Testing anomalous couplings at High Energy Colliders

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OUTLINE

• Testing anomalous Higgs couplings via vector boson scattering at LHC

• Testing anomalous gauge couplings of the Higgs boson at high energy photon colliders

• Testing fermion anomalous coupling at future ILC
Testing anomalous Higgs couplings via vector boson scattering on LHC

EWSBM is not known yet. In EW theory, all particle masses come from the VEV. Probing EWSBM concerns the understanding of the original of all particle masses.

- SM Higgs sector suffers from triviality and unnaturalness. It is currently accepted that there should be new physics beyond the SM above certain high energy scale $\Lambda$.

- New physics:
  - SUSY
  - TC
  - top seesaw
  - extra dim.
  - little Higgs

\[ \downarrow \]

\[ \text{nature} = ? \]

no-lose probe of \textit{EWSBM} and \textit{new physics} is expected!
No-lose Probe of New Physics

Effective theory is a way of No-lose Probe of New Physics:

Effective Lagrangian provide a model-independent description and the anomalous couplings reflect the effect of the new physics.

Finding sensitive processes to measure the coefficients in general EL is needed to obtain effective Lagrangian reflecting the nature.

Testing effective anomalous gauge couplings of Higgs boson can discriminate the EWSB sector of the new physics model from SM
ANOMALOUS HVV COUPLINGS

We take linear realization as example.

**SM:** only dim-4 $\Phi \nabla \nabla$ couplings,

**New physics:** can contain extra dim-6 $\Phi \nabla \nabla$ couplings.

M. C. Gonzalez-Garcia [Int. J. Mod. Phys. A14, 3121 (1999)]:

$$L^{(6)}_{\text{eff}} = \sum_{n} \frac{f_n}{\Lambda^2} O_n(\Phi, W^a, B)$$

\[\begin{align*}
O_{\Phi,1} &= (D_\mu \Phi)^\dagger \Phi^\dagger \Phi (D^\mu \Phi), \\
O_{DW} &= \text{Tr} \left( \left[ D_\mu, \hat{W}_{\nu \rho} \right] \left[ D^\mu, \hat{W}^{\nu \rho} \right] \right), \\
O_{\Phi,2} &= \frac{1}{2} \partial^\mu (\Phi^\dagger \Phi) \partial_\mu (\Phi^\dagger \Phi), \\
O_{WWW} &= \text{Tr} \left[ \hat{W}_{\mu \nu} \hat{W}^{\nu \rho} \hat{W}^\mu_\rho \right], \\
O_{BB} &= \Phi^\dagger \hat{B}_{\mu \nu} \hat{B}^{\mu \nu} \Phi, \\
O_{B} &= (D_\mu \Phi)^\dagger \hat{B}^{\mu \nu} (D_\nu \Phi), \\
O_{BW} &= \Phi^\dagger \hat{B}_{\mu \nu} \hat{W}^{\mu \nu} \Phi, \\
O_{DB} &= -\frac{g'^2}{2} (\partial_\mu B_{\nu \rho}) (\partial^\mu B^{\nu \rho}), \\
O_{\Phi,3} &= \frac{1}{3} (\Phi^\dagger \Phi)^3, \\
O_{WW} &= \Phi^\dagger \hat{W}_{\mu \nu} \hat{W}^{\mu \nu} \Phi, \\
O_{W} &= (D_\mu \Phi)^\dagger \hat{W}^{\mu \nu} (D_\nu \Phi), \\
\hat{B}_{\mu \nu} &= ig' B_{\mu \nu}, \quad \hat{W}_{\mu \nu} = ig \sigma^a W^a_{\mu \nu}.
\end{align*}\]
Anomalous HVV Couplings

relations between anomalous Higgs coupling constants $g_n$’s and linear realized effective coupling constants $f_n$’s

\[ L_{\text{eff}}^{\text{HVV}} = g_{H\gamma\gamma} H A_{\mu\nu} A^{\mu\nu} + g_{HZ\gamma}^{(1)} A_{\mu\nu} Z^{\mu} \partial^{\nu} H + g_{HZ\gamma}^{(2)} H A_{\mu\nu} Z^{\mu\nu} \\
+ g_{HZZ}^{(1)} Z_{\mu\nu} Z^{\mu} \partial^{\nu} H + g_{HZZ}^{(2)} H Z_{\mu\nu} Z^{\mu\nu} \\
+ g_{HWW}^{(1)} (W^+_{\mu\nu} W^-_{\nu\alpha} H + h.c.) + g_{HWW}^{(2)} H W^+_{\mu\nu} W^-_{\mu\nu}, \]

\[ g_{H\gamma\gamma} = - \left( \frac{g M_W}{\Lambda^2} \right) \frac{s^2 (f_{BB} + f_{WW})}{2}, \quad g_{HZ\gamma}^{(1)} = \left( \frac{g M_W}{\Lambda^2} \right) \frac{s (f_W - f_B)}{2c}, \]

\[ g_{HZ\gamma}^{(2)} = \left( \frac{g M_W}{\Lambda^2} \right) \frac{s [s^2 f_{BB} - c^2 f_{WW}]}{c}, \quad g_{HZZ}^{(1)} = \left( \frac{g M_W}{\Lambda^2} \right) \frac{c^2 f_W + s^2 f_B}{2c^2}, \]

\[ g_{HZZ}^{(2)} = - \left( \frac{g M_W}{\Lambda^2} \right) \frac{s^4 f_{BB} + c^4 f_{WW}}{2c^2}, \quad g_{HWW}^{(1)} = \left( \frac{g M_W}{\Lambda^2} \right) \frac{f_W}{2}, \]

\[ g_{HWW}^{(2)} = - \left( \frac{g M_W}{\Lambda^2} \right) f_{WW}, \quad s \equiv \sin \theta_W, \quad c \equiv \cos \theta_W. \]
Testing HVV Anomalous Couplings

Now, a light Higgs candidate is found, is it a Higgs in SM or new physics?

$-\text{testing the HVV couplings } (V = W, Z, \gamma)$.

$$\mathcal{L}_{\text{eff}}(H, V) = \sum_{V=W,Z,\gamma} \sum_{i=1}^{2} g_{HVV}^{(i)} \mathcal{O}_{HVV}^{(i)}(H, V, \varphi).$$

$W^+W^+$ scattering is the most sensitive test of $g_{HVV}^{(i)}$ at the LHC.

Testing HVV via WW Scatterings

Why sensitive?

Scattering amplitude contains two parts: (a) $T(V)$, (b) $T(H)$. 

\[ T(V): \quad W_L \quad \text{+} \quad W_L \quad \text{+} \quad W_L \quad \text{+} \quad \text{cross diagrams} \]

\[ T(H): \quad W_L \quad \text{+} \quad W_L \quad \text{+} \quad \text{cross diagrams} \]
Why sensitive?

Consider $W_L W_L \rightarrow W_L W_L$.

- In SM, $HVV$ coupling constant is $g$. At high energies,
  $$T(V) \sim E^2, \quad T(H) \sim E^2$$
  $$\Downarrow \quad \text{standard } HVV$$
  $$T(V) + T(H) \sim E^0$$

  guaranteeing unitarity of the $S$-matrix.

- With anomalous $g^{(i)}_{HVV}$ due to new physics, $E < \Lambda$,
  $$T(V) \sim E^2, \quad T(H) \sim E^2$$
  $$\Downarrow \quad \text{anomalous } HVV$$
  $$T(V) + T(H) \sim E^2$$

Cross section is sensitive to anomalous $g^{(i)}_{HVV}$. 
WW Scatterings on LHC

- **Signal**: $W, Z \rightarrow \text{leptons}$ (gold-plated mode)

- **Backgrounds** [J. Bagger et al., PRD 49 (1994) 1246]:
  - (a), QCD bkgd:
  - (b), top quark bkgd:
  - (c), EW bkgd:
Numerical results

We calculated the full tree level cross sections of

\[ pp \rightarrow Z_L Z_L jj \rightarrow l^+ l^- l^+ l^- jj, l^+ l^- \nu \bar{\nu} jj, \]
\[ pp \rightarrow W_L^+ W_L^- jj \rightarrow l^+ \nu l^- \bar{\nu} jj, \]
\[ pp \rightarrow W_L^+ W_L^+ jj \rightarrow l^+ \nu l^+ \nu jj, \]
\[ pp \rightarrow W_L^- W_L^- jj \rightarrow l^- \bar{\nu} l^- \bar{\nu} jj, \]
\[ pp \rightarrow Z_L W_L^+ jj \rightarrow l^+ l^- l^+ \nu jj, \]
\[ pp \rightarrow Z_L W_L^- jj \rightarrow l^+ l^- l^- \bar{\nu} jj. \]

Our result shows: \( W^+ W^+ \rightarrow W^+ W^+ \) is the most sensitive channel.
Numerical results

Obtained 1σ detectable limit (sensitivity):

\[ \frac{f_{WW}}{\Lambda^2} > 0.85 \text{ TeV}^{-2}, \quad \frac{f_{WW}}{\Lambda^2} < -1.0 \text{ TeV}^{-2}, \]

or \[ \frac{f_{WW}}{\Lambda^2} > 1.6 \text{ TeV}^{-2} \quad \frac{f_{WW}}{\Lambda^2} < -1.6 \text{ TeV}^{-2}. \]

Correspondingly (in units of TeV\(^{-1}\)),

\[ 1\sigma : \quad |g_{HWW}^{(1)}| > 0.026, \quad |g_{HZZ}^{(1)}| > 0.026, \]

\[ |g_{HWW}^{(2)}| > 0.083, \quad |g_{HZZ}^{(2)}| > 0.032, \quad |g_{HZ\gamma}^{(2)}| > 0.018, \]

More sensitive than the existing LHC 1σ limit:

\[ 1\sigma : \quad |g_{HWW}^{(2)}| \geq 0.1 \text{ TeV}^{-1} \quad (pp \rightarrow HX, \ H \rightarrow \gamma\gamma, \ \tau^+\tau^-). \]
Numerical results

Obtained 2σ detectable limit (sensitivity):

\[
f_W/\Lambda^2 > 1.2 \ \text{TeV}^{-2}, \quad f_W/\Lambda^2 < -1.4 \ \text{TeV}^{-2},
\]

or

\[
f_{WW}/\Lambda^2 > 2.2 \ \text{TeV}^{-2} \quad f_{WW}/\Lambda^2 < -2.2 \ \text{TeV}^{-2}.
\]

Correspondingly (in units of TeV\(^{-1}\)),

\[
2\sigma : \ |g_{HWW}^{(1)}| > 0.036, \quad |g_{HZZ}^{(1)}| > 0.036,
\]

\[
|g_{HWW}^{(2)}| > 0.11, \quad |g_{HZZ}^{(2)}| > 0.044, \quad |g_{HZ\gamma}^{(2)}| > 0.024,
\]

Close to the sensitivity of the existing LC 2σ limit:

\[
|g_{HZZ}^{(i)}|, \quad |g_{HZ\gamma}^{(i)}| \geq 10^{-3} - 10^{-2} \ \text{TeV}^{-1}.
\]
Semi-leptonic mode in WW scatterings

the most sensitive test at the LHC is via the pure leptonic mode in W+W+ scattering, However, the required integrated luminosity $300 \text{ fb}^{-1}$.

$$pp \rightarrow W^+W^+ j_1^f j_2^f \rightarrow l^+\nu_l l^+\nu_l j_1^f j_2^f$$

we study the possibility of taking the Semi-leptonic mode which can have a larger cross section.

$$pp \rightarrow W^+W^\pm j_1^f j_2^f \rightarrow l^+\nu_l j_1 j_2 j_1^f j_2^f$$
Semileptonic Mode

Semileptonic mode which can have a larger cross section. Since it is not possible to distinguish $W^+ \rightarrow jj$ and $W^- \rightarrow jj$ experimentally, we have to study the scatterings

$$pp \rightarrow W^+W^\pm j_1^f j_2^f \rightarrow l^+\nu_l j_1 j_2 j_1^f j_2^f$$

Now the final state contains four jets, so that the parton level study is not sufficient for finding out the suitable kinematic cuts to suppress the large backgrounds.

We worked at the hadron level, calculating the full tree level contributions to the signal and backgrounds using the helicity amplitude methods and the package PYTHIA with its default fragmentation model.
Semileptonic Mode

The required integrated luminosity for reaching the $3\sigma$ sensitivity can be reduced to $100 \text{ fb}^{-1}$, reduced by a factor of 3.

If the anomalous couplings in nature are actually not small, about $1\sigma$, the experiment can start the test for an integrated luminosity of $50 \text{ fb}^{-1}$. 
Testing anomalous gauge couplings of the Higgs boson at high energy photon colliders

At the LHC, the most sensitive constraints on $f_W/\Lambda^2$ and $f_{WW}/\Lambda^2$ will be from the measurement of the gauge-boson scattering $W^+W^+ \rightarrow W^+W^+$. Those processes are insensitive to $f_B/\Lambda^2$ and $f_{BB}/\Lambda^2$.

At $e^+e^-$ linear colliders on the other hand, the anomalous couplings $g_{HZZ}^{(1)}$ and $g_{HZZ}^{(2)}$ can be constrained at the 2σ sensitivity to $(10^{-3} - 10^{-2})$ TeV$^{-1}$ from the Higgs-strahlung process $e^+e^- \rightarrow Z^* \rightarrow ZH$.

At photon colliders, the sensitivities to probe those couplings can be improved.
With the anomalous $H\gamma\gamma$ ($g_{H\gamma\gamma} \propto f_{BB} + f_{WW}$) coupling, the signal process $\gamma\gamma \rightarrow ZZ$ can have a tree level contribution.

The contribution of the anomalous $HZZ$ interactions in $\bullet$ to the cross section is only a few percent, thus the process $\gamma\gamma \rightarrow ZZ$ mainly tests the anomalous couplings $g_{H\gamma\gamma}$.
The signal is in \( \gamma^+\gamma^+ \to Z_LZ_L \) channel, and raise with energy. The main background is \( \gamma\gamma(\gamma^+\gamma^+, \gamma^+\gamma^-) \to Z_TZ_T \) loop contributions.
Z pair center-of-mass energy distribution of signal (LL) and background (TT) on $2\lambda_e P_c = -1$ polarized $\sqrt{s_{ee}} = 1$ TeV photon collider

\[ M_{ZZ} > 0.65 \sqrt{s_{ee}}. \]
Final $Z$ bosons outgoing angle distribution

$-0.5 < \cos \theta_z < 0.5$
the energy difference between the two leptons (jets) is

\[ \Delta E_{\parallel(jj)} \equiv |q_1^0 - q_2^0| = |P_Z| \cos \theta'. \]

\( \theta' \) distribution for \( Z_T \) decay \( \propto (1 \pm \cos \theta')^2 \), while
\( \theta' \) distribution for \( Z_L \) decay \( \propto \sin^2 \theta' \).
So the leptons (jets) from \( Z_T \) decay are mainly in the region near \( \theta' = 0 \) or \( \pi \),
while the leptons (jets) from \( Z_L \) decay are mainly in the region near \( \theta' = \pi/2 \).

If we make a cut on the upper limit of \( \Delta E_{\parallel(jj)} \) which picks up the
region near \( \theta' = \pi/2 \), the \( Z_T \) decay mode will be suppressed. So
we propose our third cut requiring

\[ \Delta E_{\parallel(jj)} < \frac{1}{2} |P_Z|. \]
The efficiency of cuts

\[ \sigma(\ell\ell, f_{WW}/\Lambda^2 = f_{BB}/\Lambda^2 = 2 \text{ TeV}^{-2}) \text{ as signal on } \sqrt{s_{ee}} = 1 \text{ TeV}, \]
\[ \sigma(\ell\ell, f_{WW}/\Lambda^2 = f_{BB}/\Lambda^2 = 0.67 \text{ TeV}^{-2}) \text{ as signal on } \sqrt{s_{ee}} = 3 \text{ TeV and} \]
\[ \sigma(TT, f_{WW}/\Lambda^2 = f_{BB}/\Lambda^2 = 0 \text{ TeV}^{-2}) \text{ as background} \]

<table>
<thead>
<tr>
<th></th>
<th>without cut</th>
<th>(M_{ZZ}) cut</th>
<th>(+ \theta_Z) cut</th>
<th>(+ \Delta E_{jj(ll)}) cut</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sqrt{s_{ee}} = 1 \text{ TeV})</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\sigma_{LL} \text{ (fb)})</td>
<td>22</td>
<td>20</td>
<td>9.2</td>
<td>4.9</td>
</tr>
<tr>
<td>(\sigma_{TT} \text{ (fb)})</td>
<td>115</td>
<td>72</td>
<td>16</td>
<td>3.5</td>
</tr>
<tr>
<td>efficiency</td>
<td></td>
<td>90%(^{0.63})% (\approx 1.4)</td>
<td>47%(^{0.22})% (\approx 2.1)</td>
<td>54%(^{0.22})% (\approx 2.5)</td>
</tr>
</tbody>
</table>

|                      |             |                |                     |                            |
| \(\sqrt{s_{ee}} = 3 \text{ TeV}\) |             |                |                     |                            |
| \(\sigma_{LL} \text{ (fb)}\) | 27          | 23             | 9.9                 | 5.6                       |
| \(\sigma_{TT} \text{ (fb)}\)  | 190         | 85             | 6.5                 | 1.1                       |
| efficiency            |             | 85%\(^{0.45}\)% \(\approx 1.9\) | 43%\(^{0.76}\)% \(\approx 5.7\) | 57%\(^{0.17}\)% \(\approx 3.4\) |


CONCLUSION

The process $\gamma \gamma \rightarrow ZZ$ at polarized photon colliders based on $e^+ e^-$ linear colliders of 500 GeV, 1 TeV, and 3 TeV is sensitive to test the $H\gamma \gamma$ anomalous coupling. With an integrated luminosity of 1000 fb$^{-1}$, the $2\sigma$ testing sensitivities are

$$\sqrt{s_{ee}} = 500 \text{ GeV} :$$

$$-0.021 \text{ TeV}^{-1} < g_{H\gamma \gamma} < 0.0078 \text{ TeV}^{-1}.$$ 

$$\sqrt{s_{ee}} = 1 \text{ TeV} :$$

$$-0.0068 \text{ TeV}^{-1} < g_{H\gamma \gamma} < 0.0048 \text{ TeV}^{-1}.$$ 

$$\sqrt{s_{ee}} = 3 \text{ TeV} :$$

$$-0.0015 \text{ TeV}^{-1} < g_{H\gamma \gamma} < 0.0015 \text{ TeV}^{-1}.$$
Testing fermion anomalous coupling at future ILC

- The flavor physics about leptons, neutrino mass and flavor mixing has become a hot topic.

- Many theoretical models introduced the heavy neutrinos or the fourth generation leptons.

- The effects of those extra massive particles can be reflected in the anomalous couplings of leptons and gauge bosons.

- Detecting these leptonic anomalous gauge couplings on colliders can also test and verify these electroweak new physical models.
The Leptonic Anomalous Gauge Couplings In Effective Lagrangian

\[ \mathcal{O}_7^{VF} = i\bar{L}\gamma_{\mu}W^{\mu\nu}\bar{D}_\nu L, \]
\[ \mathcal{O}_{11}^{VF} = i\bar{L}\gamma_{\mu}B^{\mu\nu}\bar{D}_\nu L, \]
\[ \mathcal{O}_{13}^{VF} = i\bar{E}\gamma_{\mu}B^{\mu\nu}\bar{D}_\nu E, \]
\[ \mathcal{O}_{24}^{VF} = \bar{L}\gamma_{\mu}(D_\nu W^{\mu\nu})L, \]
\[ \mathcal{O}_{26}^{VF} = \bar{L}\gamma_{\mu}\partial_\nu B^{\mu\nu}L, \]
\[ \mathcal{O}_{27}^{VF} = \bar{E}\gamma_{\mu}\partial_\nu B^{\mu\nu}E, \]

the operator \( \mathcal{O}_7 \) and \( \mathcal{O}_{24} \) can affect all the vertices. Therefore, the process \( e^+e^- \rightarrow W^+W^- \) is much more sensitive to operators \( \mathcal{O}_7 \) and \( \mathcal{O}_{24} \) than others.
$e^+e^- \rightarrow W^+W^-$ at the future linear collider (ILC)

In the standard model $e^+e^- \rightarrow W^+W^-$ process, the $E^2$ terms in the amplitude cancel each other between different Feynman diagrams.

The constraints from LEP2 are very weak:

\[-2.6 \text{ TeV}^{-2} < \frac{f_7}{\Lambda^2} < 2.6 \text{ TeV}^{-2},\]
\[-9.8 \text{ TeV}^{-2} < \frac{f_{24}}{\Lambda^2} < 1.8 \text{ TeV}^{-2},\]
\[-11 \text{ TeV}^{-2} < \frac{f_{26}}{\Lambda^2} < 33 \text{ TeV}^{-2},\]
\[-13 \text{ TeV}^{-2} < \frac{f_{27}}{\Lambda^2} < 30 \text{ TeV}^{-2}.\]
the detection sensitivities of the anomalous coupling at 500 GeV ILC

\[-0.18 \text{ TeV}^{-2} < \frac{f_7}{\Lambda^2} < 0.18 \text{ TeV}^{-2},\]
\[-0.045 \text{ TeV}^{-2} < \frac{f_{24}}{\Lambda^2} < 0.15 \text{ TeV}^{-2},\]
\[-2.5 \text{ TeV}^{-2} < \frac{f_{26}}{\Lambda^2} < 8.8 \text{ TeV}^{-2},\]
\[-3.3 \text{ TeV}^{-2} < \frac{f_{27}}{\Lambda^2} < 6.2 \text{ TeV}^{-2}.\]

the detection sensitivities of the anomalous coupling at 1 TeV ILC

\[-2.0 \times 10^{-2} \text{ TeV}^{-2} < \frac{f_7}{\Lambda^2} < 2.0 \times 10^{-2} \text{ TeV}^{-2},\]
\[-2.5 \times 10^{-3} \text{ TeV}^{-2} < \frac{f_{24}}{\Lambda^2} < 8.0 \times 10^{-3} \text{ TeV}^{-2},\]
\[-0.6 \text{ TeV}^{-2} < \frac{f_{26}}{\Lambda^2} < 2.3 \text{ TeV}^{-2},\]
\[-1.0 \text{ TeV}^{-2} < \frac{f_{27}}{\Lambda^2} < 1.6 \text{ TeV}^{-2}.\]
After $W$ angle cut, the ILC detection sensitivities are increased by $3-4$ orders of magnitude than the LEP2.

<table>
<thead>
<tr>
<th>$\cos\theta$ cut</th>
<th>$f_7/\Lambda^2(10^{-3}\text{TeV}^{-2})$</th>
<th>$f_{24}/\Lambda^2(10^{-4}\text{TeV}^{-2})$</th>
<th>$f_{26}/\Lambda^2(10^{-1}\text{TeV}^{-2})$</th>
<th>$f_{27}/\Lambda^2(10^{-1}\text{TeV}^{-2})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>none</td>
<td>$-20 \sim 20$</td>
<td>$-25 \sim 80$</td>
<td>$-6 \sim 23$</td>
<td>$-9 \sim 15$</td>
</tr>
<tr>
<td>$\cos\theta &lt; 0.75$</td>
<td>$-9.5 \sim 9.5$</td>
<td>$-6.7 \sim 10$</td>
<td>$-1.9 \sim 2.3$</td>
<td>$-3.3 \sim 10$</td>
</tr>
<tr>
<td>$\cos\theta &lt; 0.5$</td>
<td>$-6.5 \sim 6.5$</td>
<td>$-4.2 \sim 5.0$</td>
<td>$-1.1 \sim 1.4$</td>
<td>$-2.0 \sim 9.2$</td>
</tr>
<tr>
<td>$\cos\theta &lt; 0$</td>
<td>$-4.6 \sim 4.6$</td>
<td>$-2.8 \sim 2.9$</td>
<td>$-0.75 \sim 0.78$</td>
<td>$-1.4 \sim 2.5$</td>
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</table>
THANKS
The Polarization Scheme

Right handed polarized initial electrons are useful to detect the anomalous coupling between lepton and Z boson.

For the polarized $\sqrt{s_{ee}} = 500$ GeV ILC, the detection sensitivity is:

$$-3.5 \times 10^{-2} \text{ TeV}^{-2} < \frac{f_{27}}{\Lambda^2} < 3.5 \times 10^{-2} \text{ TeV}^{-2},$$

And for $\sqrt{s_{ee}} = 1$ TeV polarized ILC, the detection sensitivity is:

$$-9 \times 10^{-3} \text{ TeV}^{-2} < \frac{f_{27}}{\Lambda^2} < 9 \times 10^{-3} \text{ TeV}^{-2},$$