

Physics of CP Violation (IV)

Special Lecture at

Tsinghua University, Beijing, China

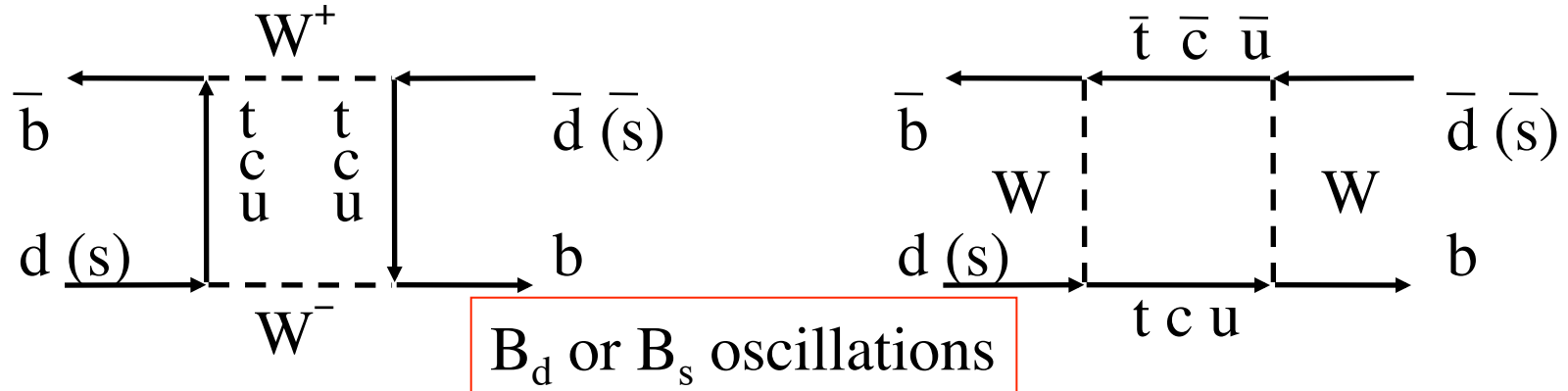
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B meson system

The short distance effect dominates in the oscillations.



$$H_{\text{effective}}^{\Delta B=2} = \frac{G_F^2 m_W^2}{16\pi^2} \sum_{\substack{i,j= \\ u,c,t}} \lambda_i^q \lambda_j^q \left(\langle B^0 | M_{V-A}^q | \bar{B}^0 \rangle B_{ij} + \langle B^0 | M_{S-P}^q | \bar{B}^0 \rangle C_{ij} \right)$$

$q = d$ for B_d and s for B_s

CKM elements

$$\lambda_i^q = V_{ib} V_{iq}^*$$

quark operators

$$M_{V-A} = [\bar{q} \gamma_\mu (1 - \gamma_5) b] [\bar{q} \gamma^\mu (1 - \gamma_5) b]$$

$$M_{S-P} = [\bar{q} (1 - \gamma_5) b] [\bar{q} (1 - \gamma_5) b]$$

integrating out W

$$\left. \begin{matrix} B_{ij} \\ C_{ij} \end{matrix} \right\} \text{Real part} \rightarrow M_{12}, \text{Imaginary part} \rightarrow \Gamma_{12}$$

$$|\lambda_t^d|^2 = |V_{tb}^2 V_{td}^{*2}| \approx A^2 \lambda^6 \quad S_0(x_t) \approx 2.5 \quad \leftarrow \text{dominating}$$

$$|\lambda_c^d|^2 = |V_{cb}^2 V_{cd}^{*2}| \approx A^2 \lambda^6 \quad S_0(x_c) \approx 0.00024$$

$$|\lambda_c^d \lambda_t^d| = |V_{cb} V_{cd}^* V_{tb} V_{td}^*| \approx A^2 \lambda^6 \quad S_0(x_c, x_t) \approx 0.0021$$

$$|\lambda_t^s|^2 = |V_{tb}^2 V_{ts}^{*2}| \approx A^2 \lambda^4 \quad S_0(x_t) \approx 2.5 \quad \leftarrow \text{dominating}$$

$$|\lambda_c^s|^2 = |V_{cb}^2 V_{cs}^{*2}| \approx A^2 \lambda^4 \quad S_0(x_c) \approx 0.00024$$

$$|\lambda_c^s \lambda_t^s| = |V_{cb} V_{cs}^* V_{tb} V_{ts}^*| \approx A^2 \lambda^4 \quad S_0(x_c, x_t) \approx 0.0021$$

top-loop dominate for M_{12} \rightarrow m_b can be neglected, i.e. $C_{tt}=0$

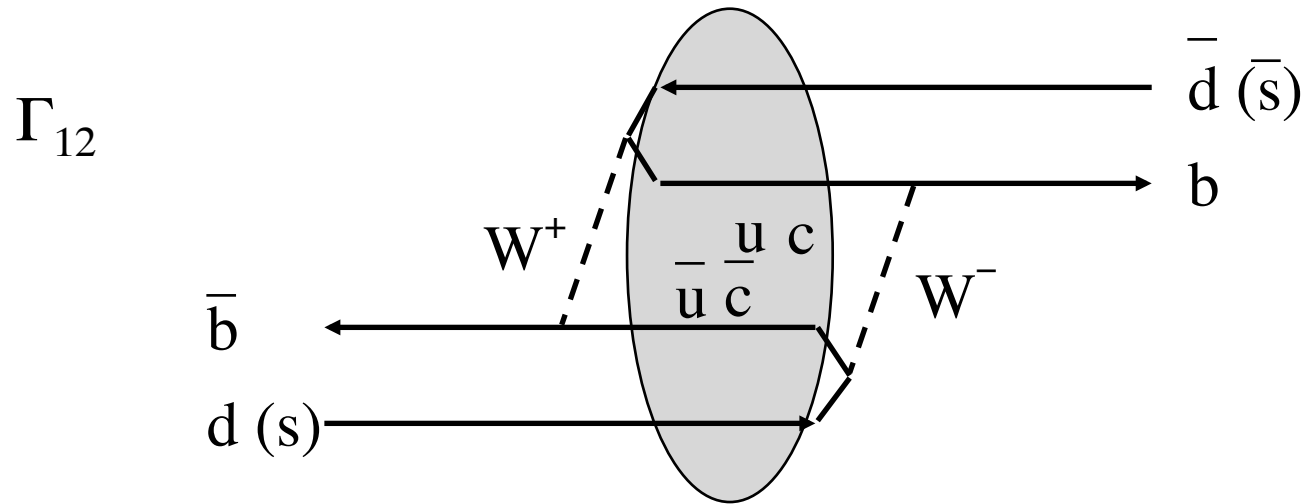
$$M_{12}^q = \frac{G_F^2}{12\pi^2} f_{B_q}^2 B_{B_q} m_{B_q} m_W^2 \lambda_t^{q2} \eta_B S_0(x_t) e^{i(\pi - \theta_{CP})}$$

$$\eta_B = 0.55$$

QCD correction

$$B_B f_B^2 = 0.83(223 \pm 40)^2 \text{ MeV}^2 \text{ for } B_d \text{ lattice calculations}$$

(no experimental measurement on f_B)



absorptive terms (no top)

m_b can not be neglected, i.e. $C_{ij} \neq 0$

$$\lambda_u = -\lambda_c - \lambda_t \quad \text{unitarity relation}$$

$$\Gamma_{12}^q \approx \frac{G_F^2}{8\pi} f_B^2 B_B m_B m_b^2 \left[\lambda_t^{q2} + \frac{8}{3} \lambda_t^q \lambda_c^q z_c + O(z_c^2) \right] e^{-i\theta_{CP}}$$

$$z_c = (m_c/m_b)^2 \approx 0.09$$

$$\frac{\Gamma_{12}^q}{M_{12}^q} = -\frac{3\pi x_b}{2S_0(x_t)} \left[1 + \frac{8\lambda_c^q}{3\lambda_t^q} z_c + O(z_c^2) \right]$$

where,

$$\frac{\lambda_c^d}{\lambda_t^d} = \frac{(\hat{\rho} - 1) + i\hat{\eta}}{(\hat{\rho} - 1)^2 + \hat{\eta}^2} \qquad \frac{\lambda_c^s}{\lambda_t^s} = -(1 + i\lambda^2\eta)$$

$$\Longrightarrow \frac{|\Gamma_{12}|}{|M_{12}|} \approx 4 \times 10^{-3} \quad \text{i.e. small for both } B_d \text{ and } B_s$$

$$\xi = \sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} + \frac{i}{2}\Gamma_{12}}} \approx \left(1 - \frac{1}{2} \text{Im} \frac{\Gamma_{12}}{M_{12}} \right) e^{-i\phi_M} \qquad \text{for } |\Gamma_{12}/M_{12}| \ll 1$$

$$\phi_M = \arg M_{12}$$

$$\text{Im}\left(\frac{\Gamma_{12}}{M_{12}}\right) = -\frac{4\pi x_b z_c}{S_0(x_t)} \text{Im}\left(\frac{\lambda_c^q}{\lambda_t^q}\right) = \begin{cases} -\frac{4\pi x_c}{S_0(x_t)} \frac{\hat{\eta}}{(\hat{\rho}-1)^2 + \hat{\eta}^2} \approx -6 \times 10^{-4} & B_d \\ \frac{4\pi x_c}{S_0(x_t)} \lambda^2 \eta \approx 2 \times 10^{-5} & B_s \end{cases}$$

→ $|\zeta| \approx 1$ i.e. CP violation in oscillation is small!

$$\arg \Gamma_{12} - \arg M_{12} \approx \begin{cases} \pi + 0.15 & B_d \\ \pi - 0.005 & B_s \end{cases}$$

Weak eigenstates with definite masses and decay widths:

$$|B_{\pm}\rangle = \frac{1}{\sqrt{1+|\zeta|^2}} \left(|B^0\rangle \pm \zeta |\bar{B}^0\rangle \right)$$

$$m_{\pm} = M \pm |M_{12}| \quad \Gamma_{\pm} = \Gamma \mp |\Gamma_{12}| \cos(\arg \Gamma_{12} - \arg M_{12}) \quad \text{for } |\Gamma_{12}/M_{12}| \ll 1$$

$$\Delta m = m_+ - m_- = 2|M_{12}| \quad \Delta\Gamma = \Gamma_- - \Gamma_+ = 2|\Gamma_{12}| \times \begin{cases} 1 - O(10^{-2}) & B_d \\ 1 - O(10^{-5}) & B_s \end{cases}$$

$$|B_+\rangle \equiv |B_h\rangle, |B_-\rangle \equiv |B_l\rangle$$

B-heavy decays slower

For B_d ,

experimental knowledge:

$$\Delta m = 0.472 \times 10^{12} \hbar s^{-1}, \tau = 1.55 \times 10^{-12} \text{ s}$$

$$\frac{\Delta\Gamma}{\Gamma} = \frac{|\Gamma_{12}|}{|M_{12}|} (\Delta m \times \tau) \approx 3 \times 10^{-3}$$

$$\text{SM} \\ \sim 4 \times 10^{-3}$$

Negligible decay widths difference.

For B_s ,

experimental knowledge:

$$\Delta m = 17.77 \times 10^{12} \hbar s^{-1}, \tau = 1.44 \times 10^{-12} \text{ s}$$

$$\frac{\Delta\Gamma}{\Gamma} = \frac{|\Gamma_{12}|}{|M_{12}|} (\Delta m \times \tau) > 7 \times 10^{-2}$$

$$\text{SM} \\ \sim 4 \times 10^{-3}$$

Non negligible decay widths difference.

K

$$|\mathbf{K}_{S(L)}\rangle = \frac{1}{\sqrt{2}(1 - \text{Re } \varepsilon)} \left[|\mathbf{K}^0\rangle + (-)(1 - 2\varepsilon)e^{-i\phi_\Gamma} |\bar{\mathbf{K}}^0\rangle \right]$$

$$\varepsilon = \frac{|M_{12}||\Gamma_{12}|\Delta_{M-\Gamma}}{4|M_{12}|^2 + |\Gamma_{12}|^2} \left(1 + i \frac{2|M_{12}|}{|\Gamma_{12}|} \right) \quad \phi_\Gamma = \arg \Gamma_{12}.$$

ε : complex

heavier meson decays little slower

For B

$$|\mathbf{B}_{1(h)}\rangle = \frac{1}{\sqrt{2}(1 - \varepsilon_B)} \left[|\mathbf{B}^0\rangle - (+)(1 - 2\varepsilon_B)e^{-i\phi_M} |\bar{\mathbf{B}}^0\rangle \right]$$

$$\varepsilon_B = \frac{1}{4} \text{Im} \frac{\Gamma_{12}}{M_{12}} \quad \phi_M = \arg M_{12}.$$

ε_B : real

heavier meson decays slower

B^0 at $t = 0$

$$|B^0(t)\rangle = f_+(t)|B^0\rangle + (1 - 2\varepsilon_B)e^{-i\phi_M} f_-(t)|\bar{B}^0\rangle$$

$$f_{\pm}(t) = \frac{1}{2} \left(e^{-i\lambda_h t} \pm e^{-i\lambda_l t} \right)$$

$$= \frac{1 + \varepsilon_B}{\sqrt{2}} \left(e^{-i\lambda_h t} |B_h\rangle + e^{-i\lambda_l t} |B_l\rangle \right)$$

\bar{B}^0 at $t = 0$

$$|\bar{B}^0(t)\rangle = (1 + 2\varepsilon_B)e^{i\phi_M} f_-(t)|B^0\rangle + f_+(t)|\bar{B}^0\rangle$$

$$= \frac{(1 + \varepsilon_B)e^{i\phi_M}}{\sqrt{2}} \left(e^{-i\lambda_h t} |B_l\rangle - e^{-i\lambda_l t} |B_h\rangle \right)$$

Useful to know...

$$|f_{\pm}(t)|^2 = \frac{1}{2} e^{-\Gamma t} \left(\cosh \frac{\Delta\Gamma}{2} t \pm \cos \Delta m t \right)$$

$$f_+(t)f_-^*(t) = \frac{1}{2} e^{-\Gamma t} \left(\sinh \frac{\Delta\Gamma}{2} t + i \sin \Delta m t \right)$$

Time dependent decay rates

Four kinds of decay final states:

- unique CP eigenstate,

e.g. K and $B \rightarrow \pi^+\pi^-$, $B \rightarrow J/\psi K_S$

- mixed CP eigenstate,

e.g. K and $B \rightarrow \pi^+\pi^-\pi^0$, $B_s \rightarrow J/\psi \phi$

- flavour specific, semileptonic and **hadronic**

e.g. K and $B \rightarrow \ell^+ \nu X^-$ vs K and $\bar{B} \rightarrow \ell^- \nu X^+$

$B \rightarrow K^+ \pi^-$ vs $\bar{B} \rightarrow K^- \pi^+$

- **flavour non-specific,**

e.g. $B \rightarrow D^- \pi^+, \rightarrow D^+ \pi^-$ vs $\bar{B} \rightarrow D^- \pi^+, \rightarrow D^+ \pi^-$

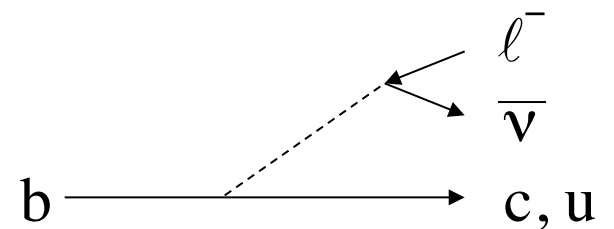
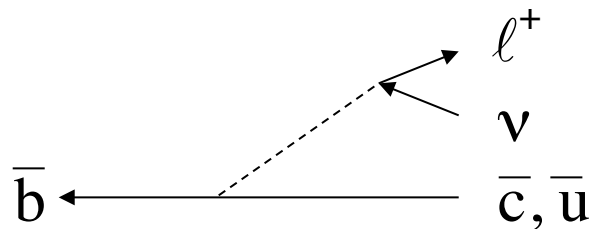
$B_s \rightarrow D_s^- K^+, \rightarrow D_s^+ K^-$ vs $\bar{B}_s \rightarrow D_s^- K^+, \rightarrow D_s^+ K^-$

-not possible in the kaon system-

Semileptonic decays:

$$A_+ = \langle \ell^+ X_h^- \nu | H_W | B^0 \rangle, \quad \bar{A}_- = \langle \ell^- X_h^+ \bar{\nu} | H_W | \bar{B}^0 \rangle$$

Like for the kaon system



$$\langle \ell^- X_h^+ \bar{\nu} | H_W | B^0 \rangle = \langle \ell^+ X_h^- \nu | H_W | \bar{B}^0 \rangle = 0$$

No strong phase and only one diagram

$$|A_+| = |\bar{A}_-|$$

B^0 at $t = 0 \rightarrow \ell^- X_h \bar{\nu}$ at t

$$\begin{aligned}
 R_-(t) &= \left| \langle \ell^- X_h \bar{\nu} | H_W | B^0(t) \rangle \right|^2 \\
 &= \left| f_+(t) \langle \ell^- X_h \bar{\nu} | H_W | B^0 \rangle + (1 - 2\varepsilon_B) e^{-i\phi_M} \langle \ell^- X_h \bar{\nu} | H_W | B^0 \rangle f_-(t) \right|^2 \\
 &= \left| f_+(t) A_- + (1 - 2\varepsilon_B) e^{-i\phi_M} \bar{A}_- f_-(t) \right|^2 \\
 &= (1 - 4\varepsilon_B) |\bar{A}_-|^2 |f_-(t)|^2
 \end{aligned}$$

\bar{B}^0 at $t = 0 \rightarrow \ell^+ X_h \nu$ at t

$$\begin{aligned}
 \bar{R}_+(t) &= \left| \langle \ell^+ X_h \nu | H_W | \bar{B}^0(t) \rangle \right|^2 \\
 &= (1 + 4\varepsilon_B) |A_+|^2 |f_-(t)|^2
 \end{aligned}$$

CP violation in B - \bar{B} mixing

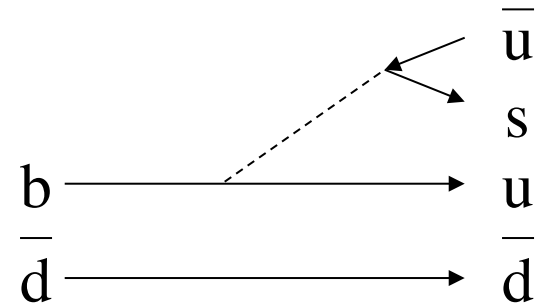
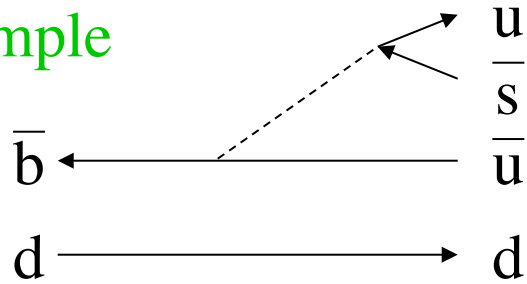
$$\begin{aligned}
 A_{CP}^\ell(t) &= \frac{\bar{R}_+(t) - R_-(t)}{\bar{R}_+(t) + R_-(t)} \\
 &= 4\varepsilon_B = \text{Im} \frac{\Gamma_{12}}{M_{12}}
 \end{aligned}$$

$\sim 10^{-3}$ (B_d)
to $\sim 10^{-5}$ (B_s)

Flavour specific hadronic decays:

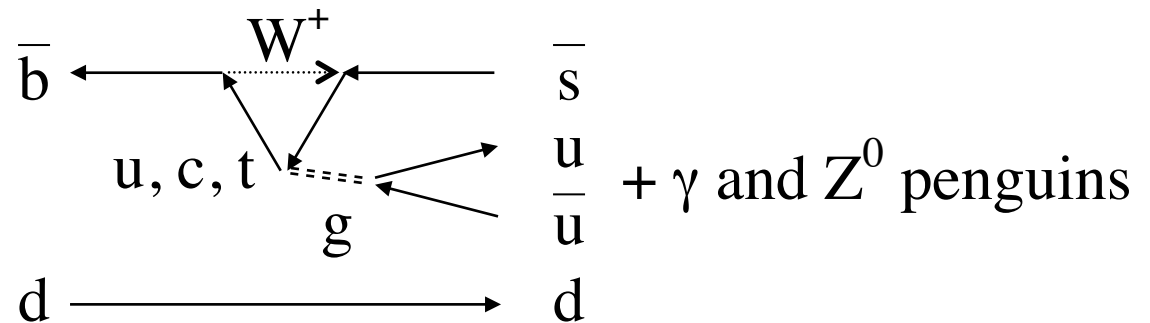
$$A_K = \langle K^+ \pi^- | H_W | B^0 \rangle, \bar{A}_K = \langle K^- \pi^+ | H_W | \bar{B}^0 \rangle$$

Example



$$\langle K^- \pi^+ | H_W | B^0 \rangle = \langle K^+ \pi^- | H_W | \bar{B}^0 \rangle = 0$$

$K\pi$: $I = 3/2$ and $1/2$



Two isospin eigenstates and tree + penguin contributions

most likely $\rightarrow |A_K| \neq |\bar{A}_K|$ ~~CP~~ in decay amplitudes

B^0 at $t = 0 \rightarrow K^- \pi^+$ at t

$$R_{K^- \pi^+}(t) = (1 - 4\varepsilon_B) |\bar{A}_K|^2 |f_-(t)|^2$$

\bar{B}^0 at $t = 0 \rightarrow K^+ \pi^-$ at t

$$\bar{R}_{K^+ \pi^-}(t) = (1 + 4\varepsilon_B) |A_K|^2 |f_-(t)|^2$$

CP violation in B - \bar{B} mixing
and CP violation in decay
amplitudes are mixed...

~~CP~~ in decay amplitudes only can be extracted by ...

B^0 at $t = 0 \rightarrow K^+ \pi^-$ at t

$$R_{K^+ \pi^-}(t) = |A_K|^2 |f_+(t)|^2$$

\bar{B}^0 at $t = 0 \rightarrow K^- \pi^+$ at t

$$\bar{R}_{K^- \pi^+}(t) = |\bar{A}_K|^2 |f_+(t)|^2$$

$$A_{CP}^K(t) = \frac{\bar{R}_{K^- \pi^+}(t) - R_{K^+ \pi^-}(t)}{\bar{R}_{K^- \pi^+}(t) + R_{K^+ \pi^-}(t)} \\ = \frac{|\bar{A}_K|^2 - |A_K|^2}{|\bar{A}_K|^2 + |A_K|^2}$$

A_{CP}^K BABAR $-0.133 \pm 0.030 \pm 0.009$ BELLE $-0.101 \pm 0.025 \pm 0.005$

Observation of ~~CP~~ in decay amplitudes: **much faster progress than K**

CP eigenstates

$$CP|f^{\text{CP}}\rangle = \sigma_{CP}|f^{\text{CP}}\rangle \quad \sigma_{CP}: \text{CP eigenvalue of } f^{\text{CP}}$$

$$A_{f^{\text{CP}}} = \langle f^{\text{CP}}|H_W|B^0\rangle, \quad \bar{A}_{f^{\text{CP}}} = \langle f^{\text{CP}}|H_W|\bar{B}^0\rangle$$

B^0 at $t = 0 \rightarrow f^{\text{CP}}$ at t

$$\begin{aligned} R_{f^{\text{CP}}}(t) &= \left| \langle f^{\text{CP}}|H_W|B^0(t)\rangle \right|^2 \\ &= \left| f_+(t) \langle f^{\text{CP}}|H_W|B^0\rangle + (1 - 2\varepsilon_B) e^{-i\phi_M} \langle f^{\text{CP}}|H_W|\bar{B}^0\rangle f_-(t) \right|^2 \\ &= |\bar{A}_{f^{\text{CP}}}|^2 (1 - 2\varepsilon_B)^2 \left| (1 + 2\varepsilon_B) e^{i\phi_M} \frac{A_{f^{\text{CP}}}}{\bar{A}_{f^{\text{CP}}}} f_+(t) + f_-(t) \right|^2 \\ &= |\bar{A}_{f^{\text{CP}}}|^2 (1 - 4\varepsilon_B) \left| L_{f^{\text{CP}}} f_+(t) + f_-(t) \right|^2 \\ &= |\bar{A}_{f^{\text{CP}}}|^2 (1 - 4\varepsilon_B) \left\{ \left| L_{f^{\text{CP}}} \right|^2 |f_+(t)|^2 + |f_-(t)|^2 + 2 \text{Re} \left[L_{f^{\text{CP}}} f_+(t) f_-(t)^* \right] \right\} \end{aligned}$$

$$L_f^{\text{CP}} = (1 + 4\varepsilon_B) e^{i\phi_M} \frac{A_{f^{\text{CP}}}}{\bar{A}_{f^{\text{CP}}}}$$

$$|f_{\pm}(t)|^2 = \frac{1}{2} e^{-\bar{\Gamma}t} \left(\cosh \frac{\Delta\Gamma}{2} t \pm \cos \Delta mt \right)$$

$$\begin{aligned} & |L_{f^{\text{CP}}}|^2 |f_+(t)|^2 + |f_-(t)|^2 \\ &= \frac{1}{2} e^{\bar{\Gamma}t} \left[\left(1 + |L_{f^{\text{CP}}}|^2 \right) \cosh \frac{\Delta\Gamma}{2} t - \left(1 - |L_{f^{\text{CP}}}|^2 \right) \cos \Delta mt \right] \end{aligned}$$

$$f_+(t)f_-^*(t) = \frac{1}{2} e^{-\bar{\Gamma}t} \left(\sinh \frac{\Delta\Gamma}{2} t + i \sin \Delta mt \right)$$

$$\begin{aligned} & 2 \operatorname{Re} \left[L_{f^{\text{CP}}} f_+(t) f_-(t)^* \right] \\ &= 2 \operatorname{Re} L_{f^{\text{CP}}} \operatorname{Re} \left[f_+(t) f_-(t)^* \right] - 2 \operatorname{Im} L_{f^{\text{CP}}} \operatorname{Im} \left[f_+(t) f_-(t)^* \right] \\ &= e^{-\bar{\Gamma}t} \left(\operatorname{Re} L_{f^{\text{CP}}} \sinh \frac{\Delta\Gamma}{2} t - \operatorname{Im} L_{f^{\text{CP}}} \sin \Delta mt \right) \end{aligned}$$

$$R_{f^{CP}}(t) = \frac{|\bar{A}_{f^{CP}}|^2 (1 - 4\varepsilon_B)}{2} e^{-\bar{\Gamma}t} \left\{ I_+^{CP}(t) - I_-^{CP}(t) \right\}$$

$$I_+^{CP}(t) = \left(1 + |L_f^{CP}|^2 \right) \cosh \frac{\Delta\Gamma}{2} t + 2 \operatorname{Re} L_f^{CP} \sinh \frac{\Delta\Gamma}{2} t$$

$$I_-^{CP}(t) = \left(1 - |L_f^{CP}|^2 \right) \cos \Delta m t + 2 \operatorname{Im} L_f^{CP} \sin \Delta m t$$

\bar{B}^0 at $t = 0 \rightarrow f^{CP}$ at t

$$\bar{R}_{f^{CP}}(t) = \frac{|\bar{A}_{f^{CP}}|^2}{2} e^{-\bar{\Gamma}t} \left\{ I_+^{CP}(t) + I_-^{CP}(t) \right\}$$

CP violating term

“CP conserving” part

$$I_+^{CP}(t) = \left(1 + |L_f^{CP}|^2\right) \cosh \frac{\Delta\Gamma}{2} t + 2 \operatorname{Re} L_f^{CP} \sinh \frac{\Delta\Gamma}{2} t$$

“CP violating” part

$$I_-^{CP}(t) = \left(1 - |L_f^{CP}|^2\right) \cos \Delta m t + 2 \operatorname{Im} L_f^{CP} \sin \Delta m t$$

$$L_f^{CP} = (1 + 4\varepsilon_B) e^{i\phi_M} \frac{A_{f^{CP}}}{\bar{A}_{f^{CP}}}$$

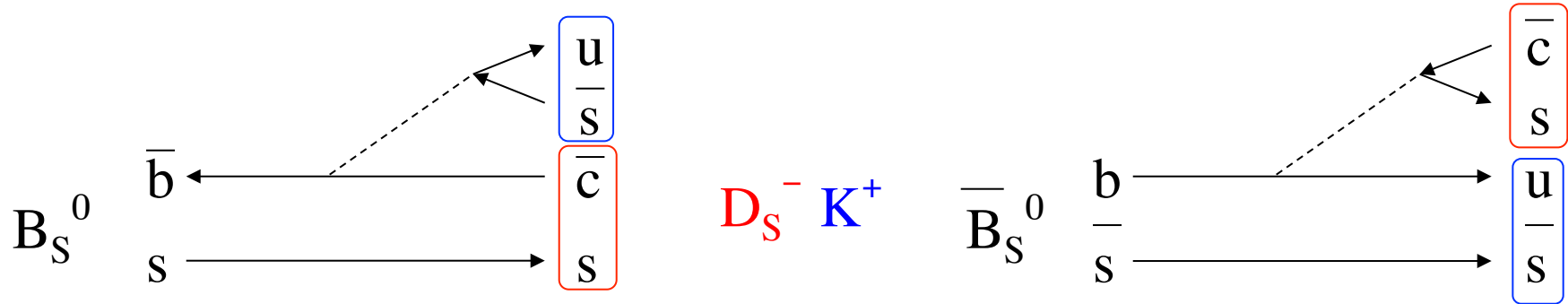
Mixed CP eigenstates

$$|f^{CP}\rangle = |f_+^{CP}\rangle + |f_-^{CP}\rangle, \quad CP|f^{CP}\rangle = |f_+^{CP}\rangle - |f_-^{CP}\rangle$$

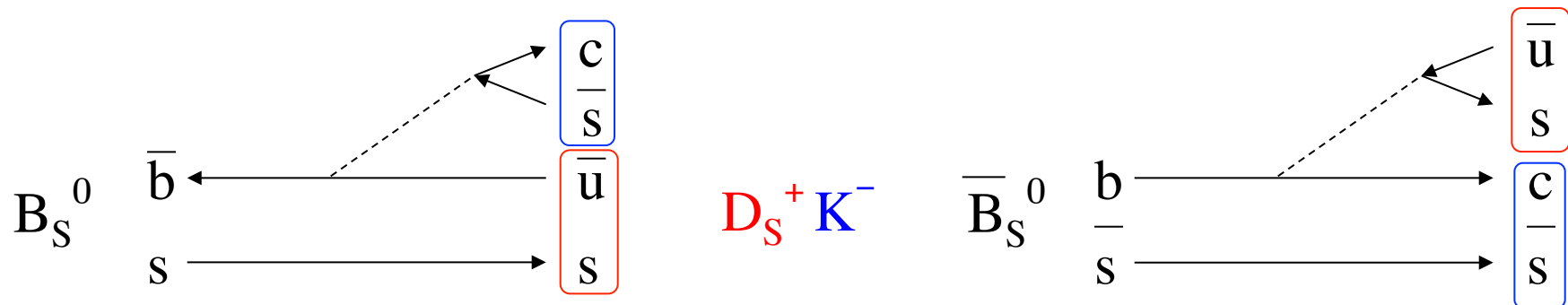
Flavour non specific hadronic decays:

Example

$$A_f = \langle D_S^- K^+ | H_W | B_S^0 \rangle, \quad \bar{A}_f = \langle D_S^- K^+ | H_W | \bar{B}_S^0 \rangle$$



$$A_{\bar{f}} = \langle D_S^+ K^- | H_W | B_S^0 \rangle, \quad \bar{A}_{\bar{f}} = \langle D_S^+ K^- | H_W | \bar{B}_S^0 \rangle$$



B^0 at $t = 0 \rightarrow f$ at t

$$\begin{aligned}
 R_f(t) &= \left| \langle f | H_W | B^0(t) \rangle \right|^2 \\
 &= \left| f_+(t) \langle f | H_W | B^0 \rangle + (1 - 2\varepsilon_B) e^{-i\phi_M} \langle f | H_W | \bar{B}^0 \rangle f_-(t) \right|^2 \\
 &= \frac{|A_f|^2}{2} e^{-\bar{\Gamma}t} \left\{ I_+^f(t) + I_-^f(t) \right\}
 \end{aligned}$$

$$I_+^f(t) = \left(1 + |L_f|^2 \right) \cosh \frac{\Delta\Gamma}{2} t + 2 \operatorname{Re} L_f \sinh \frac{\Delta\Gamma}{2} t$$

$$I_-^f(t) = \left(1 - |L_f|^2 \right) \cos \Delta m t + 2 \operatorname{Im} L_f \sin \Delta m t$$

$$L_f = (1 - 4\varepsilon_B) e^{-i\phi_M} \frac{\bar{A}_f}{A_f}$$

\bar{B}^0 at $t = 0 \rightarrow f$ at t

$$\begin{aligned}
 \bar{R}_f(t) &= \left| \langle f | H_W | \bar{B}^0(t) \rangle \right|^2 \\
 &= \frac{|A_f|^2}{2} e^{-\bar{\Gamma}t} \left\{ I_+^f(t) - I_-^f(t) \right\}
 \end{aligned}$$

B^0 at $t = 0 \rightarrow \bar{f}$ at t

$$R_{\bar{f}}(t) = \frac{|A_{\bar{f}}|^2}{2} e^{-\bar{\Gamma}t} \{I_{+}^{\bar{f}}(t) + I_{-}^{\bar{f}}(t)\}$$

\bar{B}^0 at $t = 0 \rightarrow \bar{f}$ at t

$$\bar{R}_{\bar{f}}(t) = \frac{|A_{\bar{f}}|^2}{2} e^{-\bar{\Gamma}t} \{I_{+}^{\bar{f}}(t) - I_{-}^{\bar{f}}(t)\}$$

$$I_{+}^{\bar{f}}(t) = \left(1 + |L_{\bar{f}}|^2\right) \cosh \frac{\Delta\Gamma}{2} t + 2 \operatorname{Re} L_{\bar{f}} \sinh \frac{\Delta\Gamma}{2} t$$

$$I_{-}^{\bar{f}}(t) = \left(1 - |L_{\bar{f}}|^2\right) \cos \Delta m t + 2 \operatorname{Im} L_{\bar{f}} \sin \Delta m t$$

$$L_{\bar{f}} = (1 - 4\varepsilon_B) e^{-i\phi_M} \frac{\bar{A}_{\bar{f}}}{A_{\bar{f}}}$$

Four time dependent decay rates.

Standard model predictions

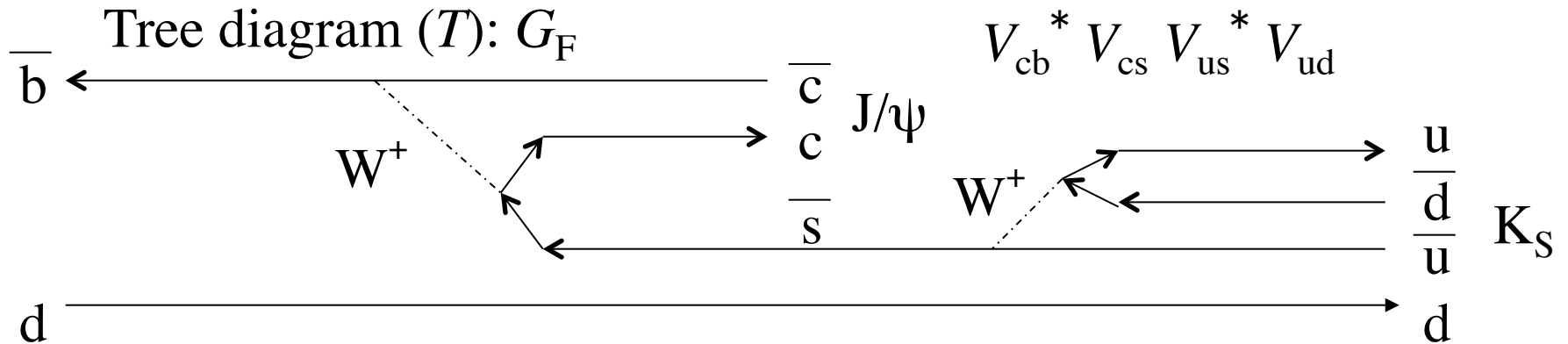
CP eigenstates generated by $b \rightarrow c + W$ decays

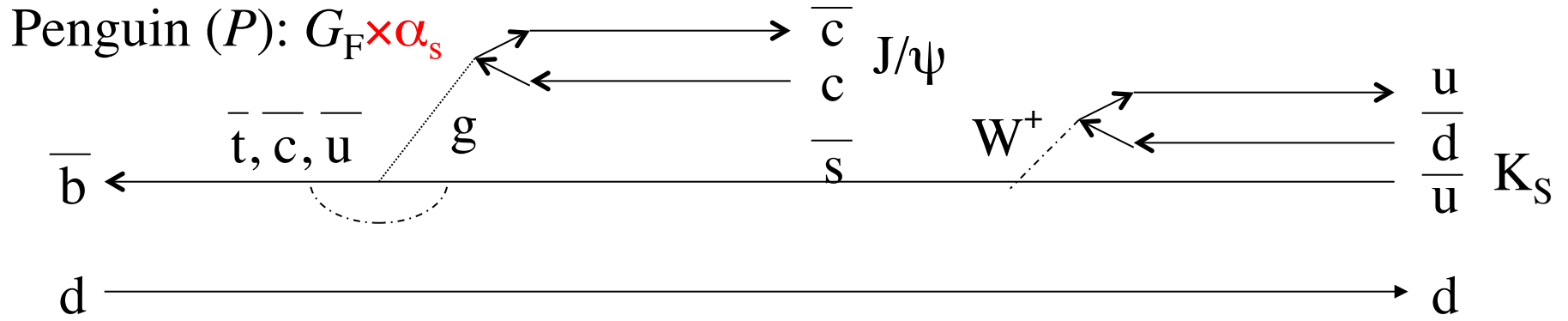
$$B^0 \rightarrow J/\psi K_S$$

$$CP(J/\psi K_S) = CP(J/\psi) \times CP(K_S) \times (-1)^l = -1$$

$$I = 1/2$$

B^0 decay





$$\left\{ V_{tb}^* V_{ts} F(m_t^2) + V_{cb}^* V_{cs} F(m_c^2) + V_{ub}^* V_{us} F(m_u^2) \right\} V_{us}^* V_{ud}$$

$$F(m_c^2) \approx F(m_u^2) \equiv F$$

$$\left\{ V_{tb}^* V_{ts} F(m_t^2) + [V_{cb}^* V_{cs} + V_{ub}^* V_{us}] F \right\} V_{us}^* V_{ud}$$

$$V_{tb}^* V_{ts} + V_{cb}^* V_{cs} + V_{ub}^* V_{us} = 0$$

$$V_{tb}^* V_{ts} [F(m_t^2) - F] V_{us}^* V_{ud}$$

$$A_{J/\psi K_S} \propto [V_{cb}^* V_{cs} + V_{tb}^* V_{ts} P/T] V_{us}^* V_{ud}$$

Final state interaction between J/ψ and K_S are elastic (???)

Only one state ($I=1/2$) is involved.

$$A_{J/\psi K_S} \propto [V_{cb}^* V_{cs} + V_{tb}^* V_{ts} P/T] V_{us}^* V_{ud} e^{i\delta}$$

$$\bar{A}_{J/\psi K_S} \propto - [V_{cb} V_{cs}^* + V_{tb} V_{ts}^* P/T] V_{us} V_{ud}^* e^{i\delta} e^{-i\theta_{CP}}$$

CP($J/\psi K_S$)

$$\longrightarrow |A_{J/\psi K_S}| = |\bar{A}_{J/\psi K_S}|$$

$$\text{Unitarity: } V_{tb}^* V_{ts} + V_{cb}^* V_{cs} + V_{ub}^* V_{us} = 0$$

$$|V_{tb}^* V_{ts}| \approx |V_{cb}^* V_{cs}| = A\lambda^2 \ll |V_{ub}^* V_{us}| = A\lambda^4 \sqrt{\rho^2 + \eta^2}$$

$$V_{tb}^* V_{ts} = -V_{cb}^* V_{cs} + O(\lambda^4)$$

$$\arg V_{tb}^* V_{ts} \approx \arg V_{cb}^* V_{cs} + \pi + O(\lambda^2)$$

Penguin is suppressed by the strong interactions: $P/T < 1$

$$\begin{aligned}
\frac{A_{J/\psi K_s}}{\overline{A}_{J/\psi K_s}} &= -\frac{V_{cb}^* V_{cs} V_{us}^* V_{ud} \left(1 - e^{iO(10^{-3})}\right)}{V_{cb} V_{cs}^* V_{us} V_{ud}^* \left(1 - e^{-iO(10^{-3})}\right)} e^{i\theta_{CP}} \\
&\approx -\frac{V_{cb}^* V_{cs} V_{us}^* V_{ud}}{V_{cb} V_{cs}^* V_{us} V_{ud}^*} e^{i\theta_{CP}} \\
&= -e^{i(\theta_{CP} + 2 \arg V_{cs} + 2 \arg V_{ud} - 2 \arg V_{cb} - 2 \arg V_{us})}
\end{aligned}$$

From

$$M_{12}^q = \frac{G_F^2}{12\pi^2} f_{B_q}^2 B_{B_q} m_{B_q} m_W^2 \lambda_t^{q^2} \eta_B S_0(x_t) e^{i(\pi - \theta_{CP})}$$

$$\begin{aligned}
\arg M_{12} &= \pi - \theta_{CP} + 2 \arg \lambda_t^d \\
&= \pi - \theta_{CP} + 2 \arg V_{tb} - 2 \arg V_{td}
\end{aligned}$$

$$L_f^{CP} = (1 + 4\varepsilon_B) e^{i\phi_M} \frac{A_f^{CP}}{\overline{A}_f^{CP}}$$

$$\begin{aligned} L_{J/\psi K_S}^{CP} &= -1 \times e^{i\pi} \times \\ &(1 + 4\varepsilon_B) e^{i(2 \arg V_{tb} - 2 \arg V_{td} - \theta_{CP})} e^{i(\theta_{CP} + 2 \arg V_{cs} + 2 \arg V_{ud} - 2 \arg V_{cb} - 2 \arg V_{us})} \\ &= (1 + 4\varepsilon_B) e^{i2(\arg V_{tb} - \arg V_{cb} + \arg V_{cs} - \arg V_{us} + \arg V_{ud} - \arg V_{td})} \end{aligned}$$

This phase is invariant under
the redefinition of the quark phases:
 $\equiv \beta$ or ϕ_1

With an accuracy of $\sim 10^{-3}$

$$L_{J/\psi K_S}^{CP} = e^{i2\beta}$$

Interpretation of β :

$$\beta = \arg V_{tb} - \arg V_{cb} + \arg V_{cs} - \arg V_{us} + \arg V_{ud} - \arg V_{td}$$

The first 2×2 submatrix is unitary to λ^4

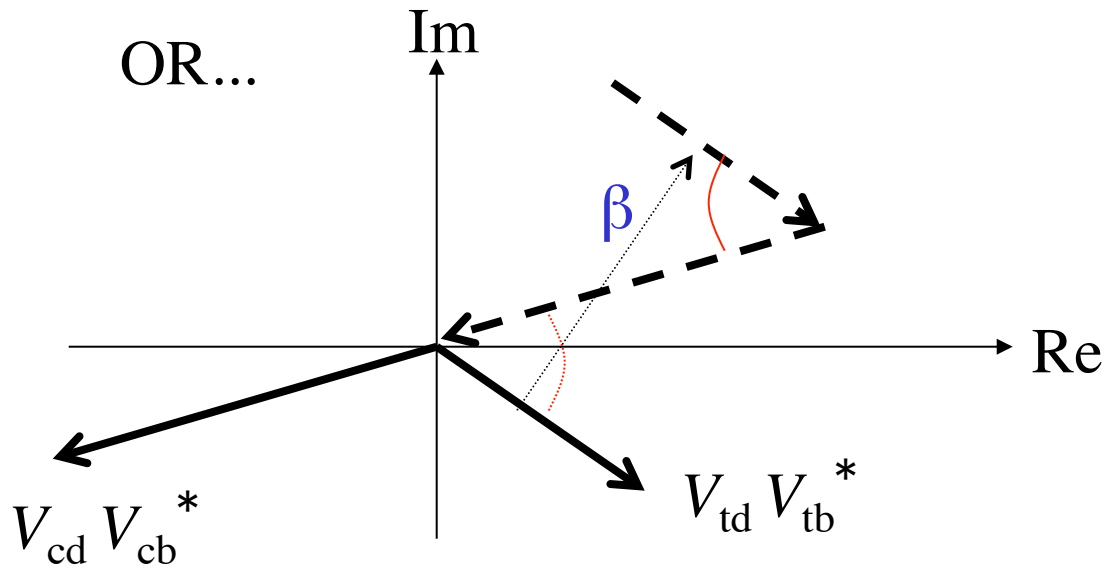
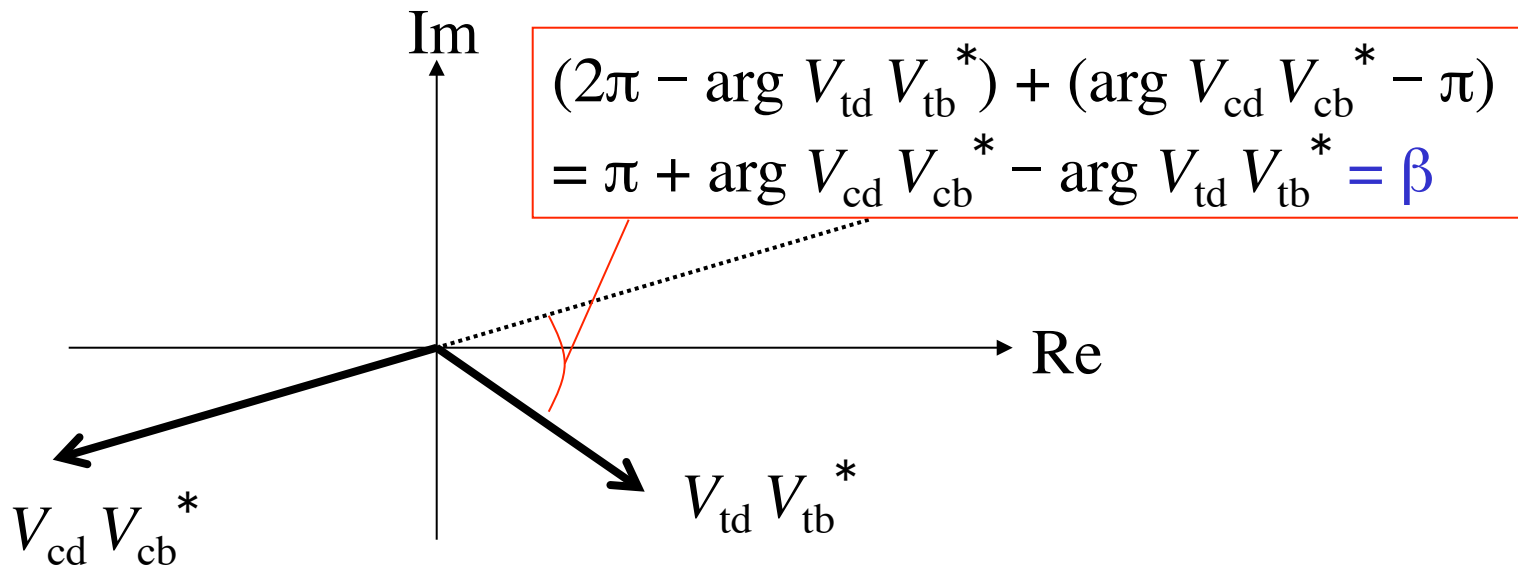
$$V_{ud} V_{us}^* + V_{cd} V_{cs}^* = 0 + O(\lambda^5) \sim 10^{-4}$$

$$(\arg V_{ud} - \arg V_{us}) - (\arg V_{cd} - \arg V_{cs}) \approx \pi$$

$$\arg V_{cs} \approx \pi - \arg V_{ud} + \arg V_{us} + \arg V_{cd}$$

$$\begin{aligned} \beta &\approx \pi + \arg V_{tb} - \arg V_{cb} + \arg V_{cd} - \arg V_{td} \\ &= \pi + (\arg V_{cd} - \arg V_{cb}) - (\arg V_{td} - \arg V_{tb}) \\ &= \pi + \arg V_{cd} V_{cb}^* - \arg V_{td} V_{tb}^* \end{aligned}$$

$$\beta = \pi + \arg V_{cd} V_{cb}^* - \arg V_{td} V_{tb}^* + O(10^{-4})$$



“CP conserving” part

$$I_+^{CP}(t) = \left(1 + |L_f^{CP}|^2\right) \cosh \frac{\Delta\Gamma}{2} t + 2 \operatorname{Re} L_f^{CP} \sinh \frac{\Delta\Gamma}{2} t$$
$$\approx 2$$

“CP violating” part

$$I_-^{CP}(t) = \left(1 - |L_f^{CP}|^2\right) \cos \Delta mt + 2 \operatorname{Im} L_f^{CP} \sin \Delta mt$$
$$\approx 2 \sin 2\beta \sin \Delta mt$$

Time dependent decay rates	}	Initial B^0	$R_{J/\psi K_S}(t) \propto e^{-\bar{\Gamma}t} \{1 - \sin 2\beta \sin \Delta mt\}$
		Initial \bar{B}^0	$\bar{R}_{J/\psi K_S}(t) \propto e^{-\bar{\Gamma}t} \{1 + \sin 2\beta \sin \Delta mt\}$

Correct to an accuracy of 10^{-3}

Another way to look at it...

B^0 at $t = 0 \rightarrow J/\psi K_S$ at t

$$\begin{aligned} R_{J/\psi K_S}(t) &= \left| \langle J/\psi K_S | H_W | B^0(t) \rangle \right|^2 \\ &= \frac{1}{2} \left| e^{-i\lambda_h t} \langle J/\psi K_S | H_W | B_h \rangle + e^{-i\lambda_l t} \langle J/\psi K_S | H_W | B_l \rangle \right|^2 \\ &\propto e^{-\bar{\Gamma}t} \left| 1 + e^{i\Delta m t} \eta_{J/\psi K_S} \right|^2, \quad \eta_{J/\psi K_S} = \frac{A_{J/\psi K_S}^l}{A_{J/\psi K_S}^h} \\ &\propto e^{-\bar{\Gamma}t} \left(1 + \left| \eta_{J/\psi K_S} \right|^2 + 2 \operatorname{Re} \eta_{J/\psi K_S} \cos \Delta m t - 2 \operatorname{Im} \eta_{J/\psi K_S} \sin \Delta m t \right) \end{aligned}$$

$\eta_{J/\psi K_S}$ is a CP violation parameter like η_{+-} for the kaon system

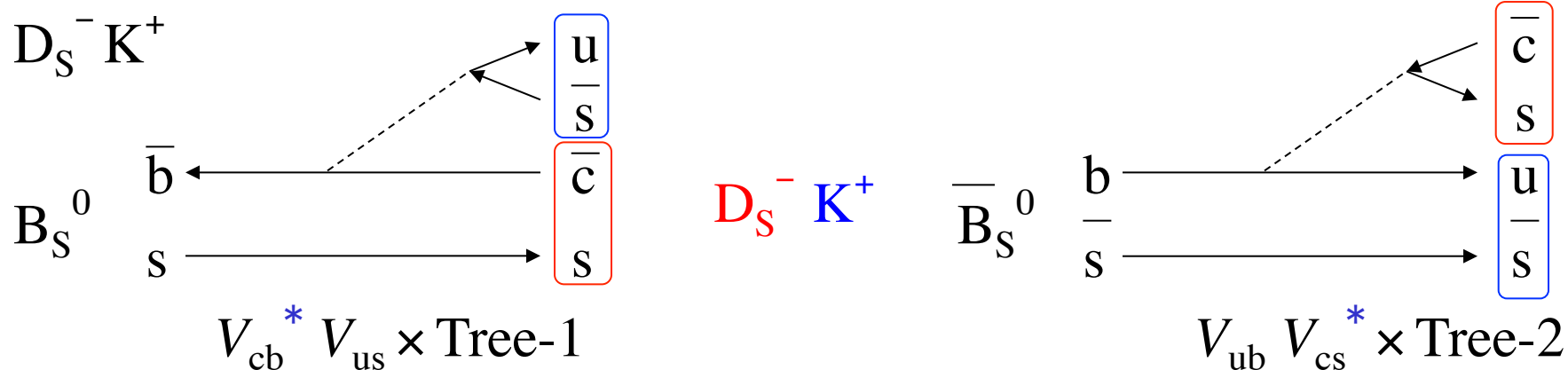
$$\begin{aligned}
\eta_{J/\psi K_S} &= \frac{A_{J/\psi K_S}^l}{A_{J/\psi K_S}^h} \\
&= \frac{A_{J/\psi K_S} - (1 - 2\varepsilon_B)e^{-i\phi_M} \bar{A}_{J/\psi K_S}}{A_{J/\psi K_S} + (1 - 2\varepsilon_B)e^{-i\phi_M} \bar{A}_{J/\psi K_S}} \\
&= \frac{L_{J/\psi K_S} - 1}{L_{J/\psi K_S} + 1} \\
&\approx \frac{e^{2i\beta} - 1}{e^{2i\beta} + 1} \\
&= \frac{i \sin 2\beta}{1 + \cos 2\beta}
\end{aligned}$$

$\eta_{J/\psi K_S}$ is imaginary to $\sim 10^{-3}$

$$\begin{aligned}
R_{J/\psi K_S}(t) &\propto e^{-\bar{\Gamma}t} \left(1 + |\eta_{J/\psi K_S}|^2 + 2 \operatorname{Re} \eta_{J/\psi K_S} \cos \Delta mt - 2 \operatorname{Im} \eta_{J/\psi K_S} \sin \Delta mt \right) \\
&= e^{-\bar{\Gamma}t} \left(1 + \frac{\sin^2 2\beta}{(1 + \cos 2\beta)^2} - \frac{2 \sin 2\beta}{1 + \cos 2\beta} \sin \Delta mt \right) \\
&= \frac{e^{-\bar{\Gamma}t}}{1 + \cos 2\beta} (1 - \sin 2\beta \sin \Delta mt)
\end{aligned}$$

$$\bar{R}_{J/\psi K_S}(t) \propto \frac{e^{-\bar{\Gamma}t}}{1 + \cos 2\beta} (1 + \sin 2\beta \sin \Delta mt)$$

If $\sin 2\beta \approx O(1)$, B_1 and B_h are far from the CP eigenstates, (different from K_S, K_L), although $\varepsilon_B \approx 0$!



$$\frac{\overline{A}_{D_s^- K^+}}{A_{D_s^- K^+}} = \frac{V_{ub} V_{cs}^* \times \text{Tree-2}}{V_{cb}^* V_{us} \times \text{Tree-1}}$$

$$= r \times e^{i(\arg V_{ub} - \arg V_{us} + \arg V_{cb} - \arg V_{cs})} e^{i\Delta} e^{-i\theta_{CP}}$$

$$r = \left| \frac{V_{ub} V_{cs}}{V_{cb} V_{us}} \right| \times \left| \frac{\text{Tree-2}}{\text{Tree-1}} \right|$$

$$= \frac{A\lambda^3 \sqrt{\rho^2 + \eta^2}}{A\lambda^3} \times \left| \frac{\text{Tree-2}}{\text{Tree-1}} \right|$$

~ 0.4

Δ : Tree-1 Tree-2 “strong phase” difference

Only D_s -K elastic scattering

From

$$M_{12}^q = \frac{G_F^2}{12\pi^2} f_{B_q}^2 B_{B_q} m_{B_q} m_W^2 \lambda_t^{q^2} \eta_B S_0(x_t) e^{i(\pi - \theta_{CP})}$$

$$\begin{aligned} \arg M_{12} &= \pi - \theta_{CP} + 2 \arg \lambda_t^s \\ &= \pi - \theta_{CP} + 2 \arg V_{tb} - 2 \arg V_{ts} \end{aligned}$$

$$L_f = (1 - 4\varepsilon_B) e^{-i\phi_M} \frac{\bar{A}_f}{A_f}$$

$$\begin{aligned} L_{D_s^- K^+} &= -(1 - 4\varepsilon_B) e^{i(\theta_{CP} + 2 \arg V_{ts} - 2 \arg V_{tb})} \\ &\quad \times r e^{i(\arg V_{ub} - \arg V_{us} + \arg V_{cb} - \arg V_{cs} - \theta_{CP})} e^{i\Delta} \\ &= -(1 - 4\varepsilon_B) r e^{i(\arg V_{ub} - \arg V_{us} + \arg V_{cb} - \arg V_{cs} + 2 \arg V_{ts} - 2 \arg V_{tb})} e^{i\Delta} \end{aligned}$$

This phase is invariant under the redefinition of the quark phases:

$$\equiv -\gamma + 2\delta\gamma \text{ or } -\phi_3 + 2\delta\phi_3$$

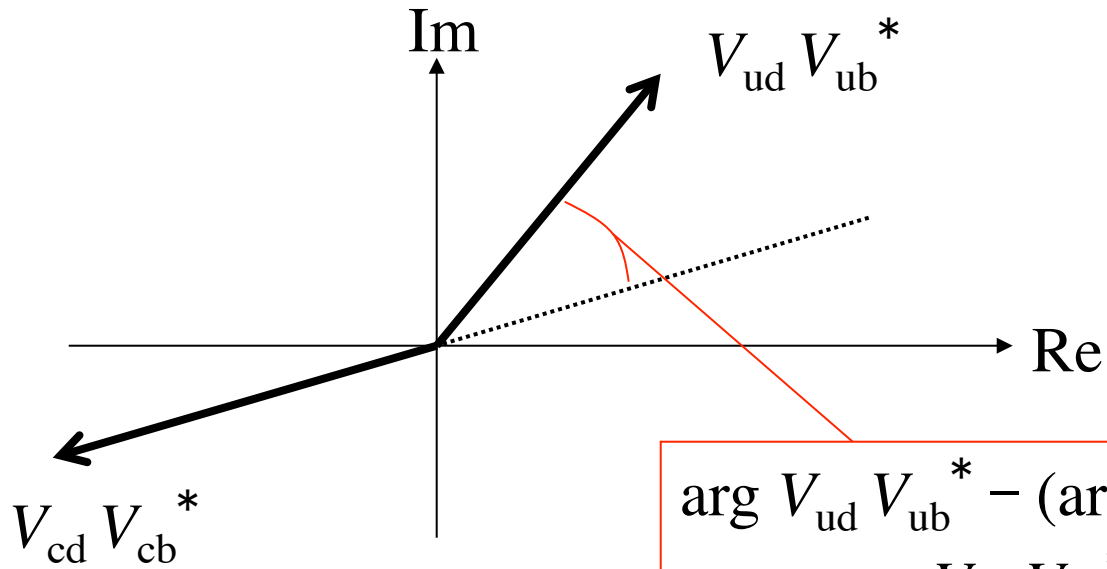
$$= -r e^{i(-\gamma + 2\delta\gamma + \Delta)}$$

with an accuracy of $\sim 10^{-4}$

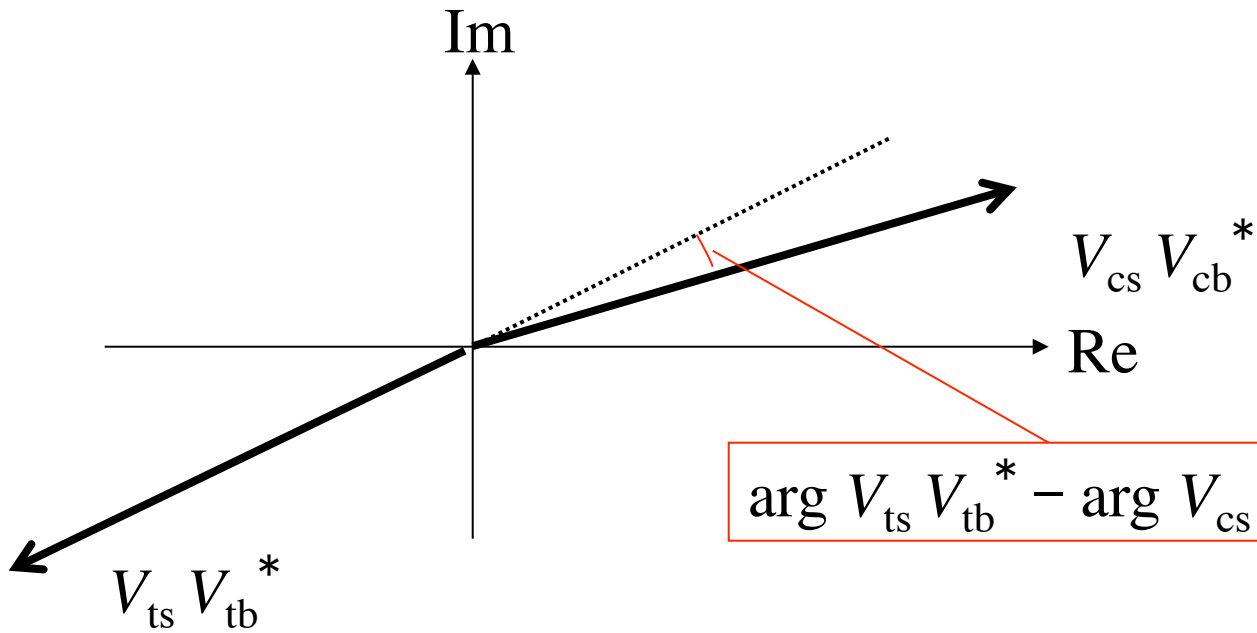
$$\begin{aligned}
& \arg V_{ub} - \arg V_{us} + \arg V_{cb} - \arg V_{cs} + 2\arg V_{ts} - 2\arg V_{tb} \\
&= \arg V_{ub} - \arg V_{us} - \arg V_{cb} + \arg V_{cs} \\
&\quad + 2(\arg V_{cb} - \arg V_{cs} + \arg V_{ts} - \arg V_{tb})
\end{aligned}$$

$$\arg V_{cs} \approx -\pi - \arg V_{ud} + \arg V_{us} + \arg V_{cd}$$

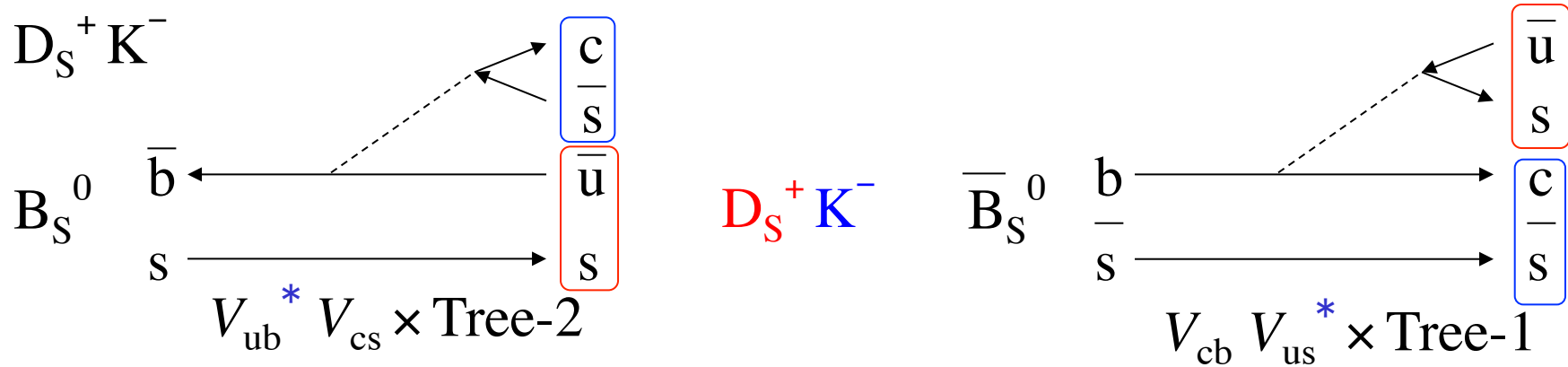
$$\begin{aligned}
&= \arg V_{ub} - \arg V_{ud} - \arg V_{cb} + \arg V_{cd} - \pi \\
&\quad + 2(\arg V_{cb} - \arg V_{cs} + \arg V_{ts} - \arg V_{tb}) \\
&= \arg V_{cd} V_{cb}^* - \arg V_{ud} V_{ub}^* - \pi + 2(\arg V_{ts} V_{tb}^* - \arg V_{cs} V_{cb}^*) \\
&= 2\delta\gamma - \gamma
\end{aligned}$$



$$\arg V_{ud} V_{ub}^* - (\arg V_{cd} V_{cb}^* - \pi) = \pi + \arg V_{ud} V_{ub}^* - \arg V_{cd} V_{cb}^* = \gamma$$



$$\arg V_{ts} V_{tb}^* - \arg V_{cs} V_{cb}^* = \delta\gamma$$



$$\frac{\bar{A}_{D_s^+ K^-}}{A_{D_s^+ K^-}} = \frac{V_{cb} V_{us}^* \times \text{Tree-1}}{V_{ub}^* V_{cs} \times \text{Tree-2}}$$

$$= \frac{1}{r} \times e^{i(\arg V_{cb} - \arg V_{us} + \arg V_{ub} - \arg V_{cs})} e^{-i\Delta} e^{-i\theta_{CP}}$$

$$L_{D_s^+ K^-} \approx -e^{i(\theta_{CP} + 2 \arg V_{ts} - 2 \arg V_{tb})}$$

$$\times \frac{1}{r} e^{i(\arg V_{cb} - \arg V_{us} + \arg V_{ub} - \arg V_{cs} - \theta_{CP})} e^{-i\Delta}$$

$$= -\frac{1}{r} e^{i(\arg V_{ub} - \arg V_{us} + \arg V_{cb} - \arg V_{cs} + 2 \arg V_{ts} - 2 \arg V_{tb})} e^{-i\Delta}$$

$$= \frac{1}{r} e^{i(-\gamma + 2\delta\gamma - \Delta)}$$

B_s^0 at $t = 0 \rightarrow D_S^- K^+$ at t

$$R_{D_s^- K^+}(t) = \frac{|A_{D_s^- K^+}|^2}{2} e^{-\bar{\Gamma}t} \left\{ I_+^{D_s^- K^+}(t) + I_-^{D_s^- K^+}(t) \right\}$$

$$I_+^{D_s^- K^+}(t) = (1 + r^2) \cosh \frac{\Delta\Gamma}{2} t - 2r \cos(\gamma - 2\delta\gamma - \Delta) \sinh \frac{\Delta\Gamma}{2} t$$

$$I_-^{D_s^- K^+}(t) = (1 - r^2) \cos \Delta mt + 2r \sin(\gamma - 2\delta\gamma - \Delta) \sin \Delta mt$$

\bar{B}_s^0 at $t = 0 \rightarrow D_S^- K^+$ at t

$$\bar{R}_{D_s^- K^+}(t) = \frac{|A_{D_s^- K^+}|^2}{2} e^{-\bar{\Gamma}t} \left\{ I_+^{D_s^- K^+}(t) - I_-^{D_s^- K^+}(t) \right\}$$

B_s^0 at $t = 0 \rightarrow D_S^+ K^-$ at t

$$R_{D_s^+ K^-}(t) = \frac{|A_{D_s^+ K^-}|^2}{2} e^{-\bar{\Gamma}t} \left\{ I_+^{D_s^+ K^-}(t) + I_-^{D_s^+ K^-}(t) \right\}$$

$$I_+^{D_s^+ K^-}(t) = \left(1 + \frac{1}{r^2} \right) \cosh \frac{\Delta\Gamma}{2} t - 2 \frac{1}{r} \cos(\gamma - 2\delta\gamma + \Delta) \sinh \frac{\Delta\Gamma}{2} t$$

$$I_-^{D_s^+ K^-}(t) = \left(1 - \frac{1}{r^2} \right) \cos \Delta mt + 2 \frac{1}{r} \sin(\gamma - 2\delta\gamma + \Delta) \sin \Delta mt$$

\bar{B}_s^0 at $t = 0 \rightarrow D_S^+ K^-$ at t

$$\bar{R}_{D_s^+ K^-}(t) = \frac{|A_{D_s^+ K^-}|^2}{2} e^{-\bar{\Gamma}t} \left\{ I_+^{D_s^+ K^-}(t) - I_-^{D_s^+ K^-}(t) \right\}$$

$r, \gamma - 2\delta\gamma + \Delta, \gamma - 2\delta\gamma - \Delta$ can be determined: $\rightarrow \gamma - 2\delta\gamma$

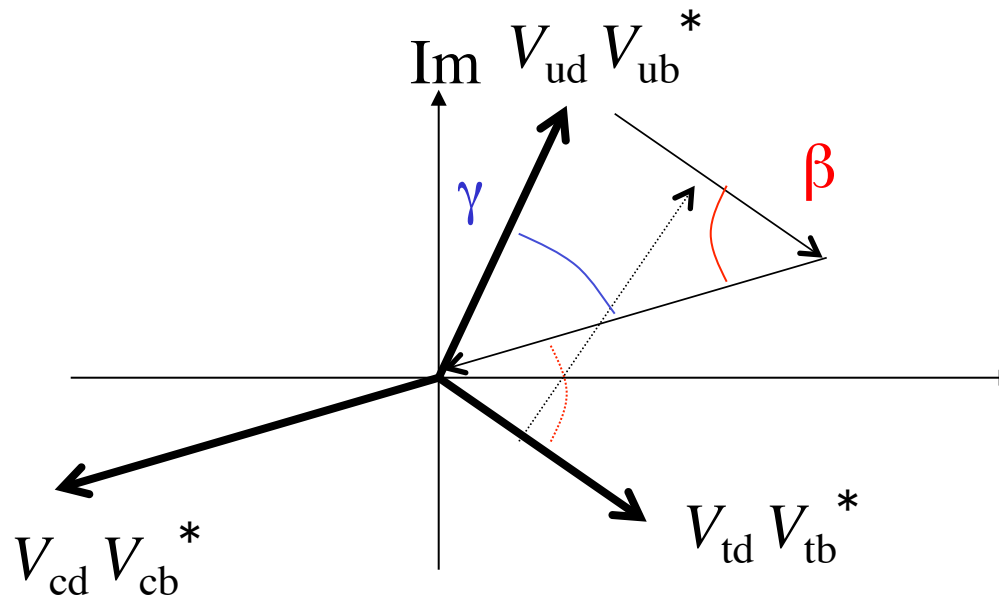
$$B_d \rightarrow J/\psi K_S: 2\beta$$

$$B_d \rightarrow D^{*\pm} \pi^\mu, \gamma + 2\beta$$

$$B_s \rightarrow J/\psi \phi: -2\delta\gamma$$

$$B_s \rightarrow D_s^\pm K^\mu, \gamma - 2\delta\gamma$$

γ and β can be extracted.



CKM parameters

Measurements

Δm_d (B^0 - \bar{B}^0 oscillations)

Δm_s (B_s^0 - \bar{B}_s^0 oscillations)

$\Gamma(B \rightarrow D^* \ell \nu)$

$\Gamma(B \rightarrow \rho \ell \nu)$

CKM elements

$|V_{tb} V_{td}^*|$

$|V_{tb} V_{ts}^*|$

$|V_{cb}|$

$|V_{ub}|$

n, π, K decays $\rightarrow \sin\theta_c = \lambda = 0.22$

unitarity $\rightarrow |V_{td}|=1$

Wolfenstein parametrization

$$|V_{cb}| = A \lambda^2$$

$$|V_{ub}| = A \lambda^3 \sqrt{\rho^2 + \eta^2}$$

$$|V_{td}| = A \lambda^3 \sqrt{(1 - \tilde{\rho})^2 + \tilde{\eta}^2}$$

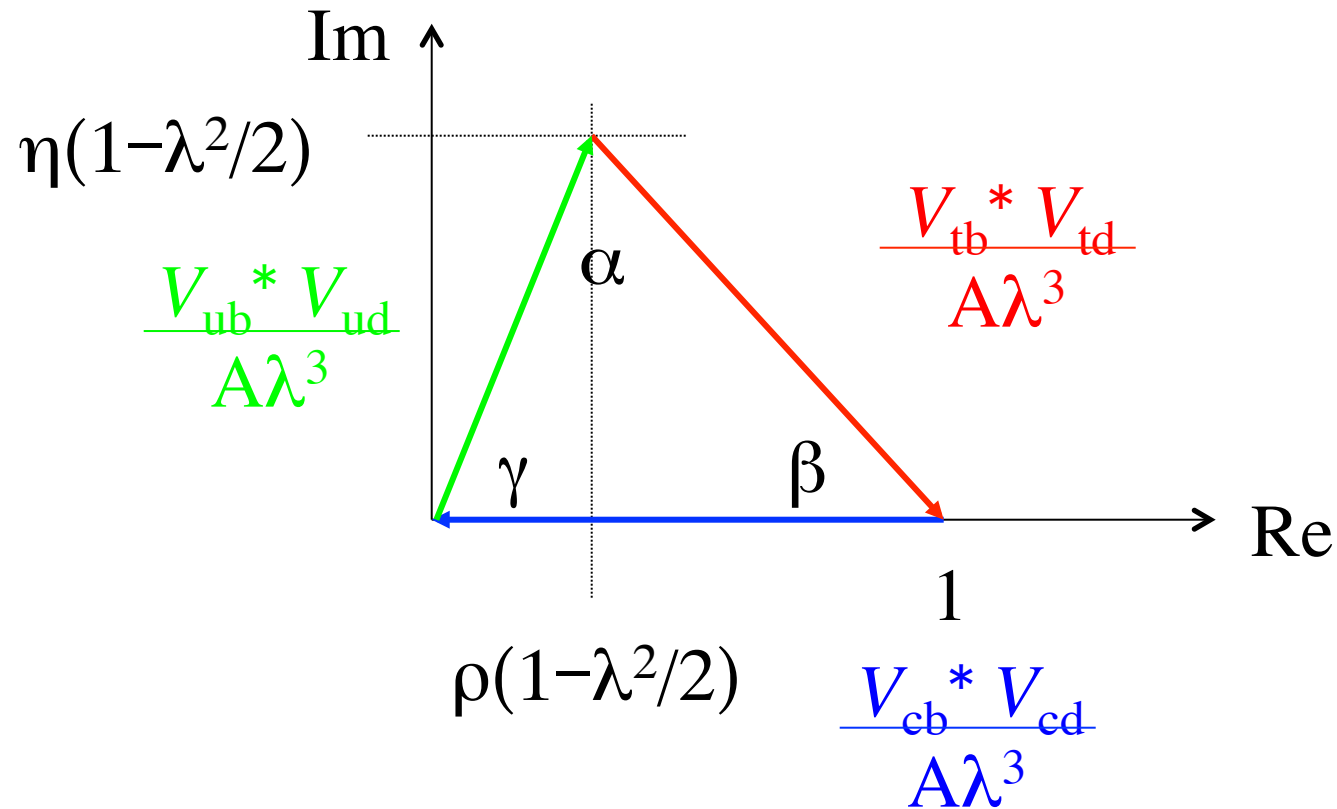
$$|V_{ts}| = A \lambda^2$$

$$\tilde{\eta} = \eta (1 - \lambda^2/2)$$

$$\tilde{\rho} = \rho (1 - \lambda^2/2)$$

$$1) V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$$

Using the Wolfenstein's parametrization (λ, A, ρ, η)



$$\begin{aligned}\arg V_{td} &= -\beta \\ \arg V_{ub} &= -\gamma \\ \arg V_{ts} &= \delta\gamma + \pi\end{aligned}$$

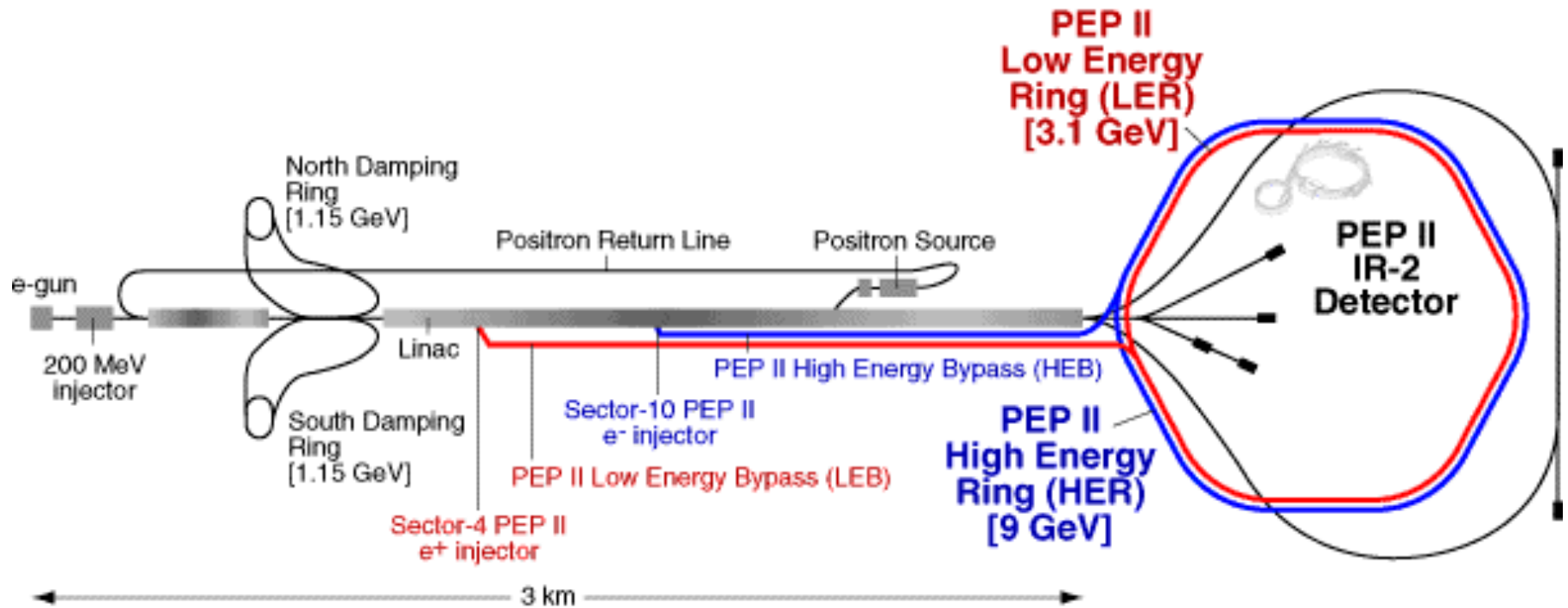
$$\beta = \tan^{-1} \frac{\eta}{1-\rho} \left[1 - \frac{\lambda^2}{2(1-\rho)} \right]$$

$$\gamma = \tan^{-1} \frac{\eta}{\rho}$$

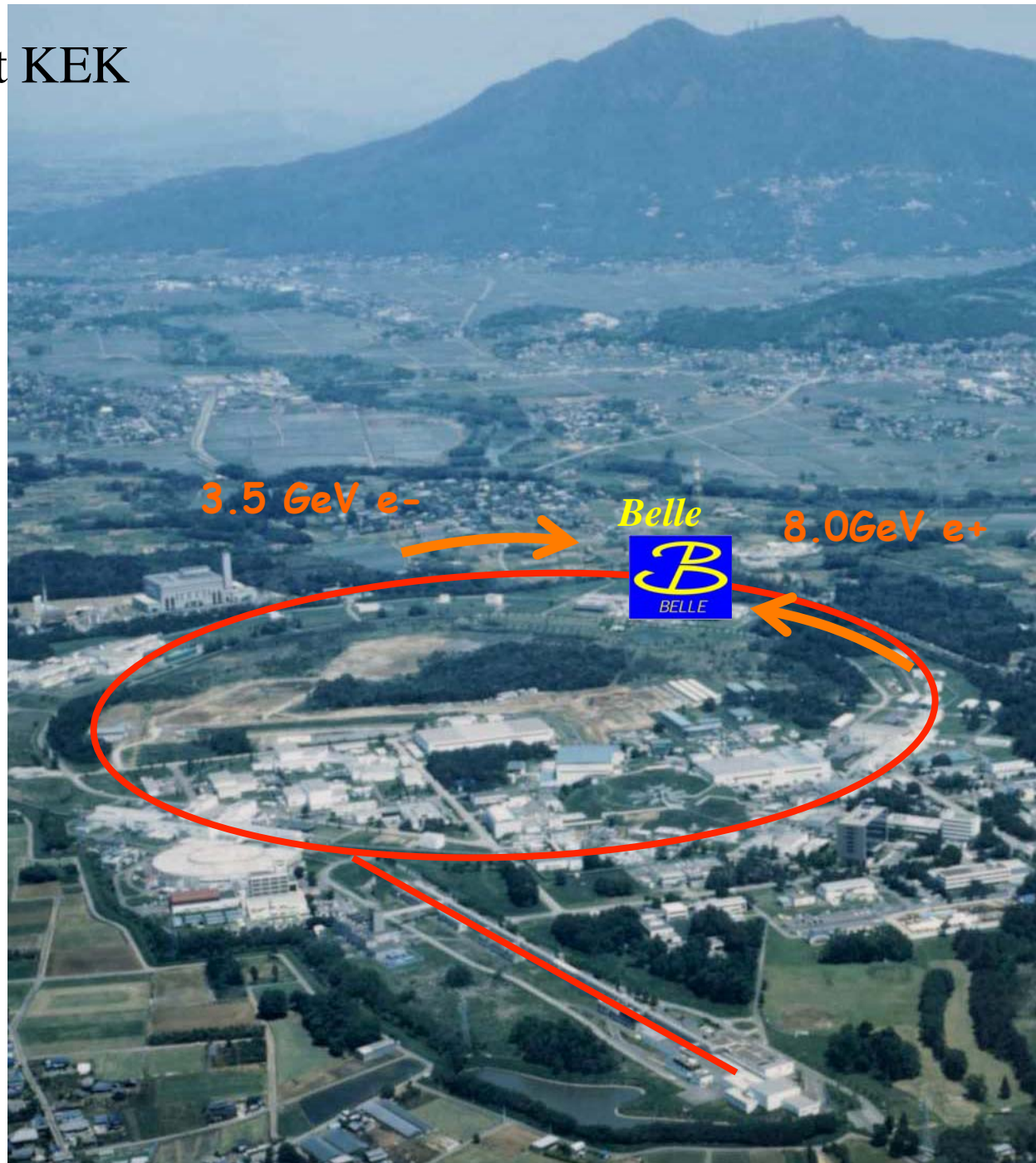
$$\delta\gamma = \eta\lambda^2$$

If we ignore the λ^2 correction, 1) and 2) are degenerate.

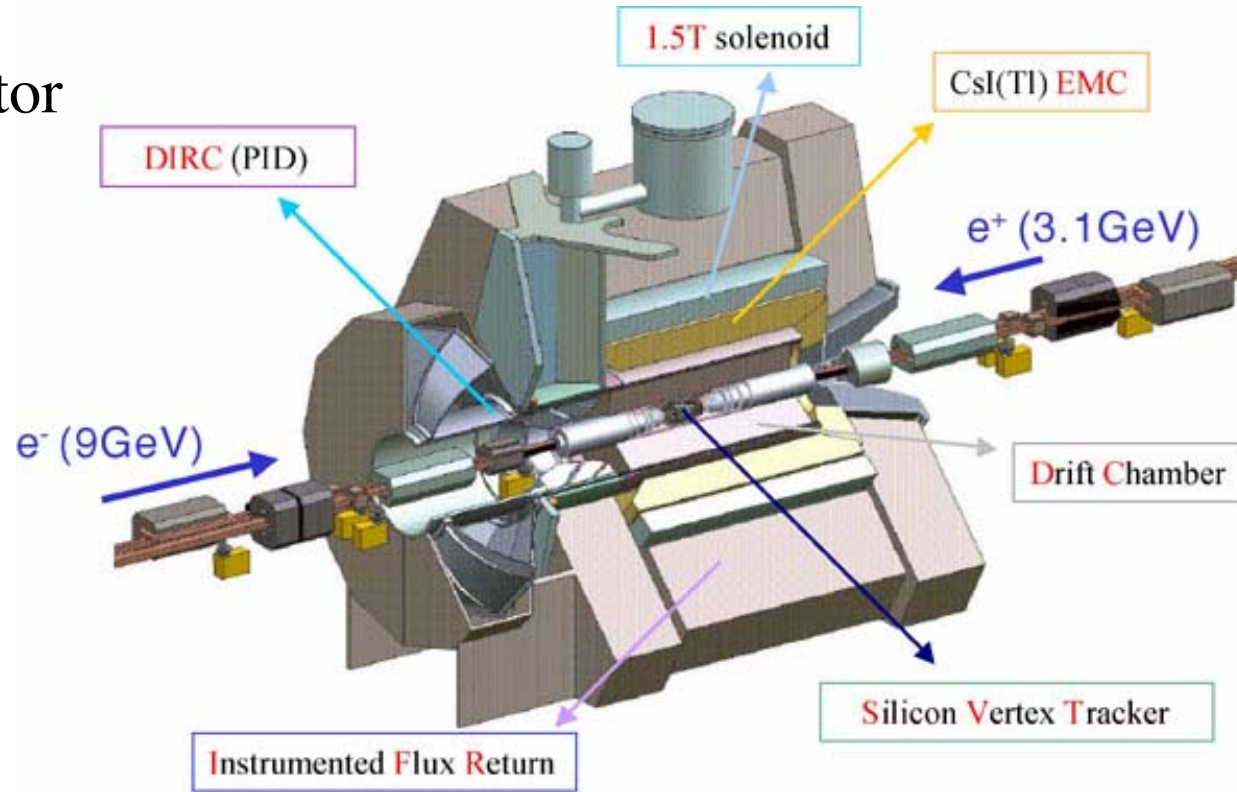
PEP II at SLAC



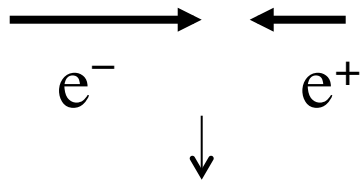
KEKB at KEK



BABAR detector

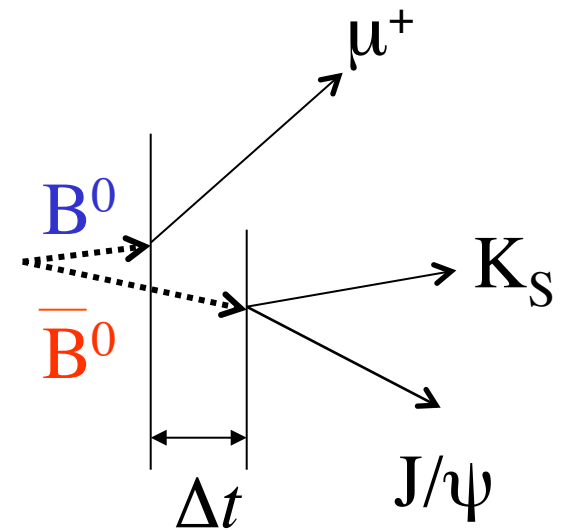


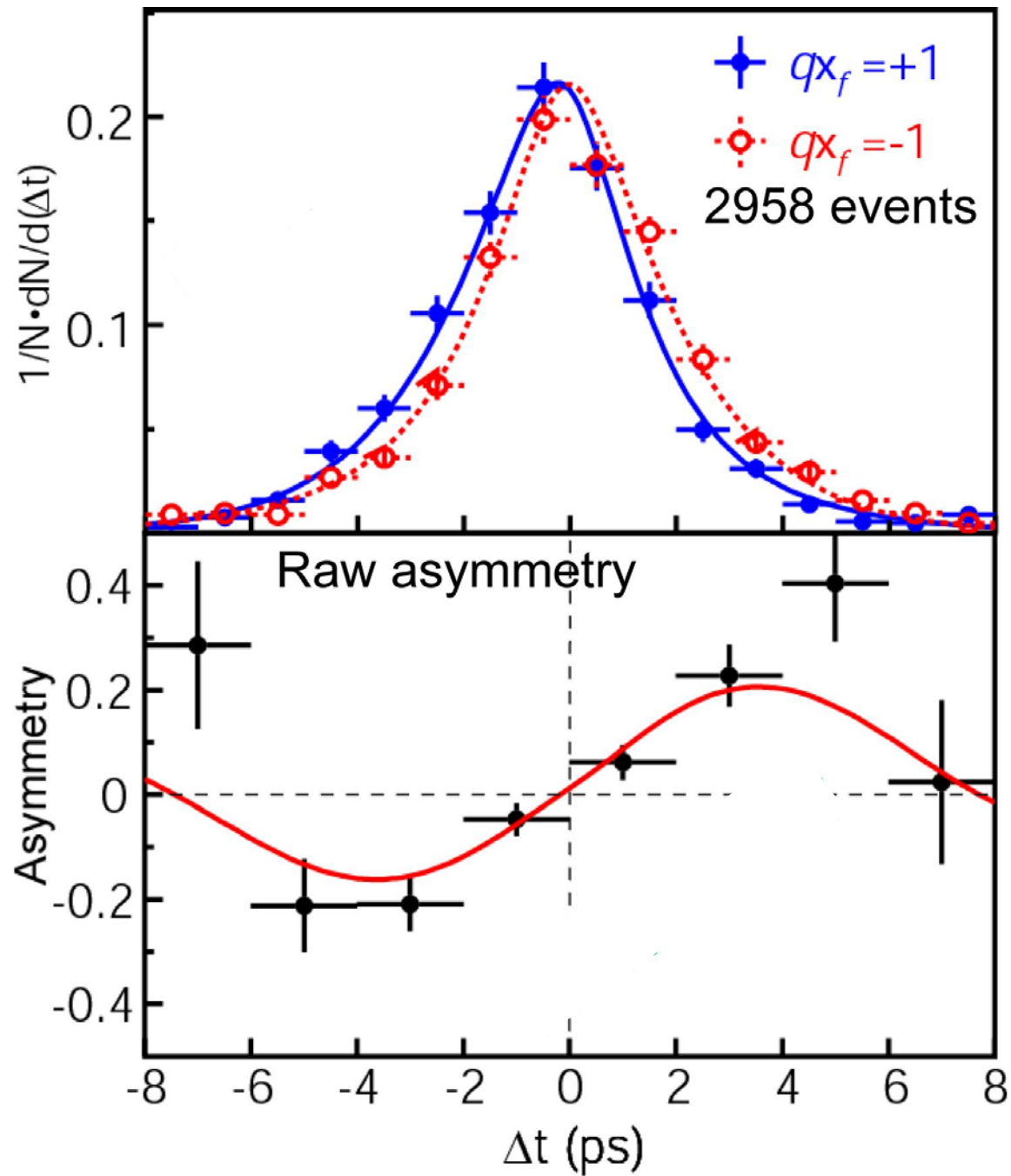
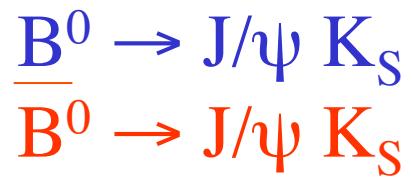
in the lab. frame

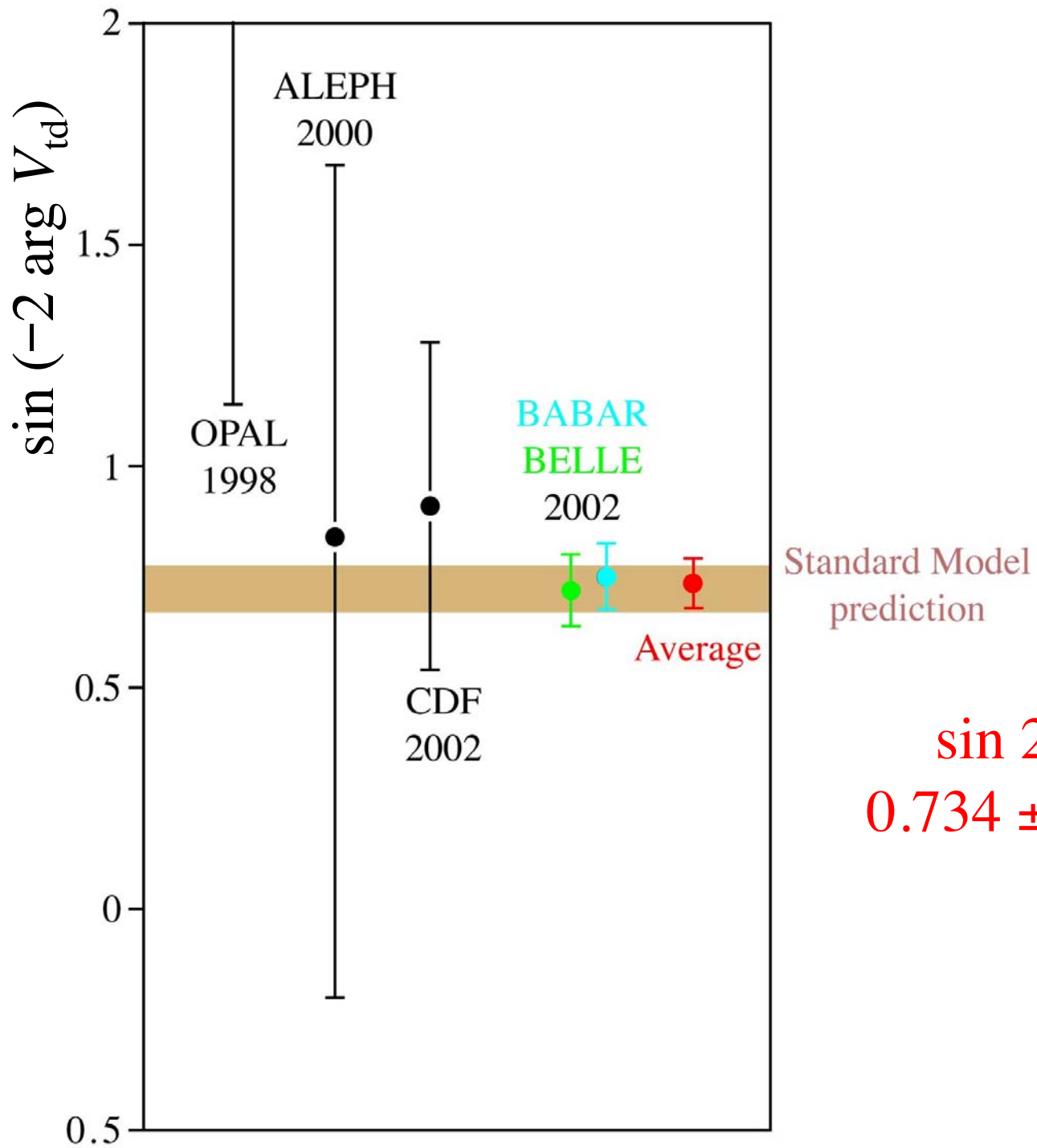


$\Upsilon(4S)$
 \Rightarrow
 boost

boosted
 $\bar{B}^0 B^0$ system
 (remains always $\bar{B}^0 B^0$)

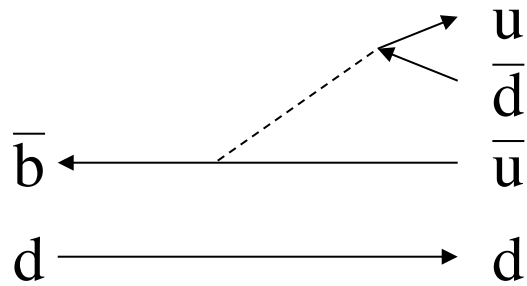




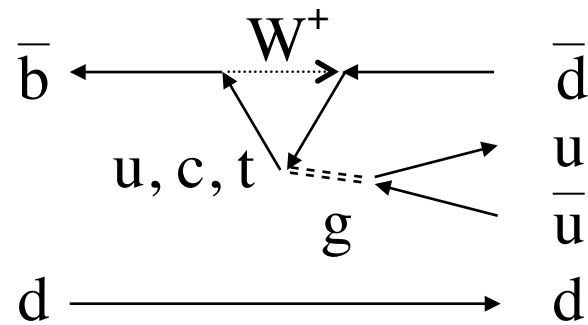


$$\sin 2\beta = 0.734 \pm 0.055$$

$B_d \rightarrow \pi\pi, \rho\pi, \rho\rho$



$V_{ub} \rightarrow \gamma$



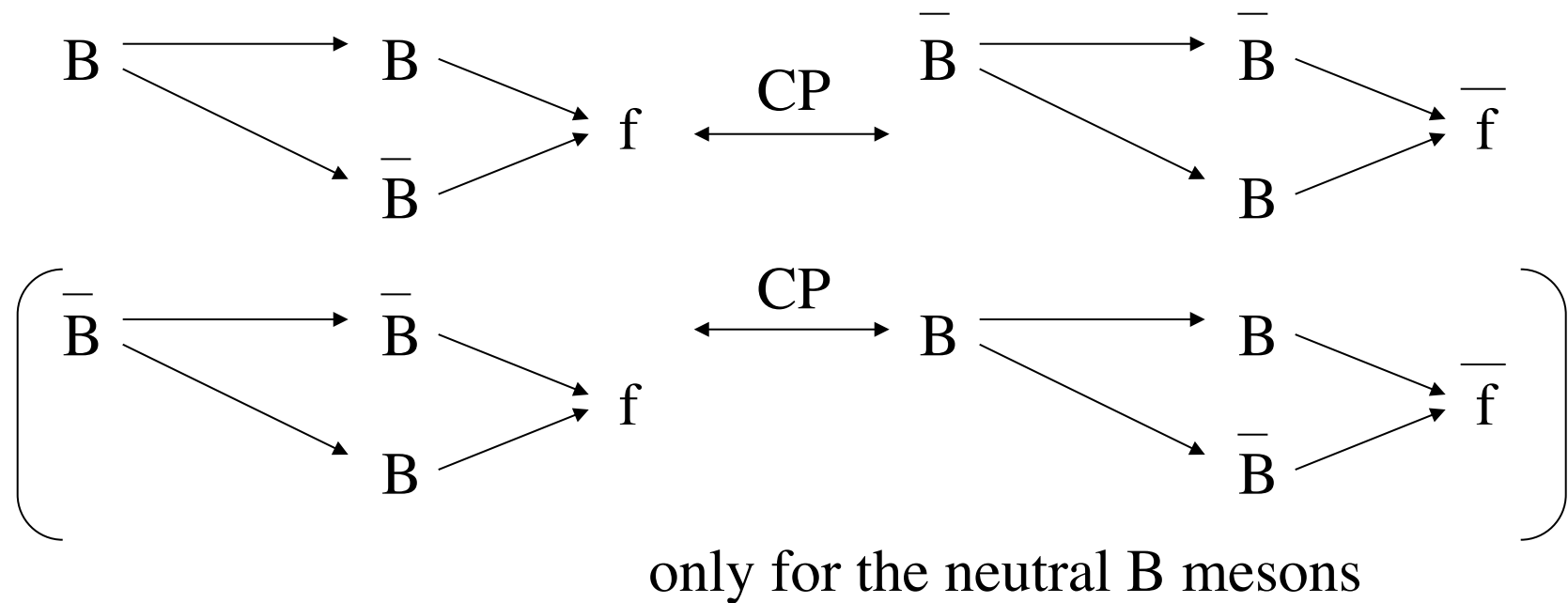
$+ \gamma$ and Z^0 penguins

$V_{td} \rightarrow \beta$

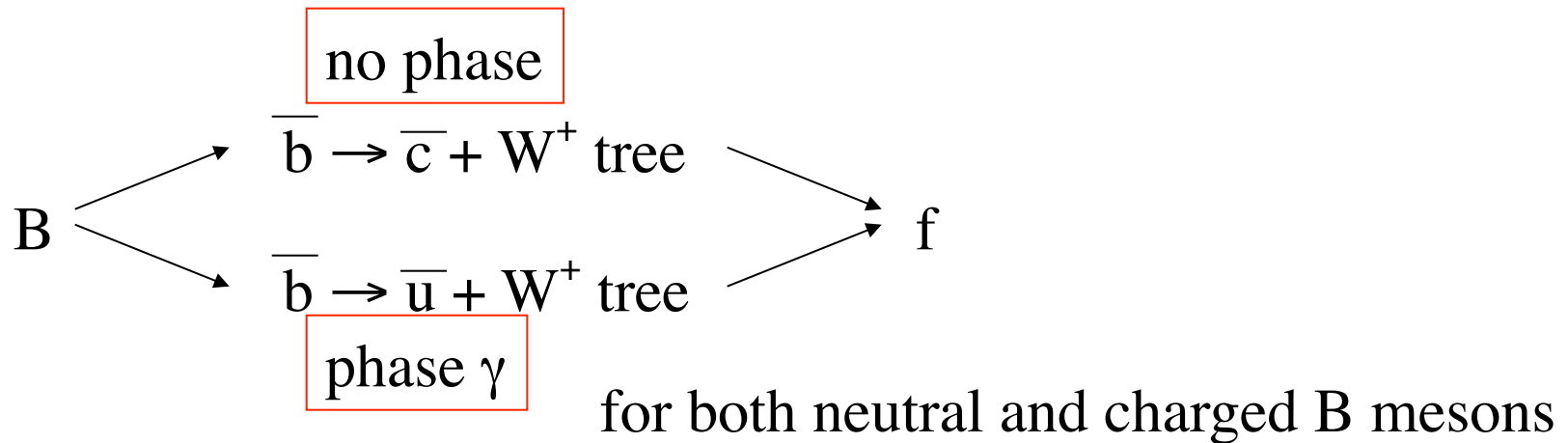
$\pi\pi, \rho\pi$: penguin contribution cannot be neglected

$\rho\rho$: penguin contribution seems to be very small

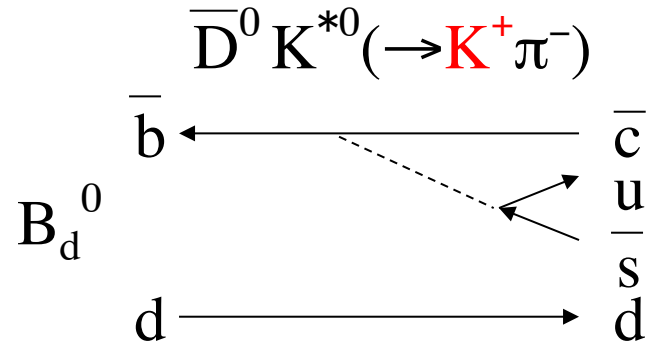
Processes we have looked at are:



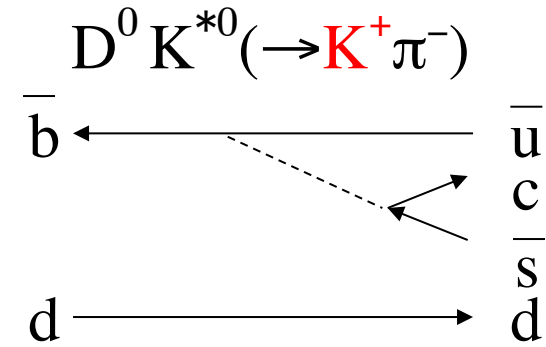
Another way to mix two decay processes



Another way to determine γ without new physics contamination



$$V_{cb}^* V_{us} \times \text{Tree-1}$$



$$V_{ub}^* V_{cs} \times \text{Tree-2}$$

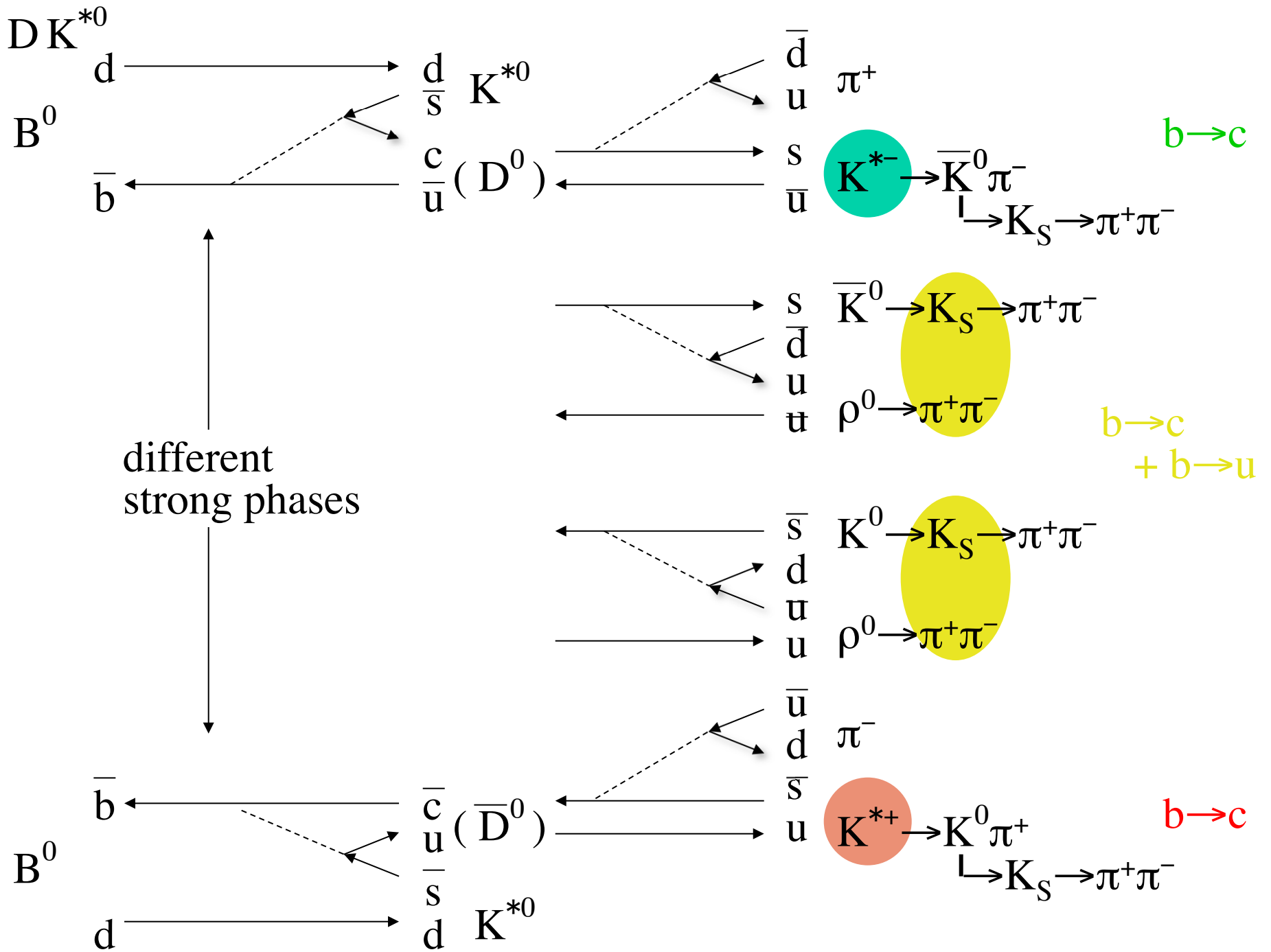
$$\bar{D}^0 \rightarrow K_S \pi^+ \pi^- \leftarrow D^0 \quad \text{K-}\bar{\text{K}} \text{ mixing (GLW)}$$

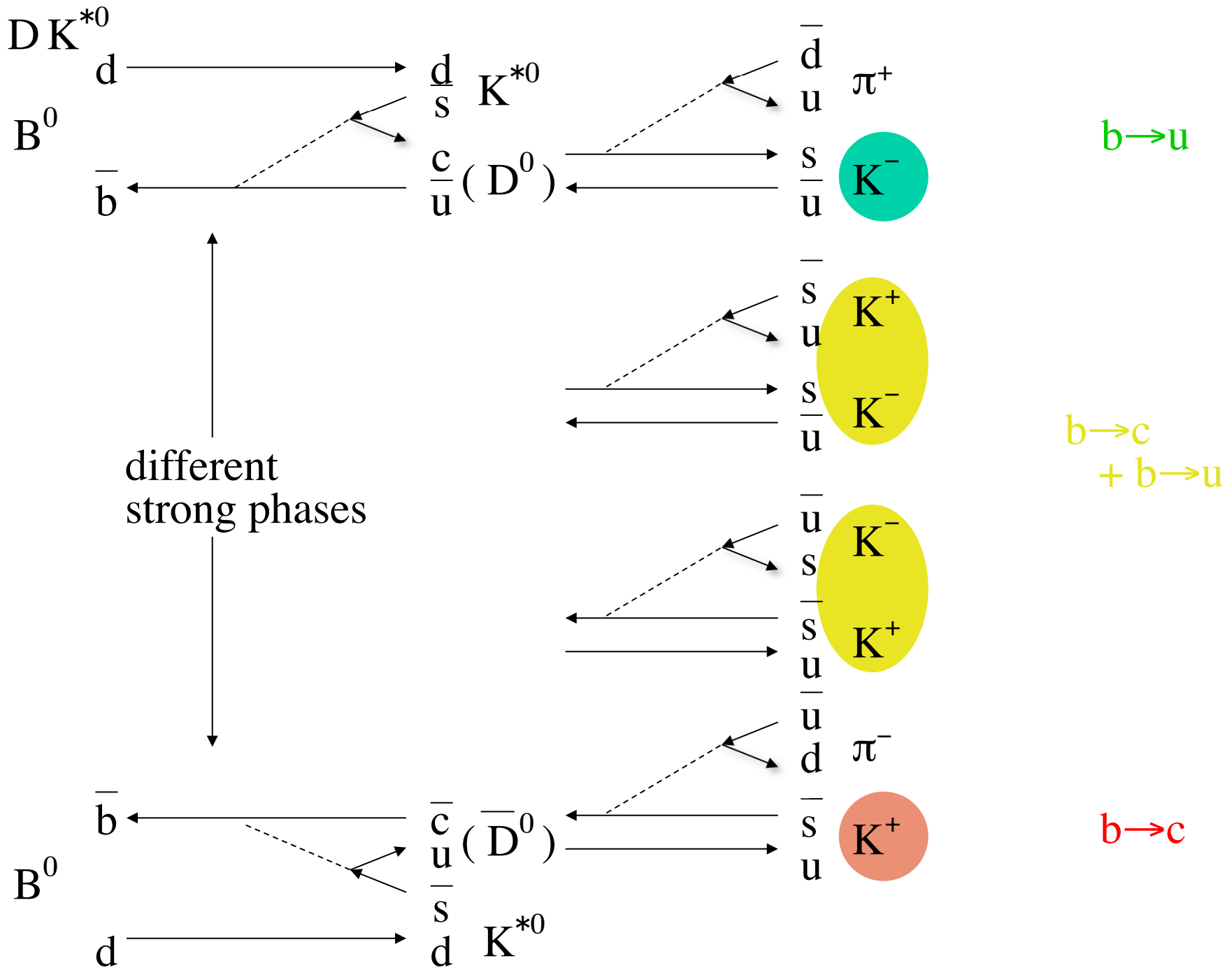
$$\bar{D}^0 \rightarrow K^+ K^-, \pi^+ \pi^- \leftarrow D^0 \quad \text{D-}\bar{\text{D}} \text{ mixing (GGSZ)}$$

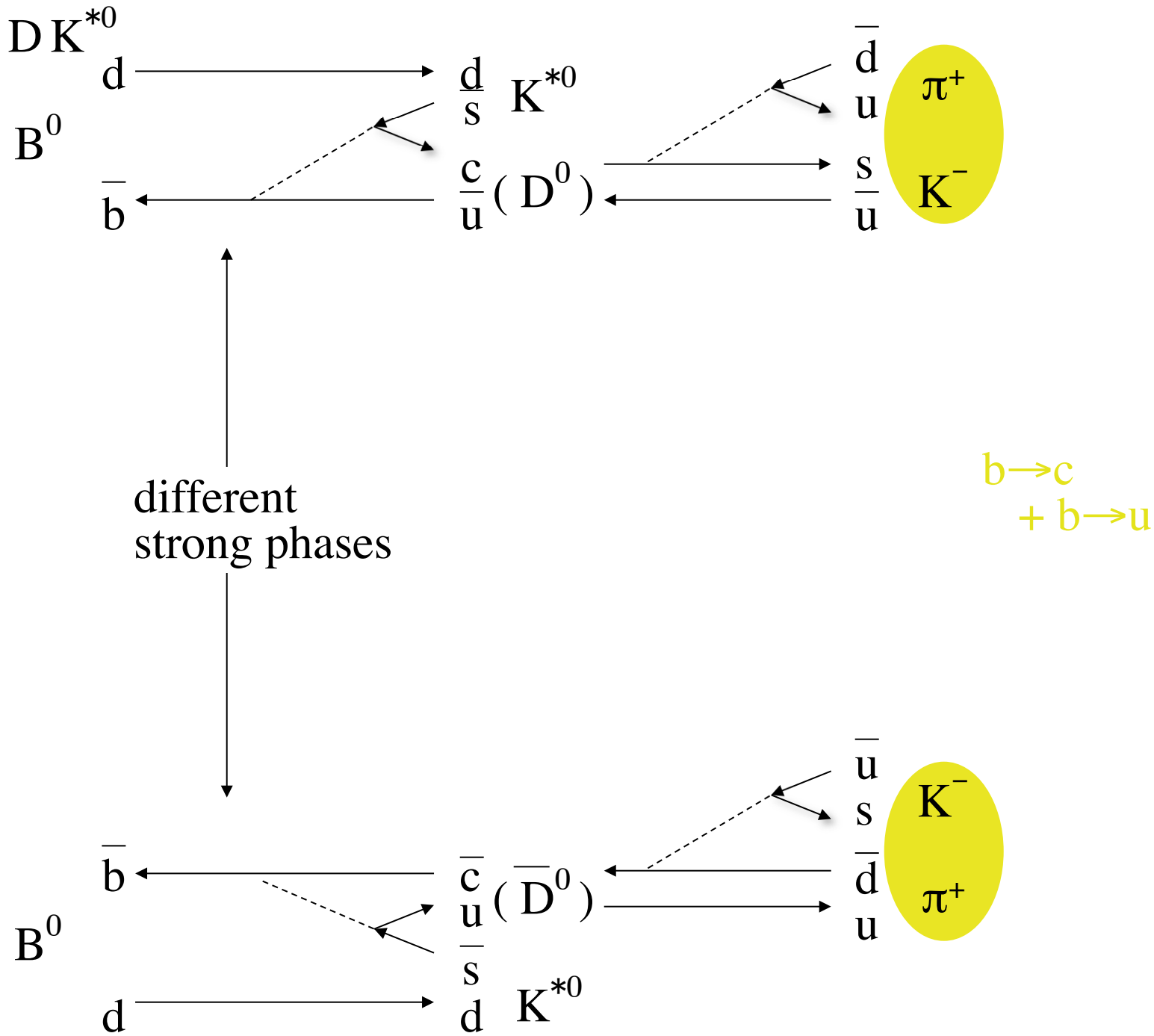
$$\bar{D}^0 \rightarrow K^+ \pi^-, K^+ \pi^- \pi^+ \pi^- \leftarrow D^0 \quad \text{Doubly Cabibbo Suppressed Decay (ADS)}$$

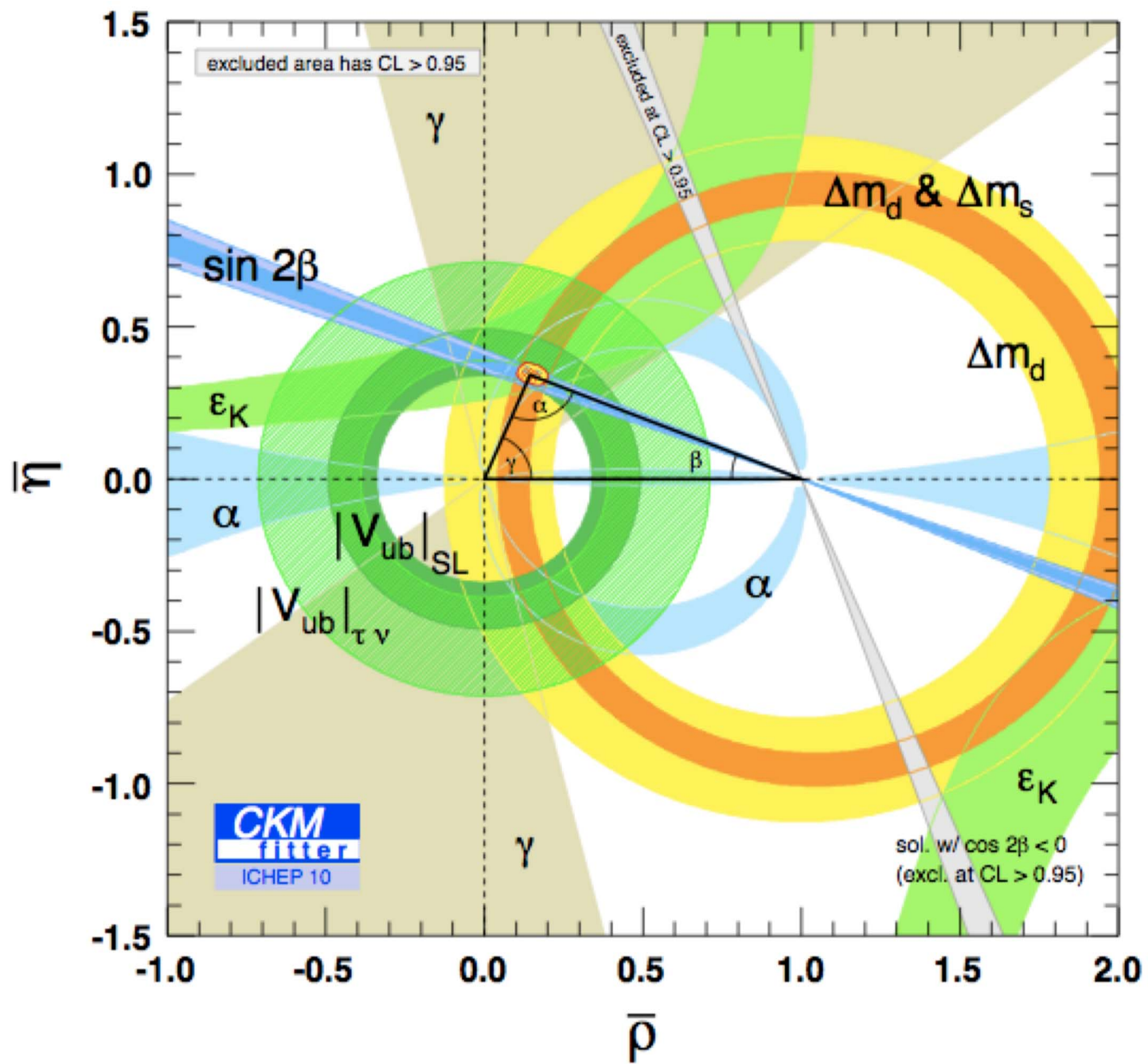
$$V_{cb}^* V_{us} \times \text{Tree-1} + \overbrace{V_{ub}^* V_{cs} \times \text{Tree-2}}^{\text{strong phase}}$$

weak phase γ



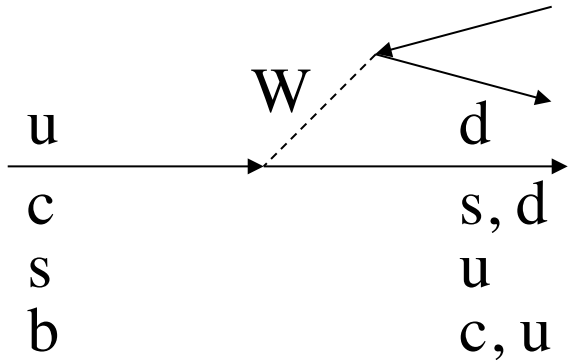




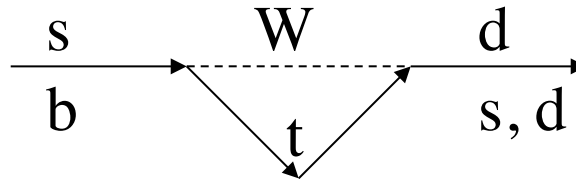


Possible effect of new physics

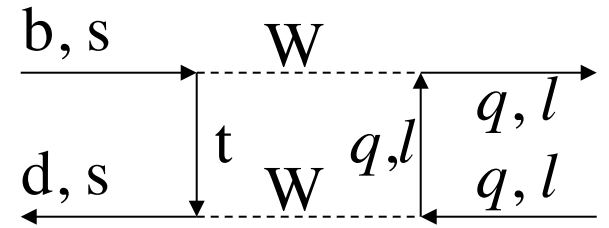
Tree level decays



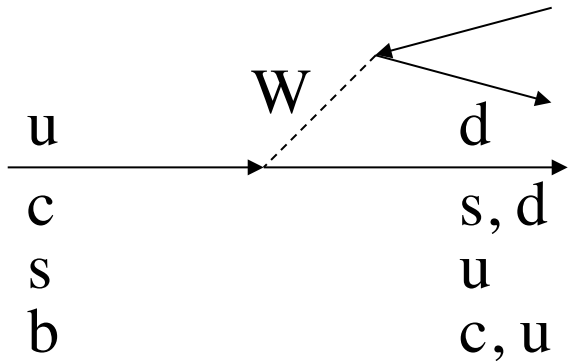
Penguin level decays



Box level decays

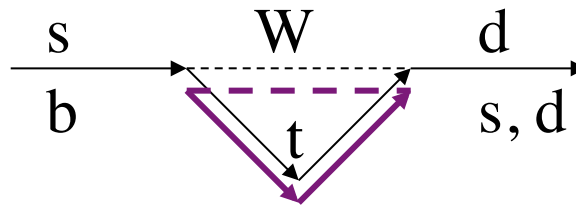


Tree level decays



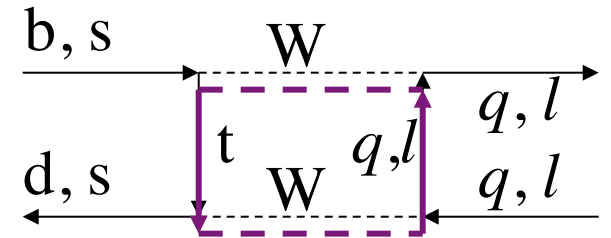
unchanged

Penguin level decays



+ new particles

Box level decays



Phases

Lorentz structure of the amplitude modified

Absolute values

New phases in new physics loops...

- (ρ, η) determination with loop diagrams
 $\Delta m_d + \text{CPV}(B \rightarrow J/\psi K_S, \varepsilon_K)$

and

(ρ, η) determination with tree diagrams
 $\text{CPV}(B \rightarrow DK)$ and $\Gamma(b \rightarrow u/\nu)$

would be different

Better low energy QCD theory for $B_B f_B^2, B_K, \dots$
and

More statistics for $\text{CPV}(B \rightarrow DK)$ γ determination
 \Rightarrow LHCb experiments

- $\text{CPV}(B_s \rightarrow J/\psi \phi)$ does not agree with the CKM fit
- $\text{CPV}(B_s \rightarrow J/\psi \phi)$ and $\text{CPV}(B_s \rightarrow \phi \phi)$ are different

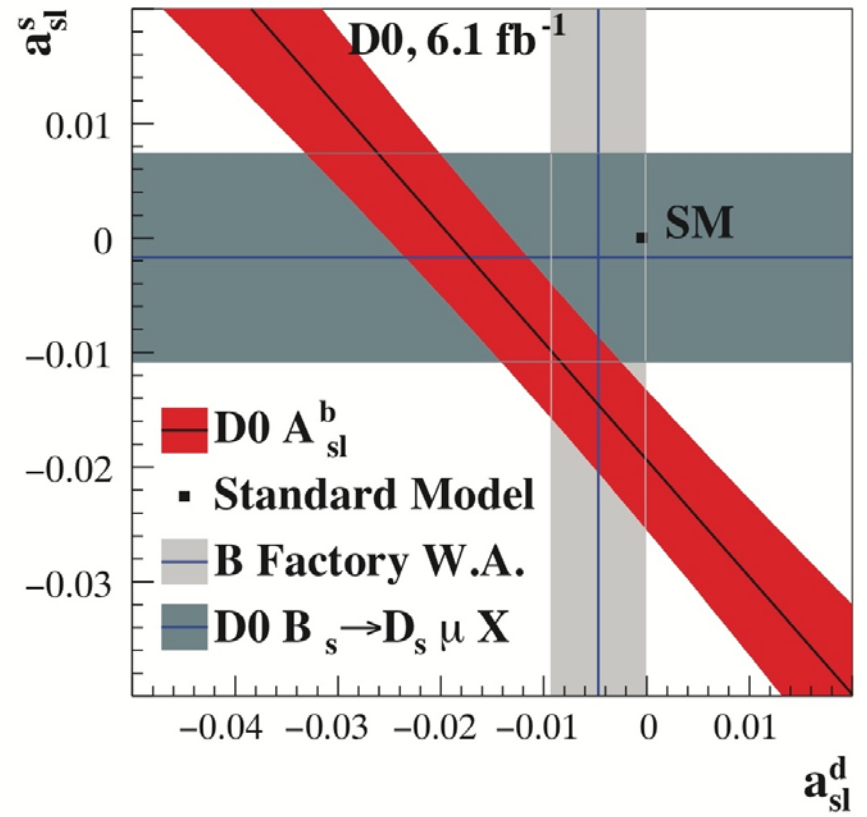
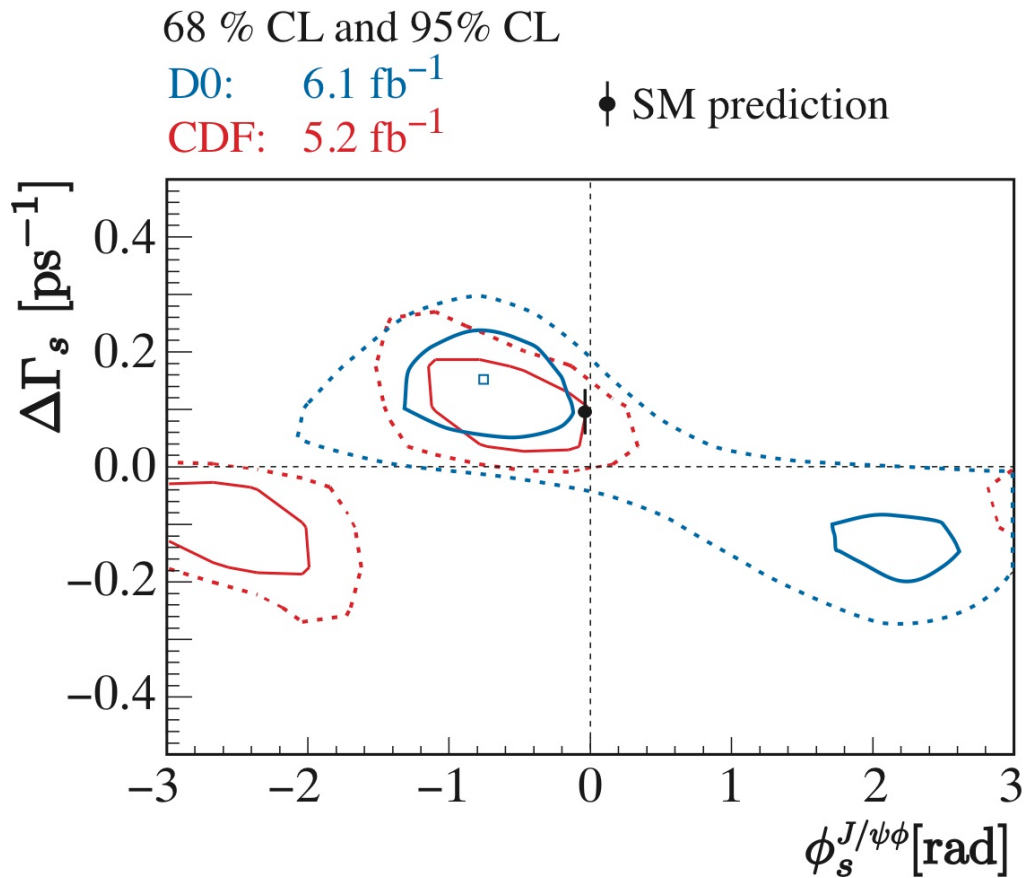
New Lorentz structure in new physics loops...

- Angular distribution of the decay particles in $B_d \rightarrow K^{*0} \mu^+ \mu^-$ different from the SM: e.g. A_{FS} of $\mu^+ \mu^-$
- $CPV(B_s \rightarrow \phi \gamma)$

And enhanced rare decays

- $\text{Br}(\text{B}_s \rightarrow \mu^+ \mu^-), \text{Br}(\text{B}_d \rightarrow \mu^+ \mu^-)$

Are they really signs of new physics?



LHCb will answer.