How to get a Very Small Cosmological Constant

戴自海

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November 13, 2012
7th Workshop of TeV Physics Working Group
Tsinghua University, Beijing, China

Happy Birthday to Professor Yu-Ping Kuang

Hong Kong University of Science and Technology

Institute for Advanced Study of HKUST

This talk is based on work with **Yoske Sumitomo** :

arXiv:1204.5177 (JCAP **1208** (2012) 032) and
arXiv:1209.5086

Applied to :
Large Volume Flux Compactification Scenario in Type IIB String Theory

in particular : M. Rummel and A. Westphal, arXiv:1107.2115

Also : A. Aazami and R. Easther, hep-th/0512102;
X. Chen, G. Shiu, Y. Sumitomo and S.-H.H. Tye, arXiv:1112.3338;
T. Bachlechner, D. Marsh, L. McAllister and T. Wrase, arXiv:1207.2763.

Background

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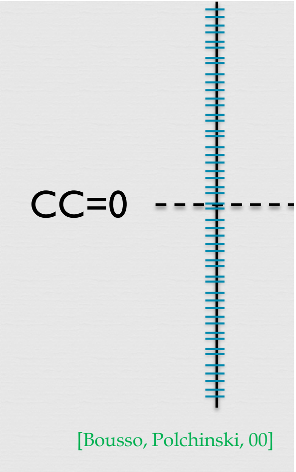
- There is strong evidence that our universe has gone through an inflationary period, when the vacuum energy is below the Planck scale but much higher than the TeV scale.
- Given the scale of the underlying theory, how the observed value emerges ?
  E.g., String theory has string scale M_S, so it must generate both M_P and Λ from M_S.

The situation in string theory : J types of 4-form fluxes $F^i_{\mu\nu\rho\sigma}$ (Brown and Teitelbaum)



$$\lambda = \lambda_{\rm bare} + \frac{1}{2}\sum_{i=1}^{J} n_i^2 q_i^2$$

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Why nature picks such a very small positive Λ ?

- We present a possible Stringy Mechanism why a very small Λ may be preferred.
- We use a simple but non-trivial model to illustrate the main idea.
- The key points can be understood with little knowledge about string theory.
- The key idea may be applied to other hierarchy problems.

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This peaking behavior of $P(\Lambda)$ at $\Lambda = 0$ is quiet generic.

The Basic Idea is very simple :

It is based on the properties of the probability distribution of functions of random variables.

Does Λ has the right functional form ? Do the random parameters have the right range and distribution ?

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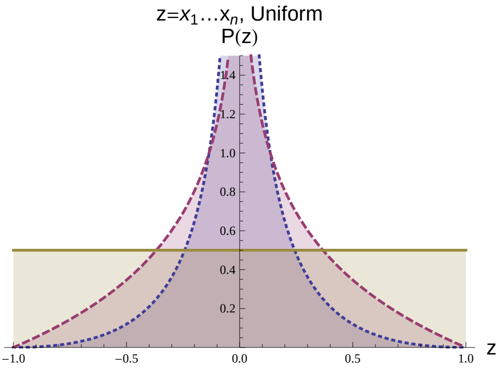
An example :

Consider a set of random variables x_i ($i = 1, 2, ..., n$). Let the probability distribution of each x_i be uniform in the range $[-1, +1]$. *What is the probability distribution of their product z ?*

Probability distribution of $z = x_1 x_2$ and $z = x_1 x_2 x_3$



$$P(z) = \frac{1}{2(n-1)!}\left(\ln\frac{1}{|z|}\right)^{n-1}$$

$z = x_1 x_2$

Let x_j to have a uniform distribution $P(x_j) = 1$ between 0 and 1.
What is the probability distribution $P(z)$ of the product $z = x_1 x_2$?

$$P(z) = \int_0^1 dx_1 \int_0^1 dx_2 \, \delta(x_1 x_2 - z) = \int_z^1 dx_1 \frac{1}{x_1} = \ln\left(\frac{1}{z}\right)$$

for $0 \leq z \leq 1$.

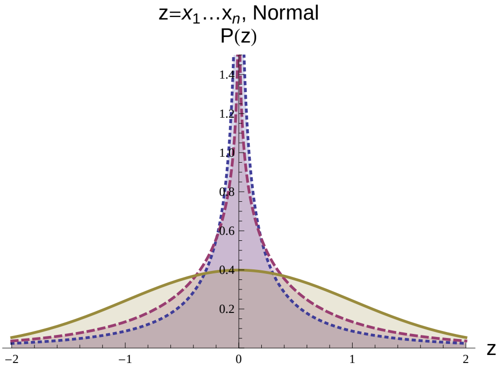
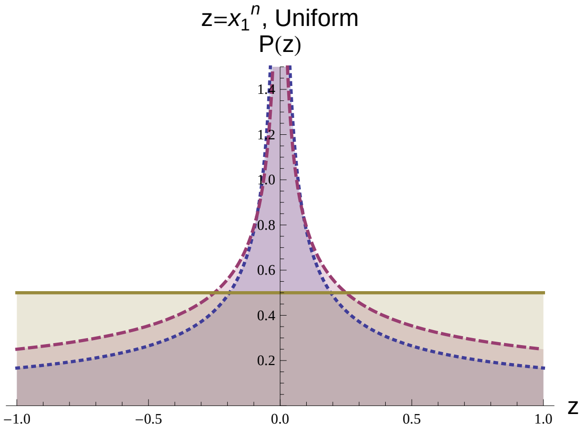


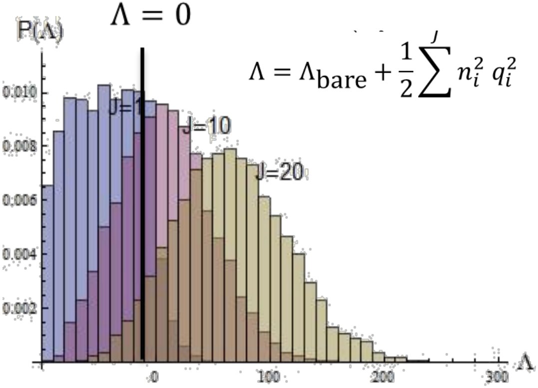
Figure: The product distribution $P(z)$ is for $z = x_1$ (solid brown curve for normal distribution), $z = x_1 x_2$ (red dashed curve), and $z = x_1 x_2 x_3$ (blue dotted curve), respectively. In general, the curves are given by the Meijer-G function.

Probability distribution $P(z)$ for $z = x_1^n$



$$P(z) \sim z^{-1+1/n}$$

No peaking behavior for $P(\Lambda)$ if Λ is a sum of terms.



Typical Manifolds Studied

$$\chi(M) = 2(h^{1,1} - h^{2,1})$$

Manifold	$N_K = h^{1,1}$	$N_{cs} = h^{2,1}$	χ
$\mathcal{P}^4_{[1,1,1,6,9]}$	2	272	-540
$\mathcal{F}_{11}$	3	111	-216
$\mathcal{F}_{18}$	5	89	-168
$\mathcal{CP}^4_{[1,1,1,1,1]}$	1	$\mathcal{O}(100)$	$\mathcal{O}(-200)$

$$V = e^K \left(K^{I\bar{J}} D_I W D_{\bar{J}} \bar{W} - 3|W|^2 \right),$$

$$K = -2\ln(\mathcal{V} + \hat{\xi}/2) - \ln(S + \bar{S}) - \sum_j \ln(U_j + \bar{U}_j)$$

$$\mathcal{V} = Vol/\alpha'^3 = \gamma_1 (T_1 + \bar{T}_1)^{3/2} - \sum_{i=2} \gamma_i (T_i + \bar{T}_i)^{3/2},$$

$$\hat{\xi} = -\frac{\zeta(3)\chi(M)}{4\sqrt{2}(2\pi)^3} \left(\frac{S + \bar{S}}{2} \right)^{3/2},$$

$$W = W_0(U_i, S) + \sum_{i=1}^{N_K} A_i e^{-a_i T_i},$$

$$W_0(U_i, S) = c_1 + \sum_j b_j U_j - s(c_2 + \sum_j d_j U_j)$$

Approach

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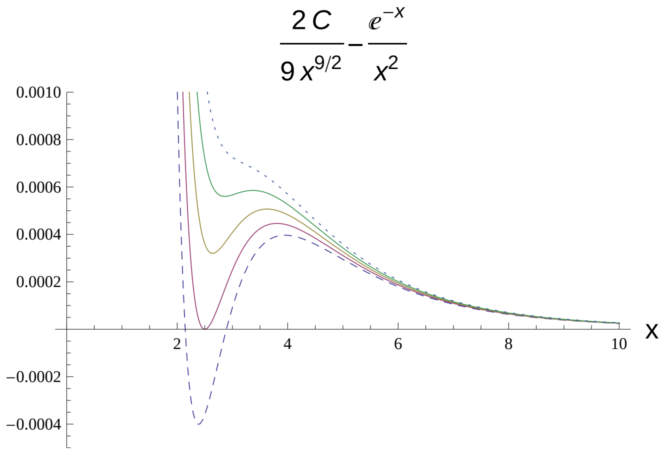
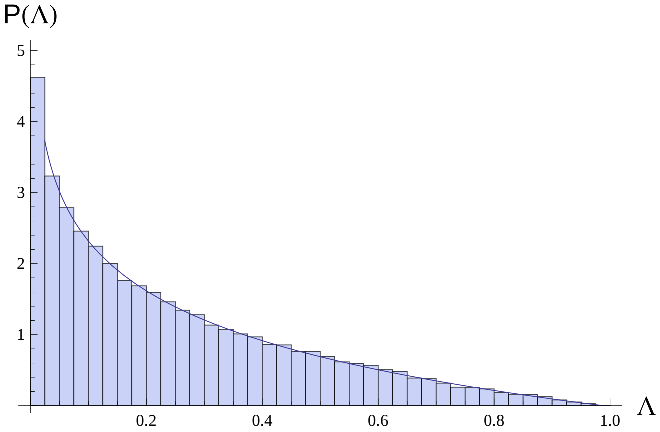
- All parameters introduced are treated as random variables with some probability distributions.
- Find the supersymmetric solution $w_0 = W_0|_{\min}$ of W_0 for the complex structure moduli and insert this w_0 into V to stabilize the Kähler moduli.
- The functional form of $\Lambda = V_{\min}$ (and $w_0 = W_0|_{\min}$) in terms of the parameters are non-trivial.

The form of $V(x)$ with $W_0 A_1 \leq 0$



$P(\Lambda) \propto \ln(1/|\Lambda|)$ at $\Lambda \sim 0$



$$D_S W_0 = \partial_S W_0 + K_S W_0 = 0, \qquad D_i W_0 = 0$$

$$W_0(u_i, s) = c_1 + \sum_j b_j u_j - s(c_2 + \sum_j d_j u_j)$$

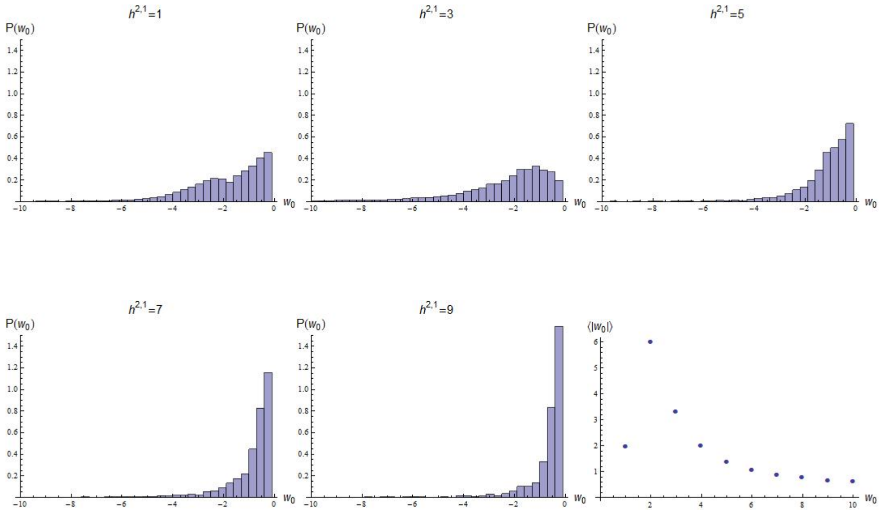
Solution :

$$(N_{cs} - 2)\frac{c_1 + sc_2}{c_1 - sc_2} = \sum_{i=1}^{N_{cs}} \frac{b_i + sd_i}{b_i - sd_i}$$

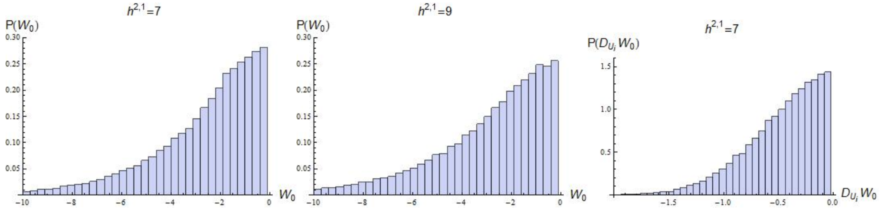
$$w_0 = W_0|_{\min} = \frac{2(c_1 + sc_2)\Pi_1^n(b_i - sd_i)}{\sum_i (b_i + sd_i)\Pi_{j\neq i}(b_j - sd_j)}$$

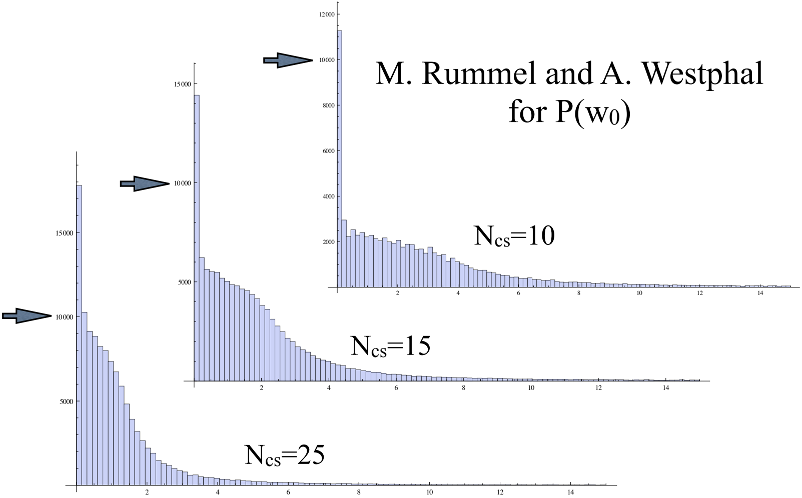
Then insert w_0 into the V for the Kähler moduli and solve :

$$\Lambda = \frac{e^{-5/2}}{9}\left(\frac{2}{5}\right)^2 \frac{-w_0 a_1^3 A_1}{\gamma_1^2}\left(x_m - \frac{5}{2}\right)$$


$h^{2,1}$=1
$h^{2,1}$=3
$h^{2,1}$=5
$h^{2,1}$=7
$h^{2,1}$=9
P(w_0)
w_0
⟨|w_0|⟩

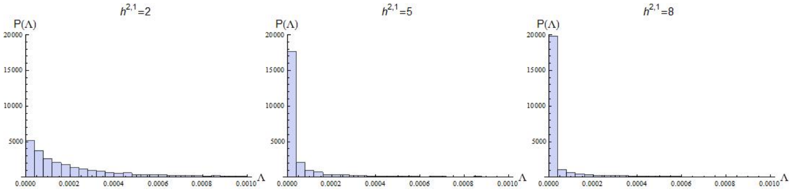
If $P(W_0)$ and $P(D_i W_0)$ are truly independent :




M. Rummel and A. Westphal
for P(w0)
Ncs=10
Ncs=15
Ncs=25

$P(\Lambda)$ as a function of $h^{2,1}$

Imposing the conditions $V_{barrier} \leq 1$, $s > 1$ and $u_i \geq 0$, for meta-stable vacua :



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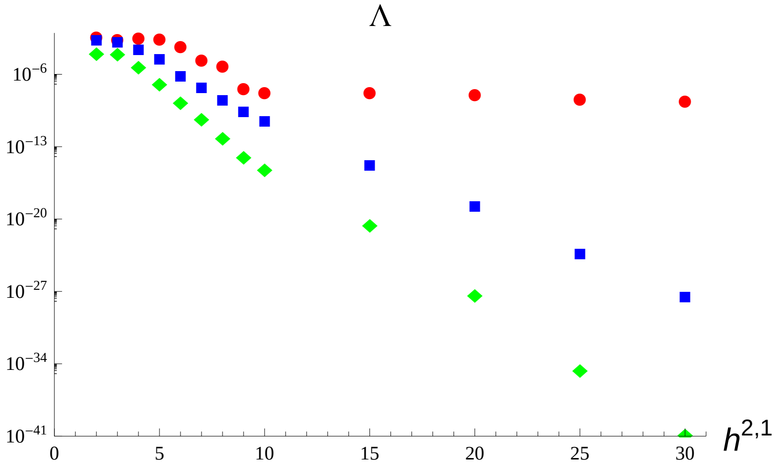
So we ask : what is the cut-off $\Lambda_{Y\%}$ if $\int_0^{\Lambda_{Y\%}} P(\Lambda)\, d\Lambda = Y\%$?

That is, there is a $Y\%$ chance that $\Lambda \leq \Lambda_{Y\%}$.

In the above example, $\Lambda_{99\%} = \Lambda_{10\%} = 10^{-100}$.

Likely value of Λ as a function of $h^{2,1}$



$< \Lambda >$ versus $\Lambda_{10\%}$

There is a $Y\%$ probability that $\Lambda_{Y\%} \geq \Lambda \geq 0$.

At $h^{2,1} = 10$,
$< \Lambda > \sim 10^{-8}$ while $\Lambda_{80\%} \sim 10^{-10}$ and $\Lambda_{10\%} \sim 10^{-19}$

At $h^{2,1} = 30$,
$< \Lambda > \sim 10^{-9}$ while $\Lambda_{80\%} \sim 10^{-29}$ and $\Lambda_{10\%} \sim 10^{-41}$

That is, for 30 complex structure moduli, there is a 10% chance that Λ is smaller than 10^{-41}.

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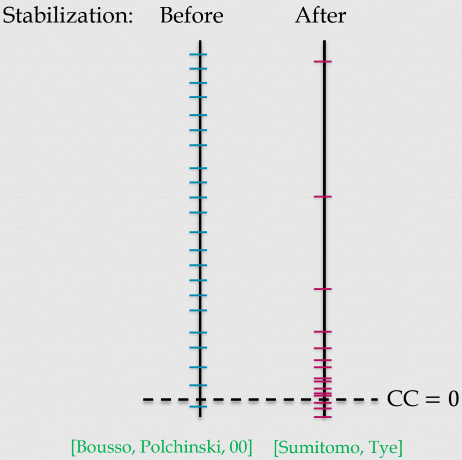
For $h^{2,1} > 5$, $\Lambda_{50\%} \sim 10^{-h^{2,1}}$

Summary

- At high vacuum energies, no meta-stable vacua (because most extrema are unstable)
- At very low vacuum energies, meta-stable vacua begin to appear


Stabilization:
Before
After
CC = 0
[Bousso, Polchinski, 00]
[Sumitomo, Tye]

Summary and Remarks

- At high vacuum energies, no stable vacua (because most extrema are unstable)
- At very low vacuum energies, meta-stable vacua begin to appear

Many technical questions to be further studied :

- What is the back-reaction due to SUSY breaking ?
- What about higher (α' and loop) corrections ?
- How about the cosmological light moduli problem ?

The picture is very encouraging: many directions to be explored.

One can apply this statistical property to other hierarchy problems.