Physics of CP Violation (III)

Special Lecture at
Tsinghua Univiersity, Beijing, China
21-25 March 2011

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Experimental observation of CP violation in the decay amplitudes

From: $\mathrm{Re}\,\varepsilon \neq 0$ ~~CP~~ in oscillations

From: $\mathrm{Im}\,\eta_{+-} \neq 0$ ~~CP~~ in interplay between decay and oscillations

From: $\mathrm{Re}\,\eta_{+-} \neq 0$ ~~CP~~ in decay amplitudes is inconclusive...

$\rightarrow$dedicated experiments to look for this...

Comparing η_{+-} and η_{00}.

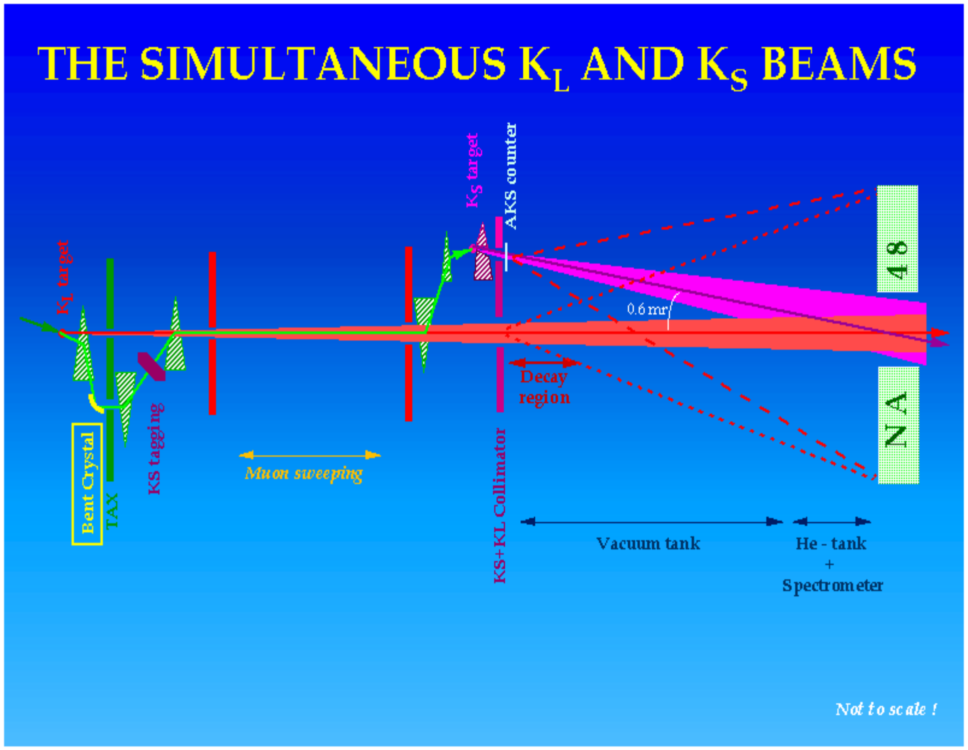
$$\left|\eta_{+-}\right|^2 = \frac{\left|A\left(\mathrm{K_L} \to \pi^+\pi^-\right)\right|^2}{\left|A\left(\mathrm{K_S} \to \pi^+\pi^-\right)\right|^2} = \frac{N_{\mathrm{S}}^{+-}}{N_{\mathrm{L}}^{+-}} \frac{N\left(\mathrm{K_L} \to \pi^+\pi^-\right)}{N\left(\mathrm{K_S} \to \pi^+\pi^-\right)} = \varepsilon + \varepsilon'$$

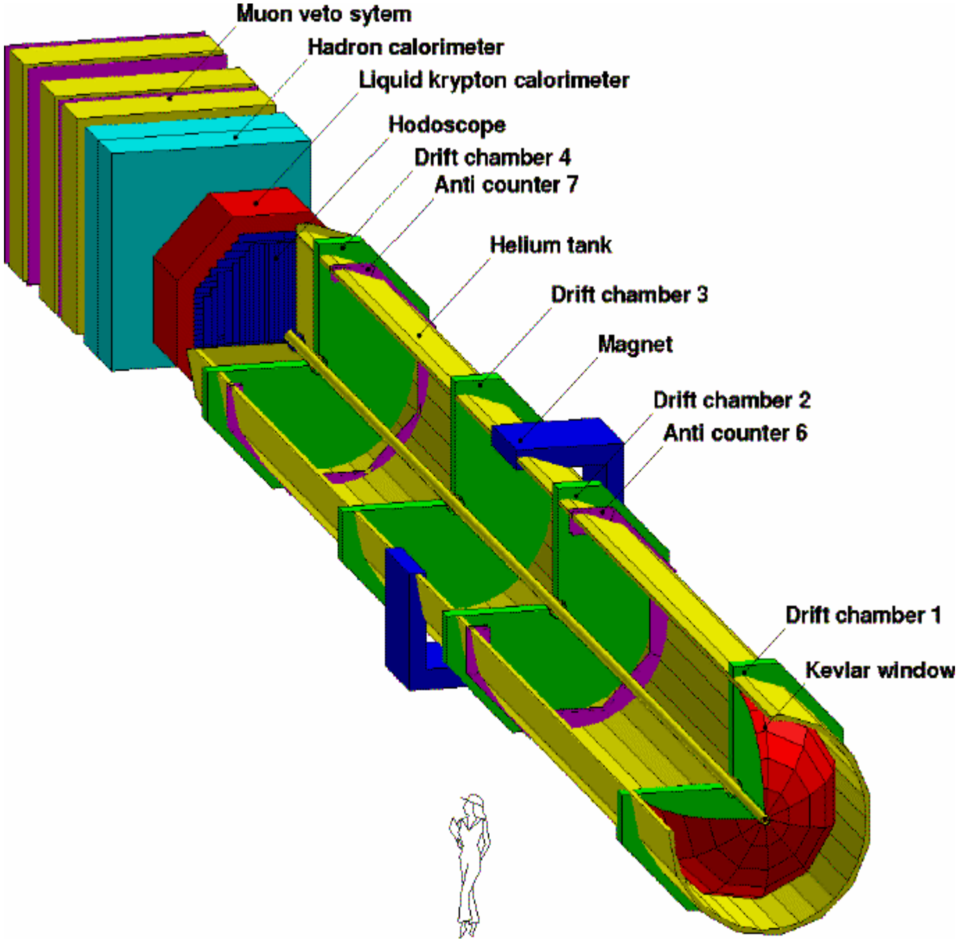
$$\left|\eta_{00}\right|^2 = \frac{\left|A\left(\mathrm{K_L} \to \pi^0\pi^0\right)\right|^2}{\left|A\left(\mathrm{K_S} \to \pi^0\pi^0\right)\right|^2} = \frac{N_{\mathrm{S}}^{00}}{N_{\mathrm{L}}^{00}} \frac{N\left(\mathrm{K_L} \to \pi^0\pi^0\right)}{N\left(\mathrm{K_S} \to \pi^0\pi^0\right)} = \varepsilon - 2\varepsilon'$$

$$\frac{\left|\eta_{00}\right|^2}{\left|\eta_{+-}\right|^2} = 1 - 6\,\mathrm{Re}\frac{\varepsilon'}{\varepsilon}$$

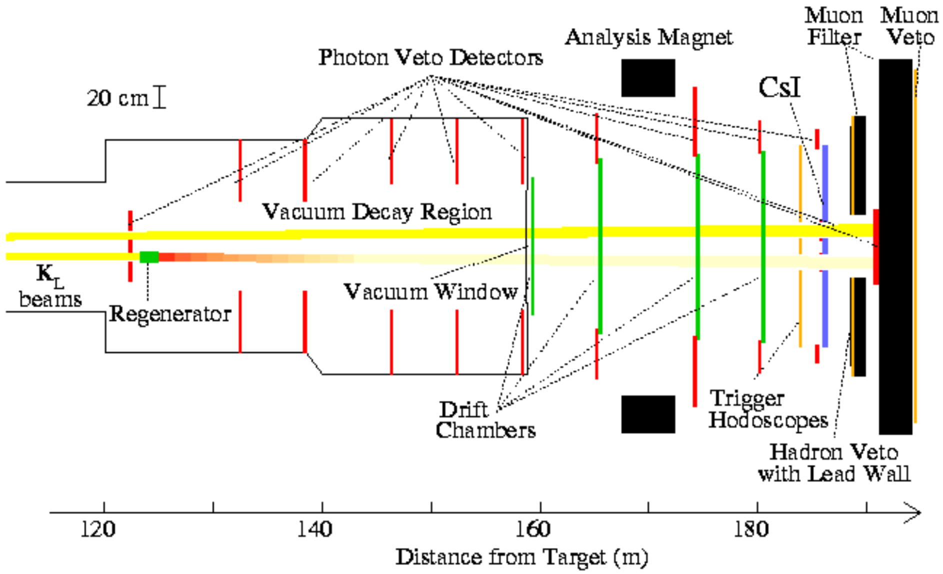
$$= \frac{N_{\mathrm{S}}^{00} N_{\mathrm{L}}^{+-}}{N_{\mathrm{L}}^{00} N_{\mathrm{S}}^{+-}} \frac{N\left(\mathrm{K_L} \to \pi^0\pi^0\right) N\left(\mathrm{K_S} \to \pi^+\pi^-\right)}{N\left(\mathrm{K_S} \to \pi^0\pi^0\right) N\left(\mathrm{K_L} \to \pi^+\pi^-\right)}$$

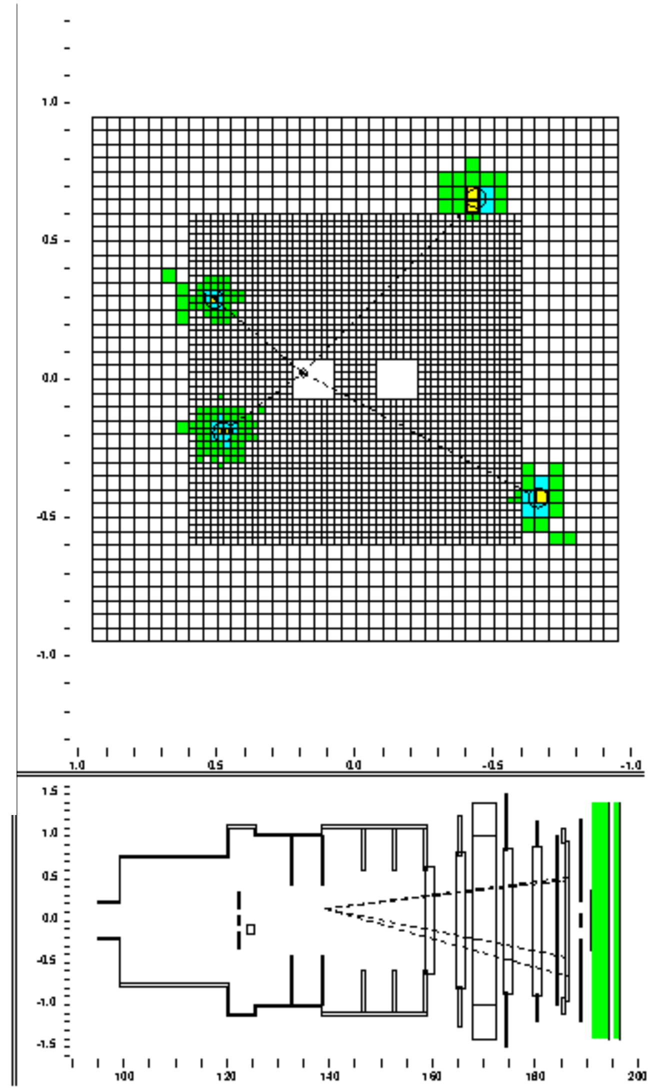
NA48

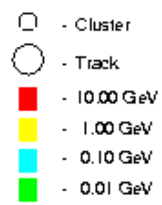

THE SIMULTANEOUS K_L AND K_S BEAMS
K_L target
Bent Crystal
TAX
KS tagging
Muon sweeping
K_S target
AKS counter
KS+KL Collimator
Decay region
0.6 mr
Vacuum tank
He - tank
+
Spectrometer
N A
4 8
Not to scale !


Muon veto sytem
Hadron calorimeter
Liquid krypton calorimeter
Hodoscope
Drift chamber 4
Anti counter 7
Helium tank
Drift chamber 3
Magnet
Drift chamber 2
Anti counter 6
Drift chamber 1
Kevlar window

KTeV


Photon Veto Detectors
Analysis Magnet
Muon Filter
Muon Veto
20 cm
CsI
Vacuum Decay Region
K$_L$ beams
Regenerator
Vacuum Window
Drift Chambers
Trigger Hodoscopes
Hadron Veto with Lead Wall
120
140
160
180
Distance from Target (m)



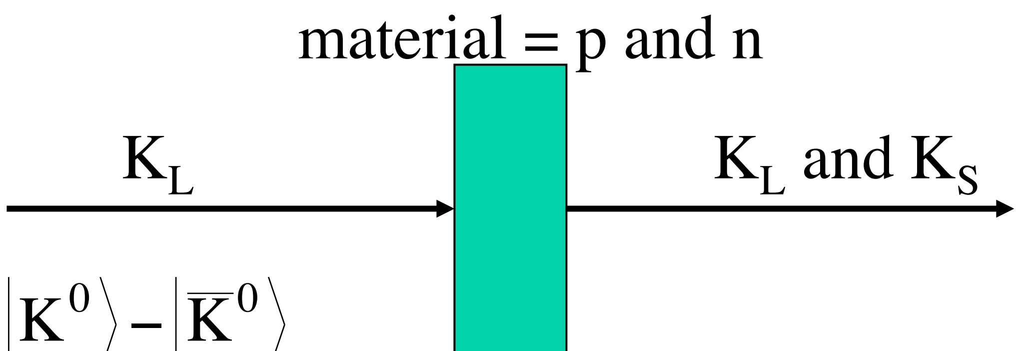

- Cluster
- Track
- 10.00 GeV
- 1.00 GeV
- 0.10 GeV
- 0.01 GeV

Regeneration

$$\sigma_{\overline{K}n}, \sigma_{\overline{K}p} > \sigma_{Kn}, \sigma_{Kp}$$

$$K^0 = (d\bar{s})$$
$$\overline{K}^0 = (\bar{d}s)$$

$$p = (uud),\ n = (udd)$$



$$\left|K^0\right\rangle - \left|\overline{K}^0\right\rangle$$

$$\left|K^0\right\rangle - \alpha\left|\overline{K}^0\right\rangle$$
$$= \frac{1-\alpha}{2}\left(\left|K^0\right\rangle + \left|\overline{K}^0\right\rangle\right) + \frac{1+\alpha}{2}\left(\left|K^0\right\rangle - \left|\overline{K}^0\right\rangle\right)$$
$$= \frac{1-\alpha}{2}\left|K_S\right\rangle + \frac{1+\alpha}{2}\left|K_L\right\rangle$$

$$\alpha = \sqrt{\frac{\sigma_{\overline{K}N}}{\sigma_{KN}}} \neq 1$$

Measure

$\pi^+\pi^-$ and $\pi^0\pi^0$ at the same time: $N_S^{00} = N_S^{+-}, N_L^{00} = N_L^{+-}$

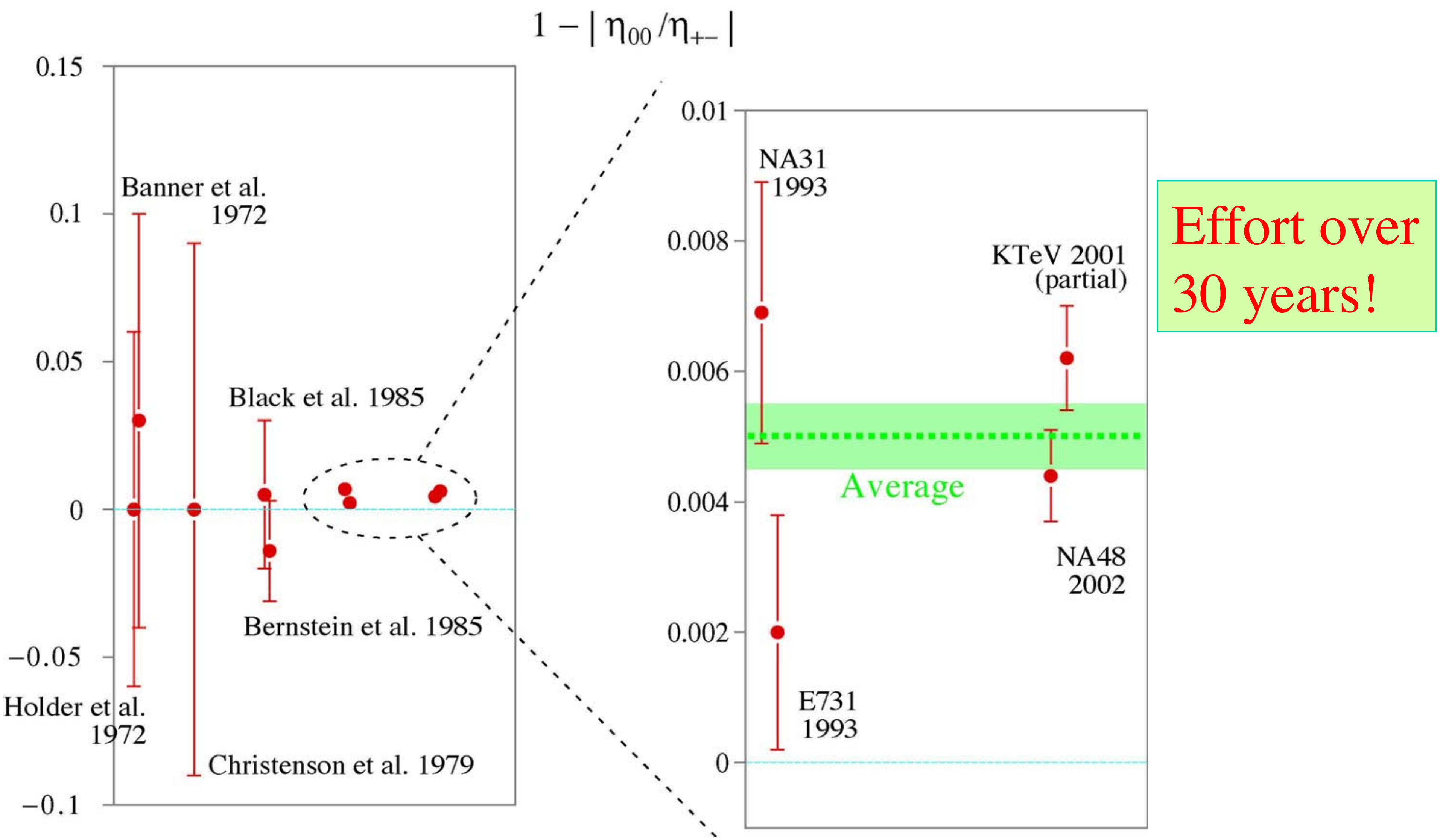
NA31, NA48

K_L is regenerated from K_S: $N_L^{00} = rN_S^{00}, N_L^{+-} = rN_S^{+-}$

E731, KTeV

No normalization is required,

but efficiencies, acceptances etc. have to be corrected…



$$0.0050 \pm 0.0005 \neq 0 \;\Rightarrow\; |\eta_{+-}| \neq |\eta_{00}|$$

Note; $3\ \mathrm{Re}\ \dfrac{\varepsilon'}{\varepsilon} = |\eta_{+-}/\eta_{00}| - 1$, i.e. $\mathrm{Re}\ \dfrac{\varepsilon'}{\varepsilon} \neq 0$

Kaon interferometer

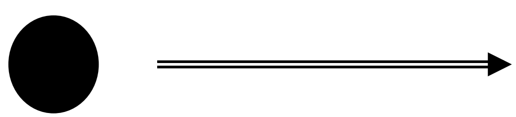
e^+e^- (@1GeV) $\rightarrow$ virtual $\gamma \rightarrow \phi(1020) \rightarrow K\,\bar{K}$ (charged or neutral)

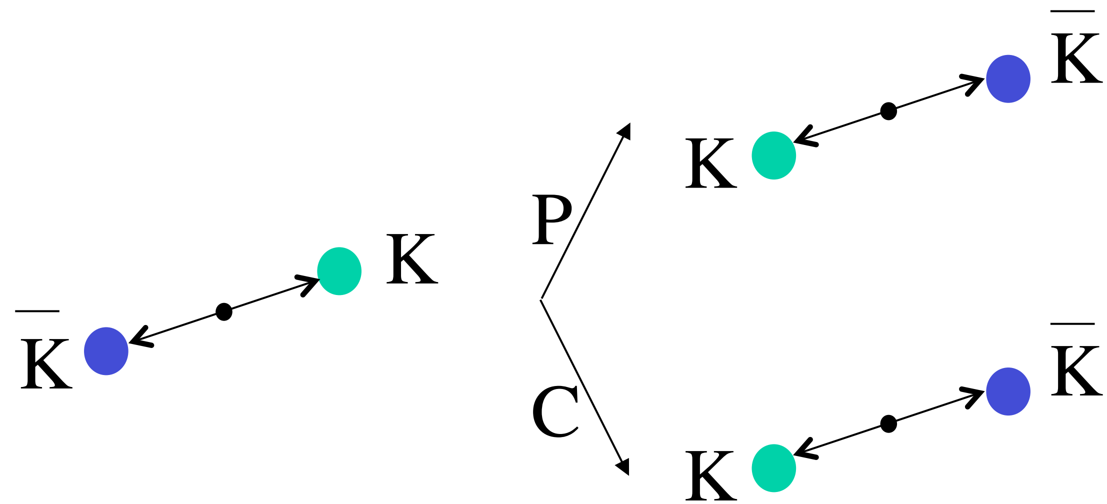
all due to electromagnetic interactions

$\phi(1020) \rightarrow K\,\bar{K}$

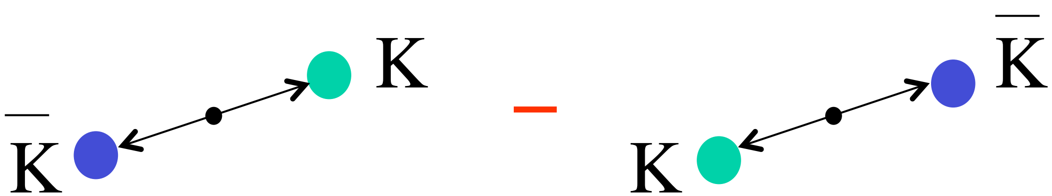
$C = -1$

$P = -1$





$K\bar{K}$ is a quantum superposition of

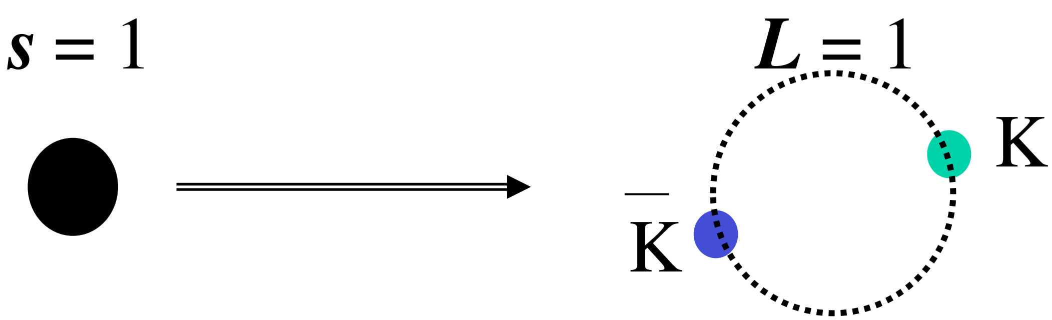


$C = -1$

$P = -1$

Or...

$$\phi(1020) \rightarrow \mathrm{K}\,\overline{\mathrm{K}}$$



$$\left|\mathrm{K}\overline{\mathrm{K}}_{L=1}\right\rangle = \int d\Omega \sum_{m=-1}^{1} Y_{1m}(\vec{e}\,')\left|\mathrm{K}_{\vec{e}\,'}\right\rangle\left|\overline{\mathrm{K}}_{-\vec{e}\,'}\right\rangle$$

Projection to a momentum state

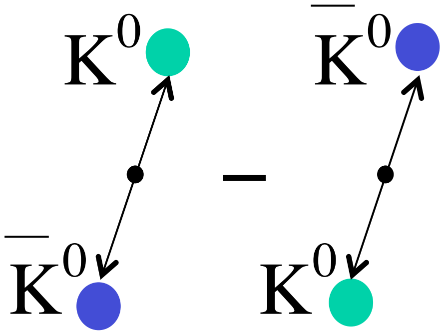
$$\sum_{m=-1}^{1}\left\{Y_{1m}(\vec{e})\left|\mathrm{K}_{\vec{e}}\right\rangle\left|\overline{\mathrm{K}}_{-\vec{e}}\right\rangle + Y_{1m}(-\vec{e})\left|\mathrm{K}_{-\vec{e}}\right\rangle\left|\overline{\mathrm{K}}_{\vec{e}}\right\rangle\right\}$$

$$= \sum_{m=-1}^{1} Y_{1m}(\vec{e})\left\{\left|\mathrm{K}_{\vec{e}}\right\rangle\left|\overline{\mathrm{K}}_{-\vec{e}}\right\rangle - \left|\mathrm{K}_{-\vec{e}}\right\rangle\left|\overline{\mathrm{K}}_{\vec{e}}\right\rangle\right\}$$

For neutral kaons, they oscillates, but....

$$\left|K\overline{K}_{L=1}(t)\right\rangle = \int d\Omega \sum_{m=-1}^{1} Y_{1m}(\vec{e}\,')\left|K_{\vec{e}\,'}(t)\right\rangle\left|\overline{K}_{-\vec{e}\,'}(t)\right\rangle$$

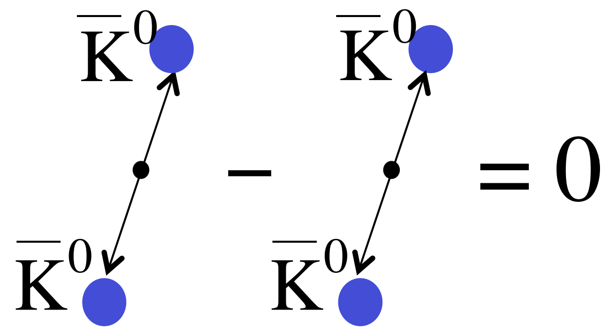
$t = 0$

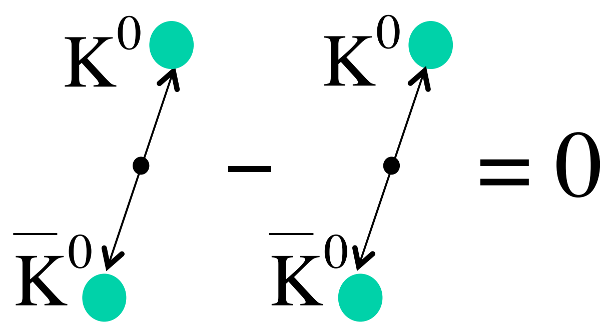


$K^0 \rightarrow \overline{K}^0$

$\overline{K}^0 \rightarrow K^0$

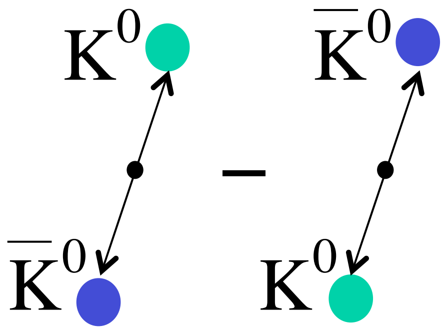
at time t





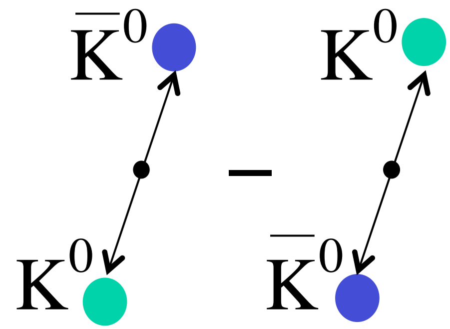
Only the allowed oscillations are

$t = 0$



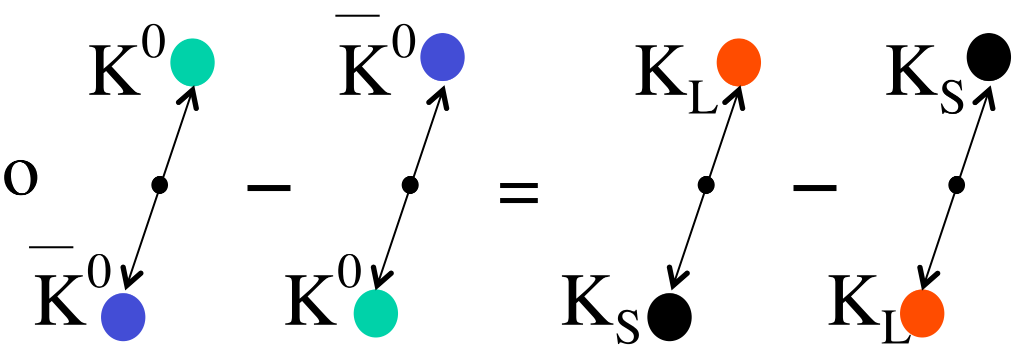
$$\left\{ \begin{array}{c} K^0 \to \bar{K}^0 \\ \bar{K}^0 \to K^0 \end{array} \right\}$$

sometime later...



One kaon seems to know what the other does!!

Also



but no $K_S K_S$ or $K_L K_L$

$$\left|K^0\overline{K}^0{}_{L=1}(t)\right\rangle \propto \left|K_{S\,\vec{p}}\right\rangle e^{-i\lambda_S t}\left|K_{L\,-\vec{p}}\right\rangle e^{-i\lambda_L t} - \left|K_{L\,\vec{p}}\right\rangle e^{-i\lambda_L t}\left|K_{S\,-\vec{p}}\right\rangle e^{-i\lambda_S t}$$

It cannot decay into a same final state, e.g. $\pi^+\pi^-$ at the **same time.**

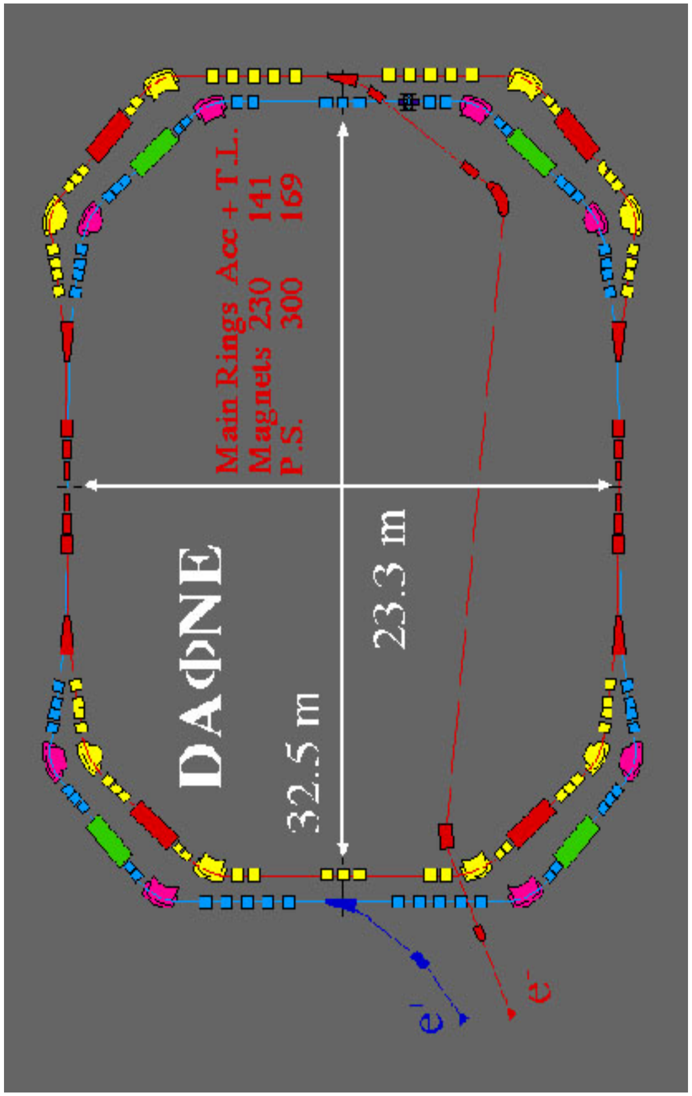
$$\left\langle\left(\pi^+\pi^-\right)_{\vec{p}}\left(\pi^+\pi^-\right)_{-\vec{p}}\left|H_W\right|K^0\overline{K}^0{}_{L=1}(t)\right\rangle \propto A_{S\,\vec{p}}^{+-}A_{S\,-\vec{p}}^{+-}\left(\eta_{+-}-\eta_{+-}\right)$$

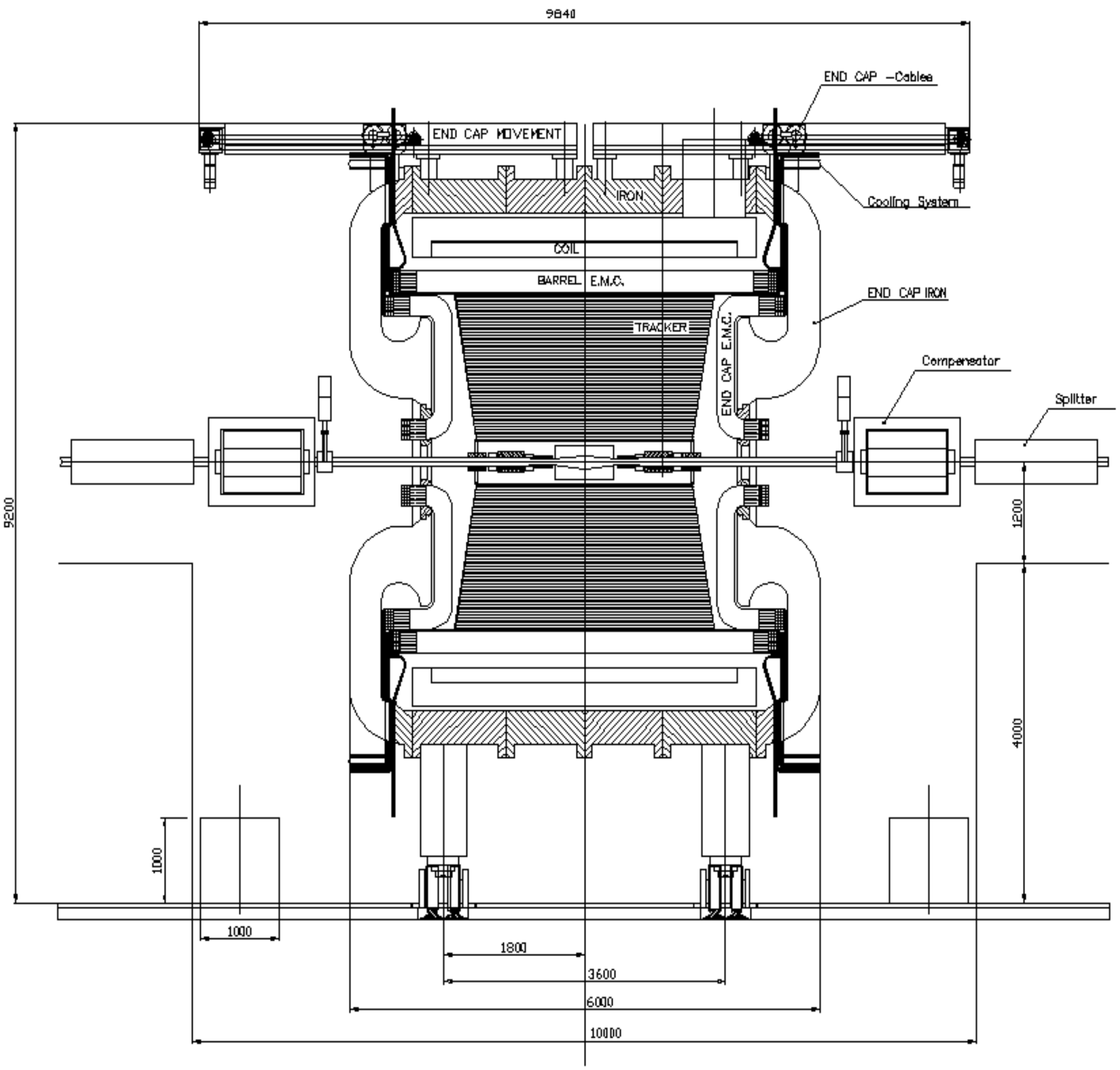
$$= 0$$

It can decay into $\pi^+\pi^-$ and $\pi^0\pi^0$ **at the same time only if $\eta_{+-} \neq \eta_{00}$.**

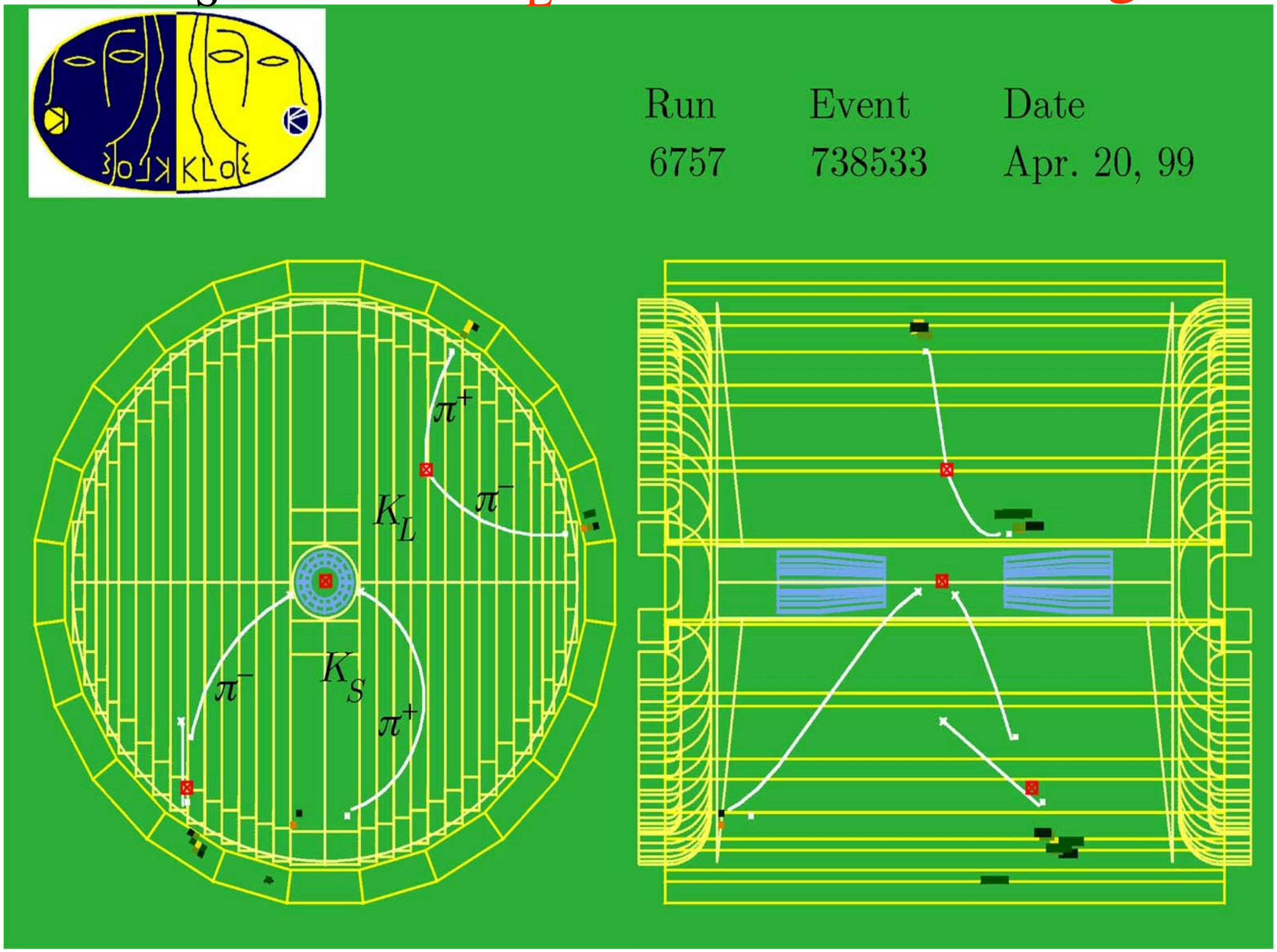
$$\left\langle\left(\pi^+\pi^-\right)_{\vec{p}}\left(\pi^0\pi^0\right)_{-\vec{p}}\left|H_W\right|K^0\overline{K}^0{}_{L=1}(t)\right\rangle \propto A_{S\,\vec{p}}^{+-}A_{S\,-\vec{p}}^{00}\left(\eta_{+-}-\eta_{00}\right)$$

KLOE experiment at DAFNE storage ring (@Frascati)





$K_S \to \pi^+\pi^-$, $K_L \to \pi^+\pi^-$ CP violating decays!!



An ideal way to produce K_S beam

1) Identify K_L decay with the decay time.

2) opposite side is K_S.

To summarise

CP violation can be generated through three interference mechanisms:

Interfering processes	**~~CP~~ phase**	**CP phase**
Dispersive (M_{12}) and absorptive (Γ_{12}) paths in P-$\overline{\text{P}}$ oscillations	Weak interaction phases	Dispersion vs absorption (*i*)

CP violation in the oscillation

Different decay amplitudes	Weak interaction phases	Strong interaction phases

CP violation in the decay amplitudes

Decay with and without oscillations	Weak interaction phases	Time evolution functions

CP violation in the interplay between the decay and oscillation

All three mechanisms seen in the neutral kaon system:
Re ε, Re ε', Im η_{+-}, Im η_{00}

Standard Model and CP violation

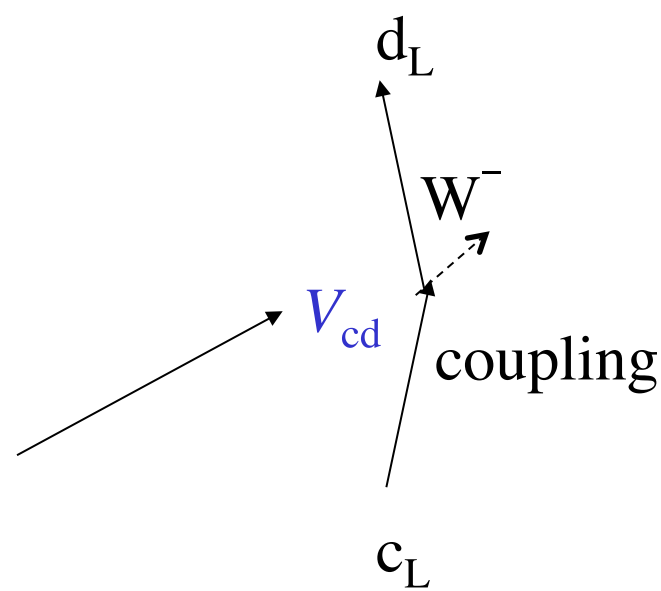
Strong interaction — gluons

Electromagnetic interaction — photons

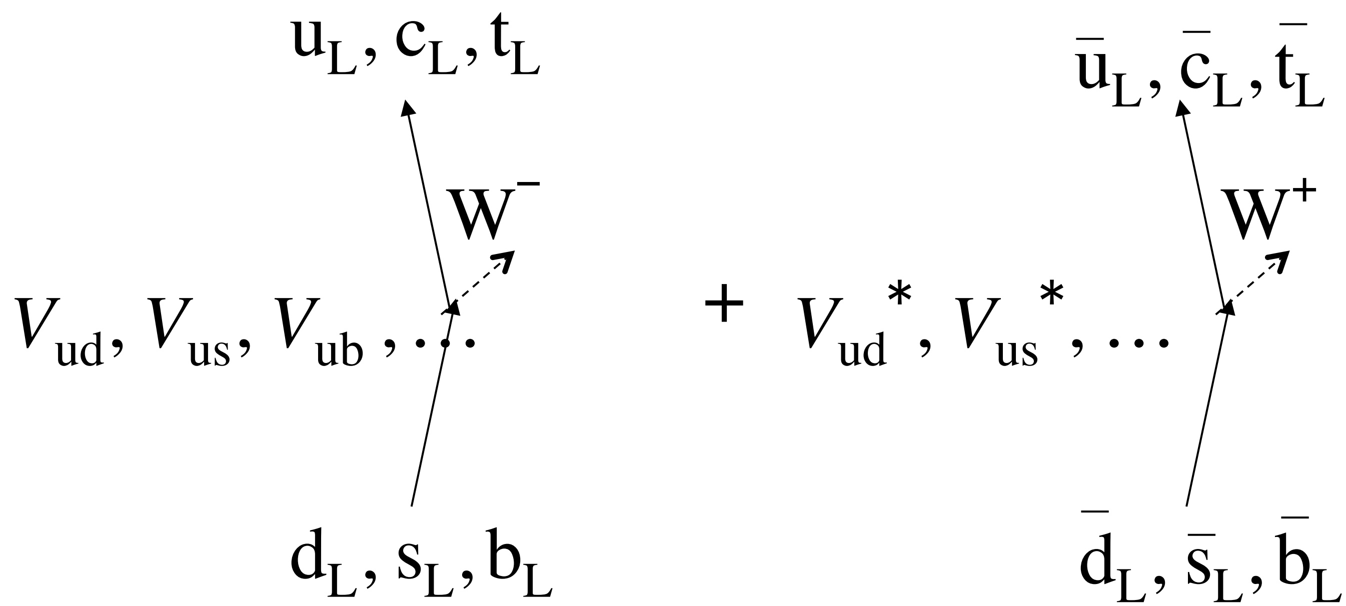
Weak interaction — neutral current; charged current: $W^{\pm}$

Up type quark spinor field $Q = 2/3$	Down type quark spinor field $Q = -1/3$
$\boldsymbol{U} = \begin{pmatrix} u \\ c \\ t \end{pmatrix}$	$\boldsymbol{D} = \begin{pmatrix} d \\ s \\ b \end{pmatrix}$

example



there are 3×3 = 9 V's



$$L \propto V_{ij}\, \bar{U}_i\, \gamma^\mu(1-\gamma_5)\, D_j\, W_\mu^\dagger + V_{ij}^*\, \bar{D}_i\, \gamma^\mu(1-\gamma_5)\, U_j\, W_\mu$$

CP conjugation

$$L_{CP} \propto V_{ij}\, \bar{D}_i\, \gamma^\mu(1-\gamma_5)\, U_j\, W_\mu + V_{ij}^*\, \bar{U}_i\, \gamma^\mu(1-\gamma_5)\, D_j\, W_\mu^\dagger$$

If $V_{ij}^* = V_{ij} \rightarrow L = L_{CP}$: i.e. CP conservation

Let us look at now: $V_{ij}\,\overline{U}_i\,\gamma^\mu(1-\gamma_5)\,D_j$

One family

$\boldsymbol{V}$ 1 free phase
1 free modula $= |V|\,e^{i\phi}$

$$|V|\,e^{i\phi}\,\overline{u}\,\gamma^\mu(1-\gamma_5)\,d \longrightarrow |V|\,\overline{u}\,\gamma^\mu(1-\gamma_5)\,d$$

Changing u quark phase: $u \rightarrow u\,e^{i\phi}$

Unitarity: $\boldsymbol{V}^\dagger V = \boldsymbol{V}\boldsymbol{V}^\dagger = \boldsymbol{E}$ (one constraint)

$$|V|^2 = 1$$

0 free phase
0 free modula

NO ~~C~~P

Two families

$$V = \begin{pmatrix} V_{ud} & V_{us} \\ V_{cd} & V_{cs} \end{pmatrix}$$ 4 free phase
4 free moduli (or rotation angles)

1) The phase of V_{ij} can be absorbed by adjusting the phase differences between i- and j- quark

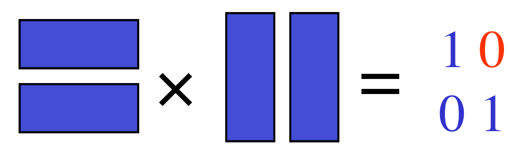
4 quarks = 3 phase differences

4 − 3 = 1 phase left

2) Unitarity $\boldsymbol{V}^{\dagger}\boldsymbol{V} = \boldsymbol{V}\boldsymbol{V}^{\dagger} = \boldsymbol{E}$: four constraints:

1 off-diagonal constraint for the phase

1 − 1 = 0 phase left



three constraint for the rest

4 − 3 = 1 rotation angle left

V is real, i.e. no $\not{CP}$.

Explicit demonstration

$$|V_{ud}| e^{i\phi_{ud}}\, \bar{u}\, \gamma^{\mu}(1-\gamma_5)\, d + |V_{us}| e^{i\phi_{us}}\, \bar{u}\, \gamma^{\mu}(1-\gamma_5)\, s$$
$$+ |V_{cd}| e^{i\phi_{cd}}\, \bar{c}\, \gamma^{\mu}(1-\gamma_5)\, d + |V_{cs}| e^{i\phi_{cs}}\, \bar{c}\, \gamma^{\mu}(1-\gamma_5)\, s$$

$$u \rightarrow u\, e^{i\phi_{ud}}$$

$$|V_{ud}|\, \bar{u}\, \gamma^{\mu}(1-\gamma_5)\, d + |V_{us}| e^{i(\phi_{us} - \phi_{ud})}\, \bar{u}\, \gamma^{\mu}(1-\gamma_5)\, s$$
$$+ |V_{cd}| e^{i\phi_{cd}}\, \bar{c}\, \gamma^{\mu}(1-\gamma_5)\, d + |V_{cs}| e^{i\phi_{cs}}\, \bar{c}\, \gamma^{\mu}(1-\gamma_5)\, s$$

$$s \rightarrow s\, e^{-i(\phi_{us} - \phi_{ud})}, \quad c \rightarrow c\, e^{i(\phi_{cs} - \phi_{us} + \phi_{ud})}$$

$$|V_{ud}|\, \bar{u}\, \gamma^{\mu}(1-\gamma_5)\, d + |V_{us}|\, \bar{u}\, \gamma^{\mu}(1-\gamma_5)\, s$$
$$+ |V_{cd}| e^{i\delta}\, \bar{c}\, \gamma^{\mu}(1-\gamma_5)\, d + |V_{cs}|\, \bar{c}\, \gamma^{\mu}(1-\gamma_5)\, s$$

Out of four quark, three quark phases can be adjusted:

4 free phase → 1 free phase

Unitarity: $\boldsymbol{V}^{\dagger}\boldsymbol{V} = \boldsymbol{V}\boldsymbol{V}^{\dagger} = \boldsymbol{E}$ (4 constraints) $\begin{pmatrix} V_{ud}^{*} & V_{cd}^{*} \\ V_{us}^{*} & V_{cs}^{*} \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} \\ V_{cd} & V_{cs} \end{pmatrix}$

$$V_{ud}^{*} V_{us} + V_{cd}^{*} V_{cs} = 0 \rightarrow |V_{ud}|\,|V_{us}| + |V_{cd}|\,|V_{cs}|\, e^{-i\delta} = 0$$

$\delta = \pi$:0 free phase

$$|V_{ud}|\,|V_{cd}| - |V_{us}|\,|V_{cs}| = 0$$

$$|V_{ud}|^2 + |V_{cd}|^2 = 1,\ |V_{us}|^2 + |V_{cs}|^2 = 1$$

1 free modula or rotation angle

$$|V_{11}| = \cos\theta,\ |V_{22}| = \cos\theta,\ |V_{12}| = \sin\theta,\ |V_{21}| = -\sin\theta$$

$$\boldsymbol{V} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

One rotation angle without phase: → **NO ~~CP~~**

(Cabibbo angle)

Three families

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

9 free phase
9 free moduli (or rotation angles)

Out of six quark, five quark phases can be adjusted:

9 free phase → 4 free phase

1 0 0
0 1 0
0 0 1

Out of nine unitarity constraints, three are for the phases

4 free phase → **1 free phase**

The rest (six) are for the rotation angles

9 free rotation angles → 3 free rotation angles

Three rotation angles with one phase:

→ **$\not{C}\not{P}$ can be generated**

CKM matrix and Wolfenstein's parameters

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{\mathrm{ud}} & V_{\mathrm{us}} & V_{\mathrm{ub}} \\ V_{\mathrm{cd}} & V_{\mathrm{cs}} & V_{\mathrm{cb}} \\ V_{\mathrm{td}} & V_{\mathrm{ts}} & V_{\mathrm{tb}} \end{pmatrix}$$

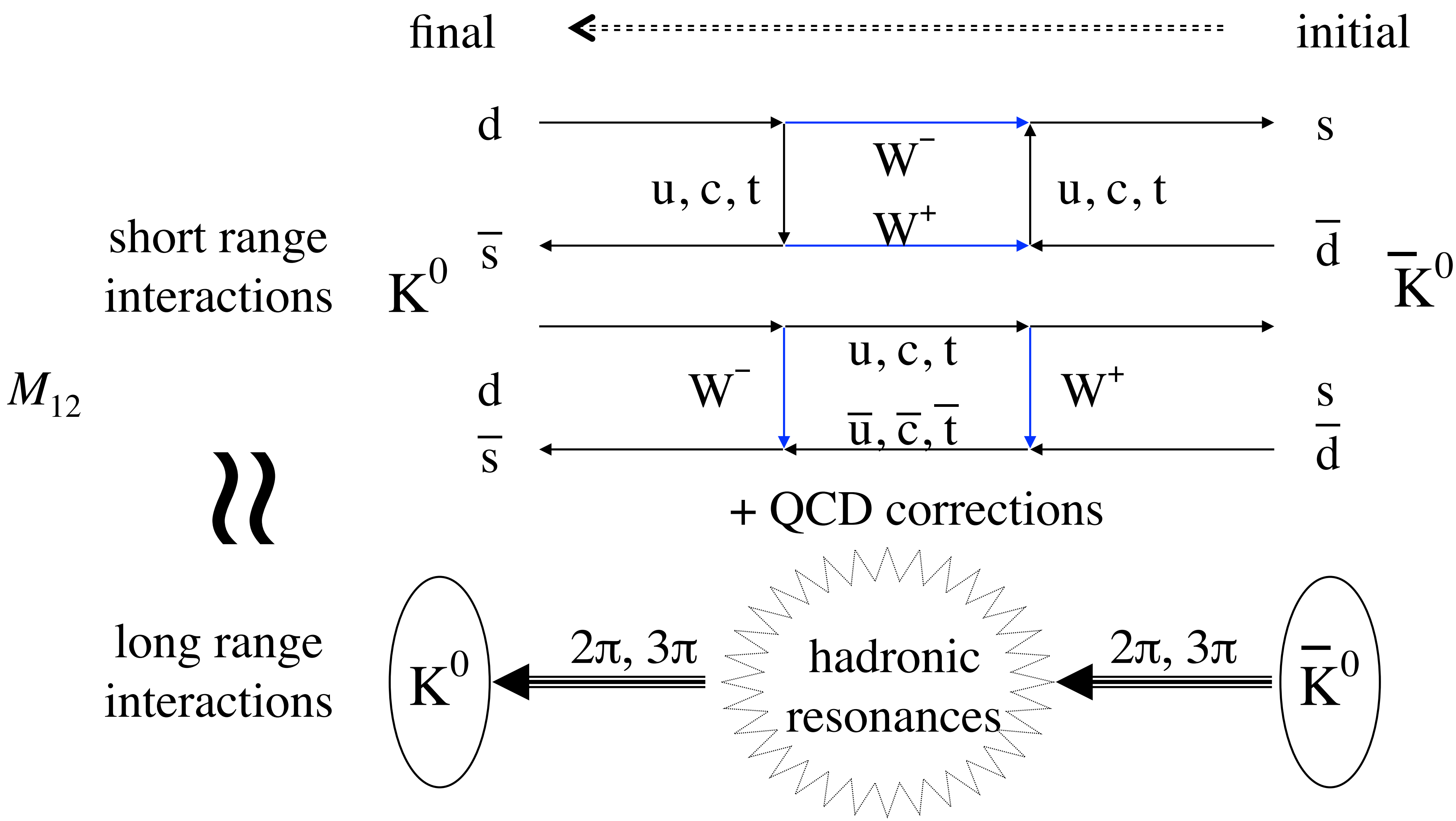
$$\approx \begin{pmatrix} 1-\dfrac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda - iA^2\lambda^5\eta & 1-\dfrac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\hat{\rho}-i\hat{\eta}) & -A\lambda^2 - iA\lambda^4\eta & 1 \end{pmatrix}$$

$$\hat{\rho} = \rho\left(1-\frac{\lambda^2}{2}\right),\ \hat{\eta} = \rho\left(1-\frac{\eta^2}{2}\right)$$

$A \sim 1, \lambda \sim 0.22$, $\rho \neq 0$ but $\eta \neq 0$???

Electroweak theory with 3 families can naturally accommodate CP violation in the charged current induced interactions through the complex Cabibbo-Kobayashi-Maskawa quark mixing matrix $\boldsymbol{V}$, with 4 parameters.

Standard Model for the kaon system



Large uncertainties in the theoretical calculations of $|M_{12}|$

Short distance part

$$H_{\text{effective}}^{\Delta S=2} = \frac{G_F^2 m_W^2}{16\pi^2} \sum_{\substack{i,j= \\ u,c,t}} \lambda_i \lambda_j \left(\langle K^0 | M_{V-A} | \overline{K}^0 \rangle B_{ij} + \langle K^0 | M_{S-P} | \overline{K}^0 \rangle C_{ij} \right)$$

G_F: Fermi constant
m_W: W mass

quark operators

$$M_{V-A} = \left[\bar{d}\gamma_\mu (1-\gamma_5) s\right]\left[\bar{d}\gamma^\mu (1-\gamma_5) s\right]$$

$$M_{S-P} = \left[\bar{d}(1-\gamma_5) s\right]\left[\bar{d}(1-\gamma_5) s\right]$$

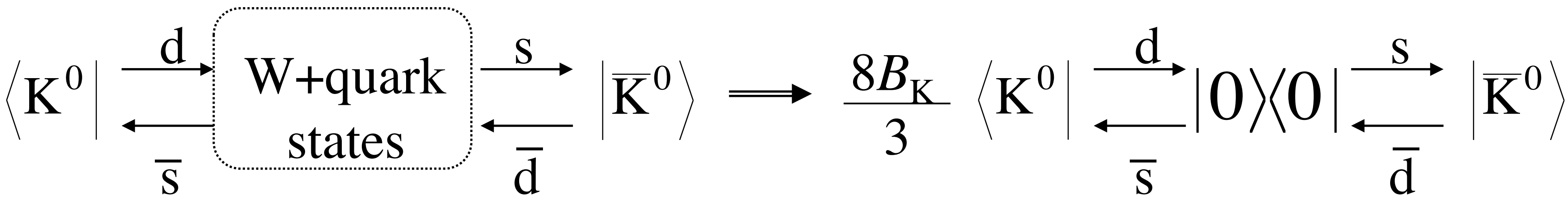
CKM elements $\lambda_i = V_{is} V_{id}^*$

integrating out W $\left.\begin{matrix} B_{ij} \\ C_{ij} \end{matrix}\right\}$ Real part $\rightarrow M_{12}$, Imaginary part $\rightarrow \Gamma_{12}$

$C_{ij} = 0$ for $m_s/m_c \ll 1$

$$\left\langle \mathrm{K}^0 \middle| M_{V-A} \middle| \overline{\mathrm{K}}^0 \right\rangle = \left\langle \mathrm{K}^0 \middle| \left[\bar{d}\gamma_\mu (1-\gamma_5) s\right]\left[\bar{d}\gamma^\mu (1-\gamma_5) s\right] \middle| \overline{\mathrm{K}}^0 \right\rangle$$

$$= \frac{8}{3} B_{\mathrm{K}} \left\langle \mathrm{K}^0 \middle| \bar{d}\gamma_\mu (1-\gamma_5) s \middle| 0 \right\rangle \left\langle 0 \middle| \bar{d}\gamma^\mu (1-\gamma_5) s \middle| \overline{\mathrm{K}}^0 \right\rangle \qquad \text{vacuum insertion}$$



B parameters have to be obtained from QCD calculations.

$$\left\langle \mathrm{K}^0 \middle| \bar{d}\gamma_\mu (1-\gamma_5) s \middle| 0 \right\rangle = \left\langle \mathrm{K}^0 \middle| (CP)^\dagger (CP) \bar{d}\gamma_\mu (1-\gamma_5) s (CP)^\dagger (CP) \middle| 0 \right\rangle$$

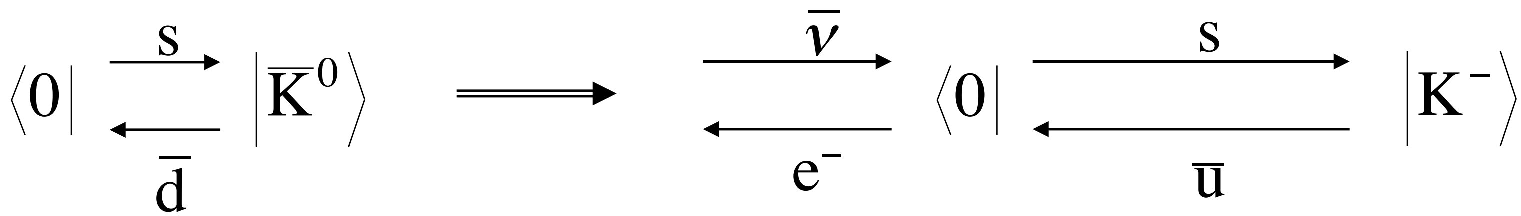
$$= \left\langle \overline{\mathrm{K}}^0 \middle| (CP) \bar{d}\gamma_\mu (1-\gamma_5) s (CP)^\dagger \middle| 0 \right\rangle e^{-i\theta_{CP}}$$

$$= -\left\langle \overline{\mathrm{K}}^0 \middle| \bar{s}\gamma_\mu (1-\gamma_5) d \middle| 0 \right\rangle e^{-i\theta_{CP}}$$

$$\langle K^0 | M_{V-A} | \overline{K}^0 \rangle = -\frac{8}{3} B_K \left| \langle 0 | \bar{d}\gamma^\mu (1-\gamma_5) s | \overline{K}^0 \rangle \right|^2 e^{-i\theta_{CP}}$$

$$= -\frac{4 B_K f_K^2 m_K}{3} e^{-i\theta_{CP}}$$

m_K: Kaon mass
f_K: decay constant



$= f_K / \sqrt{2}$

$$M_{12} = \frac{G_F^2}{12\pi^2} f_K^2 B_K m_K m_W^2 \times \left[\lambda_c^2 \eta_1 S_0(x_c) + \lambda_t^2 \eta_2 S_0(x_t) + \lambda_c \lambda_t \eta_3 S_0(x_c, x_t) \right] e^{i(\pi - \theta_{CP})}$$

$x_c = (m_c/m_W)^2$, $x_t = (m_t/m_W)^2$

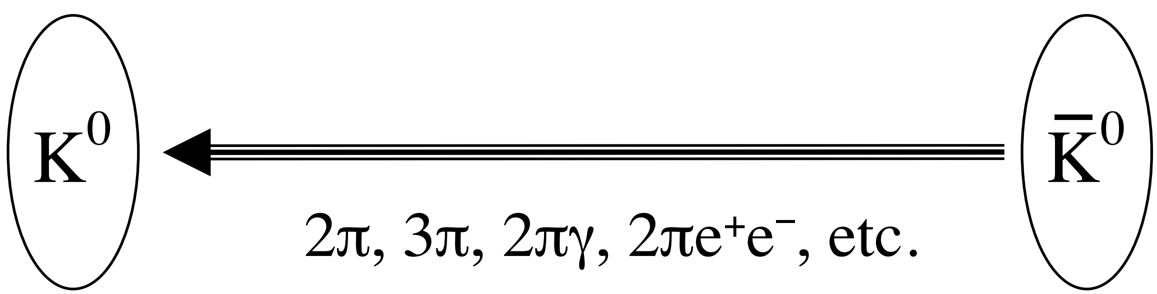
$\eta_1 = 1.38 \pm 0.20$
$\eta_2 = 0.57 \pm 0.01$
$\eta_3 = 0.47 \pm 0.04$
} QCD corrections

If no ~~CP~~ → CKM is real ⟹ $\arg M_{12} = -\theta_{CP} + \pi$ (if $B_K > 0$)

$\lvert\lambda_t^2\rvert = \lvert V_{ts}{}^2 V_{td}{}^{*2}\rvert$	$\approx A^4\lambda^{10}$	$S_0(x_t) \approx 2.5$	6.6×10^{-7}
$\lvert\lambda_c^2\rvert = V_{cs}{}^2 V_{cd}{}^{*2}$	$\approx \lambda^2$	$S_0(x_c) \approx 0.00024$	1.2×10^{-5}
$\lvert\lambda_c\lambda_t\rvert = V_{cs}V_{cd}{}^{*}V_{ts}V_{td}{}^{*}$	$\approx A^2\lambda^6$	$S_0(x_c, x_t) \approx 0.0021$	2.4×10^{-7}
			total contributions

the biggest contribution to $|M_{12}|$ is from the charm loop

Standard Model Γ_{12} for the kaon system



Theoretical calculation on Γ_{12} very difficult.

However, $|M_{12}|$. $|\Gamma_{12}|$ are measured experimentally;

$\Rightarrow$ arg M_{12} can be determined from the short distance interactions.

theoretical short distance calculation

$$\arg M_{12} = \frac{\operatorname{Im} M_{12}}{|M_{12}|} = \frac{2\operatorname{Im} M_{12}}{|m_S - m_L|}$$

experimental value

$$\arg \Gamma_{12} = -\theta_{CP}$$ in the CKM phase convention

$$\varepsilon = \frac{|M_{12}||\Gamma_{12}|\Delta_{M-\Gamma}}{4|M_{12}|^2 + |\Gamma_{12}|^2}\left(1 + i\frac{2|M_{12}|}{|\Gamma_{12}|}\right)$$

$$\Longrightarrow \quad \operatorname{Re}\varepsilon = \frac{\Delta m \Delta\Gamma}{4\Delta m^2 + \Delta\Gamma^2} \arg M_{12}$$ Standard Model

$$\mathrm{Im}\,\lambda_t^2 = \mathrm{Im}\,V_{ts}^{\ 2} V_{td}^{*2} \approx 2A^4\lambda^{10}(1-\hat{\rho})\hat{\eta} \approx 3.7\times10^{-7}(1-\hat{\rho})\hat{\eta}$$

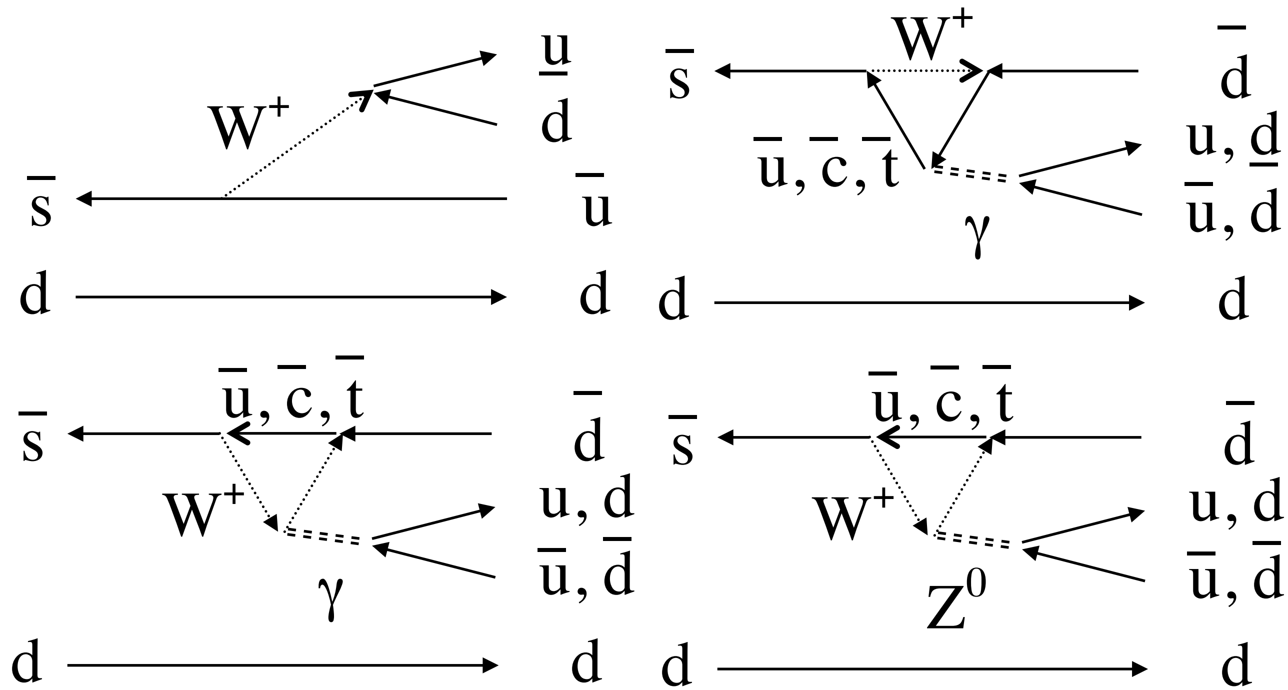
$$\mathrm{Im}\,\lambda_c^2 = \mathrm{Im}\,V_{cs}^{\ 2} V_{cd}^{*2} \approx -2A^2\lambda^6\eta \approx -1.9\times10^{-4}\,\eta$$

$$\mathrm{Im}\,\lambda_c\lambda_t = \mathrm{Im}\,V_{cs}V_{cd}^{\ *}V_{ts}V_{td}^{\ *} \approx 2A^2\lambda^6\hat{\eta} \approx 1.9\times10^{-4}\,\hat{\eta}$$

$\mathrm{S}_0(x_t) \approx 2.5$	$9.3\times10^{-7}(1-\hat{\rho})\hat{\eta}$
$\mathrm{S}_0(x_c) \approx 0.00024$	$-4.5\times10^{-8}\,\eta$
$\mathrm{S}_0(x_c, x_t) \approx 0.0021$	$4.0\times10^{-7}\,\hat{\eta}$

The t-t and t-c box diagrams dominate Im M_{12}

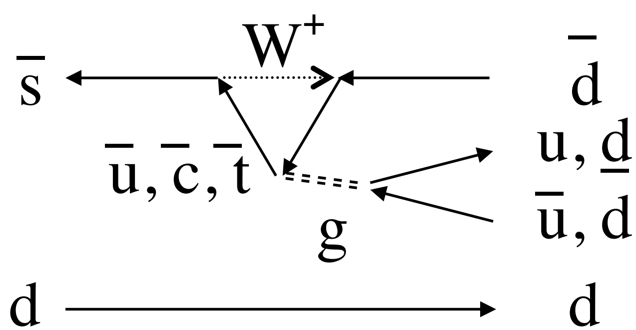
The ε′ in the Standard Model



$I = 0$ and $I = 2$

$V_{ud}V_{us}^*$ tree real

$V_{ud}V_{us}^*$, $V_{cd}V_{cs}^*$, $V_{td}V_{ts}^*$ } γ and Z^0 penguin complex



$I = 0$

$V_{ud}V_{us}^*$, $V_{cd}V_{cs}^*$, $V_{td}V_{ts}^*$ } g penguin complex

Phase difference between $I = 0$ and $I = 2$ weak amplitudes

QCD and non-perturbative effects
very very complicated calculations !!

Current Standard Model prediction

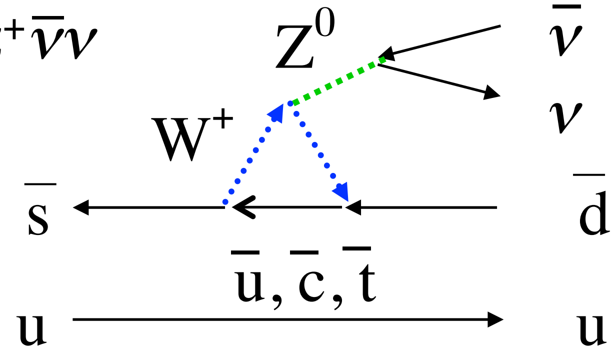
$$\mathrm{Re}\,\frac{\varepsilon'}{\varepsilon} = (10 \pm 2\,^{+9}_{-6} \pm 6)\times 10^{-4} \quad \text{(Pich 04)}$$

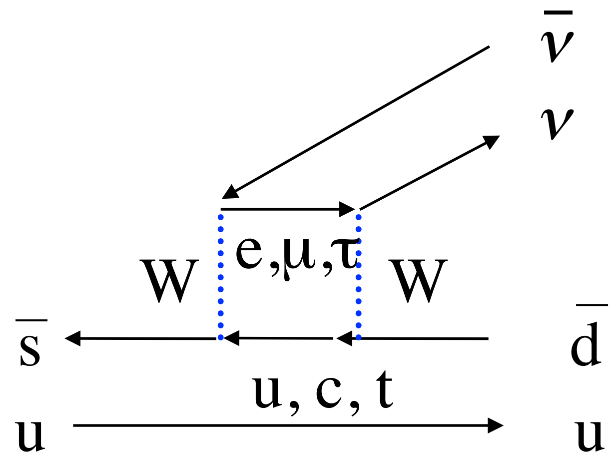
Errors are due to
- renormalization scale
- m_s
- short-long distance QCD matching

1) The Standard Model predictions are compatible with the measurement.
2) Hadronic uncertainties in the theoretical predictions are too large to make a precision test.

Other interesting kaon decays:

$K^+ \rightarrow \pi^+ \bar{\nu}\nu$





$V_{td}V_{ts}^*$
$V_{cd}V_{cs}^*$

$$A_{K\rightarrow\pi\upsilon\upsilon} = \langle \pi^+ \bar{\upsilon}\upsilon | H_{eff} | K^+ \rangle$$

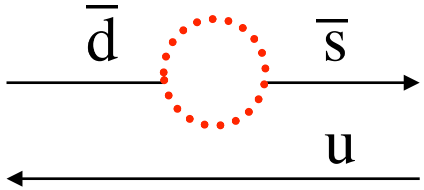
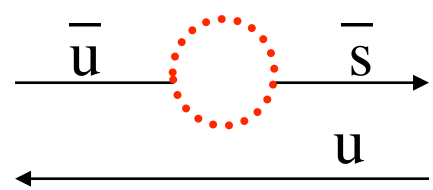
$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2\theta_W} \sum_{l=e,\mu,\tau} \left(\lambda_c^* X_{NL}^l + \lambda_t^* X(x_t)\right)\left(\bar{s}\gamma_\mu(1-\gamma_5)d\right)\left(\bar{\upsilon}_L\gamma^\mu(1-\gamma_5)\upsilon_L\right)$$

$$\text{Re}\left(\lambda_c^* X_{NL}^l\right) \approx 2\times10^{-4}, \quad \boxed{\text{Im}\left(\lambda_c^* X_{NL}^l\right) \approx 4\times10^{-7}\eta}$$ — neglected

$$\text{Re}\left(\lambda_t^* X(x_t)\right) \approx 6\times10^{-4}(1-\hat{\rho}), \quad \text{Im}\left(\lambda_t^* X(x_t)\right) \approx 6\times10^{-4}\hat{\eta}$$

real part ≈ imaginary part

Use $K^+ \to \pi^0 e^+ \nu$ (data) for the hadronic matrix element.

$\langle \pi^+ |$  $| K^+ \rangle$ $= \sqrt{2}$ $\langle \pi^0 |$  $| K^+ \rangle$

$\bar{s}\gamma_\mu(1-\gamma_5)d$ isospin relation $\bar{s}\gamma_\mu(1-\gamma_5)d$

→ *Br* prediction with a relatively small theoretical uncertainty.

Current Standard Model predictions:

$$\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu}) = (0.80 \pm 0.11)\times 10^{-10}$$

(isospin breaking taken into account)

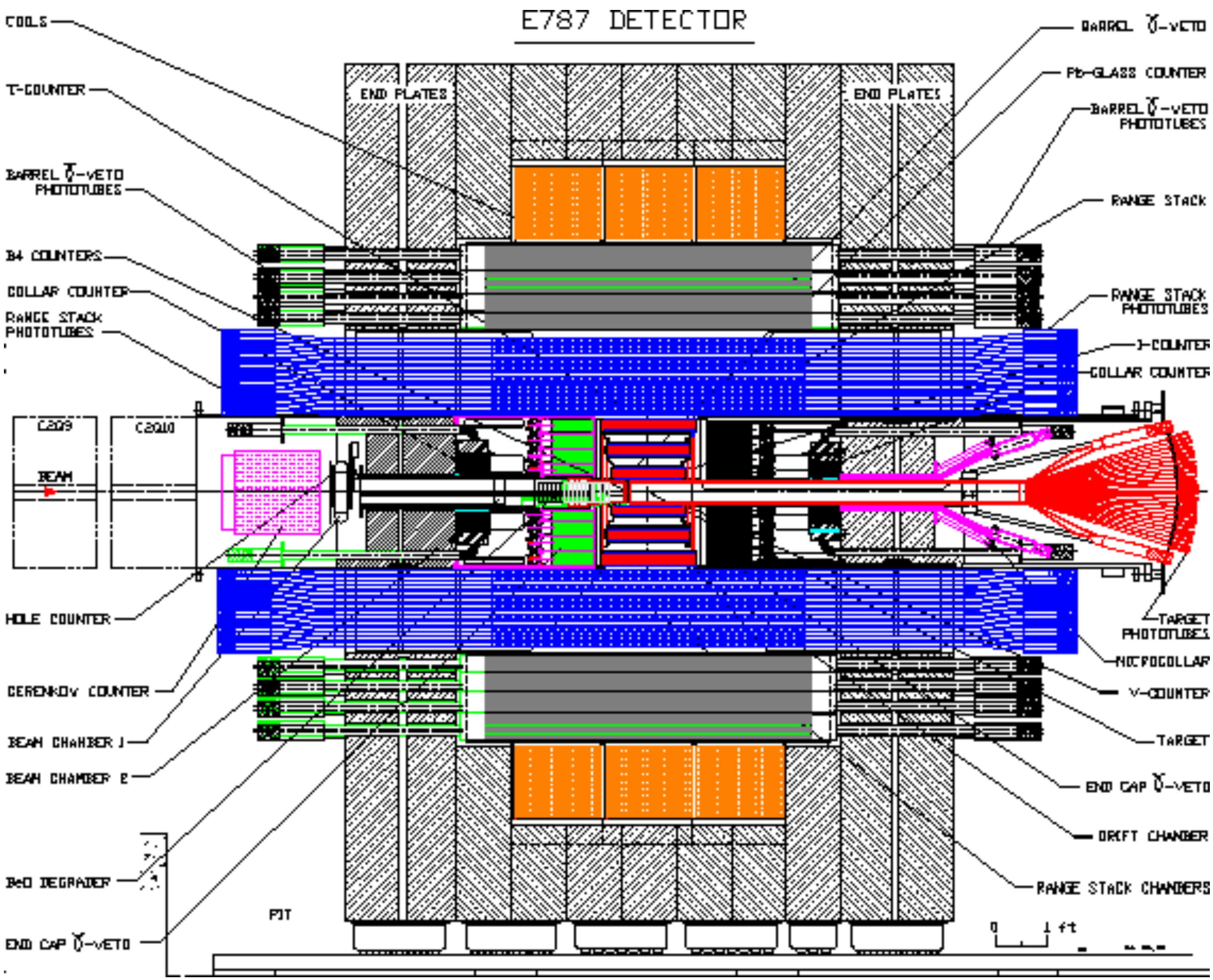
BNL787, 1995 data: $(4.2\,^{+9.7}_{-3.5})\times 10^{-10}$ PRL 97
based on one event

More data taken, no new candidate

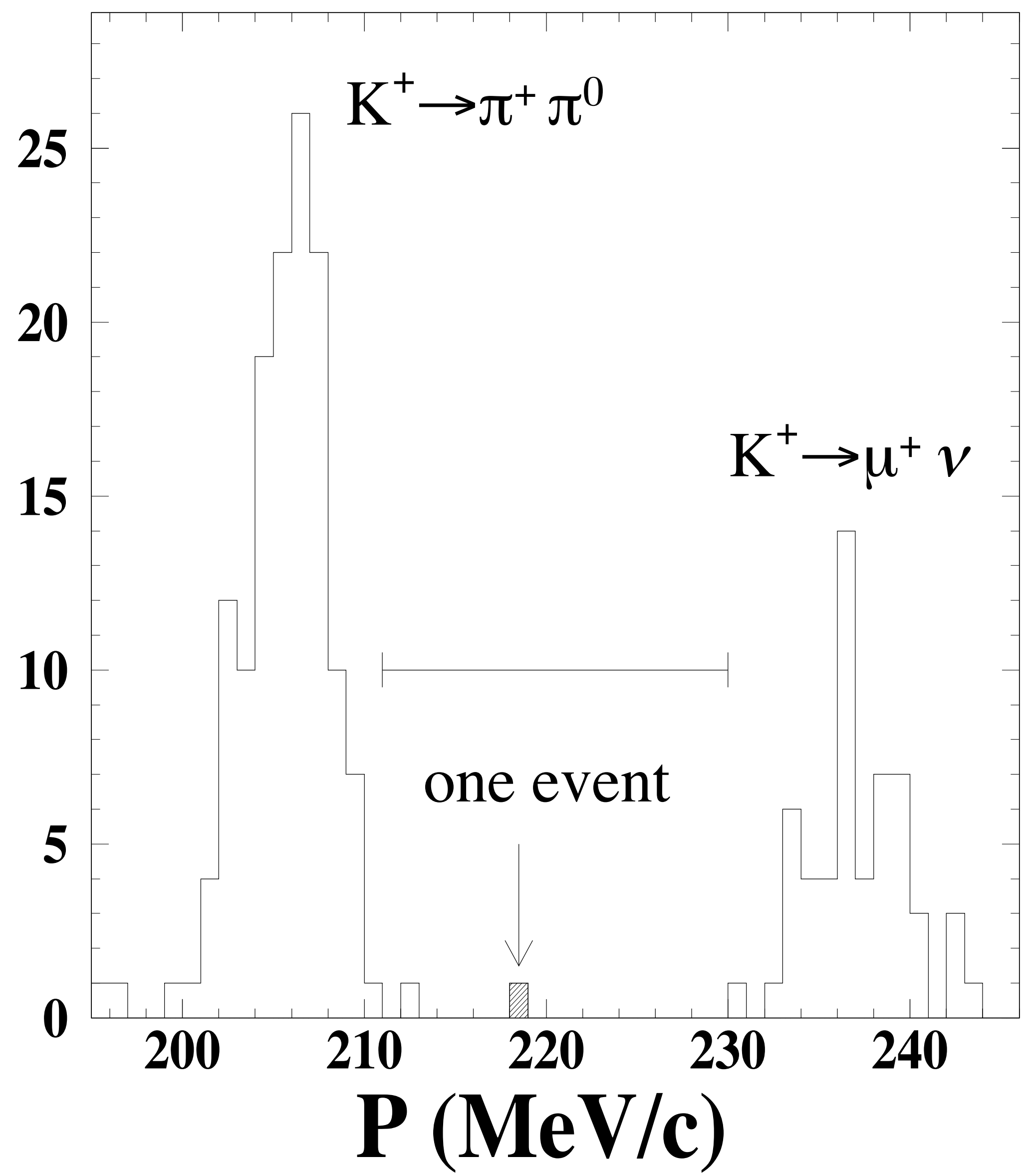
BNL787, 1995-97: $(1.5\,^{+3.4}_{-1.2})\times 10^{-10}$ PRL 2000

Further data taken, one more candidate

BNL787, 1995-98: $(1.57\,^{+1.75}_{-0.82})\times 10^{-10}$ PRL 2002



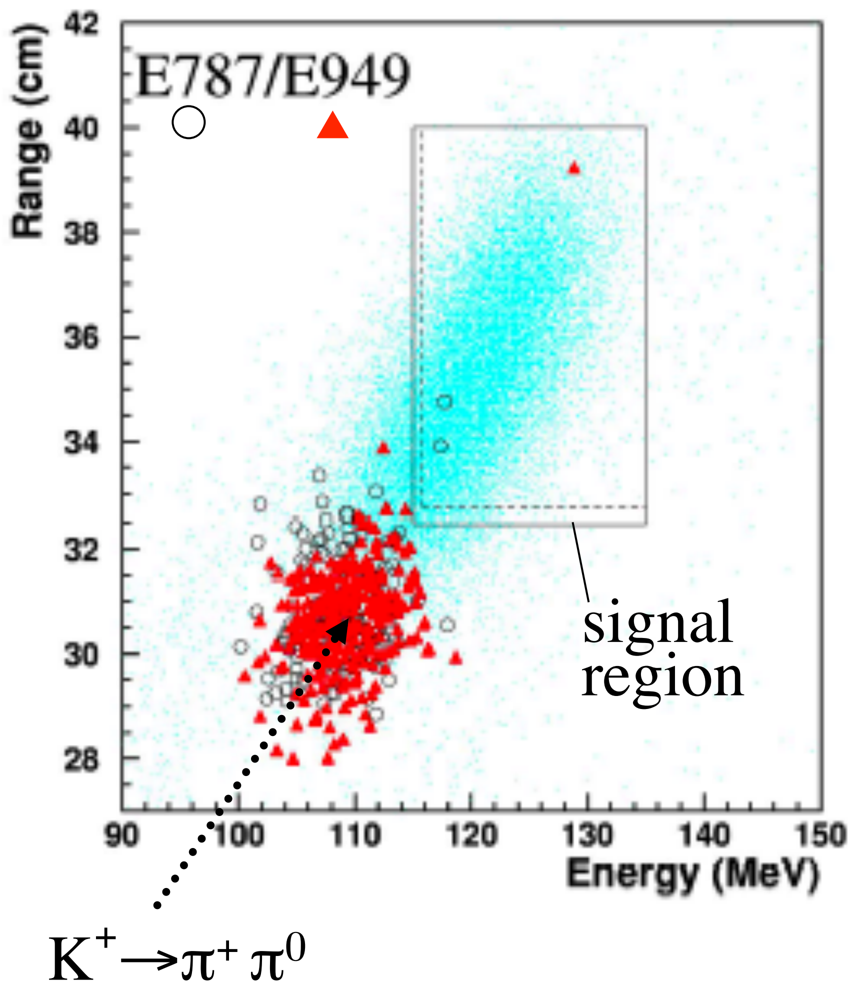
E787 DETECTOR
COILS
T-COUNTER
BARREL γ-VETO PHOTOTUBES
B4 COUNTERS
COLLAR COUNTER
RANGE STACK PHOTOTUBES
C2Q9
C2Q10
BEAM
HOLE COUNTER
CERENKOV COUNTER
BEAM CHAMBER 1
BEAM CHAMBER 2
BeO DEGRADER
END CAP γ-VETO
PIT
END PLATES
END PLATES
BARREL γ-VETO
Pb-GLASS COUNTER
BARREL γ-VETO PHOTOTUBES
RANGE STACK
RANGE STACK PHOTOTUBES
I-COUNTER
COLLAR COUNTER
TARGET PHOTOTUBES
MICROCOLLAR
V-COUNTER
TARGET
END CAP γ-VETO
DRIFT CHAMBER
RANGE STACK CHAMBERS
0
1 ft



K⁺→π⁺π⁰
K⁺→μ⁺ν
one event
0
5
10
15
20
25
200
210
220
230
240
P (MeV/c)

E787(95-98)/E949(02) combined results

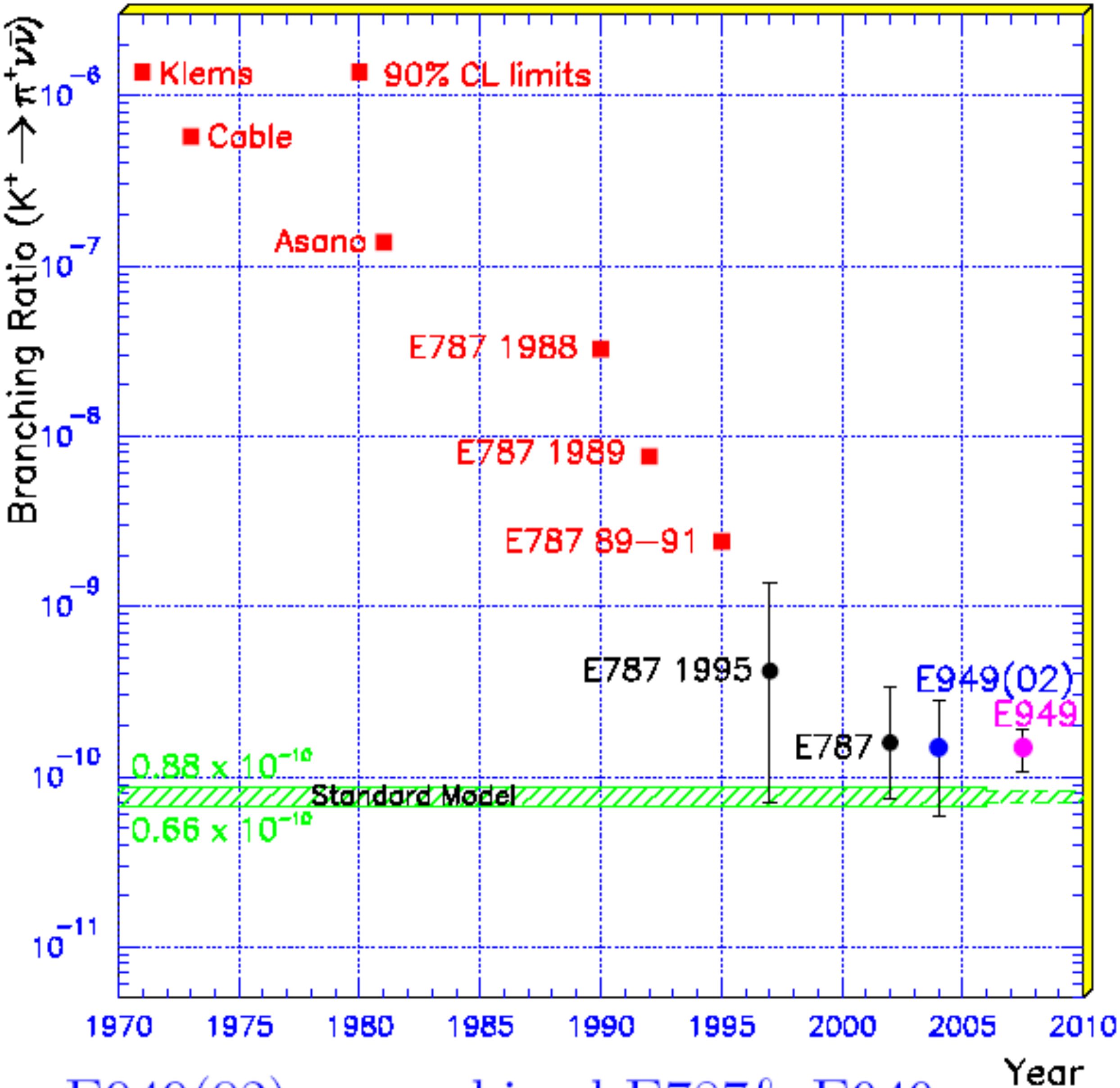
$$\left(1.47 \begin{matrix} +1.30 \\ -0.89 \end{matrix}\right)\times 10^{-10}$$ PRL 2004



Standard Model prediction
$(8.8 \pm 1.2)\times 10^{-11}$

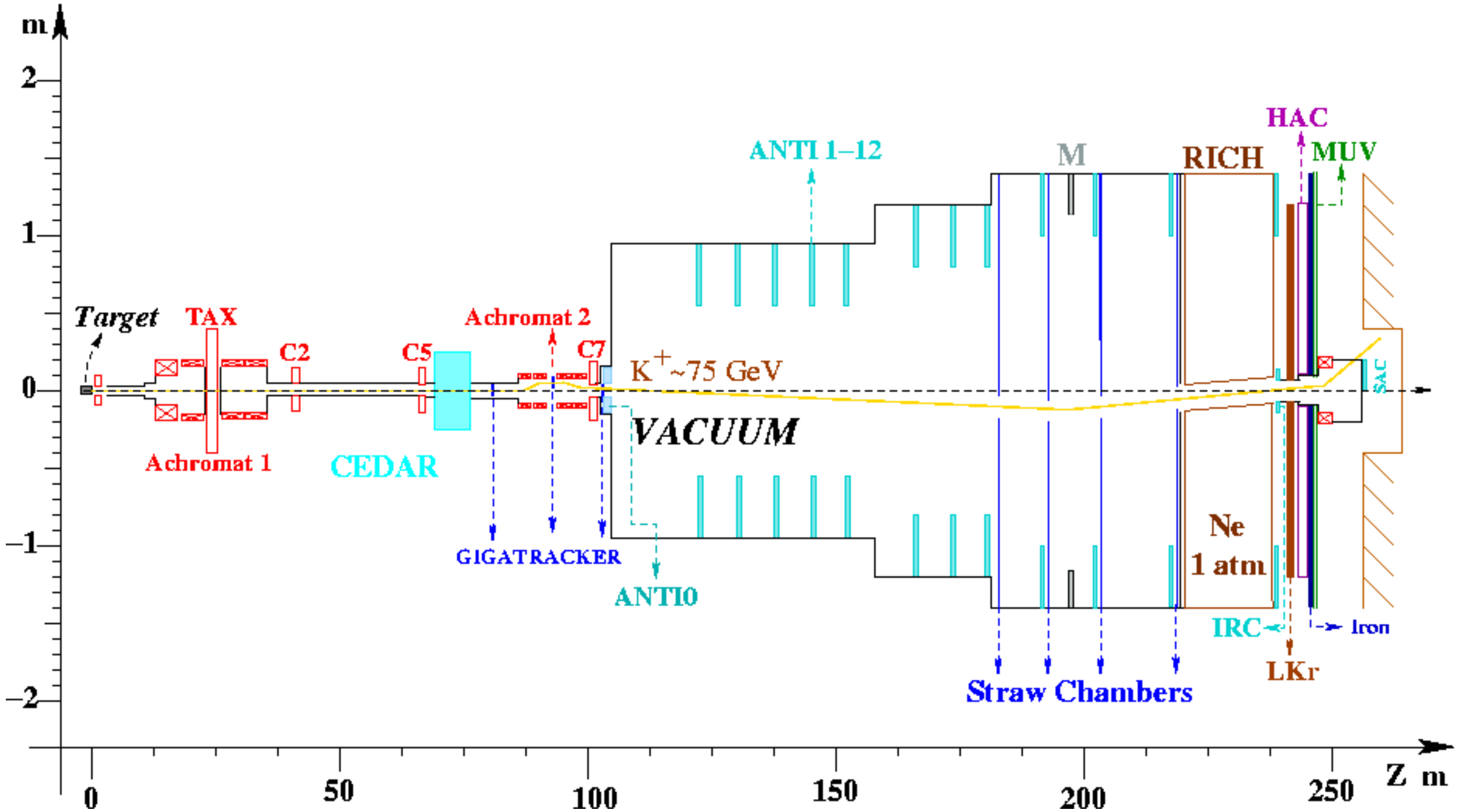
The data is compatible with the Standard Model prediction.

Could become a way to extract (ρ, η) from the kaon decays.



E949(02) = combined E787& E949.

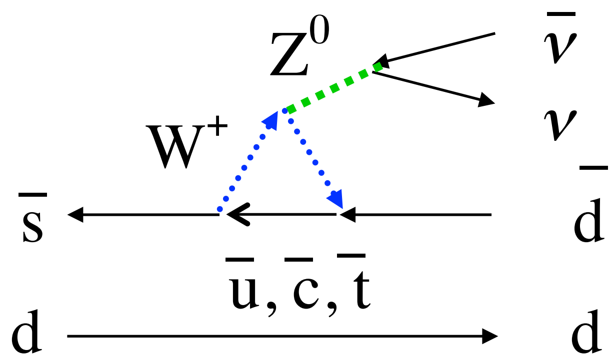
E949 projection with full running period.

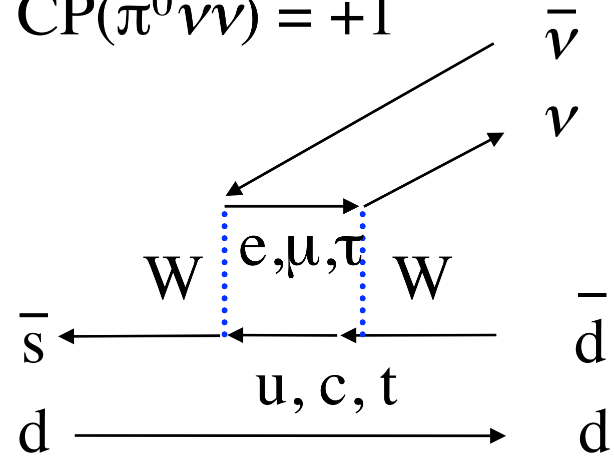


CERN NA62 under construction

expect ~100 $K^+ \to \pi^+ \nu \bar{\nu}$ events

$K_L^0 \to \pi^0 \nu \overline{\nu}$ CP violating decay $\Leftarrow$ $CP(\pi^0 \nu \overline{\nu}) = +1$





$$\left\langle \pi^0 \overline{\upsilon} \upsilon \middle| H_{eff} \middle| K^0 \right\rangle \equiv a$$

$$\left\langle \pi^0 \overline{\upsilon} \upsilon \middle| H_{eff} \middle| \overline{K}^0 \right\rangle = \left\langle \pi^0 \overline{\upsilon} \upsilon \middle| (\overrightarrow{CPT})^\dagger (\overrightarrow{CPT}) H_W (\overrightarrow{CPT})^\dagger (\overrightarrow{CPT}) \middle| \overline{K}^0 \right\rangle$$

$$= a^* e^{i(\theta_{CP} - \overline{\theta}_T)}$$

CPT symmetry
No hadronic final state interactions.

As for the K^+ decay,

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2\theta_W} \sum_{l=e,\mu,\tau} \left(\lambda_c^* X_{NL}^l + \lambda_t^* X(x_t)\right)\left(\bar{s}\gamma_\mu(1-\gamma_5)d\right)\left(\bar{\upsilon}_L \gamma^\mu (1-\gamma_5)\upsilon_L\right)$$

$$\mathrm{Re}\left(\lambda_c^* X_{NL}^l\right) \approx 2\times 10^{-4}, \quad \boxed{\mathrm{Im}\left(\lambda_c^* X_{NL}^l\right) \approx 4\times 10^{-7}\eta}$$ — neglected

$$\mathrm{Re}\left(\lambda_t^* X(x_t)\right) \approx 6\times 10^{-4}(1-\hat{\rho}), \quad \mathrm{Im}\left(\lambda_t^* X(x_t)\right) \approx 6\times 10^{-4}\hat{\eta}$$

- real part ≈ imaginary part
- imaginary part is given by λ_t

$$\left\langle \pi^0 \bar{\nu}\nu \middle| H_W \middle| K_L \right\rangle = \frac{1}{\sqrt{2}}\left[a_{\pi\bar{\upsilon}\upsilon} - (1-2\varepsilon)e^{-i\phi_\Gamma}\bar{a}_{\pi\bar{\upsilon}\upsilon}\right]$$

$$= \frac{a_{\pi\bar{\upsilon}\upsilon}}{\sqrt{2}}\left[1-(1-2\varepsilon)e^{-i(2\phi_{\pi\bar{\upsilon}\upsilon}+\theta_T-\theta_{CP}+\phi_\Gamma)}\right]$$

$$= \frac{a_{\pi\bar{\upsilon}\upsilon}}{\sqrt{2}}\left[1-(1-2\varepsilon)e^{-i(2\phi_{\pi\bar{\upsilon}\upsilon}-2\phi_0)}\right]$$

$$\boxed{\Delta\phi_{\pi\bar{\upsilon}\upsilon} = \phi_{\pi\bar{\upsilon}\upsilon} - \phi_0}$$

$$= \frac{a_{\pi\bar{\upsilon}\upsilon}}{\sqrt{2}}\left[1-\cos 2\Delta\phi_{\pi\bar{\upsilon}\upsilon} + i\sin 2\Delta\phi_{\pi\bar{\upsilon}\upsilon} - 2\varepsilon e^{-i2\Delta\phi_{\pi\bar{\upsilon}\upsilon}}\right]$$

$$= \sqrt{2}a_{\pi\bar{\upsilon}\upsilon}\left[\sin^2\Delta\phi_{\pi\bar{\upsilon}\upsilon} + i\sin\Delta\phi_{\pi\bar{\upsilon}\upsilon}\cos\Delta\phi_{\pi\bar{\upsilon}\upsilon} + o(\varepsilon)\right]$$

$$= \sqrt{2}ia_{\pi\bar{\upsilon}\upsilon}\sin\Delta\phi_{\pi\bar{\upsilon}\upsilon}\left[\cos\Delta\phi_{\pi\bar{\upsilon}\upsilon} - i\sin\Delta\phi_{\pi\bar{\upsilon}\upsilon} + o(\varepsilon)\right]$$

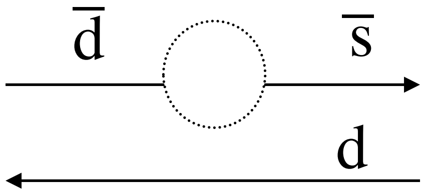
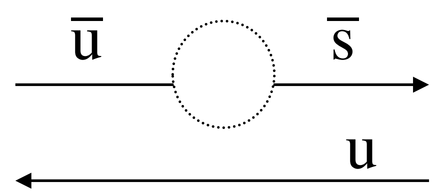
$$\approx \sqrt{2}i\left|a_{\pi\bar{\upsilon}\upsilon}\right|e^{i\phi_{\pi\bar{\upsilon}\upsilon}}\sin\Delta\phi_{\pi\bar{\upsilon}\upsilon}e^{-i\Delta\phi_{\pi\bar{\upsilon}\upsilon}} = \sqrt{2}i\left|a_{\pi\bar{\upsilon}\upsilon}\right|\sin\Delta\phi_{\pi\bar{\upsilon}\upsilon}e^{i\phi_0}$$

$$\boxed{\left|\left\langle \pi^0 \bar{\nu}\nu \middle| H_W \middle| K_L \right\rangle\right| = \sqrt{2}\left|a_{\pi\bar{\upsilon}\upsilon}\right|\sin\Delta\phi_{\pi\bar{\upsilon}\upsilon} \equiv \sqrt{2}\,\mathrm{Im}\,a_{\pi\bar{\upsilon}\upsilon} \text{ (CKM phase convention)}}$$

~~CP~~ due to the interplay between decays and oscillations
much bigger than ~~CP~~ in the oscillations ↔ different from 2π

Imaginary part can come only from $\bar{s} \to \bar{t} \to \bar{d}$.

hadronic part: $K^0 \to \pi^0 \bar{\nu}\nu = K^+ \to \pi^0 e^+ \nu$

$\langle \pi^0 |$  $| K^0 \rangle$ = $\langle \pi^0 |$  $| K^+ \rangle$

$\bar{s}\gamma_\mu(1-\gamma_5)d$ $\qquad$ $\bar{s}\gamma_\mu(1-\gamma_5)d$

$$Br_{K_L \to \pi^0 \upsilon\bar{\upsilon}} = Br_{K^+ \to \pi^0 e^+ \upsilon} \frac{\tau_L}{\tau_+} \frac{3\alpha^2}{|V_{us}|^2 2\pi^2 \sin^4 \theta_W} \left[\mathrm{Im}\left(V_{td} V_{ts}^*\right) X(x_t) \right]^2$$

$$\approx (3.0 \pm 0.6) \times 10^{-11}$$

Only one term → simpler than K^+ decays

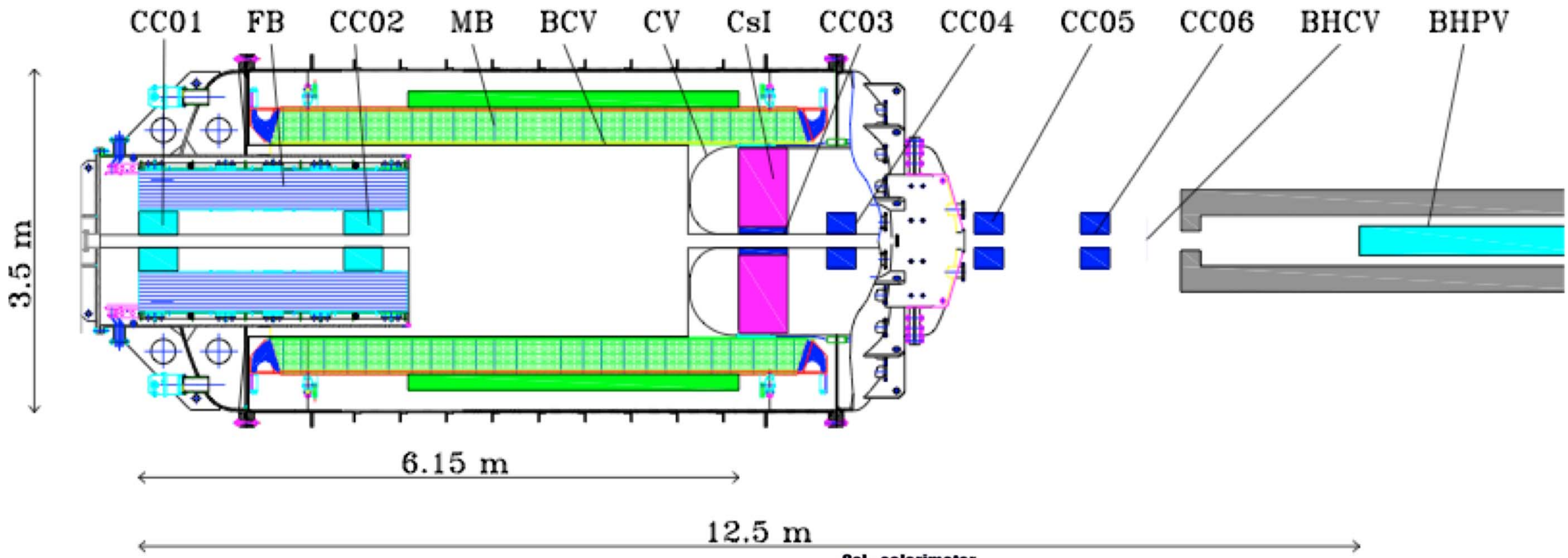
⇒ more reliable calculation

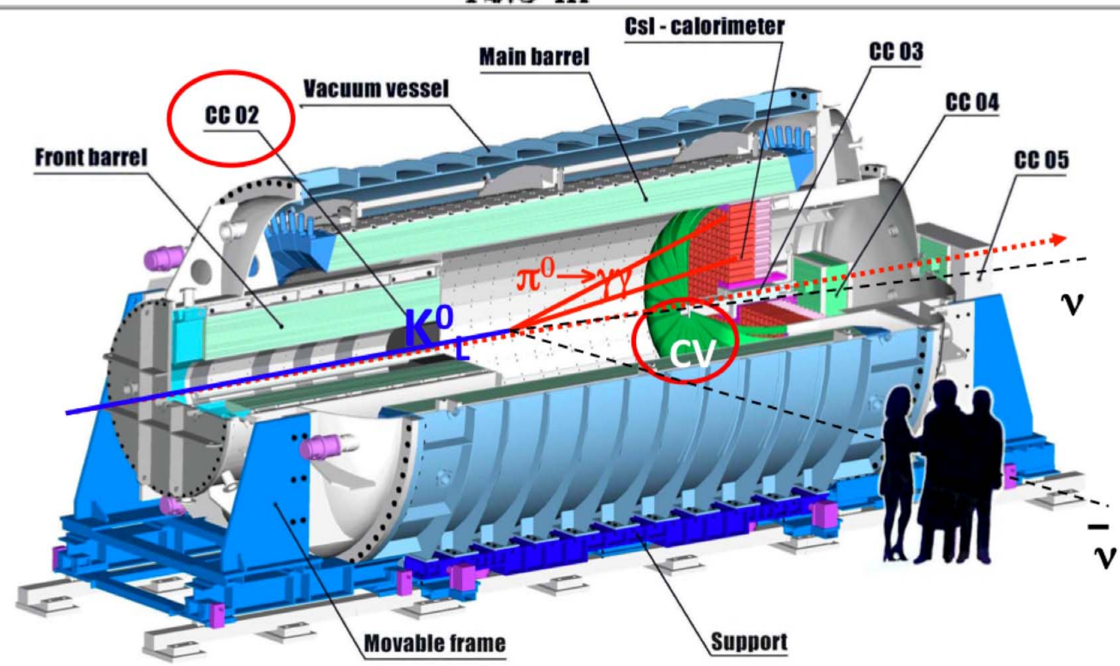
$Br(K^0 \to \pi^0 \bar{\nu}\nu) < 1.6\times10^{-6}$ 90% CL (KTeV, PLB 99) $\quad \pi^0 \to \gamma\gamma$

5.9×10^{-7} $\quad \pi^0 \to ee\gamma$

future experiment planned but a tough experiment

JPARC E14: few $K^0 \to \pi^0 \nu\bar{\nu}$ if SM





Note:

$$\langle \pi^0 \bar{\nu}\nu | H_{eff} | K_S \rangle = \frac{a_{\pi\bar{\upsilon}\upsilon}}{\sqrt{2}}\left[1+(1-2\varepsilon)e^{-i(2\phi_{\pi\bar{\upsilon}\upsilon}+\theta_T-\theta_{CP}+\phi_\Gamma)}\right]$$

$$= \frac{a_{\pi\bar{\upsilon}\upsilon}}{\sqrt{2}}\left[1+(1-2\varepsilon)e^{-i(2\phi_{\pi\bar{\upsilon}\upsilon}-2\phi_0)}\right]$$

$$= \frac{a_{\pi\bar{\upsilon}\upsilon}}{\sqrt{2}}\left[1+\cos 2\Delta\phi_{\pi\bar{\upsilon}\upsilon} - i\sin 2\Delta\phi_{\pi\bar{\upsilon}\upsilon} - 2\varepsilon e^{-i2\Delta\phi_{\pi\bar{\upsilon}\upsilon}}\right]$$

$$= \sqrt{2}a_{\pi\bar{\upsilon}\upsilon}\left[\cos^2\Delta\phi_{\pi\bar{\upsilon}\upsilon} - i\sin\Delta\phi_{\pi\bar{\upsilon}\upsilon}\cos\Delta\phi_{\pi\bar{\upsilon}\upsilon} + o(\varepsilon)\right]$$

$$\approx \sqrt{2}a_{\pi\bar{\upsilon}\upsilon}\cos\Delta\phi_{\pi\bar{\upsilon}\upsilon}\left[\cos\Delta\phi_{\pi\bar{\upsilon}\upsilon} - i\sin\Delta\phi_{\pi\bar{\upsilon}\upsilon}\right]$$

$$= \sqrt{2}\left|a_{\pi\bar{\upsilon}\upsilon}\right|\cos\Delta\phi_{\pi\bar{\upsilon}\upsilon}e^{i\phi_0}$$

$$\boxed{\eta_{\pi^0\bar{\nu}\nu} \equiv \frac{\langle \pi^0\bar{\nu}\nu | H_{eff} | K_L\rangle}{\langle \pi^0\bar{\nu}\nu | H_{eff} | K_S\rangle} = i\tan\Delta\phi_{\pi\bar{\upsilon}\upsilon} \approx O(1)}$$

$\Longleftrightarrow$ $\eta_{+-} \approx \varepsilon = O(10^{-3})$

Theoretical accuracy of the Standard Model predictions
in the kaon sector will be limited to >10%
(may be) except $K^0 \to \pi^0 \nu \bar{\nu}$ which will be experimentally challenging!