Measuring Top-Quark Polarization in Top-Pair + Missing-Energy Events

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Reference:
Why top-quark?

- Electroweak triangle
Why top-quark?

- Electroweak triangle
Top-quark pair plus missing energy

- Typical collider signature in several NP models

- Minimal Supersymmetric extension of the Standard Model (MSSM)

- Little Higgs Model with T-parity (LHT)

- Universal Extra Dimension Model (UED)
Our goal

• is to find a method to measure top-quark polarization without reconstructing top-quark kinematics.

• Advantages of our method:
  ✓ It is sensitive to the top-quark polarization.
  ✓ It is not sensitive to the mass splitting between a heavy resonance parent and the DM candidate, provided that this splitting is not too small.
  ✓ The difference between \( t_L \) and \( t_R \) is not sensitive to the spin of a heavy parent resonance or to the collider energy.
Top quark is very special

- Large mass: 173 GeV ~ VEV (246 GeV) $y_t \sim O(1)$

- Short lifetime:

  $\frac{1}{m_t} = 5 \times 10^{-27} \text{ s}$
  $\frac{1}{\Gamma_t} = 5 \times 10^{-25} \text{ s}$
  $\frac{1}{\Lambda_{QCD}} = 3 \times 10^{-24} \text{ s}$

- "bare" quark:

  spin info well kept among its decay products
Measuring $t$-polarization

- Traditional method of measuring top-polarization is through the angle between the charged lepton and top-quark spin.

The charged-lepton tends to follow the top-quark spin direction.
Charged lepton distribution

- In the rest frame of the top-quark

\[
\frac{d\Gamma}{dx d\cos \theta} = \frac{\alpha_W^2 m_t}{32\pi A B} x (1 - x) \text{Arctan} \left[ \frac{Ax}{B - x} \right] \frac{1 + s_t \cos \theta}{2}
\]

\[x \equiv 2E_\ell/m_t\]

\[\lambda_t = + \quad \text{right-handed}\]

\[\lambda_t = - \quad \text{left-handed}\]

Top-quark momentum has to be known.

\[Czarnecki, Jezabek, Kuhn, NPB351,70 (1991)\]
Top-quark reconstruction

- The charged leptons produced always in association with an **invisible** neutrino

\[ p_x^\nu = E_T(x) \quad p_y^\nu = E_T(y) \quad m_\nu = 0 \]

\[ p_z^\nu \text{ unknown} \]

- **W**-boson on-shell condition

\[ m_W^2 = (p_\ell + p_\nu)^2 \]

\[ p_z^\nu = \frac{1}{2(p_T^e)^2} \left[ A p_z^e \pm E_e \sqrt{A^2 - 4(p_T^e)^2 \vec{E}_T^2} \right] \]

\[ A = m_W^2 + 2 \vec{p}_T^e \cdot \vec{E}_T \]
Difficulty in $t\bar{t} + E_T$ events

- It is impossible to reconstruct a top-quark in the leptonic-decay mode. Angular distribution of the charged-lepton cannot be used.

Masses and spins of $\tilde{t}$ and $\tilde{\chi}_0$ are not known.
Charged lepton distribution

- In the rest frame of the top-quark

\[
\frac{d\Gamma}{dxd\cos\theta} = \frac{\alpha_W^2 m_t}{32\pi AB} x(1-x) \arctan \left( \frac{Ax}{B - x} \right) \frac{1 + s_t \cos\theta}{2}
\]

\[
\frac{1}{s_t} \left( \frac{d\Gamma}{d\cos\theta} \right)_{\text{hel}}
\]

\[
\lambda_t = + \quad \text{right-handed}
\]

\[
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\]

The energy and angle are correlated once top is boosted.

Czarnecki, Jezabek, Kuhn, NPB351,70 (1991)
Lepton energy and top-quark polarization

Lepton energy distribution is sensitive to top quark polarization.

\[ \frac{d\Gamma(\hat{s}_t)}{dx} = \frac{\alpha_W^2 m_t}{64\pi AB} \int_{z_{\min}}^{z_{\max}} x\gamma^2 \left[ 1 - x\gamma^2 (1 - z\beta) \right] 
\times \left( 1 + \hat{s}_t \frac{z - \beta}{1 - z\beta} \right) \text{Arctan} \left[ \frac{Ax\gamma^2 (1 - z\beta)}{B - x\gamma^2 (1 - z\beta)} \right] dz \]

\[ A = \frac{\Gamma_W}{m_W} \quad B = \frac{m_W^2}{m_t^2} \approx 0.216 \]
\[ \gamma = \frac{E_t}{m_t} \quad \beta = \sqrt{1 - 1/\gamma^2} \]
\[ z_{\min} = \max \left[ \left( 1 - 1/\gamma^2 x \right) / \beta, -1 \right] \]
\[ z_{\max} = \min \left[ \left( 1 - B/\gamma^2 x \right) / \beta, 1 \right] \]

Schmidt, Peskin, PRL69, 410 (1992)
Lepton energy and top-quark polarization

- Define a variable $\mathcal{R}$ to quantify the difference between $t_L$ and $t_R$

$$\mathcal{R}(x_c) \equiv \frac{\text{Area}(x_\ell < x_c)}{\text{Area}(\text{tot})} = \text{Area}(x_\ell < x_c)$$

![Graph showing $\mathcal{R}(x_c)$ for different lepton energies and top-quark polarization](a)
\[ R(x_c) = \frac{3x_c(1 - \lambda_t)}{2(1 + 2B)} - \frac{3\lambda_t x_c^2(1 - B + \ln B)}{2(1 + 2B)(1 - B)^2} \]

\[ R(x_c) = \frac{B^2(2B - 3)}{(1 + 2B)(1 - B)^2} + \frac{3x_c(1 - \lambda_t)}{2(1 - B)^2(1 + 2B)} \]

\[ -\frac{3x_c^2[1 + 2\lambda_t \ln(x_c/2)]}{4(1 - B)^2(1 + 2B)} + \frac{x_c^3(1 + 3\lambda_t)}{8(1 - B)^2(1 + 2B)} \]

\[ x_c = \frac{2E_\ell}{E_t} \]
Lepton energy and top-quark polarization

- Identical decay chains

\[ x'_{\ell} = \frac{2E_{\ell^+}}{E_{\tilde{t}}} \]
Toy model mimicking MSSM

- MSSM like:

\[ \mathcal{L}_{\tilde{t}t\tilde{\chi}} = g_{\text{eff}} \tilde{t}\tilde{\chi}(\cos \theta_{\text{eff}} P_L + \sin \theta_{\text{eff}} P_R) t \]

Collider signature: \( b\bar{b}jj\ell^+ E_T \)

- Major SM backgrounds
Collider simulation

- **Basic selection cuts**
  \[ p_T^\ell > 20 \text{ GeV} \quad p_T^j > 25 \text{ GeV} \]
  \[ \mathbb{E}_T > 25 \text{ GeV} \quad \Delta R_{jj,\ell j} > 0.4 \]
  \[ |\eta_{\ell,j}| < 2.5 \]

- **Hard cuts**
  \[ \mathbb{E}_T > 100 \text{ GeV} \quad H_T > 500 \text{ GeV} \]
  \[ H_T = p_T^\ell + p_T^{j_1} + p_T^{j_2} + p_T^b + p_T^{\bar{b}} + \mathbb{E}_T \]

- **\bar{t} \to 3j** reconstruction (Minimal-\(\chi^2\) method)
  
  Loop over all jet combinations and pick up the one minimize

  \[ \chi^2 = \frac{(m_W - m_{jj})^2}{\Delta m_W^2} + \frac{(m_t - m_{jjj})^2}{\Delta m_t^2} \]

  \[ m_\bar{t} = 360 \text{ GeV} \quad m_{\tilde{\chi}} = 50 \text{ GeV} \]
Signal versus Backgrounds

- Cross section (fb) of signal and backgrounds at 14TeV LHC

<table>
<thead>
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<th>Basic</th>
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- $E_T$ solution cut
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  $$A = m_W^2 + 2 \vec{p}_T^e \cdot \vec{E}_T$$

  **Implicit condition:** $A^2 - 4(p_T^e)^2 E_T^2 \geq 0$
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$$A = m_W^2 + 2 p_T^e \cdot E_T$$

$$A^2 - 4 (p_T^e)^2 E_T^2 \leq 0$$

Han, Mahbubani, Walker, Wang, JHEP 0905, 117 (2009)
$\mathcal{R}(x_c)$ versus $\mathcal{R}'(x_c)$

$x_\ell = 2E_{\ell^+}/E_L \quad \Rightarrow \quad x'_\ell = 2E_{\ell^+}/E_{\bar{L}}$

(a) $t_L$ and $t_R$

- $t_L$: $x_\ell$ in lab
- $t_R$: $x'_\ell$ in lab
- $\mathcal{R}(x_c)$
- $\mathcal{R}'(x_c)$
$R'$ distribution

- $t_L$ and $t_R$ are separated

LHC: 14 TeV, 100 fb$^{-1}$
The result holds for $T$-prime pair production in the LHT.
Summary

- Conventional method of measuring top-quark polarization in the charged lepton angle distribution failed in $t\bar{t} + E_T$ events.

- The long ignored lepton energy could also be used to measure top-quark polarization without reconstructing the top-quark kinematics.

- The information of the mass and spin of new heavy particles in the intermediate state is no longer needed.

$\rightarrow$ Probe the interaction before mass and spin
THANK YOU!