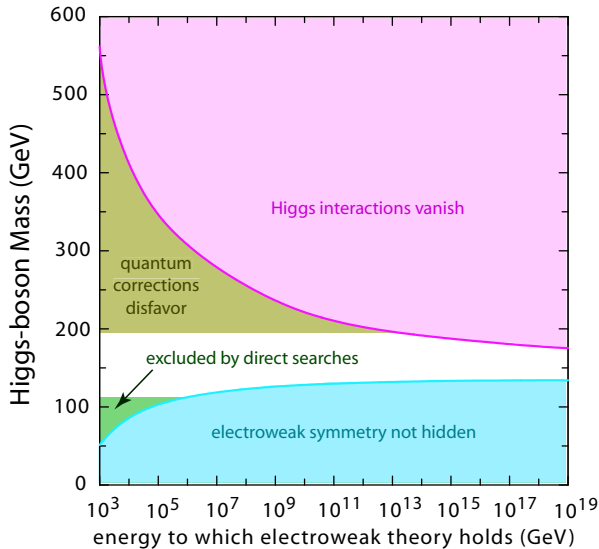
If EW theory is to make sense all the way up to a unification scale $\Lambda^\star = 10^{16}$ GeV, then 134 GeV/$c^2 \lesssim M_H \lesssim 177$ GeV

Higgs-Boson Properties

$$\Gamma(H \to f\bar{f}) = \frac{G_F m_f^2 M_H}{4\pi\sqrt{2}} \cdot N_c \cdot \left(1 - \frac{4m_f^2}{M_H^2}\right)^{3/2}$$

$\propto M_H$ in the limit of large Higgs mass; $\propto \beta^3$ for scalar

$$\Gamma(H \to W^+ W^-) = \frac{G_F M_H^3}{32\pi\sqrt{2}}(1-x)^{1/2}(4 - 4x + 3x^2) \quad x \equiv 4M_W^2/M_H^2$$

$$\Gamma(H \to Z^0 Z^0) = \frac{G_F M_H^3}{64\pi\sqrt{2}}(1-x')^{1/2}(4 - 4x' + 3x'^2) \quad x' \equiv 4M_Z^2/M_H^2$$

asymptotically $\propto M_H^3$ and $\frac{1}{2}M_H^3$, respectively   ($\frac{1}{2}$ from weak isospin)

$2x^2$ and $2x'^2$ terms $\Leftrightarrow$ decays into transverse gauge bosons
Dominant decays for large M_H: pairs of longitudinal weak bosons
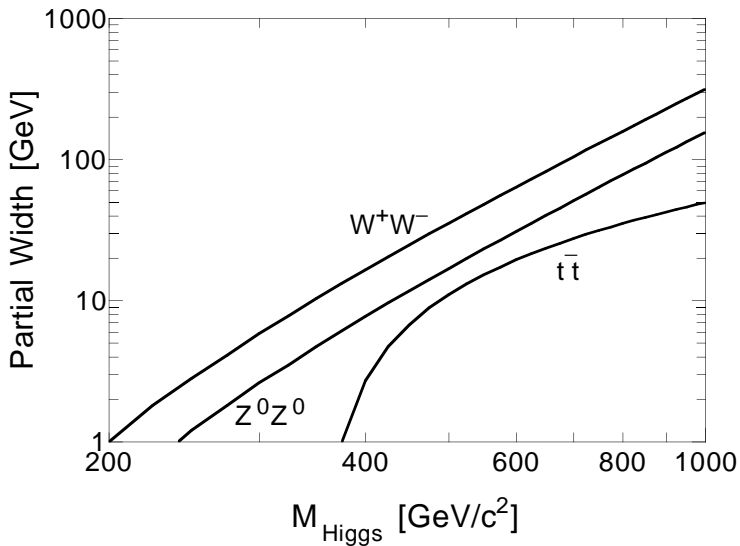
For $M_H \rightarrow 1$ TeV, Higgs boson is *ephemeral:* $\Gamma_H \rightarrow M_H$.

SM Higgs branching ratios (HDECAY)

m_H (GeV/c^2)
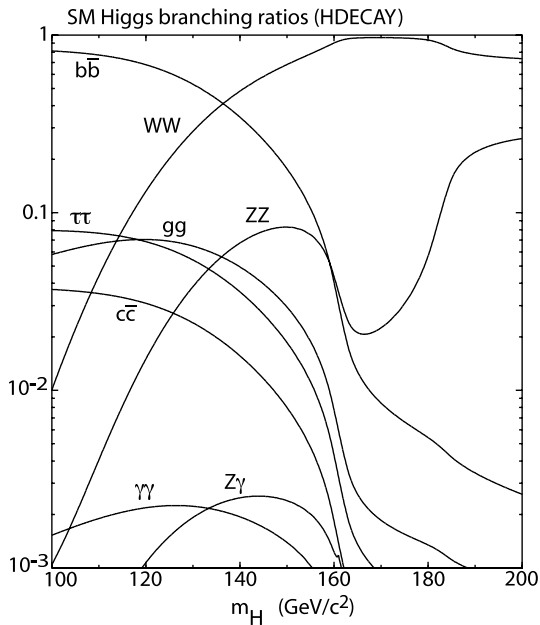
ILC would measure light Higgs-boson couplings precisely
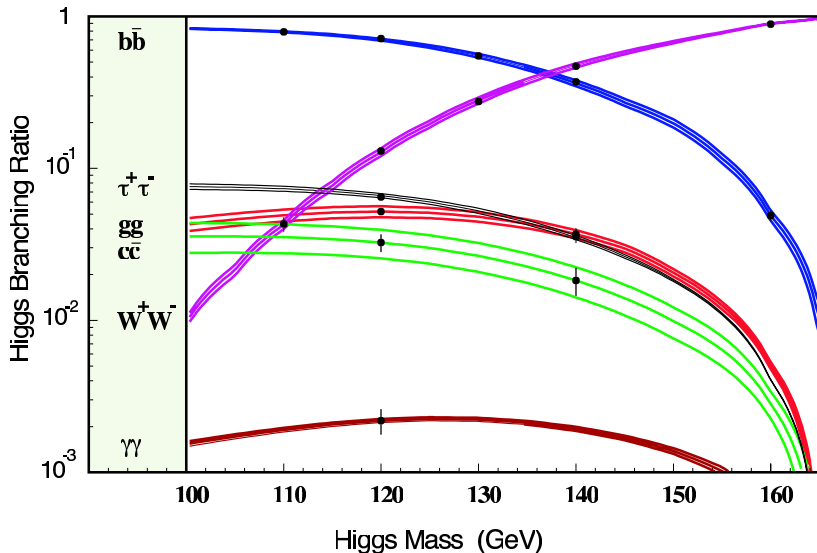
Points: 500 fb^{-1} @ 350 GeV   Bands: theory uncertainty (m_b)
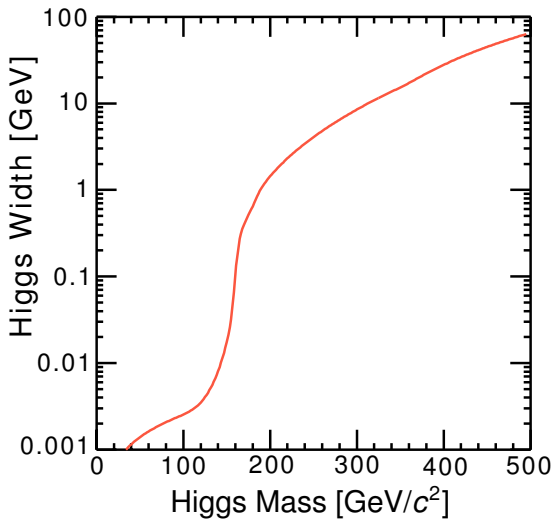
Below $W^+ W^-$ threshold, $\Gamma_H \lesssim 1$ GeV

Far above $W^+ W^-$ threshold, $\Gamma_H \propto M_H^3$

Experimental clues to the Higgs-boson mass

Sensitivity of EW observables to m_t gave early indications for massive top
Quantum corrections to SM predictions for M_W and M_Z arise from
different quark loops
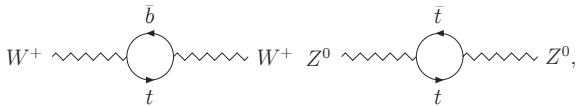
$$\ldots \text{alter the link} \underbrace{M_W^2}_{(80.392 \pm 0.029 \text{ GeV})^2} = \underbrace{M_Z^2 \left(1 - \sin^2\theta_W\right)}_{(80.939 \text{ GeV})^2} (1 - \Delta\rho)$$

where $\Delta\rho \approx \Delta\rho^{(\text{quarks})} = 3 G_F m_t^2 / 8\pi^2 \sqrt{2}$

Strong dependence on m_t^2 accounts for precision of m_t estimates derived
from EW observables

*Tevatron: $\delta m_t / m_t \approx 1.28\%$ ... Look beyond quark loops to next
most important quantum corrections:* $\boxed{\text{Higgs-boson effects}}$

Global fits to precision EW measurements

- precision improves with time / calculations improve with time
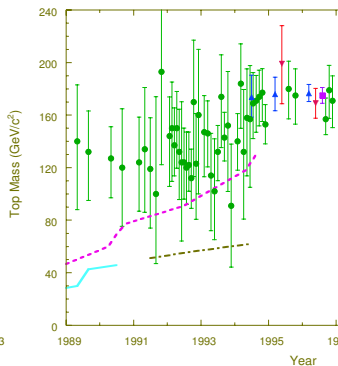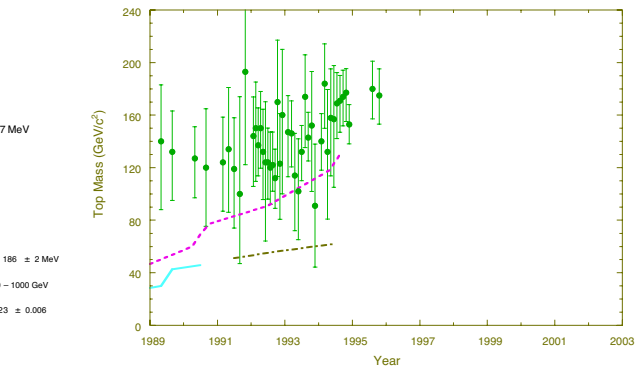
11.94, LEPEWWG: $m_t = 178 \pm 11^{+18}_{-19}$ GeV/c^2

Direct measurements: $m_t = 171.4 \pm 2.2$ GeV/c^2

H quantum corrections smaller than t corrections, exhibit more subtle dependence on M_H than the m_t^2 dependence of the top-quark corrections

$$\Delta\rho^{(\text{Higgs})} = \mathcal{C} \cdot \ln\left(\frac{M_H}{v}\right)$$

M_Z known to 23 ppm, m_t and M_W well measured
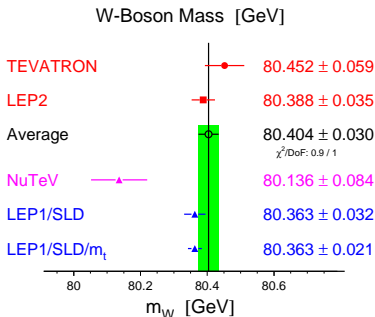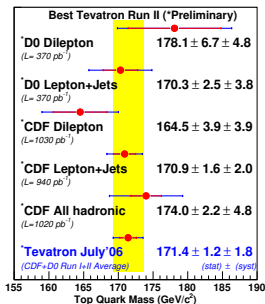
... so examine dependence of M_W upon m_t and M_H

CDF's top-mass projections . . .

CDF Top Mass Uncertainty
(l+l and l+j channels combined)

1 fb^{-1} 2 fb^{-1} 4 fb^{-1} 8 fb^{-1}

ΔM/M < 1%

▼ CDF Results

★ Run IIa goal (TDR 1996)

—— Scale Δ(stat) / $\sqrt{L}$, Fix Δ(syst)
(assumes no improvements)

-·-·- Scale Δ(total) / $\sqrt{L}$
(improvements required)

Δ M$_t$(total) GeV/c^2

Integrated Luminosity (pb^{-1})
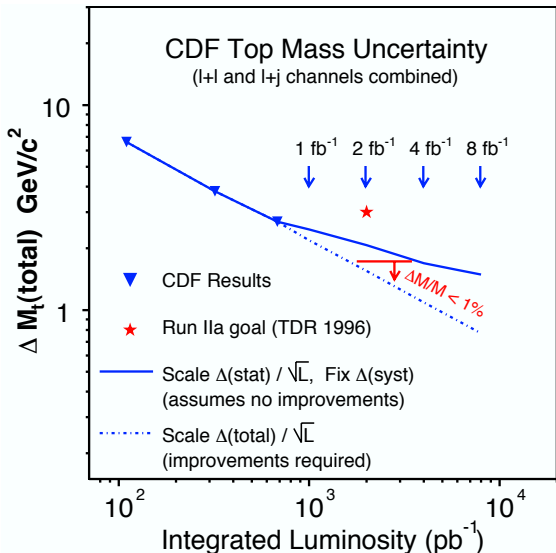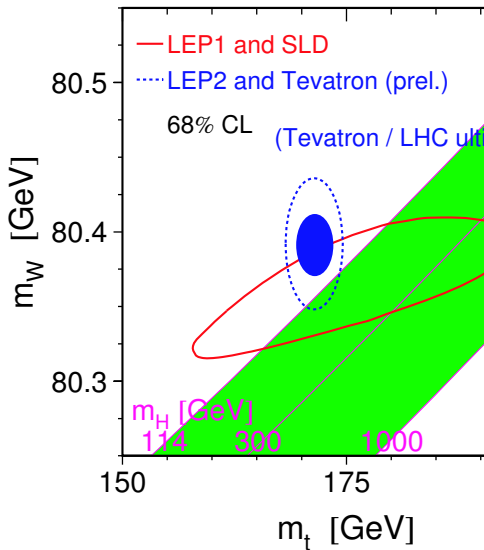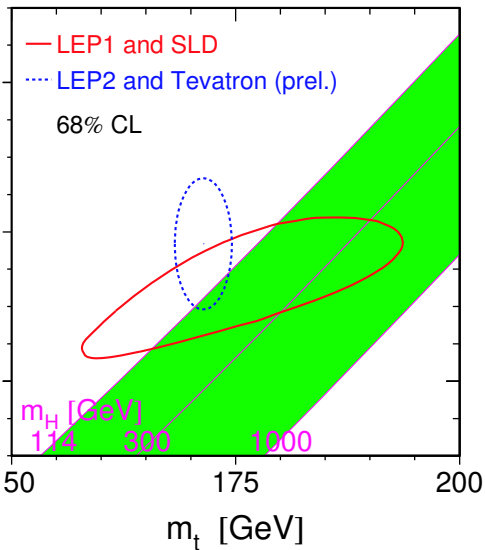
Direct, indirect determinations agree reasonably

Both favor a light Higgs boson, ... *within framework of SM analysis.*

Fit to a universe of data
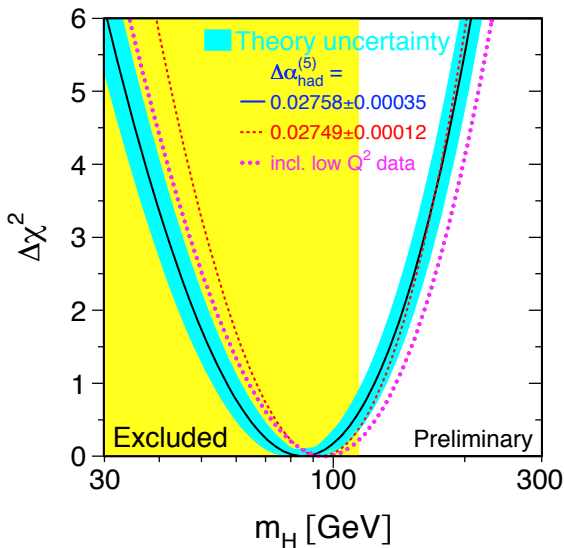
Standard-Model $M_H \lesssim 200$ GeV at 95% CL

Fit to a universe of data ...

- *Within SM,* LEP EWWG deduce a 95% CL upper limit, $M_H \lesssim 200$ GeV/c^2
- Direct searches at LEP $\Rightarrow M_H > 114.4$ GeV/c^2, excluding much of the favored region
- Either the Higgs boson is just around the corner, or SM analysis is misleading

Things will soon be popping!

A Cautionary Note

- A_{FB}^b, which exerts the greatest "pull" on the global fit [slide 35], is most responsible for raising M_H above the range excluded by direct searches [slide 77].

- Leptonic and hadronic observables point to different best-fit values of M_H

- Many subtleties in experimental and theoretical analyses

M. Chanowitz, *Phys. Rev. Lett.* **87**, 231802 (2001); *Phys. Rev. D***66**, 073002 (2002); hep-ph/0304199;
http://phyweb.lbl.gov/~chanowitz/rpm-10-06.pdf

Introduction to global analyses: J. L. Rosner, hep-ph/0108195; hep-ph/0206176

χ^2 Distributions: Leptonic Asymmetries

Leptonic Asymmetries

A_L Combined

A_{LR}

A_{FB}^1

P_τ

$m_H(\text{GeV})$
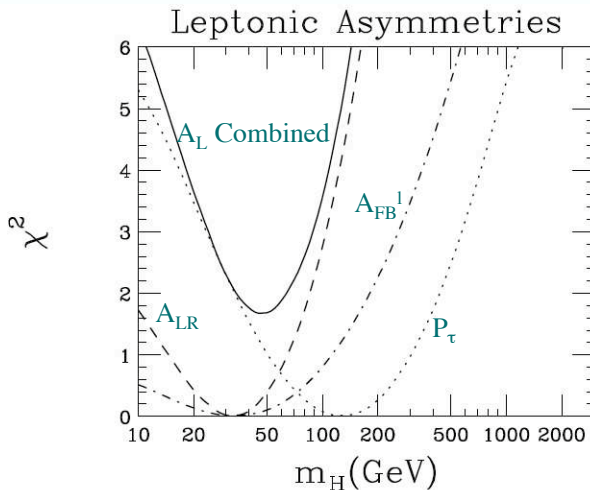
M. Chanowitz   RPM 10/26/06   20

χ² Distributions: Hadronic Asymmetries

Hadronic Asymmetries

A_H combined

$A_{FB}{}^c$

$A_{FB}{}^b$

Q_{FB}

χ^2

$m_H(GeV)$
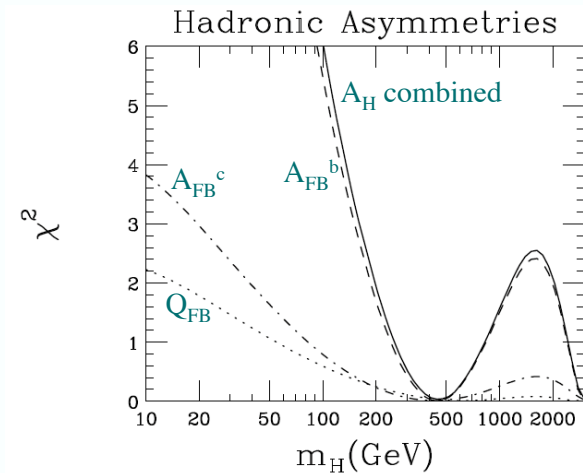
- Tevatron, LHC measurements will determine m_t within 1 or 2 GeV
  . . . and improve δM_W to about 15 MeV
- As the Tevatron's integrated luminosity approaches 10 fb^{-1}, CDF and DØ will explore the region of M_H not excluded by LEP
- ATLAS and CMS will carry on the exploration of the Higgs sector at the LHC;
  could require a few years, at low mass;
  full range accessible, $\gamma\gamma, \ell\ell\nu\nu, b\bar{b}, \ell^+\ell^-\ell^+\ell^-, \ell\nu jj, \tau\tau$ channels.

A few words on Higgs production . . .

$e^+e^- \rightarrow H$: hopelessly small
$\mu^+\mu^- \rightarrow H$: scaled by $(m_\mu/m_e)^2 \approx 40\,000$
$e^+e^- \rightarrow HZ$: prime channel

Hadron colliders:
$gg \rightarrow H \rightarrow b\bar{b}$: background ?!
$gg \rightarrow H \rightarrow \gamma\gamma$: rate ?!

$\bar{p}p \rightarrow H(W, Z)$: prime Tevatron channel

At the LHC:
Many channels become accessible, expect sensitive search
up to 1 TeV

H couples to gluons through quark loops
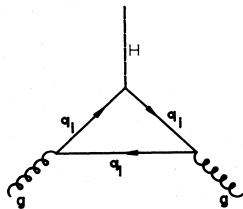
Only heavy quarks matter:
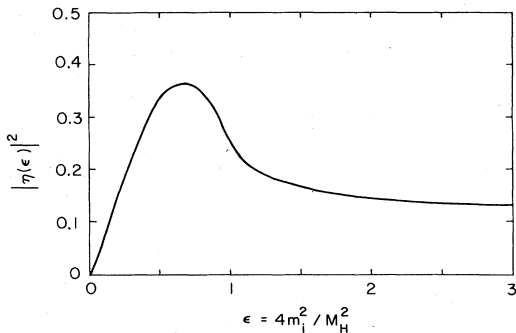
Higgs search in e^+e^- collisions

$\sigma(e^+e^- \to H \to \text{all})$ is *minute*, $\propto m_e^2$

Even narrowness of low-mass H is not enough to make it visible . . . Sets aside a traditional strength of e^+e^- machines—*pole physics*

Most promising:
associated production $e^+e^- \to HZ$
(has no small couplings)
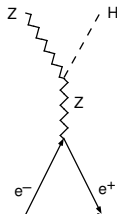
$$\sigma = \frac{\pi\alpha^2}{24\sqrt{s}} \frac{K(K^2 + 3M_Z^2)[1 + (1 - 4x_W)^2]}{(s - M_Z^2)^2 \; x_W^2(1 - x_W)^2}$$

K: c.m. momentum of H $\qquad$ $x_W \equiv \sin^2\theta_W$

$\ell^+\ell^- \rightarrow X \ldots$
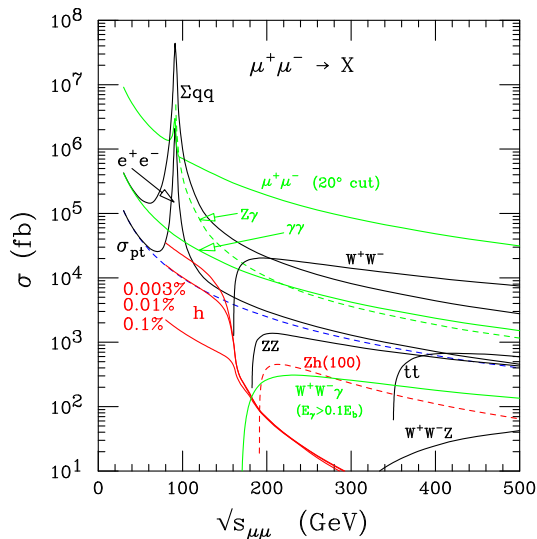
$$\sigma(e^+e^- \rightarrow H) = (m_e/m_\mu)^2 \sigma(\mu^+\mu^- \rightarrow H) \approx \sigma(\mu^+\mu^- \rightarrow H)/40\,000$$
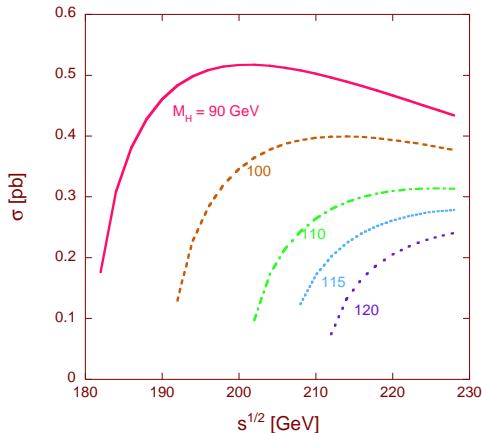
$+$ important effect of ISR

LEP 2: sensitive nearly to kinematical limit $\boxed{M_H^{\max} = \sqrt{s} - M_Z}$

LC: sensitive for $M_H \lesssim 0.7\sqrt{s}$

& measure excitation curve to determine

$$\delta M_H \approx 60 \text{ MeV } \sqrt{100 \text{ fb}^{-1}/\mathcal{L}} \text{ for } M_H = 100 \text{ GeV}$$

Tevatron Search Strategies

- $gg \to H \to b\bar{b}$ is swamped by QCD production of $b\bar{b}$.
  Even with 30 fb^{-1}, only < 1-σ excess.
  By-product: $Z^0 \to b\bar{b}$ observable in Run II.

- Special topologies improve signal/background and significance:

$$\bar{p}p \quad \to \quad HW + \text{anything}$$
$$\llcorner\!\to \ell\nu$$
$$\llcorner\!\to b\bar{b}$$
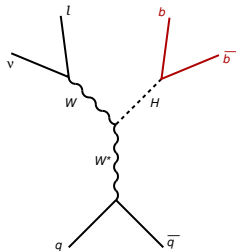
$$\bar{p}p \quad \to \quad HZ + \text{anything}$$
$$\llcorner\!\to \ell^+\ell^- + \nu\bar{\nu}$$
$$\llcorner\!\to b\bar{b}$$
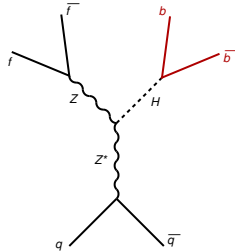
A. Stange, W. Marciano and S. Willenbrock, *Phys. Rev. D***49**, 1354 (1994); *Phys. Rev. D***50**, 4491 (1994).

Higgs-boson production at the Tevatron

SM Higgs cross section [pb]

$gg \to H$

WH

ZH

m_H (GeV/c^2)
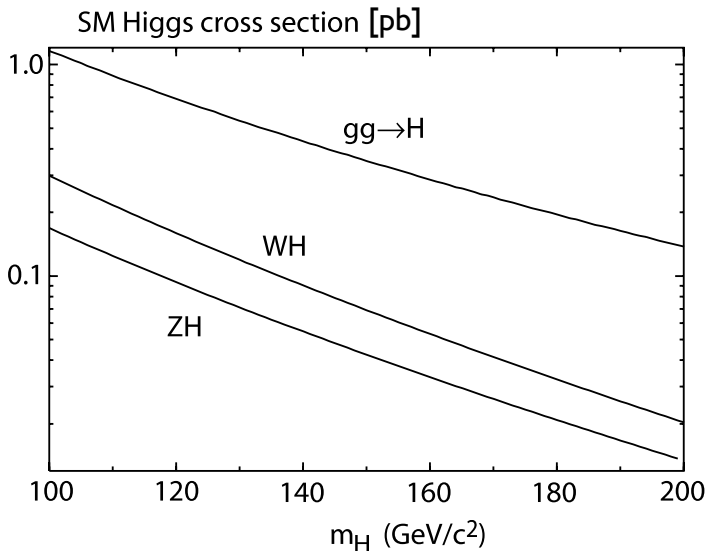
Current Tevatron Sensitivity
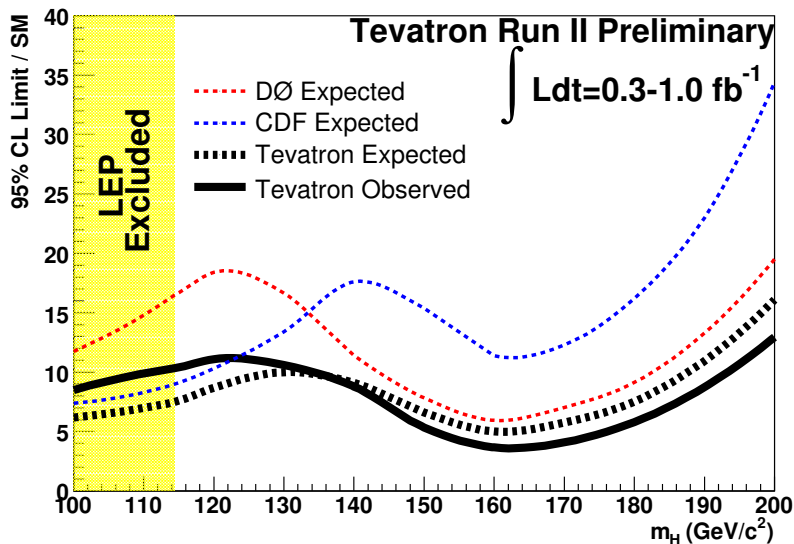
combining experiments, channels

The agent of electroweak symmetry breaking represents
a novel fundamental interaction
at an energy of a few hundred GeV.

We do not know the nature of the new force.

Inspired by the Meissner effect, we describe the EWSB
interaction as an analogue of the Ginzburg–Landau
picture of superconductivity.

light Higgs boson $\Leftrightarrow$ perturbative dynamics
heavy Higgs boson $\Leftrightarrow$ strong dynamics

What is the nature of the mysterious new force that hides electroweak symmetry?

- A fundamental force of a new character, based on interactions of an elementary scalar
- A new gauge force, perhaps acting on undiscovered constituents
- A residual force that emerges from strong dynamics among the weak gauge bosons
- An echo of extra spacetime dimensions

We have explored examples of all four, theoretically.

Which path has Nature taken?

Essential step toward understanding the new force that shapes our world:

Find the Higgs boson and explore its properties.

- Is it there? How many?
- Verify $J^{PC} = 0^{++}$
- Does H generate mass for gauge bosons, fermions?
- How does H interact with itself?

Finding the Higgs boson starts a new adventure!

Assessment

$SU(2)_L \otimes U(1)_Y$: 25 years of confirmations

- neutral currents;
- $W^\pm$, Z^0
- charm

(+ experimental guidance)

- τ, ν_τ
- b, t

+ experimental surprises

- narrowness of ψ, ψ'
- long B lifetime; large B^0–$\bar{B}^0$ mixing
- large B^0–$\bar{B}^0$ mixing
- heavy top
- neutrino oscillations

10 years precise measurements: no significant deviations

Quantum corrections tested at $\pm 10^{-3}$

No "new physics" ... yet!

Theory tested at distances from 10^{-17} cm to $\sim 10^{22}$ cm

origin — Coulomb's law (tabletop experiments)

smaller $\begin{cases} \text{Atomic physics} \rightarrow \text{QED} \\ \text{high-energy expts.} \rightarrow \text{EW theory} \end{cases}$

larger — $M_\gamma \approx 0$ in planetary ... measurements

Is EW theory true? Is it complete ??

A crack in the standard model?

Update for ICHEP-Tau06 (Preliminary)

$a_\mu^{had}[ee] = (690.9 \pm 4.4) \times 10^{-10}$

$a_\mu[ee] = (11\ 659\ 180.5 \pm 4.4_{had} \pm 3.5_{LBL} \pm 0.2_{QED+EW}) \times 10^{-10}$

including:

Hadronic HO	$-(9.8 \pm 0.1) \times 10^{-10}$
Hadronic LBL	$+(12.0 \pm 3.5) \times 10^{-10}$
Electroweak	$(15.4 \pm 0.2) \times 10^{-10}$
QED	$(11\ 658\ 471.9 \pm 0.1) \times 10^{-10}$
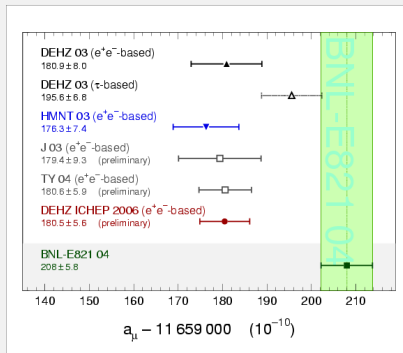
Knecht-Nyffeler, Phys.Rev.Lett. 88 (2002) 071802

Melnikov-Vainshtein, hep-ph/0312226

Davier-Marciano, Ann. Rev. Nucl. Part. Sc. (2004)

Kinoshita-Nio (2006)

BNL E821 (2004):
$a_\mu^{exp} = (11\ 659\ 208.0 \pm 6.3)\ 10^{-10}$

Observed Difference with Experiment:

$a_\mu[exp] - a_\mu[SM] = (27.5 \pm 8.4) \times 10^{-10}$

Δ 3.3 „standard deviations"

DEHZ 03 (e^+e^--based)
180.9 ± 8.0

DEHZ 03 (τ-based)
195.6 ± 6.8

HMNT 03 (e^+e^--based)
176.3 ± 7.4

J 03 (e^+e^--based)
179.4 ± 9.3 (preliminary)

TY 04 (e^+e^--based)
180.6 ± 5.9 (preliminary)

DEHZ ICHEP 2006 (e^+e^--based)
180.5 ± 5.6 (preliminary)

BNL-E821 04
208 ± 5.8

$a_\mu - 11\ 659\ 000 \quad (10^{-10})$

140   150   160   170   180   190   200   210

Tau Workshop, SNS Pisa, Sept 19-22, 2006   M. Davier – Hadronic Contribution to $(g-2)_\mu$ 30

What about gravity?

Natural to neglect gravity in particle physics . . .

$$G_{\text{Newton}} \text{ small} \iff M_{\text{Planck}} = \left(\frac{\hbar c}{G_{\text{Newton}}} \right)^{\frac{1}{2}} \approx 1.22 \times 10^{19} \text{ GeV } \text{large}$$
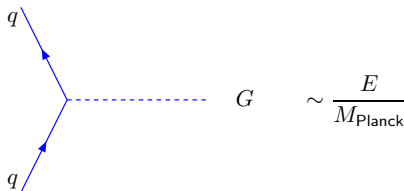
$$G \qquad \sim \frac{E}{M_{\text{Planck}}}$$

Estimate $B(K \to \pi G) \sim \left(\frac{M_K}{M_{\text{Planck}}} \right)^2 \sim 10^{-38}$

300 years after Newton: Why is gravity weak?

But gravity is not always negligible . . .

The vacuum energy problem

$$\text{Higgs potential } V(\varphi^\dagger \varphi) = \mu^2(\varphi^\dagger \varphi) + |\lambda|\,(\varphi^\dagger \varphi)^2$$

At the minimum,

$$V(\langle \varphi^\dagger \varphi \rangle_0) = \frac{\mu^2 v^2}{4} = -\frac{|\lambda|\,v^4}{4} < 0.$$

$$\text{Identify } M_H^2 = -2\mu^2$$

$V \neq 0$ contributes position-independent vacuum energy density

$$\boxed{\varrho_H \equiv \frac{M_H^2 v^2}{8} \geq 10^8 \text{ GeV}^4} \quad \approx 10^{24} \text{ g cm}^{-3}$$

Adding vacuum energy density $\varrho_{\text{vac}} \Leftrightarrow$ adding cosmological constant Λ to Einstein's equation

$$R_{\mu\nu} - \tfrac{1}{2}R g_{\mu\nu} = \frac{8\pi G_N}{c^4} T_{\mu\nu} + \Lambda g_{\mu\nu} \qquad \Lambda = \frac{8\pi G_N}{c^4} \varrho_{\text{vac}}$$

Observed vacuum energy density $\varrho_{\text{vac}} \lesssim 10^{-46}$ GeV^4

(figure by L. Verde)
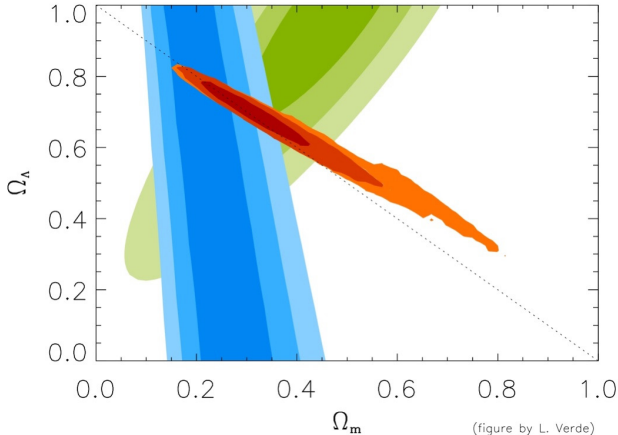
$\varrho_H \gtrsim 10^8$ GeV^4: mismatch by 10^{54}

A chronic dull headache for thirty years . . .

EWSB: another path?

Modeled EWSB on Ginzburg–Landau description of superconducting phase transition;

... had to introduce new, elementary scalars

GL is not the last word on superconductivity:

dynamical Bardeen–Cooper–Schrieffer theory

The elementary fermions – electrons – and gauge interactions – QED – needed to generate the scalar bound states are already present in the case of superconductivity.

Could a scheme of similar economy account for EWSB?

$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ + massless u and d

(treat $SU(2)_L \otimes U(1)_Y$ as perturbation)

$m_u = m_d = 0$:
QCD has exact $SU(2)_L \otimes SU(2)_R$ chiral symmetry.

At an energy scale $\sim \Lambda_{QCD}$, strong interactions become strong, fermion condensates appear, and

$$SU(2)_L \otimes SU(2)_R \rightarrow SU(2)_V$$

$\rightsquigarrow$ 3 Goldstone bosons, one for each broken generator:
3 massless pions (Nambu)

Broken generators: 3 axial currents; couplings to π measured by pion decay constant f_π.

Turn on $SU(2)_L \otimes U(1)_Y$: EW gauge bosons couple to axial currents, acquire masses of order $\sim g f_\pi$.

$$\mathcal{M}^2 = \begin{pmatrix} g^2 & 0 & 0 & 0 \\ 0 & g^2 & 0 & 0 \\ 0 & 0 & g^2 & gg' \\ 0 & 0 & gg' & g'^2 \end{pmatrix} \frac{f_\pi^2}{4} \quad (W^+, W^-, W_3, \mathcal{A})$$

same structure as standard EW theory.

Diagonalize: $M_W^2 = g^2 f_\pi^2/4$, $M_Z^2 = (g^2 + g'^2)f_\pi^2/4$, $M_A^2 = 0$, so

$$\frac{M_Z^2}{M_W^2} = \frac{(g^2 + g'^2)}{g^2} = \frac{1}{\cos^2 \theta_W}$$

Massless pions disappear from physical spectrum, to become longitudinal components of weak bosons. $\boxed{M_W \approx 30 \text{ MeV}/c^2}$ No fermion masses ...

Parameters of the Standard Model

3	coupling parameters $\alpha_s, \alpha_{\mathsf{EM}}, \sin^2\theta_W$
2	parameters of the Higgs potential
1	vacuum phase (QCD)
6	quark masses
3	quark mixing angles
1	CP-violating phase
3	charged-lepton masses
3	neutrino masses
3	leptonic mixing angles
1	leptonic CP-violating phase ($+$ Majorana ...)

26^+ arbitrary parameters

parameter count not improved by unification

The EW scale and beyond

EWSB scale, $v = (G_F\sqrt{2})^{-\frac{1}{2}} \approx 246$ GeV, sets

$$M_W^2 = g^2 v^2/2 \qquad M_Z^2 = M_W^2/\cos^2\theta_W$$

But it is not the only scale of physical interest

quasi-certain: $M_{\text{Planck}} = 1.22 \times 10^{19}$ GeV

probable: $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ unification scale $\sim 10^{15-16}$ GeV

somewhere: flavor scale

How to keep the distant scales from mixing in the face of quantum corrections?

OR

How to stabilize the mass of the Higgs boson on the electroweak scale?

OR

Why is the electroweak scale small?

"The hierarchy problem"

Higgs potential $\quad V(\phi^\dagger\phi) = \mu^2(\phi^\dagger\phi) + |\lambda|\,(\phi^\dagger\phi)^2$

$\mu^2 < 0$: $\mathsf{SU}(2)_L \otimes \mathsf{U}(1)_Y \to U(1)_{\mathrm{em}}$, as

$$\langle\phi\rangle_0 = \left(\begin{array}{c} 0 \\ \sqrt{-\mu^2/2|\lambda|} \end{array}\right) \equiv \left(\begin{array}{c} 0 \\ \underbrace{(G_F\sqrt{8})^{-1/2}}_{175\ \mathrm{GeV}} \end{array}\right)$$

Beyond classical approximation, quantum corrections to scalar mass parameters:

$$m^2(p^2) = m_0^2 + \underset{J=1}{\text{———}\mathord{\sim\!\sim\!\sim}\text{———}} + \underset{J=1/2}{\text{———}\bigcirc\text{———}} + \underset{J=0}{\text{———}\bigcirc\text{———}}$$

Loop integrals are potentially divergent

$$m^2(p^2) = m^2(\Lambda^2) + Cg^2 \int_{p^2}^{\Lambda^2} dk^2 + \cdots$$

Λ: reference scale at which m^2 is known

g: coupling constant of the theory

C: coefficient calculable in specific theory

For mass shifts induced by radiative corrections to remain under control (not greatly exceed the value measured on the laboratory scale), *either*

- Λ must be small, *or*
- New Physics must intervene to cut off integral

But natural reference scale for Λ is

$$\Lambda \approx M_{\text{Planck}} = \left(\frac{\hbar c}{G_{\text{Newton}}} \right)^{1/2} \approx 1.22 \times 10^{19} \text{ GeV}$$

for $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$

or

$$\Lambda \approx M_U \approx 10^{15}\text{-}10^{16} \text{ GeV} \quad \text{for unified theory}$$

Both $\gg v/\sqrt{2} \approx 175$ GeV $\implies$

New Physics at $E \lesssim 1$ TeV

Fine tuning the Higgs
$\Lambda = 10$ TeV
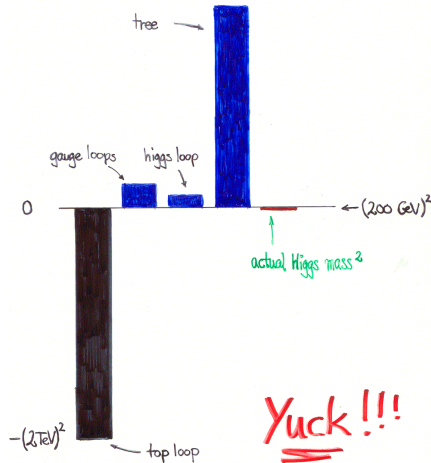
$$\delta M_H^2 = \frac{G_F \Lambda^2}{4\pi^2 \sqrt{2}}(6M_W^2 + 3M_Z^2 + M_H^2 - 12m_t^2)$$

Martin Schmaltz, ICHEP02

Only a few distinct scenarios . . .

- Supersymmetry: balance contributions of fermion loops (-1) and boson loops $(+1)$
  Exact supersymmetry,

$$\sum_{\substack{i=\text{fermions} \\ +\text{bosons}}} C_i \int dk^2 = 0$$

Broken supersymmetry, shifts acceptably small if superpartner mass splittings are not too large

$$g^2 \Delta M^2 \text{ "small enough"} \Rightarrow \widetilde{M} \lesssim 1 \text{ TeV}/c^2$$

What is supersymmetry?

- A fermion-boson symmetry that arises from new *fermionic* dimensions

- Most general symmetry of S-matrix: SUSY $+$ Poincaré invariance $+$ internal symmetries

- Relates fermion to boson degrees of freedom: roughly, each particle has a superpartner with spin offset by $\frac{1}{2}$

- SUSY relates interactions of particles, superpartners

- Known particle spectrum contains no superpartners $\Rightarrow$ SUSY doubles the spectrum

- SUSY invariance or anomaly cancellation requires two Higgs doublets to give masses to $I_3 = \pm\frac{1}{2}$ particles

SUSY doubles the spectrum

If $m_{\tilde{e}} < m_e$, no Pauli principle to dictate integrity of molecules

Dyson & Lieb: If basic constituents of matter were bosons, individual molecules would join into a

shrinking . . . insatiable . . . undifferentiated B L O B !

Supersymmetry menaces us
with an amorphous death

Full understanding of SUSY will show us why we live in a world ruled by the Exclusion Principle

![Boselab logo] Boselab

Why Supersymmetry?

- Closely approximates the standard model
- Maximal (unique) extension of Poincaré invariance
- Path to gravity: local supersymmetry $\longrightarrow$ supergravity
- Solution to naturalness problem: allows fundamental scalar at low E
- ($+$ unification) $\sin^2 \theta_W$, coupling constant unification
- ($+$ universality) Can generate SSB potential
- ($+R$-parity) LSP as dark matter candidate

MSSM closely resembles standard EW theory
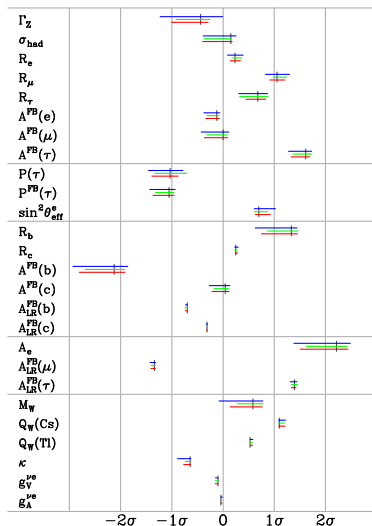
Erler & Pierce: SUSY *vs.* SM, hep-ph/9801238   Cho & Hagiwara, hep-ph/9912260

| SM   — SUGRA   — 5 ⊕ 5* GMSB   — 10 ⊕ 10* GMSB

Coupling constant unification through supersymmetry?
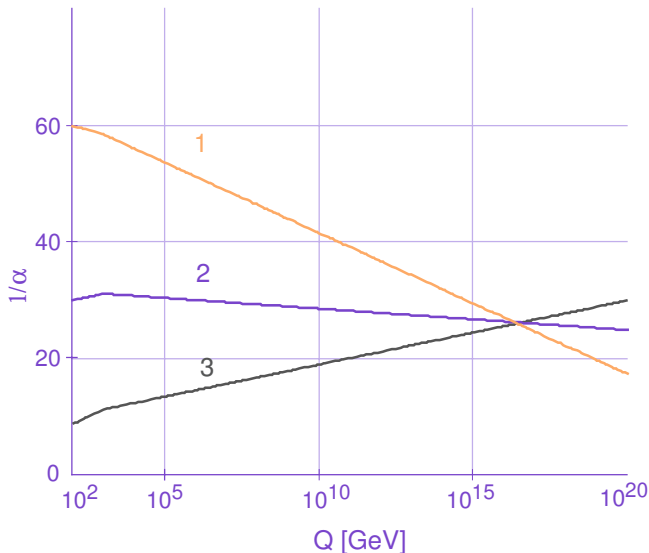
Coupling constant unification by many Higgs doublets?
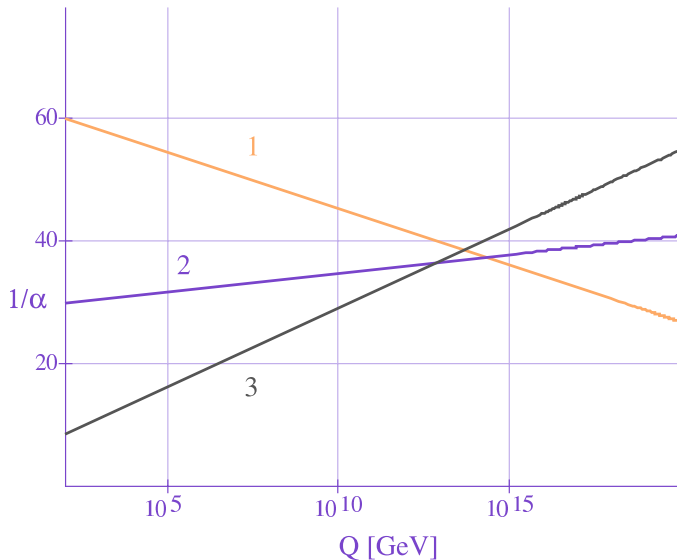
For heavy top, SSB may follow naturally in SUSY
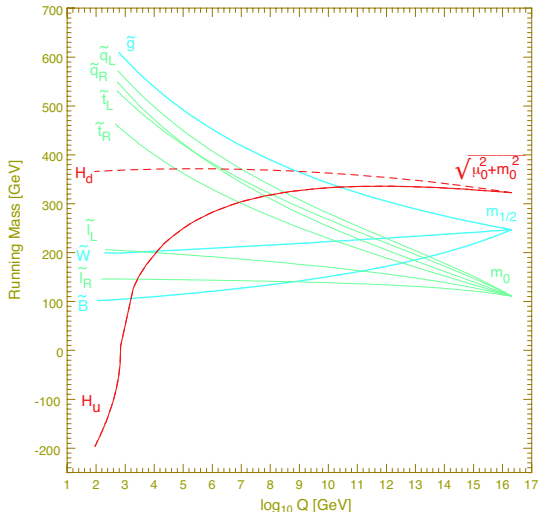
... (sign of M^2 indicated)　　　Kane, *et al.* (hep-ph/9312272, *Phys. Rev.* D**49**, 6173 (1994))

Yukawa terms consistent with SUSY induce dangerous lepton- and baryon-number violations:

$$\lambda_{ijk} L^i L^j E^k + \lambda'_{ijk} L^i Q^j \bar{D}^k + \lambda'' \bar{U}^i \bar{D}^j \bar{D}^k$$

45 free parameters ... Transitions like

$$\mathcal{L}_{LLE} = \lambda_{ijk} \, \tilde{\nu}_L^i e_L^i \bar{e}_R^k + \ldots$$

To banish these, impose symmetry under R-parity:

$$R = (-1)^{3B+L+S}$$

... even for particles, odd for superpartners.

Superpartners produced in pairs

Lightest superpartner is stable

5 physical Higgs bosons: CP even h^0, H^0; CP odd A^0; $H^{\pm}$

Upper bounds on M_h in the MSSM

$$M_h^2 = M_Z^2 \cos^2 2\beta + \frac{3g^2 m_t^4}{8\pi^2 M_W^2}\left[\log\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right) + \cdots\right] \lesssim (130 \text{ GeV}/c^2)^2$$

Upper bound on $M_h \Leftrightarrow$
large M_A limit, ($M_s = 1$ TeV)
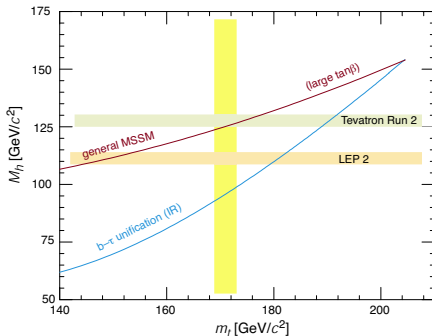
Carena, *et al.*, *Phys. Lett.* **B355**, 209 (1995)

If nonminimal SUSY Higgs couplings are perturbative up to M_U,

$$\boxed{M_h \lesssim 150 \text{ GeV}}$$

SUSY Challenges . . .

- Extra dynamics needed to break SUSY
  "Soft" SUSY breaking $\Longrightarrow$
  MSSM with $\boxed{124}$ parameters

  Contending schemes for SUSY breaking:
  - *Gravity mediation.* SUSY breaking at a very high scale, communicated to standard model by supergravity interactions
  - *Gauge mediation.* SUSY breaking nearby ($\lesssim 100$ TeV), communicated to standard model by (nonperturbative ?) gauge forces.
  - . . .

  None meets all challenges

. . . SUSY Challenges

- Weak-scale SUSY protects M_H, but does not explain the weak scale ("μ problem")

- Global SUSY must deal with the threat of FCNC

- (Like SM) Clear predictions for gauge-boson masses, not so clear for squarks and sleptons

- So far, SUSY is well hidden
  Contortions required for $M_H \gtrsim 115$ GeV

- Disappointing that SUSY didn't relate particles & forces, but doubled spectrum

- Baryon- and lepton-number violating interactions arise naturally, are abolished by decree

. . . SUSY Challenges

- SUSY introduces new sources of CP violation that are potentially too large.
- We haven't found a convincing and viable picture of the TeV superworld.

This long list of challenges doesn't mean that Supersymmetry is wrong, or even irrelevant to the 1-TeV scale.

But SUSY is not automatically right, either!

If SUSY does operate on the 1-TeV scale, then Nature must have found solutions to all these challenges . . .
. . . and we will need to find them, too.

If weak-scale SUSY is present, we should see it soon in the Higgs sector and beyond, ... and we will live in "interesting times"

Example SUSY thresholds in e^+e^-
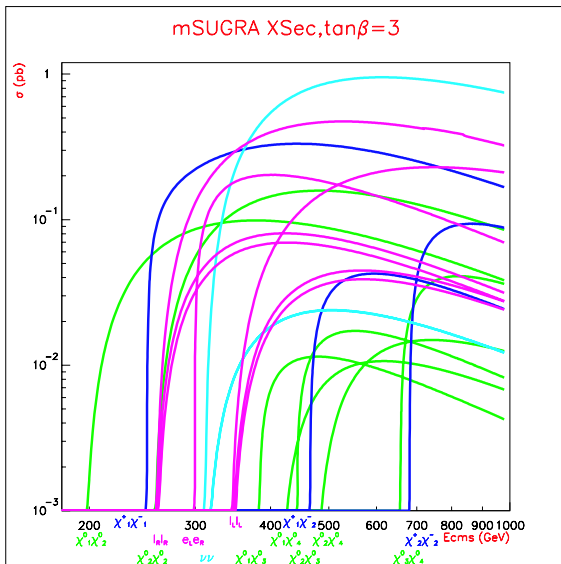
Grahame Blair

My view

Supersymmetry is (almost) certain to be true . . . . . . as a path to the incorporation of gravity

Whether SUSY resolves the problems of the 1-TeV scale is a logically separate question . . . answer less obvious

Experiment will decide

Only a few distinct scenarios . . .

- Composite scalars (technicolor): New physics arises on scale of composite Higgs-boson binding,

$$\Lambda_{\text{TC}} \simeq O(1 \text{ TeV})$$

"Form factor" cuts effective range of integration

A look at technicolor

Follow the "other path" to EWSB, but with new interaction, new constituents

$$\text{massless } u, d \text{ quarks} \quad \longrightarrow \quad \begin{array}{l} \text{new fermions} \\ \text{"technifermions"} \end{array}$$

$$\begin{array}{c} \text{QCD} \\ \text{color} \end{array} \quad \longrightarrow \quad \begin{array}{l} \text{new interaction} \\ \text{"technicolor"} \end{array}$$

Choose scale of interaction so that

$$f_\pi \quad \longrightarrow \quad F_\pi = v = (G_F \sqrt{2})^{-\frac{1}{2}}$$

Generates correct M_W, M_Z, *but* produces no Yukawa couplings, so no fermion masses

Shows possibility that gauge-boson masses & fermion masses
...have *different* origins

The problem of fermion masses

TC shows solving origin of EWSB will not necessarily give insight into fermion masses

Puzzling pattern of quark masses
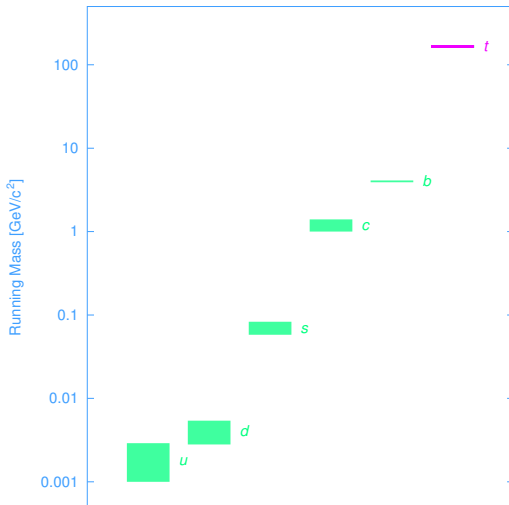
To generate fermion mass, embed technicolor in a larger extended technicolor gauge group

$$G_{ETC} \supset G_{TC}$$

that couples quarks and leptons to technifermions

If $G_{ETC} \to G_{TC}$ at scale Λ_{ETC},

then quarks and leptons may acquire masses $\boxed{m \sim \Lambda_{TC}^3/\Lambda_{ETC}^2}$

$$\langle \overline{Q}_L Q_R \rangle \approx \Lambda_{TC}^3$$

Q_L Q_R

ETC

"radiative"
mechanism

q_L q_R
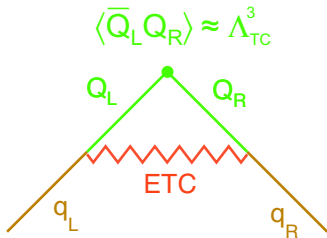
Standard ETC is challenged by problems of reproducing wide range of quark masses while avoiding FCNC traps

Consider $|\Delta S| = 2$ interactions

$$\mathcal{L}_{|\Delta S|=2} = \frac{g_{\text{ETC}}^2 \theta_{sd}^2}{M_{\text{ETC}}^2} (\bar{s}\Gamma^\mu d)(\bar{s}\Gamma'_\mu d) + \cdots$$

$\Delta M_K < 3.5 \times 10^{-12}$ MeV $\implies$ $\boxed{\dfrac{M_{\text{ETC}}^2}{g_{\text{ETC}}^2 |\theta_{sd}|^2} \text{ very large}}$

$\implies$ hard to generate heavy enough c, s, t, b

Multiscale TC (Eichten & Lane)

Many fermions (in different TC reps)
$\implies$ many technipions
light ρ_T, ω_T, π_T

Generation of fermion mass is where all the *experimental threats* to Technicolor arise:

- Flavor-changing neutral currents
- Matter content (S parameter)

> Lesson: QCD is not a good model for TC

Keep in mind: In addressing problems of fermion mass, ETC is much more ambitious than global supersymmetry

Current ideas: K. Lane, hep-ph/0202255, BUHEP-06-01

Review: Hill & Simmons, hep-ph/0203079

Only a few distinct scenarios . . .

- Strongly interacting gauge sector: *WW* resonances, multiple *W* production, probably scalar bound state "quasiHiggs" with $M < 1$ TeV
- Spontaneously broken global symmetries $\rightsquigarrow$ pseudo-Nambu–Goldstone bosons
  - ▶ Higgs boson would be massless if Nambu–Goldstone boson protected against large quantum corrections
  - ▶ Global symmetry must be large enough that left-over NGBs remain after some provide longitudinal components of massive gauge bosons
  - ▶ Explicit symmetry breaking needed for $M_H \neq 0$
    Extra gauge bosons ($M_{W'} \approx 4\pi M_H$), enlarged fermion multiplets cancel quadratic divergences in δM_H^2

  "Little Higgs" effective theories, $\Lambda \approx (4\pi)^2 M_H$

Only a few distinct scenarios . . .

- Or maybe the problem is with (our understanding of) *gravity*, not with the electroweak theory?

Large extra dimensions

Arkani-Hamed, Dimopoulos, Dvali, *Phys. Lett.* **429**, 263 (1998)

Universal extra dimensions

Appelquist, Cheng, Dobrescu, *Phys. Rev. D* **64**, 035002 (2001)

Warped extra dimensions

Randall & Sundrum, *Phys. Rev. Lett.* **83**, 3370, 4690 (1999)

Gravity follows $1/r^2$ law down to $\lesssim 1$ mm (few meV)

$$V(r) = -\int dr_1 \int dr_2 \frac{G_N \rho(r_1)\rho(r_2)}{r_{12}} \left[1 + \varepsilon_G \exp(-r_{12}/\lambda_G)\right]$$
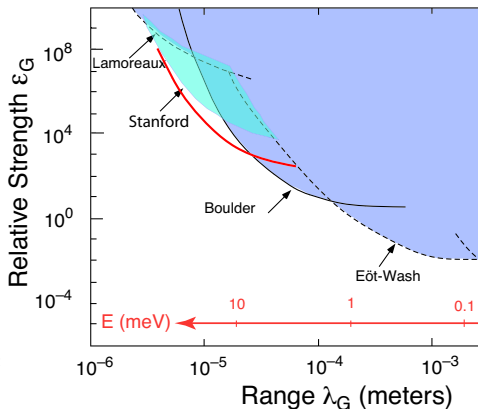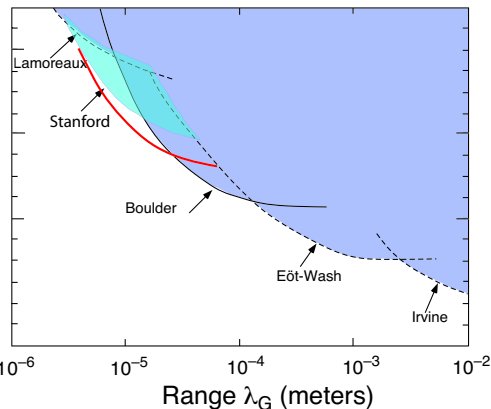
Experiment leaves us free to consider modifications to Gravity even at (nearly) macroscopic distances

Large (but compact) extra dimensions

Suppose at scale R Gravity propagates in $3 + n$ spatial dimensions
Force law changes: $F \propto 1/r^{2+n}$
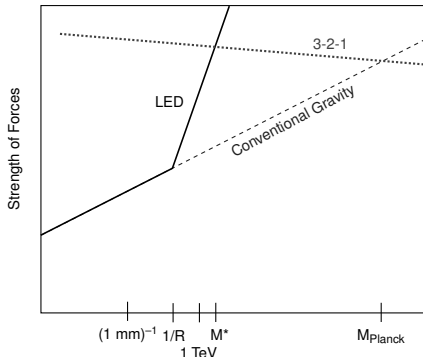
$G_N \sim M_{Pl}^{-2} \sim M^{\star -n-2} R^{-n}$ $M^{\star}$: gravity's true scale

Example: $M^{\star} = 1$ TeV $\Rightarrow R \lesssim 10^{-3}$ m for $n = 2$

M_P is a mirage (false extrapolation)!

Large extra dimensions . . .

$\triangleright$ Hierarchy problem is moot if $M^\star \ll M_{\text{Pl}}$, (small range of integration); . . . ideas of unification must change

$\triangleright$ Gravity appears weak because it propagates in the large extra dimensions; strength diluted by volume of the n extra dimensions

Other features of extra dimension pictures

▷ Much in common with new strong dynamics (Technicolor, etc.)

▷ Missing-energy signatures in high-$p_\perp$ collisions

Hewett & Spiropulu, *Ann. Rev. Nucl. Part. Sci.* **52,** 397 (2002)

▷ Kaluza–Klein excitations; dark-matter candidates

Challenge: Understanding the Everyday (bis)

*What would the world be like, without a (Higgs)
mechanism to hide electroweak symmetry and give
masses to the quarks and leptons?*

Consider the effects of all the
$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ *interactions!*

With no Higgs mechanism . . .

- Quarks and leptons would remain massless
- QCD would confine the quarks in color-singlet hadrons
- *N mass little changed,* but *p* outweighs *n*
- QCD breaks EW to EM, gives $(1/2500 \times \text{observed})$ masses to *W*, *Z*, so weak-isospin force doesn't confine
- Rapid! β-decay $\Rightarrow$ lightest nucleus is *n*; no H atom
- Some light elements in BBN (?), but ∞ Bohr radius
- No atoms (as we know them) means no chemistry, no stable composite structures like solids and liquids

. . . the character of the physical world would be profoundly changed

High expectations for the Tevatron

- Biggest changes in the way we think about LHC experiments have come from the Tevatron:

  ▷ the large mass of the top quark and
  ▷ the success of silicon vertex detectors: heavy flavors

- Top quark is a unique window on EWSB and of interest in its own right: single top production

- Entering new terrain for new gauge bosons, strong dynamics, supersymmetry, Higgs, B_s mixing, . . .

B_s - $\bar{B}_s$ Mixing
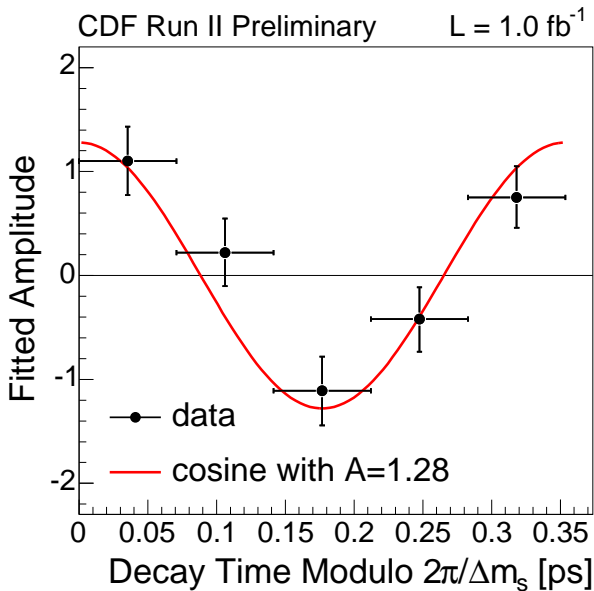
Why the LHC is so exciting (II)

- Electroweak theory (unitarity) tells us the 1-TeV scale is special: Higgs boson or other new physics (strongly interacting gauge bosons)
- Hierarchy problem $\Rightarrow$ other new physics nearby
- Our ignorance of EWSB obscures our view of other questions (e.g., identity problem). Lifting the veil at 1 TeV will change the face of physics

The cosmic connection

- Observational cosmology is like paleontology: reading the fossil record. Only a few layers are preserved, can we find more?

- Our reading of the fossil record is influenced by our world-view / theoretical framework.

- Cosmology shows us the world we must explain, provides questions and constraints; the answers will come from particle physics.

In a decade or two, we can hope to . . .

Understand electroweak symmetry breaking
Observe the Higgs boson
Measure neutrino masses and mixings
Establish Majorana neutrinos ($\beta\beta_{0\nu}$)
Thoroughly study CP violation in B decay
Exploit rare decays (K, D, . . .)
Observe n EDM, pursue e^- EDM
Use top as a tool
Observe new phases of matter
Understand hadron structure quantitatively
Uncover QCD's full implications
Observe proton decay
Understand the baryon excess
Catalogue matter & energy of universe
Measure dark energy equation of state
Search for new macroscopic forces
Determine GUT symmetry

Detect neutrinos from the universe
Learn how to quantize gravity
Learn why empty space is nearly weightless
Test the inflation hypothesis
Understand discrete symmetry violation
Resolve the hierarchy problem
Discover new gauge forces
Directly detect dark-matter particles
Explore extra spatial dimensions
Understand origin of large-scale structure
Observe gravitational radiation
Solve the strong CP problem
Learn whether supersymmetry is TeV-scale
Seek TeV dynamical symmetry breaking
Search for new strong dynamics
Explain the highest-energy cosmic rays
Formulate problem of identity
. . .

. . . learn the right questions to ask   . . . and rewrite the textbooks!

General References

- C. Quigg, "Nature's Greatest Puzzles," hep-ph/0502070.
- E. Eichten, I. Hinchliffe, K. Lane, and C. Quigg, "Supercollider Physics," *Rev. Mod. Phys.* **56,** 579 (1984).
- F. Gianotti and M. L. Mangano, "LHC physics: The first one-two year(s)," hep-ph/0504221.
- G. Altarelli and M. Grünewald, "Precision Electroweak Tests of the SM," hep-ph/0404165.
- F. Teubert, "Precision tests of the electroweak interactions," *Int. J. Mod. Phys. A* **20**, 5174 (2005).
- S. de Jong, "Tests of the Electroweak Sector of the Standard Model," *PoS* **HEP2005**, 397 (2006) [hep-ph/0512043].

.

Supplement: Details of Electroweak Theory . . .

For more, see my

- "The Electroweak Theory," in *Flavor Physics for the Millennium: TASI 2000,* edited by Jonathan L. Rosner (World Scientific, Singapore, 2001), pp. 367; hep-ph/0204104.

- *Gauge Theories of the Strong, Weak, and Electromagnetic Interactions* (Westview Press, 1997)

  http://www.perseusbooksgroup.com/perseus/book_detail.jsp?isbn=0201328321

Expand about the vacuum state

Let $\phi = \begin{pmatrix} 0 \\ (v + \eta)/\sqrt{2} \end{pmatrix}$; in *unitary gauge*

$$
\begin{aligned}
\mathcal{L}_{\text{scalar}} &= \tfrac{1}{2}(\partial^\mu \eta)(\partial_\mu \eta) - \mu^2 \eta^2 \\
&\quad + \frac{v^2}{8}[g^2 \left| b_\mu^1 - i b_\mu^2 \right|^2 + (g' \mathcal{A}_\mu - g b_\mu^3)^2] \\
&\quad + \text{interaction terms}
\end{aligned}
$$

"Higgs boson" η has acquired (mass)^2 $M_H^2 = -2\mu^2 > 0$

$$
\text{Define } W_\mu^\pm = \frac{b_\mu^1 \mp i b_\mu^2}{\sqrt{2}}
$$

$$
\frac{g^2 v^2}{8}(\left| W_\mu^+ \right|^2 + \left| W_\mu^- \right|^2) \iff M_{W^\pm} = gv/2
$$

$(v^2/8)(g'\mathcal{A}_\mu - g b_\mu^3)^2 \dots$

Now define orthogonal combinations

$$Z_\mu = \frac{-g'\mathcal{A}_\mu + g b_\mu^3}{\sqrt{g^2 + g'^2}} \qquad A_\mu = \frac{g\mathcal{A}_\mu + g' b_\mu^3}{\sqrt{g^2 + g'^2}}$$

$$M_{Z^0} = \sqrt{g^2 + g'^2}\, v/2 = M_W \sqrt{1 + g'^2/g^2}$$

$\boxed{A_\mu \text{ remains massless}}$

$$\mathcal{L}_{\text{Yukawa}} = -\zeta_e \frac{(v + \eta)}{\sqrt{2}}(\bar{e}_R e_L + \bar{e}_L e_R)$$

$$= -\frac{\zeta_e v}{\sqrt{2}}\bar{e}e - \frac{\zeta_e \eta}{\sqrt{2}}\bar{e}e$$

electron acquires $m_e = \zeta_e v/\sqrt{2}$

Higgs-boson coupling to electrons: m_e/v $\quad (\propto$ mass)

Desired particle content . . . plus a Higgs scalar
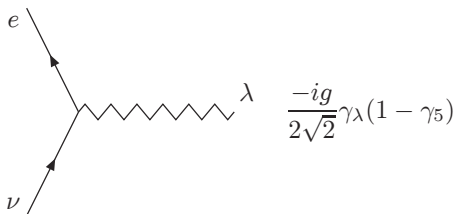
Values of couplings, electroweak scale v?

What about interactions?

Interactions . . .

$$\mathcal{L}_{W\text{-}\ell} = -\frac{g}{2\sqrt{2}}[\bar{\nu}\gamma^\mu(1-\gamma_5)eW_\mu^+ + \bar{e}\gamma^\mu(1-\gamma_5)\nu W_\mu^-]$$

$+$ similar terms for μ and τ

$$\frac{-ig}{2\sqrt{2}}\gamma_\lambda(1-\gamma_5)$$

$$= \frac{-i(g_{\mu\nu} - k_\mu k_\nu/M_W^2)}{k^2 - M_W^2}\ .$$

Compute $\nu_\mu e \rightarrow \mu \nu_e$

$$\sigma(\nu_\mu e \rightarrow \mu \nu_e) = \frac{g^4 m_e E_\nu}{16\pi M_W^4} \frac{[1 - (m_\mu^2 - m_e^2)/2m_e E_\nu]^2}{(1 + 2m_e E_\nu/M_W^2)}$$

Reproduces 4-fermion result at low energies if

$$\frac{g^4}{16 M_W^4} = 2G_F^2 \Rightarrow \frac{g}{2\sqrt{2}} = \left(\frac{G_F M_W^2}{\sqrt{2}}\right)^{\frac{1}{2}}$$

Using $M_W = gv/2$, determine the electroweak scale

$$\boxed{v = (G_F \sqrt{2})^{-\frac{1}{2}} \approx 246 \text{ GeV}}$$

$$\Rightarrow \langle \phi^0 \rangle_0 = (G_F \sqrt{8})^{-\frac{1}{2}} \approx 174 \text{ GeV}$$

W-propagator modifies HE behavior

$$\sigma(\nu_\mu e \to \mu \nu_e) = \frac{g^4 m_e E_\nu}{16\pi M_W^4} \frac{[1 - (m_\mu^2 - m_e^2)/2m_e E_\nu]^2}{(1 + 2m_e E_\nu/M_W^2)}$$

$$\lim_{E_\nu \to \infty} \sigma(\nu_\mu e \to \mu \nu_e) = \frac{g^4}{32\pi M_W^2} = \frac{G_F^2 M_W^2}{\sqrt{2}}$$
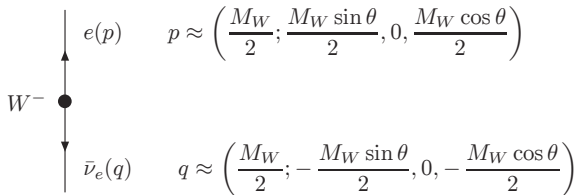
independent of energy!

Partial-wave unitarity respected for

$$s < M_W^2[\exp\left(\pi\sqrt{2}/G_F M_W^2\right) - 1]$$

W-boson properties

No prediction yet for M_W (haven't determined g)

Leptonic decay $W^- \to e^- \nu_e$

$$e(p) \qquad p \approx \left(\frac{M_W}{2}; \frac{M_W \sin\theta}{2}, 0, \frac{M_W \cos\theta}{2} \right)$$

W^-

$$\bar{\nu}_e(q) \qquad q \approx \left(\frac{M_W}{2}; -\frac{M_W \sin\theta}{2}, 0, -\frac{M_W \cos\theta}{2} \right)$$

$$\mathcal{M} = -i \left(\frac{G_F M_W^2}{\sqrt{2}} \right)^{\frac{1}{2}} \bar{u}(e,p)\gamma_\mu(1-\gamma_5)v(\nu,q)\,\varepsilon^\mu$$

$\varepsilon^\mu = (0; \hat{\varepsilon})$: W polarization vector in its rest frame

$$|\mathcal{M}|^2 = \frac{G_F M_W^2}{\sqrt{2}} \mathrm{tr}\left[\not{\varepsilon}(1-\gamma_5)\not{q}(1+\gamma_5)\not{\varepsilon}^* \not{p} \right] ;$$

$$\mathrm{tr}[\cdots] = \left[\varepsilon \cdot q\, \varepsilon^* \cdot p - \varepsilon \cdot \varepsilon^*\, q \cdot p + \varepsilon \cdot p\, \varepsilon^* \cdot q + i\epsilon_{\mu\nu\rho\sigma}\varepsilon^\mu q^\nu \varepsilon^{*\rho} p^\sigma \right]$$

$$\mathrm{tr}[\cdots] = [\varepsilon \cdot q \, \varepsilon^* \cdot p - \varepsilon \cdot \varepsilon^* \, q \cdot p + \varepsilon \cdot p \, \varepsilon^* \cdot q + i\epsilon_{\mu\nu\rho\sigma}\varepsilon^\mu q^\nu \varepsilon^{*\rho} p^\sigma]$$

decay rate is independent of W polarization; look first at longitudinal pol.
$\varepsilon^\mu = (0; 0, 0, 1) = \varepsilon^{*\mu}$, eliminate $\epsilon_{\mu\nu\rho\sigma}$

$$|\mathcal{M}|^2 = \frac{4 G_F M_W^4}{\sqrt{2}} \sin^2\theta$$

$$\frac{d\Gamma_0}{d\Omega} = \frac{|\mathcal{M}|^2}{64\pi^2} \frac{\mathcal{S}_{12}}{M_W^3}$$

$$\mathcal{S}_{12} = \sqrt{[M_W^2 - (m_e + m_\nu)^2][M_W^2 - (m_e - m_\nu)^2]} = M_W^2$$
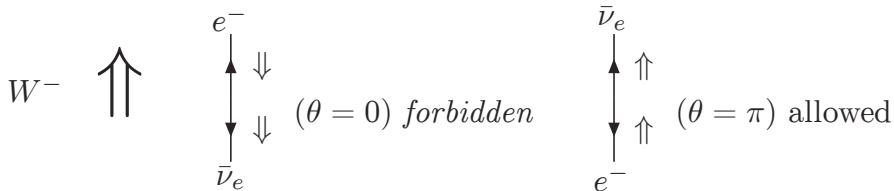
$$\frac{d\Gamma_0}{d\Omega} = \frac{G_F M_W^3}{16\pi^2 \sqrt{2}} \sin^2\theta \qquad \boxed{\Gamma(W \to e\nu) = \frac{G_F M_W^3}{6\pi\sqrt{2}}}$$

Other helicities: $\varepsilon_{\pm 1}^{\mu} = (0; -1, \mp i, 0)/\sqrt{2}$

$$\frac{d\Gamma_{\pm 1}}{d\Omega} = \frac{G_F M_W^3}{32\pi^2 \sqrt{2}}(1 \mp \cos\theta)^2$$

Extinctions at $\cos\theta = \pm 1$ are consequences of angular momentum conservation:

(situation reversed for $W^+ \to e^+ \nu_e$)

e^+ follows polarization direction of W^+

e^- avoids polarization direction of W^-

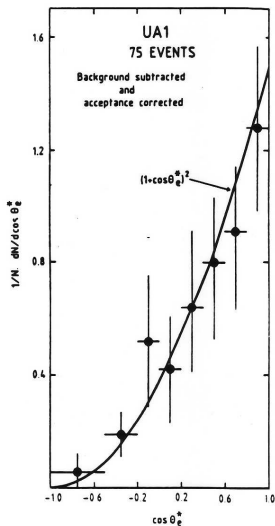*important* for discovery of $W^\pm$ in $\bar{p}p$ ($\bar{q}q$)   C violation

Fig. 2. The W decay angular distribution of the emission angle θ^* of the electron (positron) with respect to the proton (antiproton) direction in the rest frame of the W. Only those events for which the lepton charge and the decay kinematics are well determined have been used. The curve shows the $(V - A)$ expectation of $(1 + \cos\theta^*)^2$.

Interactions . . .

$$\mathcal{L}_{A\text{-}\ell} = \frac{gg'}{\sqrt{g^2 + g'^2}} \bar{e}\gamma^\mu e A_\mu$$

. . . vector interaction; $\Rightarrow A_\mu$ as γ, provided we identify

$$\boxed{gg'/\sqrt{g^2 + g'^2} \equiv e}$$

Define $g' = g \tan\theta_W$ $\qquad\qquad$ θ_W: weak mixing angle

$$
\begin{aligned}
g &= e/\sin\theta_W \geq e \\
g' &= e/\cos\theta_W \geq e
\end{aligned}
$$

$$Z_\mu = b_\mu^3 \cos\theta_W - \mathcal{A}_\mu \sin\theta_W \qquad A_\mu = \mathcal{A}_\mu \cos\theta_W + b_\mu^3 \sin\theta_W$$

$$\mathcal{L}_{Z-\nu} = \frac{-g}{4\cos\theta_W} \bar{\nu}\gamma^\mu (1 - \gamma_5)\nu \, Z_\mu$$

Purely left-handed!

Interactions . . .

$$\mathcal{L}_{Z-e} = \frac{-g}{4\cos\theta_W}\bar{e}\left[L_e\gamma^\mu(1-\gamma_5) + R_e\gamma^\mu(1+\gamma_5)\right]e\,Z_\mu$$

$$
\begin{aligned}
L_e &= 2\sin^2\theta_W - 1 = 2x_W + \tau_3 \\
R_e &= 2\sin^2\theta_W = 2x_W
\end{aligned}
$$

Z-decay calculation analogous to $W^\pm$

$$
\begin{aligned}
\Gamma(Z \to \nu\bar{\nu}) &= \frac{G_F M_Z^3}{12\pi\sqrt{2}} \\
\Gamma(Z \to e^+e^-) &= \Gamma(Z \to \nu\bar{\nu})\left[L_e^2 + R_e^2\right]
\end{aligned}
$$

Neutral-current interactions

New νe reaction, not present in $V - A$
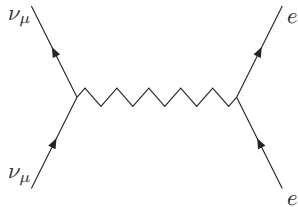
$$
\begin{aligned}
\sigma(\nu_\mu e \to \nu_\mu e) &= \frac{G_F^2 m_e E_\nu}{2\pi} \left[L_e^2 + R_e^2/3 \right] \\
\sigma(\bar{\nu}_\mu e \to \bar{\nu}_\mu e) &= \frac{G_F^2 m_e E_\nu}{2\pi} \left[L_e^2/3 + R_e^2 \right] \\
\sigma(\nu_e e \to \nu_e e) &= \frac{G_F^2 m_e E_\nu}{2\pi} \left[(L_e + 2)^2 + R_e^2/3 \right] \\
\sigma(\bar{\nu}_e e \to \bar{\nu}_e e) &= \frac{G_F^2 m_e E_\nu}{2\pi} \left[(L_e + 2)^2/3 + R_e^2 \right]
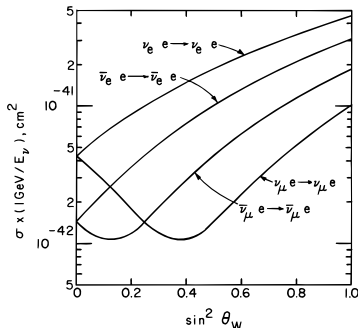\end{aligned}
$$

Gargamelle $\bar{\nu}_\mu e$ event (1973)
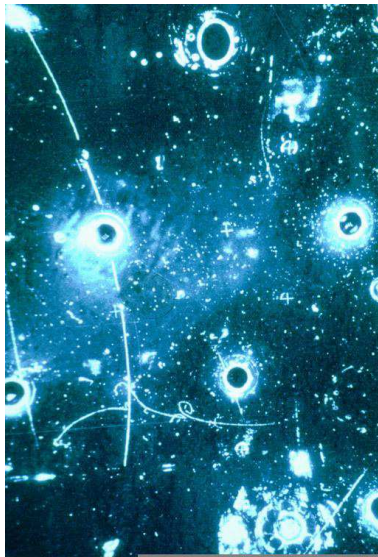
- Electromagnetism is mediated by a massless photon, coupled to the electric charge;

- Mediator of charged-current weak interaction acquires a mass $M_W^2 = \pi\alpha/G_F\sqrt{2}\sin^2\theta_W$,

- Mediator of (new!) neutral-current weak interaction acquires mass $M_Z^2 = M_W^2/\cos^2\theta_W$;

- Massive neutral scalar particle, the Higgs boson, appears, but its mass is not predicted;

- Fermions can acquire mass—values not predicted.

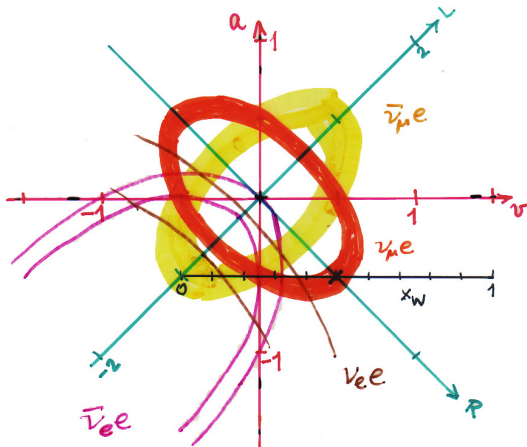Determine $\sin^2\theta_W$ to predict M_W, M_Z

"Model-independent" analysis

Measure all cross sections to determine chiral couplings L_e and R_e or traditional vector and axial couplings v and a

$$a = \tfrac{1}{2}(L_e - R_e) \qquad v = \tfrac{1}{2}(L_e - R_e)$$
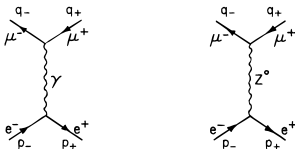$$L_e = v + a \qquad R_e = v - a$$

model-independent in V, A framework

Neutrino–electron scattering

Twofold ambiguity remains even after measuring all four cross sections: same cross sections result if we interchange $R_e \leftrightarrow -R_e$ ($v \leftrightarrow a$)

Consider $e^+ e^- \to \mu^+ \mu^-$

$$
\begin{aligned}
\mathcal{M} \;=\; & -ie^2 \bar{u}(\mu, q_-) \gamma_\lambda Q_\mu v(\mu, q_+) \frac{g^{\lambda\nu}}{s} \bar{v}(e, p_+) \gamma_\nu u(e.p_-) \\
& + \frac{i}{2} \left(\frac{G_F M_Z^2}{\sqrt{2}} \right) \bar{u}(\mu, q_-) \gamma_\lambda [R_\mu(1 + \gamma_5) + L_\mu(1 - \gamma_5)] v(\mu, q_+) \\
& \times \frac{g^{\lambda\nu}}{s - M_Z^2} \bar{v}(e, p_+) \gamma_\nu [R_e(1 + \gamma_5) + L_e(1 - \gamma_5)] u(e, p_-)
\end{aligned}
$$

muon charge $Q_\mu = -1$

$$e^+e^- \rightarrow \mu^+\mu^- \ldots$$

$$
\begin{aligned}
\frac{d\sigma}{dz} &= \frac{\pi\alpha^2 Q_\mu^2}{2s}(1+z^2) \\[2mm]
&\quad -\frac{\alpha Q_\mu G_F M_Z^2 (s - M_Z^2)}{8\sqrt{2}[(s-M_Z^2)^2 + M_Z^2\Gamma^2]} \\
&\quad \times [(R_e + L_e)(R_\mu + L_\mu)(1+z^2) + 2(R_e - L_e)(R_\mu - L_\mu)z] \\[2mm]
&\quad +\frac{G_F^2 M_Z^4 s}{64\pi[(s-M_Z^2)^2 + M_Z^2\Gamma^2]} \\
&\quad \times [(R_e^2 + L_e^2)(R_\mu^2 + L_\mu^2)(1+z^2) + 2(R_e^2 - L_e^2)(R_\mu^2 - L_\mu^2)z]
\end{aligned}
$$

Measuring A resolves ambiguity

Forward-backward asymmetry $A \equiv \dfrac{\int_0^1 \, dz \, d\sigma/dz - \int_{-1}^0 \, dz \, d\sigma/dz}{\int_{-1}^1 \, dz \, d\sigma/dz}$

$$\lim_{s/M_Z^2 \ll 1} A = \frac{3 G_F s}{16 \pi \alpha Q_\mu \sqrt{2}} (R_e - L_e)(R_\mu - L_\mu)$$

$$\approx -6.7 \times 10^{-5} \left(\frac{s}{1 \text{ GeV}^2} \right) (R_e - L_e)(R_\mu - L_\mu) = -3 G_F s a^2 / 4\pi\alpha\sqrt{2}$$
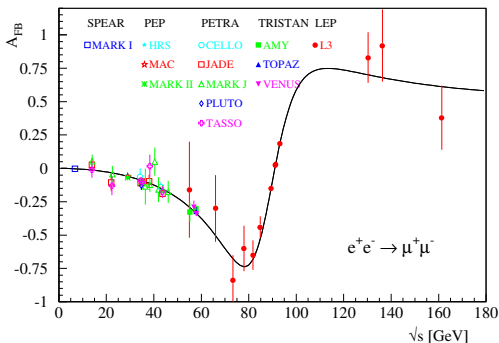
Neutrino–electron scattering $e^+e^- \rightarrow \mu^+\mu^-$
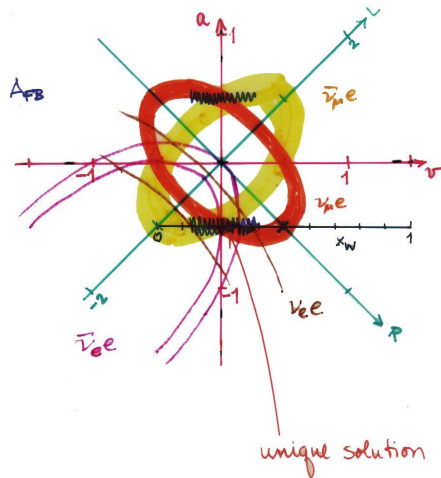
Validate EW theory, measure $\sin^2\theta_W$

With a measurement of $\sin^2\theta_W$, predict

$$M_W^2 = \pi\alpha/G_F\sqrt{2}\sin^2\theta_W \approx (37.28\ \text{GeV}/c^2)^2/\sin^2\theta_W \quad M_Z^2 = M_W^2/\cos^2\theta_W$$
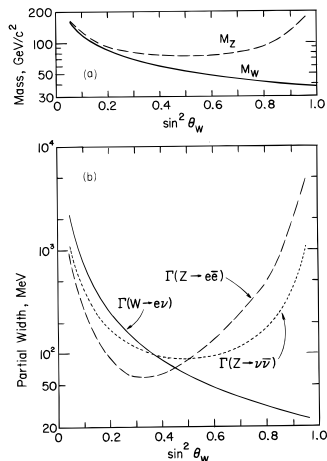
First Z from UA1

568 Intermediate Vector Bosons W^+, W^-, and Z^0

a)

b)

$p_t > 1$ GeV/c

ClibPDF - www.fastio.com   Fig. 16a,b
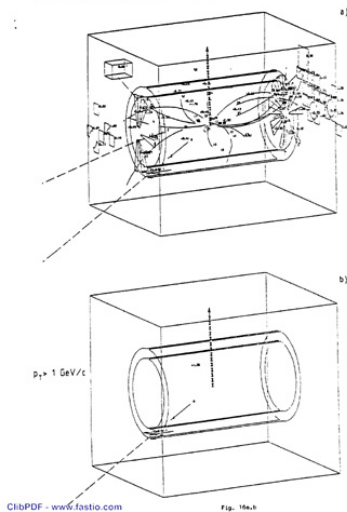
νe cross sections ...
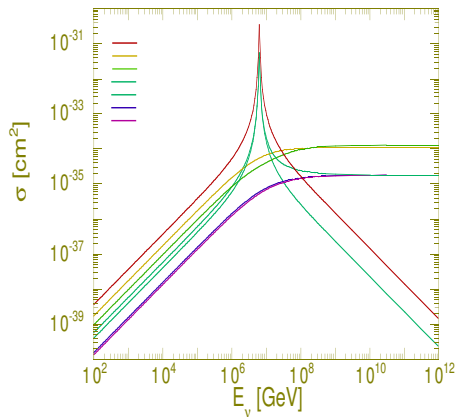
At low energies: $\sigma(\bar{\nu}_e e \rightarrow \text{ hadrons}) > \sigma(\nu_\mu e \rightarrow \mu \nu_e) > \sigma(\nu_e e \rightarrow \nu_e e) > \sigma(\overline{(}\nu_e e \rightarrow \bar{\nu}_\mu \mu) > \sigma(\bar{\nu}_e e \rightarrow \bar{\nu}_e e) > \sigma(\nu_\mu e \rightarrow \nu_\mu e) > \sigma(\bar{\nu}_\mu e \rightarrow \bar{\nu}_\mu e)$

Electroweak interactions of quarks

- Left-handed doublet

$$
\begin{array}{cccc}
 & I_3 & Q & Y = 2(Q - I_3) \\
\mathsf{L}_q = \begin{pmatrix} u \\ d \end{pmatrix}_L & \begin{array}{c} \frac{1}{2} \\ -\frac{1}{2} \end{array} & \begin{array}{c} +\frac{2}{3} \\ -\frac{1}{3} \end{array} & \frac{1}{3}
\end{array}
$$

- two right-handed singlets

	I_3	Q	$Y = 2(Q - I_3)$
$\mathsf{R}_u = u_R$	0	$+\frac{2}{3}$	$+\frac{4}{3}$
$\mathsf{R}_d = d_R$	0	$-\frac{1}{3}$	$-\frac{2}{3}$

Electroweak interactions of quarks

- CC interaction

$$\mathcal{L}_{W\text{-}q} = \frac{-g}{2\sqrt{2}} \left[\bar{u}_e \gamma^\mu (1 - \gamma_5) d \, W_\mu^+ + \bar{d} \gamma^\mu (1 - \gamma_5) u \, W_\mu^- \right]$$

identical in form to $\mathcal{L}_{W\text{-}\ell}$: universality $\Leftrightarrow$ weak isospin

- NC interaction

$$\mathcal{L}_{Z\text{-}q} = \frac{-g}{4\cos\theta_W} \sum_{i=u,d} \bar{q}_i \gamma^\mu \left[L_i (1 - \gamma_5) + R_i (1 + \gamma_5) \right] q_i \, Z_\mu$$

$$L_i = \tau_3 - 2Q_i \sin^2\theta_W \quad R_i = -2Q_i \sin^2\theta_W$$

equivalent in form (not numbers) to $\mathcal{L}_{Z\text{-}\ell}$

Trouble in Paradise

Universal $u \leftrightarrow d$, $\nu_e \leftrightarrow e$ *not quite right*

$$\text{Good:} \quad \begin{pmatrix} u \\ d \end{pmatrix}_L \rightarrow \text{Better:} \quad \begin{pmatrix} u \\ d_\theta \end{pmatrix}_L$$

$$d_\theta \equiv d \cos\theta_C + s \sin\theta_C \quad \cos\theta_C = 0.9736 \pm 0.0010$$

"Cabibbo-rotated" doublet perfects CC interaction (up to small third-generation effects) but $\Rightarrow$ serious trouble for NC

$$
\begin{aligned}
\mathcal{L}_{Z\text{-}q} &= \frac{-g}{4\cos\theta_W} Z_\mu \{ \bar{u}\gamma^\mu \left[L_u(1-\gamma_5) + R_u(1+\gamma_5) \right] u \\
&+ \bar{d}\gamma^\mu \left[L_d(1-\gamma_5) + R_d(1+\gamma_5) \right] d \cos^2\theta_C \\
&+ \bar{s}\gamma^\mu \left[L_d(1-\gamma_5) + R_d(1+\gamma_5) \right] s \sin^2\theta_C \\
&+ \bar{d}\gamma^\mu \left[L_d(1-\gamma_5) + R_d(1+\gamma_5) \right] s \sin\theta_C \cos\theta_C \\
&+ \bar{s}\gamma^\mu \left[L_d(1-\gamma_5) + R_d(1+\gamma_5) \right] d \sin\theta_C \cos\theta_C \}
\end{aligned}
$$

Strangeness-changing NC interactions highly suppressed!
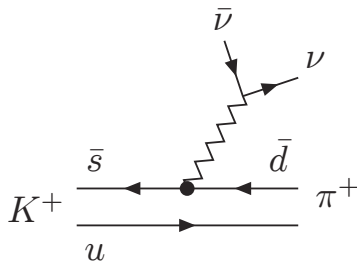
BNL E-787/E-949 has three $K^+ \to \pi^+ \nu \bar{\nu}$ candidates, with $\mathcal{B}(K^+ \to \pi^+ \nu \bar{\nu}) = 1.47^{+1.30}_{-0.89} \times 10^{-10}$
Phys. Rev. Lett. **93**, 031801 (2004)

(SM: 0.78 ± 0.11: U. Haisch, hep-ph/0605170)

Glashow–Iliopoulos–Maiani
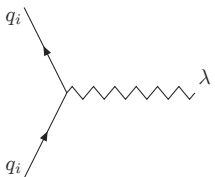
two LH doublets: $\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \quad \begin{pmatrix} u \\ d_\theta \end{pmatrix}_L \begin{pmatrix} c \\ s_\theta \end{pmatrix}_L$

$(s_\theta = s \cos\theta_C - d \sin\theta_C)$

$+$ right-handed singlets, e_R, μ_R, u_R, d_R, c_R, s_R

Required new charmed quark, c

Cross terms vanish in $\mathcal{L}_{Z\text{-}q}$,

$$\frac{-ig}{4\cos\theta_W}\gamma_\lambda[(1-\gamma_5)L_i + (1+\gamma_5)R_i] \quad ,$$

$$L_i = \tau_3 - 2Q_i \sin^2\theta_W \quad R_i = -2Q_i \sin^2\theta_W$$

flavor-diagonal interaction!

Straightforward generalization to n quark doublets

$$\mathcal{L}_{W\text{-}q} = \frac{-g}{2\sqrt{2}} \left[\bar{\Psi}\gamma^\mu(1-\gamma_5)\mathcal{O}\Psi\, W_\mu^+ + \text{h.c.} \right]$$

composite $\Psi = \begin{pmatrix} u \\ c \\ \vdots \\ d \\ s \\ \vdots \end{pmatrix}$

flavor structure $\mathcal{O} = \begin{pmatrix} 0 & U \\ 0 & 0 \end{pmatrix}$

U: unitary quark mixing matrix

Weak-isospin part: $\quad \mathcal{L}_{Z\text{-}q}^{\text{iso}} = \dfrac{-g}{4\cos\theta_W}\bar{\Psi}\gamma^\mu(1-\gamma_5)\left[\mathcal{O}, \mathcal{O}^\dagger\right]\Psi$

Since $\quad \left[\mathcal{O}, \mathcal{O}^\dagger\right] = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \propto \tau_3$

$\boxed{\Rightarrow \text{NC interaction is flavor-diagonal}}$

General $n \times n$ mixing matrix U: $n(n-1)/2$ real $\angle$, $(n-1)(n-2)/2$ complex phases

3×3 (Cabibbo–Kobayashi-Maskawa): $3 \angle + 1$ phase $\qquad \Rightarrow$ CP violation

Fermion mass is accommodated, not explained

- All fermion masses $\sim$ physics beyond the standard model!

- $\zeta_t \approx 1$ $\zeta_e \approx 3 \times 10^{-6}$ $\zeta_\nu \approx 10^{-10}$??

  What accounts for the range and values of the Yukawa couplings?

- There may be *other sources* of neutrino mass

Aside: varieties of neutrino mass

Chiral decomposition of Dirac spinor:

$$\psi = \tfrac{1}{2}(1 - \gamma_5)\psi + \tfrac{1}{2}(1 + \gamma_5)\psi \equiv \psi_L + \psi_R$$

Charge conjugate of RH field is LH:

$$\psi_L^c \equiv (\psi^c)_L = (\psi_R)^c$$

Possible forms for mass terms
Dirac connects LH, RH components of *same field*

$$\mathcal{L}_D = D(\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L) = D\bar{\psi}\psi$$

$$\implies \text{mass eigenstate } \psi = \psi_L + \psi_R$$

(invariant under global phase rotation $\nu \to e^{i\theta}\nu$, $\ell \to e^{i\theta}\ell$, so that lepton number is conserved)

Possible forms for mass terms (cont'd)

Majorana connects LH, RH components of *conjugate fields*

$$
\begin{aligned}
-\mathcal{L}_{\mathrm{MA}} &= A(\bar{\psi}_{\mathrm{R}}^{\mathrm{c}}\psi_{\mathrm{L}} + \bar{\psi}_{\mathrm{L}}\psi_{\mathrm{R}}^{\mathrm{c}}) = A\bar{\chi}\chi \\
-\mathcal{L}_{\mathrm{MB}} &= B(\bar{\psi}_{\mathrm{L}}^{\mathrm{c}}\psi_{\mathrm{R}} + \bar{\psi}_{\mathrm{R}}\psi_{\mathrm{L}}^{\mathrm{c}}) = B\bar{\omega}\omega
\end{aligned}
$$

for which the mass eigenstates are

$$
\begin{aligned}
\chi &\equiv \psi_{\mathrm{L}} + \psi_{\mathrm{R}}^{\mathrm{c}} = \chi^{\mathrm{c}} = \psi_{\mathrm{L}} + (\psi_{\mathrm{L}})^{\mathrm{c}} \\
\omega &\equiv \psi_{\mathrm{R}} + \psi_{\mathrm{L}}^{\mathrm{c}} = \omega^{\mathrm{c}} = \psi_{\mathrm{R}} + (\psi_{\mathrm{R}})^{\mathrm{c}}
\end{aligned}
$$

$\mathcal{L}_{\mathrm{M}}$ violates lepton number by two units
$\Rightarrow$ Majorana ν can mediate $\beta\beta_{0\nu}$ decays

$$(Z, A) \rightarrow (Z+2, A) + e^{-} + e^{-}$$

Detecting $\beta\beta_{0\nu}$ would offer decisive evidence for Majorana nature of ν

Parity violation in atoms

Nucleon appears elementary at very low Q^2; effective Lagrangian for nucleon β-decay

$$\mathcal{L}_\beta = -\frac{G_F}{\sqrt{2}} \bar{e} \gamma_\lambda (1 - \gamma_5) \nu \; \bar{p} \gamma^\lambda (1 - g_A \gamma_5) n \quad g_A \approx 1.26: \text{ axial charge}$$

NC interactions ($x_W \equiv \sin^2 \theta_W$):

$$\mathcal{L}ep = \frac{G_F}{2\sqrt{2}} \bar{e} \gamma_\lambda (1 - 4x_W - \gamma_5) e \; \bar{p} \gamma^\lambda (1 - 4x_W - \gamma_5) p \;,$$

$$\mathcal{L}en = \frac{G_F}{2\sqrt{2}} \bar{e} \gamma_\lambda (1 - 4x_W - \gamma_5) e \; \bar{n} \gamma^\lambda (1 - \gamma_5) n$$

▷ Regard nucleus as a noninteracting collection of Z protons and N neutrons
▷ Perform NR reduction; nucleons contribute coherently to $A_e V_N$ coupling, so dominant
P-violating contribution to eN amplitude is

$$\mathcal{M}_{pv} = \frac{-iG_F}{2\sqrt{2}} Q^W \bar{e} \rho_N(\mathbf{r}) \gamma_5 e$$

$\rho_N(\mathbf{r})$: nucleon density at e^- coordinate $\mathbf{r}$ $\quad Q^W \equiv Z(1 - 4x_W) - N$: weak charge
Bennett & Wieman (Boulder): 6S-7S transition polarizability

$$
\begin{aligned}
Q_W(\text{Cs}) &= -72.06 \pm 0.28 \;(\text{expt}) \pm 0.34 \;(\text{theory}) \\
&\rightarrow -72.71 \pm 0.29 \;(\text{expt}) \pm 0.39 \;(\text{theory}) \\
\text{Theory} &= -73.19 \pm 0.13
\end{aligned}
$$

Guéna, Lintz, Bouchiat, *Mod. Phys. Lett. A* **20**, 375 (2005)
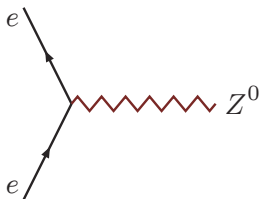
The top quark must exist

- Two families

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \qquad \begin{pmatrix} c \\ s \end{pmatrix}_L$$

don't account for CP violation. Need third family or another answer.

Given the existence of b, (τ)

- top is needed for an anomaly-free EW theory
- absence of FCNC in b decay ($b \not\to s\ell^+\ell^-$, etc.)
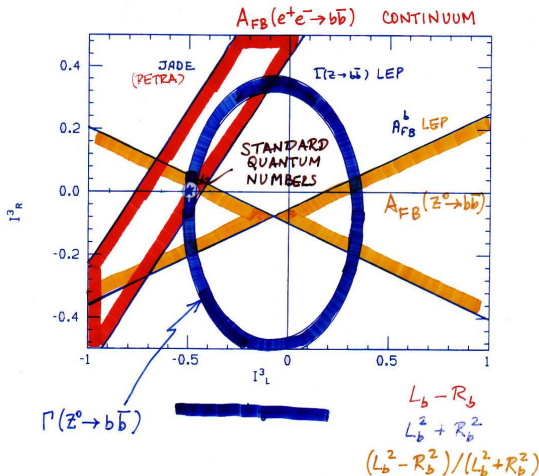- b has weak isospin $I_{3L} = -\frac{1}{2}$; needs partner

$$\begin{pmatrix} t \\ b \end{pmatrix}_L$$

$$L_b = I_{3L} - Q_b \sin^2\theta_W$$

$$R_b = I_{3R} - Q_b \sin^2\theta_W$$

Measure $I_{3L}^{(b)} = -0.490^{+0.015}_{-0.012}$ $I_{3R}^{(b)} = -0.028 \pm 0.056$

$\Gamma(Z^0 \to b\bar{b})$

$$L_b - R_b$$
$$L_b^2 + R_b^2$$
$$(L_b^2 - R_b^2)/(L_b^2 + R_b^2)$$

Needed: top with $I_{3L} = +\frac{1}{2}$

Schaile / Zerwas, *Phys. Rev. D***45**, 3262 (1992)