How to get a Very Small Cosmological Constant

戴自海

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Happy Birthday to Professor Yu-Ping Kuang
Hong Kong University of Science and Technology
Institute for Advanced Study of HKUST
This talk is based on work with Yoske Sumitomo:

arXiv:1204.5177 (JCAP 1208 (2012) 032) and
arXiv:1209.5086

Applied to:
Large Volume Flux Compactification Scenario in Type IIB String Theory

in particular: M. Rummel and A. Westphal, arXiv:1107.2115

Also: A. Aazami and R. Easther, hep-th/0512102;
T. Bachlechner, D. Marsh, L. McAllister and T. Wrase,
Introduction
The Large Volume Scenario in Type IIB String Theory
Multi-Complex Structure Moduli
Summary

10^{500} \text{ possible solutions with different } \Lambda \text{ values.}

Pressing Question
The Stringy Mechanism

Background

There is very strong evidence that we are living in a de-Sitter vacuum with a very small positive cosmological constant \( \Lambda \),

\[ \Lambda \sim +10^{-122} M_P^4 \]
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- Given the scale of the underlying theory, how the observed value emerges? E.g., String theory has string scale $M_S$, so it must generate both $M_P$ and $\Lambda$ from $M_S$. 
The situation in string theory: $J$ types of 4-form fluxes $F_{i\mu\nu\rho\sigma}$

(Brown and Teitelbaum)

$\lambda = \lambda_{\text{bare}} + \frac{1}{2} \sum_{i=1}^{J} n_i^2 q_i^2$

[CC = 0]

[Before, After Stabilization: Bousso, Polchinski, 00]
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Why nature picks such a very small positive $\Lambda$?

- We present a possible Stringy Mechanism why a very small $\Lambda$ may be preferred.
- We use a simple but non-trivial model to illustrate the main idea.
- The key points can be understood with little knowledge about string theory.
- The key idea may be applied to other hierarchy problems.
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In the SUGRA approximation, solve \( V(a_j, u_i) \) for the meta-stable vacuum, so all \( \{u_i\} \) are determined in terms of \( \{a_j\} \). Determine \( \Lambda(a_j) = V_{\text{min}}(a_j) \) in terms of \( \{a_j\} \).
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This peaking behavior of $P(\Lambda)$ at $\Lambda = 0$ is quite generic.

The Basic Idea is very simple:

It is based on the properties of the probability distribution of functions of random variables.

Does $\Lambda$ have the right functional form? Do the random parameters have the right range and distribution?
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An example:

Consider a set of random variables $x_i \ (i = 1, 2, \ldots, n)$. Let the probability distribution of each $x_i$ be uniform in the range $[-1, +1]$. What is the probability distribution of their product $z$?
Probability distribution of $z = x_1 x_2$ and $z = x_1 x_2 x_3$

$$P(z) = \frac{1}{2(n-1)!} \left( \ln \frac{1}{|z|} \right)^{n-1}$$
Let $x_j$ to have a uniform distribution $P(x_j) = 1$ between 0 and 1. What is the probability distribution $P(z)$ of the product $z = x_1 x_2$?

$$P(z) = \int_0^1 dx_1 \int_0^1 dx_2 \delta(x_1 x_2 - z) = \int_z^1 dx_1 \frac{1}{x_1} = \ln \left( \frac{1}{z} \right)$$

for $0 \leq z \leq 1$. 

$Z = X_1 X_2$
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Basic property
$P(z)$ of $z = x_1 x_2$ and $z = x_1 x_2 x_3$
Non-interacting case: e.g., Sum of terms

Figure: The product distribution $P(z)$ is for $z = x_1$ (solid brown curve for normal distribution), $z = x_1 x_2$ (red dashed curve), and $z = x_1 x_2 x_3$ (blue dotted curve), respectively. In general, the curves are given by the Meijer-G function.
Probability distribution $P(z)$ for $z = x_1^n$

$P(z) \sim z^{-1+1/n}$
No peaking behavior for $P(\Lambda)$ if $\Lambda$ is a sum of terms.

\[ \Lambda = \Lambda_{\text{bare}} + \frac{1}{2} \sum_{i} n^2_i q^2_i \]
Typical Manifolds Studied

\[ \chi(M) = 2(h^{1,1} - h^{2,1}) \]

<table>
<thead>
<tr>
<th>Manifold</th>
<th>( N_K = h^{1,1} )</th>
<th>( N_{cs} = h^{2,1} )</th>
<th>( \chi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mathcal{P}^4_{[1,1,1,6,9]} )</td>
<td>2</td>
<td>272</td>
<td>-540</td>
</tr>
<tr>
<td>( \mathcal{F}_{11} )</td>
<td>3</td>
<td>111</td>
<td>-216</td>
</tr>
<tr>
<td>( \mathcal{F}_{18} )</td>
<td>5</td>
<td>89</td>
<td>-168</td>
</tr>
<tr>
<td>( \mathcal{CP}^4_{[1,1,1,1,1]} )</td>
<td>1</td>
<td>( \mathcal{O}(100) )</td>
<td>( \mathcal{O}(-200) )</td>
</tr>
</tbody>
</table>
\[ V = e^K \left( K^{I\bar{J}} D_I W D_{\bar{J}} \bar{W} - 3 |W|^2 \right), \]
\[ K = -2 \ln(V + \hat{\xi}/2) - \ln(S + \bar{S}) - \sum_j \ln(U_j + \bar{U}_j) \]
\[ \mathcal{V} = \text{Vol}/\alpha'^3 = \gamma_1 (T_1 + \bar{T}_1)^{3/2} - \sum_{i=2} \gamma_i (T_i + \bar{T}_i)^{3/2}, \]
\[ \hat{\xi} = -\frac{\zeta(3) \chi(M)}{4\sqrt{2}(2\pi)^3} \left( \frac{S + \bar{S}}{2} \right)^{3/2}, \]
\[ W = W_0(U_i, S) + \sum_{i=1}^{N_K} A_i e^{-a_i T_i}, \]
\[ W_0(U_i, S) = c_1 + \sum_j b_j U_j - s(c_2 + \sum_j d_j U_j) \]
Consider the above simplified Large Volume Scenario (LVS) in Type II B string theory.
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Introduce the dilation $S$, $N_K = h^{1,1}$ number of Kähler moduli $T_k$, and $N_{cs} = h^{2,1}$ number of complex structure moduli $U_i$. 
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- Introduce the dilation $S$, $N_K = h^{1,1}$ number of Kähler moduli $T_k$, and $N_{cs} = h^{2,1}$ number of complex structure moduli $U_i$.

- All parameters introduced are treated as random variables with some probability distributions.

- Find the supersymmetric solution $w_0 = W_0|_{\min}$ of $W_0$ for the complex structure moduli and insert this $w_0$ into $V$ to stabilize the Kähler moduli.

- The functional form of $\Lambda = V_{\min}$ (and $w_0 = W_0|_{\min}$) in terms of the parameters are non-trivial.
The form of $V(x)$ with $W_0A_1 \leq 0$

$$\frac{2C}{9x^{9/2}} - \frac{e^{-x}}{x^2}$$
\[ P(\Lambda) \propto \ln \left( \frac{1}{|\Lambda|} \right) \] at \( \Lambda \sim 0 \)
\[ D_S \mathcal{W}_0 = \partial_S \mathcal{W}_0 + K_S \mathcal{W}_0 = 0, \quad D_i \mathcal{W}_0 = 0 \]

\[ \mathcal{W}_0(u_i, s) = c_1 + \sum_j b_j u_j - s(c_2 + \sum_j d_j u_j) \]

Solution:

\[ (N_{cs} - 2) \frac{c_1 + sc_2}{c_1 - sc_2} = \sum_{i=1}^{N_{cs}} \frac{b_i + sd_i}{b_i - sd_i} \]

\[ w_0 = \mathcal{W}_0|_{\text{min}} = \frac{2(c_1 + sc_2) \prod_{i=1}^{n}(b_i - sd_i)}{\sum_i (b_i + sd_i) \prod_{j \neq i}(b_j - sd_j)} \]

Then insert \( w_0 \) into the \( V \) for the Kähler moduli and solve:

\[ \Lambda = \frac{e^{-5/2}}{9} \left( \frac{2}{5} \right)^2 - w_0 a_1^3 A_1 \frac{x_m - \frac{5}{2}}{\gamma_1^2} \]
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Supersymmetric Solution
Probability Distribution $P(w_0)$
$P(\Lambda)$ as a function of $h^{2,1} = N$

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If $P(W_0)$ and $P(D_i W_0)$ are truly independent:
The Large Volume Scenario in Type IIB String Theory

Multi-Complex Structure Moduli

Summary

Supersymmetric Solution

Probability Distribution $P(w_0)$ as a function of $h^{2,1} = N$

M. Rummel and A. Westphal for $P(w_0)$

N_{cs}=10

N_{cs}=15

N_{cs}=25

Yoske Sumitomo and Henry Tye

How to get a Very Small $\Lambda$
Imposing the conditions $V_{\text{barrier}} \leq 1$, $s > 1$ and $u_i \geq 0$, for meta-stable vacua:
$P(\Lambda)$ is sharply peaked at $\Lambda = 0$ but with a long tail. So $<\Lambda>$ may not be a good measure of what is going on.
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Suppose there are $10^6$ data points at $\Lambda = 10^{-100}$ and 1 data point at $\Lambda = 1$. The likely value is $\Lambda = 10^{-100}$ even though $< \Lambda > \approx 10^{-6}$.
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Suppose there are \( 10^6 \) data points at \( \Lambda = 10^{-100} \) and 1 data point at \( \Lambda = 1 \). The likely value is \( \Lambda = 10^{-100} \) even though \( \langle \Lambda \rangle \approx 10^{-6} \).

So we ask: what is the cut-off \( \Lambda_{Y\%} \) if \( \int_0^{\Lambda_{Y\%}} P(\Lambda) \, d\Lambda = Y\% \)? That is, there is a \( Y\% \) chance that \( \Lambda \leq \Lambda_{Y\%} \).

In the above example, \( \Lambda_{99\%} = \Lambda_{10\%} = 10^{-100} \).
Likely value of $\Lambda$ as a function of $h^{2,1}$
< \Lambda > versus \Lambda_{10\%}

There is a \( Y\% \) probability that \( \Lambda_{Y\%} \geq \Lambda \geq 0 \).

At \( h^{2,1} = 10 \),

\[ < \Lambda > \sim 10^{-8} \text{ while } \Lambda_{80\%} \sim 10^{-10} \text{ and } \Lambda_{10\%} \sim 10^{-19} \]

At \( h^{2,1} = 30 \),

\[ < \Lambda > \sim 10^{-9} \text{ while } \Lambda_{80\%} \sim 10^{-29} \text{ and } \Lambda_{10\%} \sim 10^{-41} \]

That is, for 30 complex structure moduli, there is a 10% chance that \( \Lambda \) is smaller than \( 10^{-41} \).
There is a $\gamma\%$ probability that $\Lambda_{\gamma}\% \geq \Lambda \geq 0$.

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For $h^{2,1} > 5$, $\Lambda_{50}\% \sim 10^{-h^{2,1}}$
Summary

- At high vacuum energies, no meta-stable vacua (because most extrema are unstable)
- At very low vacuum energies, meta-stable vacua begin to appear
Before Stabilization:

\[ \text{CC} = 0 \]

[References: Bousso, Polchinski, 00; Sumitomo, Tye]

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Summary and Remarks

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Many technical questions to be further studied:
- What is the back-reaction due to SUSY breaking?
- What about higher ($\alpha'$ and loop) corrections?
- How about the cosmological light moduli problem?

The picture is very encouraging: many directions to be explored.

One can apply this statistical property to other hierarchy problems.