Physics of CP Violation (I)

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Foreseen subjects to be discussed are

CP violation
  basic description and phenomenology

Standard Model Flavour Physics
  CKM, rare decays and CP violation

Space reflection (P), Charge conjugation (C), Time reversal (T)

P transformation

x_i	coordinate	$x_i \to x'_i = -x_i$

Space reflection (P), Charge conjugation (C), Time reversal (T)

P transformation

x_i	coordinate	$x_i \rightarrow x'_i = -x_i$	
$v_i = \dfrac{dx_i}{dt}$	velocity	$v_i \rightarrow v'_i = -v_i$	
p_i	momentum	$p_i \rightarrow p'_i = -p_i$	
$L_i = \varepsilon_{ijk} x_j p_k$	angular momentum	$L_i \rightarrow L'_i = L_i$	
s_i	spin	$s_i \rightarrow s'_i = s_i$	
ρ	charge density	$\rho_i \rightarrow \rho'_i = \rho\ _i$	
$j_i = \rho v_i$	charge current	$j_i \rightarrow j'_i = -j_i$	
$E_i = -\dfrac{\partial\phi}{dx_i} - \dfrac{\partial A_i}{dt}$	electric field	$E_i \rightarrow E'_i = -E_i$	ϕ scaler potential
$B_i = \varepsilon_{ijk} \dfrac{\partial A_k}{\partial x_j}$	magnetic field	$B_i \rightarrow B'_i = B_i$	$\boldsymbol{A}$ vector potential

C transformation

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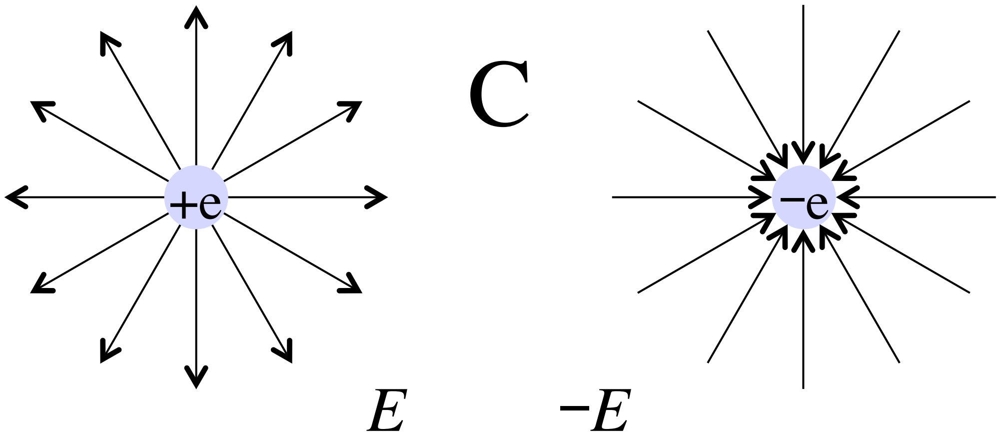
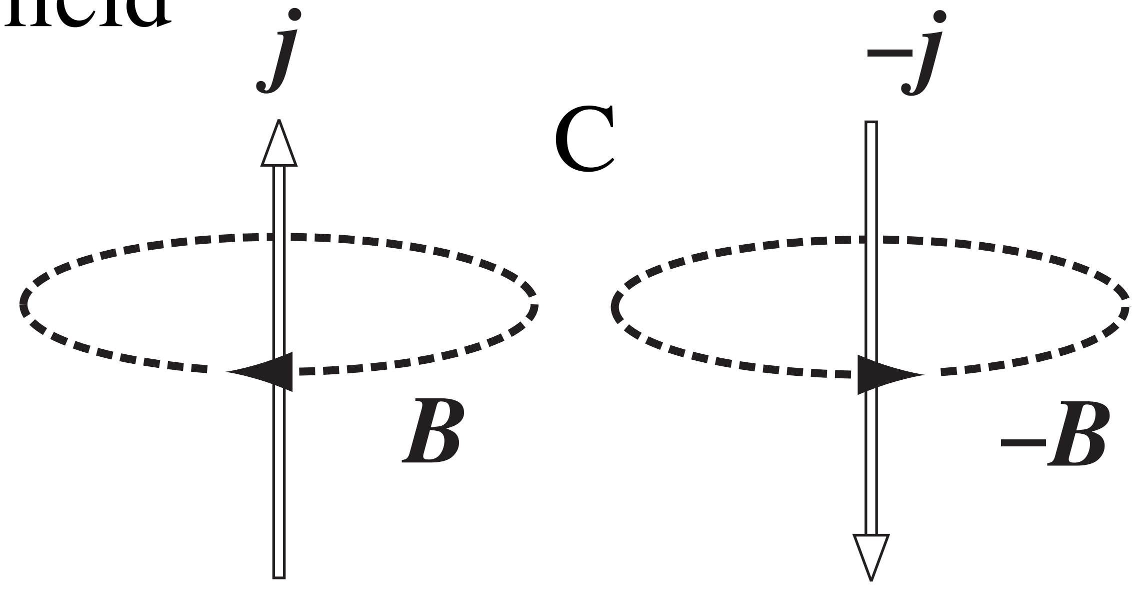
E field



B field



C transformation

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$(\phi \rightarrow \phi' = -\phi. \boldsymbol{A} \rightarrow \boldsymbol{A}' = -\boldsymbol{A})$

T transformation

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T transformation

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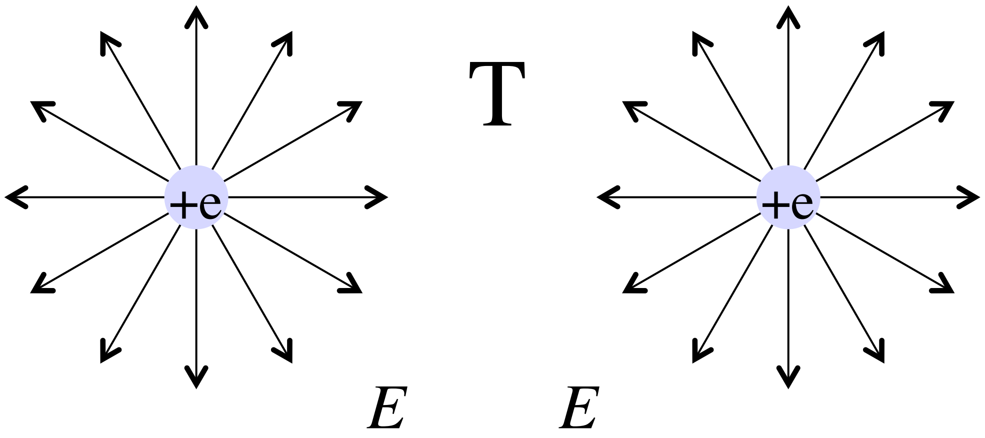
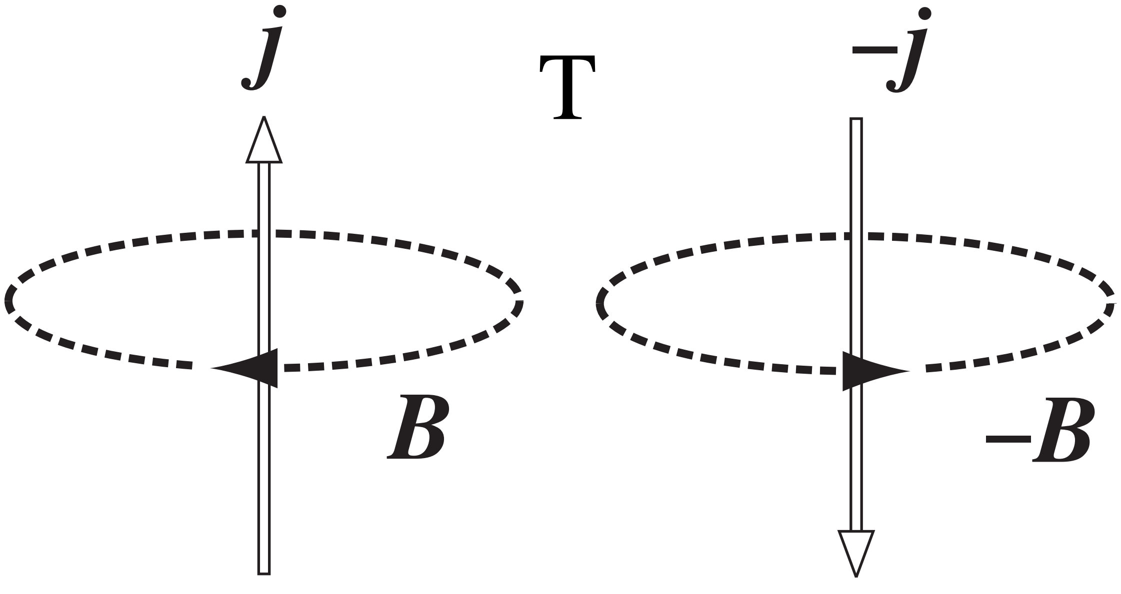


B field



T transformation

t	time	$t \rightarrow t' = -t_i$
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P, C and T transformation in covariant form in the classical world

parity			charge			time		
x^μ	$\rightarrow$	x_μ	x^μ	$\rightarrow$	x^μ	x^μ	$\rightarrow$	$-x_\mu$
p^μ	$\rightarrow$	p_μ	p^μ	$\rightarrow$	p^μ	p^μ	$\rightarrow$	p_μ
j^μ	$\rightarrow$	j_μ	j^μ	$\rightarrow$	$-j^\mu$	j^μ	$\rightarrow$	j_μ
A^μ	$\rightarrow$	A_μ	A^μ	$\rightarrow$	$-A^\mu$	A^μ	$\rightarrow$	A_μ

NB: $g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$

$$\vec{E} = -\mathrm{grad} A_0 - \frac{\partial}{\partial t}\vec{A}$$

$$\vec{H} = \mathrm{curl}\vec{A}$$

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$$\vec{E} = -\mathrm{grad} A_0 - \frac{\partial}{\partial t}\vec{A}$$

$$\vec{H} = \mathrm{curl}\vec{A}$$

$$L_{\mathrm{em}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + j^\mu A_\mu$$

Lagrangean for the electromagnetic field is invariant under P, C and T

In QM theory, P and C are unitary operators act on

position ($\boldsymbol{x}$), momentum ($\boldsymbol{p}$), angular momentum ($\boldsymbol{L}$), electric current ($\boldsymbol{j}$), electric ($\boldsymbol{E}$) and magnetic ($\boldsymbol{B}$) field operators.

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$$P\,\boldsymbol{j}\,P^{\dagger} = -\boldsymbol{j} \qquad P\,\boldsymbol{E}\,P^{\dagger} = -\boldsymbol{E} \qquad P\,\boldsymbol{B}\,P^{\dagger} = \boldsymbol{B}$$

$$C\,\boldsymbol{x}\,C^{\dagger} = \boldsymbol{x} \qquad C\,\boldsymbol{p}\,C^{\dagger} = \boldsymbol{p} \qquad C\,\boldsymbol{L}\,C^{\dagger} = \boldsymbol{L}$$

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P and C operators act on a state $|\psi(\boldsymbol{x}, t)>$ as

$$C\left|\psi(\boldsymbol{x},t)\right\rangle = \left|\psi^C(\boldsymbol{x},t)\right\rangle \qquad P\left|\psi(\boldsymbol{x},t)\right\rangle = \left|\psi^P(-\boldsymbol{x},t)\right\rangle$$

T operator **cannot** make: $T\, t\, T^{\dagger} = -t$

Because t is **not an operator** in QM but a parameter!!!

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Schrödinger equation:

$$i\frac{\partial}{\partial t}\left|\psi(\boldsymbol{x},t)\right\rangle = H\left|\psi(\boldsymbol{x},t)\right\rangle$$

$$\overrightarrow{T}\; i\; \overrightarrow{T}^{-1}\,\overrightarrow{T}\;\frac{\partial}{\partial t}\left|\psi(\boldsymbol{x},t)\right\rangle = \overrightarrow{T}\; H\; \overrightarrow{T}^{-1}\,\overrightarrow{T}\left|\psi(\boldsymbol{x},t)\right\rangle$$

→ means operating toward right.

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→ means operating toward right.

If $\vec{T}\;i\; = -\,i\;\vec{T}$ (i.e. **anti-liner operator**),

$$-i\frac{\partial}{\partial t}\vec{T}\left|\psi(\boldsymbol{x},t)\right\rangle = \vec{T}\;H\;\vec{T}^{-1}\;\vec{T}\left|\psi(\boldsymbol{x},t)\right\rangle$$

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$\rightarrow$ means operating toward right.

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By defining

$$\overrightarrow{T}\left|\psi(\boldsymbol{x},t)\right\rangle \equiv \left|\psi^{T}(\boldsymbol{x},-t)\right\rangle$$

$$\overrightarrow{T}\; H\; \overrightarrow{T}^{-1} \equiv H^{T}$$

T transformed state vector and Hamiltonian operator

$$-i\frac{\partial}{\partial t}\left|\psi^{T}(\boldsymbol{x},-t)\right\rangle = H^{T}\left|\psi^{T}(\boldsymbol{x},-t)\right\rangle$$

T transformed, $t \rightarrow -t$ Schrödinger equation:

Unitary and anti-unitary transformation:

$$\left|\hat{\alpha}\right\rangle = \overrightarrow{V}\left|\alpha\right\rangle, \ \left\langle\hat{\alpha}\right| = \left|\hat{\alpha}\right\rangle^{\dagger} = \left\langle\alpha\right|\overleftarrow{V}^{\dagger}$$

$$\left\langle\hat{\alpha}\middle|\hat{\beta}\right\rangle = \left\langle\alpha\right|\overleftarrow{V}^{\dagger}\overrightarrow{V}\left|\beta\right\rangle$$

$\overrightarrow{V}$ operator acting on right

$\overleftarrow{V}$ operator acting on left

Unitary and anti-unitary transformation:

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$\overrightarrow{V}$ operator acting on right

$\overleftarrow{V}$ operator acting on left

$$\langle\hat{\alpha}|\hat{\beta}\rangle = \langle\alpha|\overleftarrow{V}^{\dagger}\ \overrightarrow{V}|\beta\rangle$$

If $\langle\alpha|\overleftarrow{V}^{\dagger}\ \overrightarrow{V}|\beta\rangle = \langle\alpha|\overrightarrow{V}^{\dagger}\ \overrightarrow{V}|\beta\rangle$ and $\overrightarrow{V}^{\dagger}\ \overrightarrow{V} = 1$

$$\langle\hat{\alpha}|\hat{\beta}\rangle = \langle\alpha|\beta\rangle$$

unitary transformation

V: unitary operator

If $\langle\alpha|\overleftarrow{V}^{\dagger}\ \overrightarrow{V}|\beta\rangle = \left(\langle\alpha|\overrightarrow{V}^{\dagger}\ \overrightarrow{V}|\beta\rangle\right)^{*}$ and $\overrightarrow{V}^{\dagger}\ \overrightarrow{V} = 1$

$$\langle\hat{\alpha}|\hat{\beta}\rangle = \left(\langle\alpha|\beta\rangle\right)^{*} = \langle\beta|\alpha\rangle$$

anti-unitary transformation

V: anti-unitary operator

Let $|\beta\rangle = c|\alpha\rangle$ where c is a complex c-number: i.e. $\langle\beta| = c^*\langle\alpha|$

By introducing $\overrightarrow{V}c = c'\overrightarrow{V}$

$$\langle\hat{\alpha}|\hat{\beta}\rangle = \langle\hat{\alpha}|\overrightarrow{V}c|\alpha\rangle = \langle\hat{\alpha}|c'\overrightarrow{V}|\alpha\rangle = c'\langle\hat{\alpha}|\hat{\alpha}\rangle$$

As shown before

$$\langle\hat{\alpha}|\hat{\beta}\rangle = \begin{cases} \langle\alpha|\beta\rangle = c\langle\alpha|\alpha\rangle & \text{unitary} \\ \langle\beta|\alpha\rangle = c^*\langle\alpha|\alpha\rangle & \text{anti-unitary} \end{cases}$$

Therefore,

$$\boxed{\begin{array}{ll} \overrightarrow{V}c = c\overrightarrow{V} & \text{unitary operator} \\ \overrightarrow{V}c = c^*\overrightarrow{V} & \text{anti-unitary operator} \end{array}}$$

Violation of Parity

World

90%

10%

Violation of Parity

World

90%

10%

Mirror World

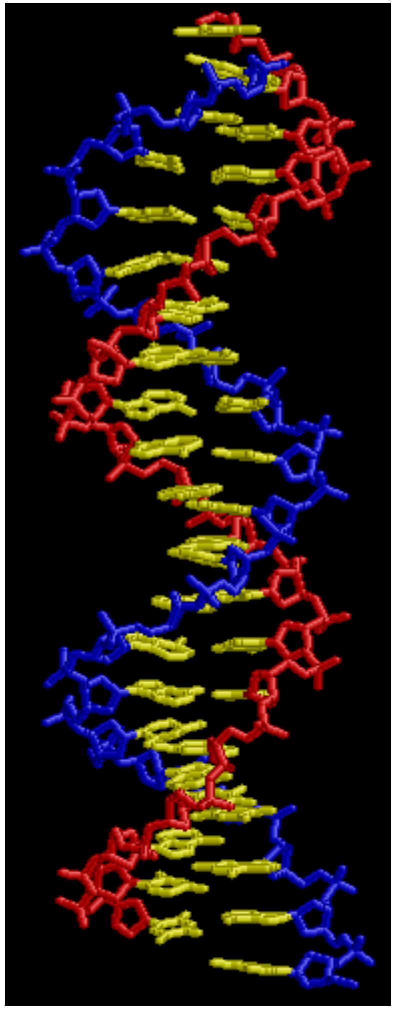
10%

90%

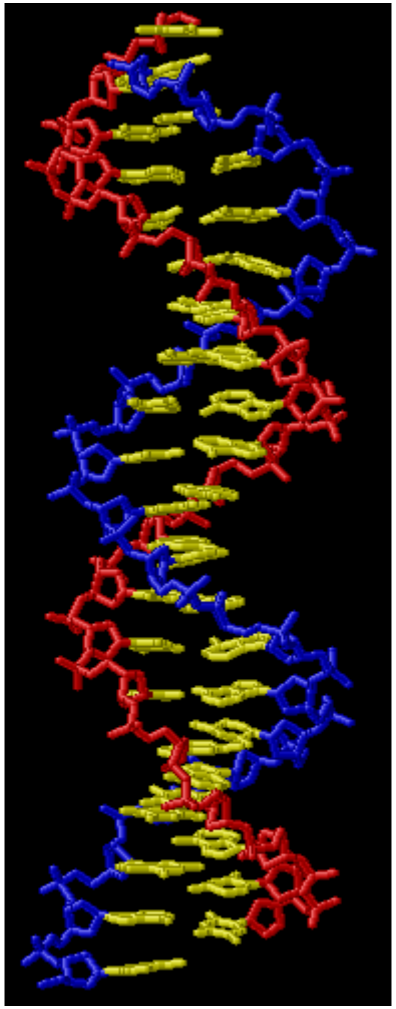
World ≠ Mirror World
(parity violation)

DNA

World

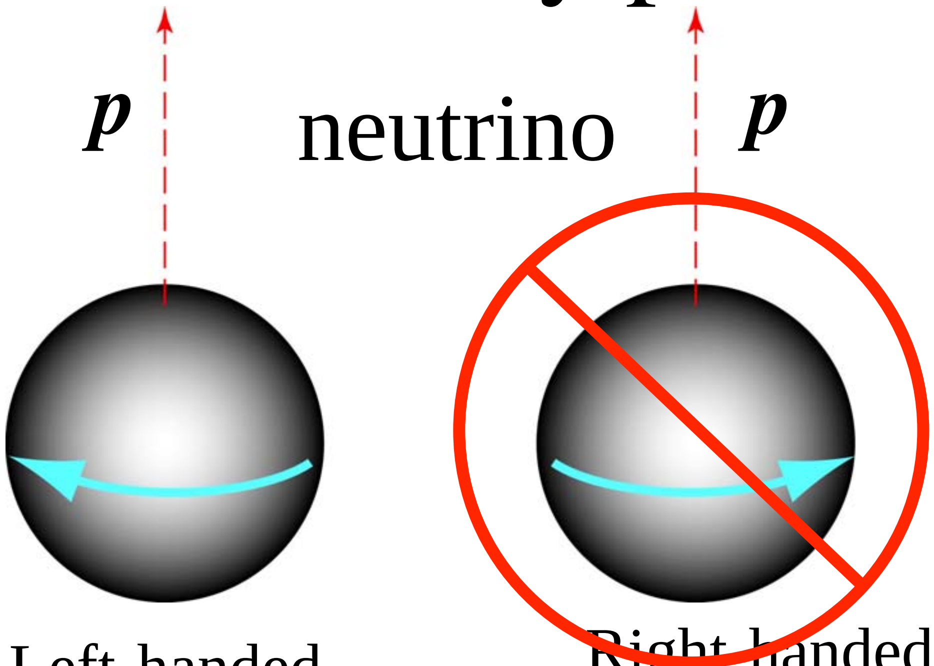


Mirror World



World ≠ Mirror World by 100%
Parity is fully violated.

And in elementary particle world



p neutrino *p*

does not exist

Left-handed

Right-handed

theory of parity violation

1956, T.D. Lee and C.N.Yang

experiment of parity violation

1957, C.S.Wu

Strong and electromagnetic interactions conserve:

flavour quantum numbers,

C, P, T, CP, CT, PT, CPT

Neutral pion decay via electromagnetic interactions

$\pi^0 \longrightarrow \gamma\gamma$ but not $\gamma\gamma\gamma$

γ and π^0 are C eigenstates: particle = anti-particle

n photon system

$$\left|n\gamma\right\rangle = \frac{1}{\sqrt{n!}}\prod_{i=1}^{n} a^{\dagger}_{\boldsymbol{p}_i\,,\,\varepsilon_i}\left|0\right\rangle$$

$a^{\dagger}{}_{\boldsymbol{p}_i,\,\varepsilon_i}$: creation operator of a photon with a momentum $\boldsymbol{p}_i$, polarization ε_{ii}

|0>: vacuum state

Since $\vec{B}, \vec{E} \xrightarrow{C} -\vec{B}, -\vec{E}$ $\Longrightarrow$ $C(\gamma) = -1$

we have $\boxed{C a^{\dagger}_{\boldsymbol{p}_i,\,\varepsilon_i} C^{\dagger} = -a^{\dagger}_{\boldsymbol{p}_i,\,\varepsilon_i}}$

It follows

$$C|n\gamma\rangle = \frac{C}{\sqrt{n!}} \prod_{i=1}^{n} a^{\dagger}_{\boldsymbol{p}_i,\,\varepsilon_i} |0\rangle$$

$$= \frac{1}{\sqrt{n!}} \left[\prod_{i=1}^{n} \left(C a^{\dagger}_{\boldsymbol{p}_i,\,\varepsilon_i} C^{\dagger} \right) \right] C|0\rangle$$

$$= \frac{(-1)^n}{\sqrt{n!}} \prod_{i=1}^{n} a^{\dagger}_{\boldsymbol{p}_i,\,\varepsilon_i} |0\rangle$$

C eigenvalue of vacuum defined to be +1: convention

i.e. $C(\gamma\gamma) = (-1)^2 = +1$, $C(\gamma\gamma\gamma) = (-1)^3 = -1$, etc..

$$\left|\pi^0\right\rangle = \frac{1}{\sqrt{2}}\left(\left|u\bar{u}\right\rangle - \left|d\bar{d}\right\rangle\right)_{n=1,L=0,s=0}$$

$$\left|u\bar{u}\right\rangle_{n=1,\,L=0,\,s=0} = \sum_{s_1,\,s_2}\int d^3p\, f_1(p) Y_0(\hat{\boldsymbol{p}})\chi_0(s_1,s_2) b^{u\dagger}_{\boldsymbol{p},\,s_1} c^{u\dagger}_{-\boldsymbol{p},\,s_2}\left|0\right\rangle$$

$b^{u\dagger}_{\boldsymbol{p},\,s_1}\left(c^{u\dagger}_{-\boldsymbol{p},\,s_2}\right)$ creation operator for u($\bar{\text{u}}$) quarks
with a momentum $\boldsymbol{p}(-\boldsymbol{p})$ and $s_z = s_1(s_2)$.

Y_0: spherical hermonics for $L = 0$ (orbital angular momentum)
χ_0; Clebsh-Gordan coefficients for $S = 0$ ($\boldsymbol{S} = \boldsymbol{s}_1 + \boldsymbol{s}_2$)

$$C\left|u\bar{u}\right\rangle_{n=1,\,L=0,\,s=0} = \sum_{s_1,\,s_2}\int d^3p\, f_1(p) Y_0(\hat{\boldsymbol{p}})\chi_0(s_1,\, s_2) C b^{u\dagger}_{\boldsymbol{p},\,s_1} c^{u\dagger}_{-\boldsymbol{p},\,s_2}\left|0\right\rangle$$

$$= \sum_{s_1,\,s_2}\int d^3p f_1(p) Y_0(\hat{\boldsymbol{p}})\chi_0(s_1,\, s_2) C b^{u\dagger}_{\boldsymbol{p},\,s_1} C^\dagger C c^{u\dagger}_{-\boldsymbol{p},\,s_2} C^\dagger C\left|0\right\rangle$$

u ↔ $\bar{\text{u}}$ under C $\Rightarrow C b^{u\dagger}_{\boldsymbol{p},\,s_1} C^\dagger = c^{u\dagger}_{\boldsymbol{p},\,s_1},\ C c^{u\dagger}_{-\boldsymbol{p},\,s_2} C^\dagger = C b^{u\dagger}_{-\boldsymbol{p},\,s_2} C^\dagger$

$$= \sum_{s_1,\,s_2}\int d^3p f_1(p) Y_0(\hat{\boldsymbol{p}})\chi_0(s_1,\, s_2) c^{u\dagger}_{\boldsymbol{p},\,s_1} b^{u\dagger}_{-\boldsymbol{p},\,s_2} C\left|0\right\rangle$$

$$\left\{ b^{u\dagger}_{\boldsymbol{p},\, s_1} ,\ c^{u\dagger}_{-\boldsymbol{p},\, s_1} \right\} = 0$$

Fermion creation operator
anti-commutation relation

$$C\left|0\right\rangle = \left|0\right\rangle$$

$$C\left|u\bar{u}\right\rangle_{n=1,\, L=0,\, s=0} = -\sum_{s_1,\, s_2} \int d^3p\, f_1(p) Y_0(\hat{\boldsymbol{p}}) \chi_1(s_1, s_2) b^{u\dagger}_{-\boldsymbol{p},\, s_2} c^{u\dagger}_{\boldsymbol{p},\, s_1} \left|0\right\rangle$$

by renaming $\boldsymbol{p}\,(s_1)$ as $-\boldsymbol{p}(s_2)$ and $-\boldsymbol{p}\,(s_2)$ as $\boldsymbol{p}(s_1)$

$$= -\sum_{s_1,\, s_2} \int d^3p\, f_1(p) Y_0(-\hat{\boldsymbol{p}}) \chi_0(s_2, s_1) b^{u\dagger}_{\boldsymbol{p},\, s_1} c^{u\dagger}_{-\boldsymbol{p},\, s_2} \left|0\right\rangle$$

since $Y_0(-\hat{\boldsymbol{p}}) = Y_0(\hat{\boldsymbol{p}})$, $\chi_0(s_2, s_1) = -\chi_0(s_1, s_2)$

$$= \sum_{s_1,\, s_2} \int d^3p\, f_1(p) Y_0(\hat{\boldsymbol{p}}) \chi_0(s_1, s_2) b^{u\dagger}_{\boldsymbol{p},\, s_1} c^{u\dagger}_{-\boldsymbol{p},\, s_2} \left|0\right\rangle$$

$$C\left|u\bar{u}\right\rangle_{n=1,\, L=0,\, s=0} = \left|u\bar{u}\right\rangle_{n=1,\, L=0,\, s=0}$$

$$\boxed{C\left|\pi^0\right\rangle = \left|\pi^0\right\rangle}$$

initial state $C(\pi^0) = +1$,

final state $C(\gamma\gamma) = (-1)^2 = +1$, $C(\gamma\gamma\gamma) = (-1)^3 = -1$

EM interaction conserves C

$\pi^0 \longrightarrow \gamma\gamma$ but not $\gamma\gamma\gamma$

Or calculating decay amplitudes

$$A_{\gamma\gamma\gamma} = <\gamma\gamma\gamma|C^{-1}C\ H\ C^{-1}C|\pi^0> = -<\gamma\gamma\gamma|C\ H\ C^{-1}|\pi^0>$$

$$= -<\gamma\gamma\gamma|\ H\ |\pi^0> = -A_{\gamma\gamma\gamma}$$

$$\mathrm{A}_{\gamma\gamma\gamma} = 0$$

How about weak interactions?

Neutrio is only left-handed

Antineutriono is only right-handed

$\Big\}\longrightarrow$ C nor P conserved

Charged pion decays via weak interaction

$$\begin{tikzcd}
\pi^+ \to \mu^+_R + \nu_L \arrow[r, "P"] \arrow[d, "C"] \arrow[dr, "CP"] & \pi^+ \to \mu^+_L + \cancel{\nu_R} \\
\pi^- \to \mu^-_R + \cancel{\overline{\nu}_L} & \pi^- \to \mu^-_L + \overline{\nu}_R
\end{tikzcd}$$

ν_R or $\overline{\nu}_L$ do not exist

P or C transformed decay processes do not exist:
→ P and C violation. (**if you can see handedness**)

However no CP violation.

Neutral Particle Anti-particle System

$|P\rangle,\ |\overline{P}\rangle$ particle, antiparticle state **at rest**

Eigenstates of strong and electromagnetic interactions and flavour-

$$(H_s + H_{em})|P\rangle = m|P\rangle,\ (H_s + H_{em})|\overline{P}\rangle = \overline{m}|\overline{P}\rangle$$

$$F|P\rangle = +|P\rangle,\ F|\overline{P}\rangle = -|\overline{P}\rangle$$

CP and T transformation act as

$$CP|P\rangle = e^{i\theta_{\mathrm{CP}}}|\overline{P}\rangle,\ CP|\overline{P}\rangle = e^{-i\theta_{\mathrm{CP}}}|P\rangle$$ so that $CP \cdot CP|P\rangle = |P\rangle$

$$T|P\rangle = e^{i\theta_{\mathrm{T}}}|P\rangle,\ T|\overline{P}\rangle = e^{i\overline{\theta}_{\mathrm{T}}}|\overline{P}\rangle$$

If strong and electromagnetic interactions conserve CPT, i.e.

$$\overrightarrow{(CPT)}(H_s + H_{em})\overrightarrow{(CPT)}^{\dagger} = H_s + H_{em}$$

$$
\begin{aligned}
m &= \left\langle P \middle| H_s + H_{em} \middle| P \right\rangle \\
&= \left\langle P \middle| (\overrightarrow{CPT})^{\dagger} (\overrightarrow{CPT})(H_s + H_{em})(\overrightarrow{CPT})^{\dagger} (\overrightarrow{CPT}) \middle| P \right\rangle \\
&= \left\langle P \middle| (\overrightarrow{CPT})^{\dagger} (H_s + H_{em})(\overrightarrow{CPT}) \middle| P \right\rangle \\
&= \left\{ \left\langle P \middle| (\overleftarrow{CPT})^{\dagger} (H_s + H_{em})(\overrightarrow{CPT}) \middle| P \right\rangle \right\}^{*} \\
&= \left\langle \overline{P} \middle| H_s + H_{em} \middle| \overline{P} \right\rangle \\
&= \overline{m}
\end{aligned}
$$

Rest masses of particle and anti-particle are identical.

We always assume this.

Time development of the particle and antiparticle

$t = 0$: only strong and electromagnetic interactions
$t > 0$: weak interactions are switched on and P and $\overline{\mathrm{P}}$ start to decay,

$|f\rangle$ weak interaction decay products
eigenstates of strong and electromagnetic interactions

$$\left(H_s + H_{em}\right)|f\rangle = E_f|f\rangle$$

A general state

$$|\psi(t)\rangle = a(t)|P\rangle + b(t)|\overline{P}\rangle + \sum_f c_f(t)|f\rangle$$

Obtained by solving Schrödeinger equation,

$$i\frac{\partial}{\partial t}|\psi(t)\rangle = \left(H_s + H_w + H_{em}\right)|\psi(t)\rangle$$

$|a(t)|^2$: fraction of P, $|b(t)|^2$: fraction of $\overline{P}$, $|c_f(t)|^2$: fraction of f

Initially at $t = 0$

$$|a(0)|^2 + |b(0)|^2 = 1$$

$$|c_f(0)|^2 = 0$$

Due to decays, $t > 0$

$$|a(t)|^2 + |b(t)|^2 = \text{decreases}$$

$$|c_f(t)|^2 = \text{increases}$$

$$|a(t)|^2 + |b(t)|^2 + \sum_f |c_f(t)|^2 = 1$$

unitarity of Hamiltonian.

$H^\dagger = H$ i.e. $\{\langle\alpha|H|\beta\rangle\}^* = \langle\beta|H^\dagger|\alpha\rangle = \langle\beta|H|\alpha\rangle$

Solution of the particle anti-particle system

By introducing $$\left|\psi(t)\right\rangle = e^{-i(H_s+H_{em})t}\left|\psi(t)\right\rangle_{\mathrm{D}}$$

the Schrödinger equation

$$i\frac{\partial}{\partial t}\left|\psi(t)\right\rangle = \left(H_s + H_w + H_{em}\right)\left|\psi(t)\right\rangle$$

becomes

$$\left(H_s + H_{em}\right)e^{-i(H_s+H_{em})t}\left|\psi(t)\right\rangle_D + e^{-i(H_s+H_{em})t} i\frac{\partial}{\partial t}\left|\psi(t)\right\rangle_D$$

$$= \left(H_s + H_w + H_{em}\right)e^{-i(H_s+H_{em})t}\left|\psi(t)\right\rangle_{\mathrm{D}}$$

By operating $e^{i(H_s+H_{em})t}$ from the left

$$i\frac{\partial}{\partial t}\left|\psi(t)\right\rangle_{\mathrm{D}} = V(t)\left|\psi(t)\right\rangle_{\mathrm{D}}$$ Dirac representation

where $$V(t) = e^{i(H_s+H_{em})t} H_w e^{-i(H_s+H_{em})t}$$

By introducing,

$$\hat{a}(t)=a(t)e^{i(H_s+H_{em})t},\ \hat{b}(t)=b(t)e^{i(H_s+H_{em})t},\ \chi_f(t)=c_f(t)e^{i(H_s+H_{em})t}$$

we get

$$\begin{aligned}\left|\psi(t)\right\rangle_{\mathrm{D}} &= e^{i(H_s+H_{em})t}\left|\psi(t)\right\rangle \\ &= e^{i(H_s+H_{em})t}\left[a(t)\left|P\right\rangle+b(t)\left|\overline{P}\right\rangle+\sum_f c_f(t)\left|f\right\rangle\right] \\ &= \hat{a}(t)\left|P\right\rangle+\hat{b}(t)\left|\overline{P}\right\rangle+\sum_f \hat{c}_f(t)\left|f\right\rangle\end{aligned}$$

and

$$i\frac{\partial}{\partial t}\left|\psi(t)\right\rangle_{\mathrm{D}}=V(t)\left|\psi(t)\right\rangle_{\mathrm{D}}$$

become

$$\begin{aligned}&i\frac{\partial}{\partial t}\hat{a}(t)\left|P\right\rangle+i\frac{\partial}{\partial t}\hat{b}(t)\left|\overline{P}\right\rangle+\sum_f i\frac{\partial}{\partial t}\chi_f(t)\left|f\right\rangle \\ &\qquad=V(t)\hat{a}(t)\left|P\right\rangle+V(t)\hat{b}(t)\left|\overline{P}\right\rangle+\sum_f V(t)\chi_f(t)\left|f\right\rangle\end{aligned}$$

By operating $\langle P|, \langle \overline{P}|, \langle f|$ from left, it follows that

$$i\frac{\partial}{\partial t}\hat{a}(t) = \langle P|H_w|P\rangle \hat{a}(t) + \langle P|H_w|\overline{P}\rangle \hat{b}(t) + \sum_f e^{i(m_0 - E_f)t}\langle P|H_w|f\rangle \chi_f(t)$$

$$i\frac{\partial}{\partial t}\hat{b}(t) = \langle \overline{P}|H_w|P\rangle \hat{a}(t) + \langle \overline{P}|H_w|\overline{P}\rangle \hat{b}(t) + \sum_f e^{i(m_0 - E_f)t}\langle \overline{P}|H_w|f\rangle \chi_f(t)$$

and

$$i\frac{\partial}{\partial t}\chi_f(t) = e^{i(E_f - m_0)t}\langle f|H_w|P\rangle \hat{a}(t) + e^{i(E_f - m_0)t}\langle f|H_w|\overline{P}\rangle \hat{b}(t)$$

$$\boxed{+\sum_{f'} e^{i(E_f - E_{f'})t}\langle f|H_w|f'\rangle \chi_{f'}(t)}$$

Wigner-Weiskopf approximation:

ignoring the weak interaction between the final states.

$$\langle f|H_w|f'\rangle = 0$$

It follows that

$$\frac{\partial}{\partial t}\chi_f(t) = -ie^{i(E_f-m_0)t}\left[\langle f|H_w|P\rangle \hat{a}(t) + \langle f|H_w|\overline{P}\rangle \hat{b}(t)\right]$$

Integration over t

$$\chi_f(t) = -i\int dt e^{i(E_f-m_0)t}\left[\langle f|H_w|P\rangle \hat{a}(t) + \langle f|H_w|\overline{P}\rangle \hat{b}(t)\right]$$

$$= \lim_{\varepsilon\to+0}\frac{e^{i(E_f-m_0)t}}{m_0-E_f+i\varepsilon}\left[\langle f|H_w|P\rangle \hat{a}(t) + \langle f|H_w|\overline{P}\rangle \hat{b}(t)\right]$$

$$\boxed{+\lim_{\varepsilon\to+0}\frac{e^{i(E_f-m_0)t}}{m_0-E_f+i\varepsilon}\left[\langle f|H_w|P\rangle \frac{\partial}{\partial t}\hat{a}(t) + \langle f|H_w|\overline{P}\rangle \frac{\partial}{\partial t}\hat{b}(t)\right]}$$

By recalling

$$i\frac{\partial}{\partial t}\hat{a}(t) = \langle P|H_w|P\rangle \hat{a}(t) + \langle P|H_w|\overline{P}\rangle \hat{b}(t) + \sum_f e^{i(m_0-E_f)t}\langle P|H_w|f\rangle \chi_f(t)$$

$$i\frac{\partial}{\partial t}\hat{b}(t) = \langle \overline{P}|H_w|P\rangle \hat{a}(t) + \langle \overline{P}|H_w|\overline{P}\rangle \hat{b}(t) + \sum_f e^{i(m_0-E_f)t}\langle \overline{P}|H_w|f\rangle \chi_f(t)$$

the second term can be truncated since $H_w{}^2$

Using

$$\chi_f(t) = \lim_{\varepsilon\to+0} \frac{e^{i(E_f-m_0)t}}{m_0 - E_f + i\varepsilon}\left[\langle f|H_w|P\rangle \hat{a}(t) + \langle f|H_w|\overline{P}\rangle \hat{b}(t)\right]$$

it follows

$$i\frac{\partial}{\partial t}\hat{a}(t) = \langle P|H_w|P\rangle \hat{a}(t) + \langle P|H_w|\overline{P}\rangle \hat{b}(t) + \sum_f e^{i(m_0-E_f)t}\langle P|H_w|f\rangle \chi_f(t)$$

$$= \langle P|H_w|P\rangle \hat{a}(t) + \langle P|H_w|\overline{P}\rangle \hat{b}(t)$$

$$+ \lim_{\varepsilon\to+0}\sum_f \frac{\langle P|H_w|f\rangle\langle f|H_w|P\rangle \hat{a}(t) + \langle P|H_w|f\rangle\langle f|H_w|\overline{P}\rangle \hat{b}(t)}{m_0 - E_f + i\varepsilon}$$

Now $\chi_f(t)$ is eliminated from the equations!

Similarly,

$$i\frac{\partial}{\partial t}\hat{b}(t) = \langle \overline{P}|H_w|P\rangle \hat{a}(t) + \langle \overline{P}|H_w|\overline{P}\rangle \hat{b}(t)$$

$$+ \lim_{\varepsilon\to+0}\sum_f \frac{\langle \overline{P}|H_w|f\rangle\langle f|H_w|P\rangle \hat{a}(t) + \langle \overline{P}|H_w|f\rangle\langle f|H_w|\overline{P}\rangle \hat{b}(t)}{m_0 - E_f + i\varepsilon}$$

By using the relation

$$\lim_{\varepsilon\to+0}\frac{1}{x+i\varepsilon}=\mathbf{P}\left(\frac{1}{x}\right)-i\pi\delta(x)$$

it follows that

$$\begin{aligned}
i\frac{\partial}{\partial t}\hat{a}(t) &= \langle P|H_w|P\rangle\hat{a}(t)+\langle P|H_w|\overline{P}\rangle\hat{b}(t)\\
&\quad+\lim_{\varepsilon\to+0}\sum_f\frac{\langle P|H_w|f\rangle\langle f|H_w|P\rangle\hat{a}(t)+\langle P|H_w|f\rangle\langle f|H_w|\overline{P}\rangle\hat{b}(t)}{m_0-E_f+i\varepsilon}\\
&= \langle P|H_w|P\rangle\hat{a}(t)+\langle P|H_w|\overline{P}\rangle\hat{b}(t)\\
&\quad+\sum_f\mathbf{P}\left[\frac{\langle P|H_w|f\rangle\langle f|H_w|P\rangle\hat{a}(t)+\langle P|H_w|f\rangle\langle f|H_w|\overline{P}\rangle\hat{b}(t)}{m_0-E_f}\right]\\
&\quad-i\pi\sum_f\left[\langle P|H_w|f\rangle\langle f|H_w|P\rangle\hat{a}(t)+\langle P|H_w|f\rangle\langle f|H_w|\overline{P}\rangle\hat{b}(t)\right]\delta\left(m_0-E_f\right)
\end{aligned}$$

$$
\begin{aligned}
& i\frac{\partial}{\partial t}\hat{a}(t) = \langle P|H_w|P\rangle \hat{a}(t) + \langle P|H_w|\overline{P}\rangle \hat{b}(t) \\
& \quad + \mathbf{P}\left(\sum_f \frac{\langle P|H_w|f\rangle\langle f|H_w|P\rangle \hat{a}(t) + \langle P|H_w|f\rangle\langle f|H_w|\overline{P}\rangle \hat{b}(t)}{m_0 - E_f}\right) \\
& \quad - i\pi\sum_f \left[\langle P|H_w|f\rangle\langle f|H_w|P\rangle \hat{a}(t) + \langle P|H_w|f\rangle\langle f|H_w|\overline{P}\rangle \hat{b}(t)\right]\delta\left(m_0 - E_f\right) \\
& = \left[\langle P|H_w|P\rangle + \sum_f \mathbf{P}\left(\frac{\langle P|H_w|f\rangle\langle f|H_w|P\rangle}{m_0 - E_f}\right) - i\pi\sum_f \langle P|H_w|f\rangle\langle f|H_w|P\rangle \delta\left(m_0 - E_f\right)\right]\hat{a}(t) \\
& + \left[\langle P|H_w|\overline{P}\rangle + \sum_f \mathbf{P}\left(\frac{\langle P|H_w|f\rangle\langle f|H_w|\overline{P}\rangle}{m_0 - E_f}\right) - i\pi\sum_f \langle P|H_w|f\rangle\langle f|H_w|\overline{P}\rangle \delta\left(m_0 - E_f\right)\right]\hat{b}(t)
\end{aligned}
$$

Master equation describing the time evolution of P, $a(t)$ and $\overline{\mathrm{P}}$, $b(t)$.

$$i\frac{\partial}{\partial t}\begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = \mathbf{\Lambda}\begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$$

$$\mathbf{\Lambda} = \mathbf{M} - \frac{i}{2}\mathbf{\Gamma} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{pmatrix}$$

- Elements of mass (M) and decay matrices (*Γ*)

$$M_{11} = m_0 + \langle P|H_W|P\rangle + \sum_f \mathbf{P}\left(\frac{\langle P|H_W|f\rangle\langle f|H_W|P\rangle}{m_0 - E_f}\right)$$

$$M_{22} = m_0 + \langle \overline{P}|H_W|\overline{P}\rangle + \sum_f \mathbf{P}\left(\frac{\langle \overline{P}|H_W|f\rangle\langle f|H_W|\overline{P}\rangle}{m_0 - E_f}\right)$$

f's are all possible P or $\overline{\mathrm{P}}$ decay states; virtual and real

$$\Gamma_{11} = 2\pi \sum_f \left| \langle P | H_W | f \rangle \right|^2 \delta\left(m_0 - E_f \right)$$

$$\Gamma_{22} = 2\pi \sum_f \left| \langle \overline{P} | H_W | f \rangle \right|^2 \delta\left(m_0 - E_f \right)$$

f's are all possible **real** decay states, i.e. Γ´s are decay widths.

$$M_{12} = \langle P | H_W | \overline{P} \rangle + \sum_f \mathbf{P} \left(\frac{\langle P | H_W | f \rangle \langle f | H_W | \overline{P} \rangle}{m_0 - E_f} \right)$$

f's are all possible decay states common to P and $\overline{\mathrm{P}}$; virtual and real

$$\Gamma_{12} = 2\pi \sum_f \langle P | H_W | f \rangle \langle f | H_W | \overline{P} \rangle \delta\left(m_0 - E_f \right)$$

f's are all possible **real** decay states, **common** to P and $\overline{\mathrm{P}}$.

NB: $\mathbf{M}^\dagger = \mathbf{M}$, $\mathbf{\Gamma}^\dagger = \mathbf{\Gamma}$, but $\mathbf{\Lambda}^\dagger = (\mathbf{M} - i\mathbf{\Lambda}/2)^\dagger \neq \mathbf{\Lambda} = \mathbf{M} - i\mathbf{\Lambda}/2$

$\Longleftrightarrow$ $|a(t)|^2 + |b(t)|^2$: not conserved

CPT, CP and T properties

If H_{W} is invariant under T transformation,

$$\left\langle P \middle| H_w \middle| \overline{P} \right\rangle = \left\langle P \middle| \vec{T}^{\dagger} \vec{T} H_w \vec{T}^{\dagger} \vec{T} \middle| \overline{P} \right\rangle$$

$$= \left\langle P \middle| \vec{T}^{\dagger} H_w \vec{T} \middle| \overline{P} \right\rangle$$

$$= \left\{ \left\langle P \middle| \overleftarrow{T}^{\dagger} H_w \vec{T} \middle| \overline{P} \right\rangle \right\}^{*}$$

$$= \left\langle \overline{P} \middle| H_w^{\dagger} \middle| P \right\rangle e^{i\left(\theta_T - \overline{\theta}_T\right)}$$

H_w is hermitian

$$= \left\langle \overline{P} \middle| H_w \middle| P \right\rangle e^{i\left(\theta_T - \overline{\theta}_T\right)}$$

For $\left\langle P \middle| H_w \middle| P \right\rangle$ we just get a trivial relation

$$\left\langle P \middle| H_w \middle| P \right\rangle = \left\langle P \middle| H_w \middle| P \right\rangle$$

$$\sum_f \langle P|H_w|f\rangle\langle f|H_w|\overline{P}\rangle = \sum_f \langle P|\vec{T}^\dagger\vec{T}H_w\vec{T}^\dagger\vec{T}|f\rangle\langle f|\vec{T}^\dagger\vec{T}H_w\vec{T}^\dagger\vec{T}|\overline{P}\rangle$$

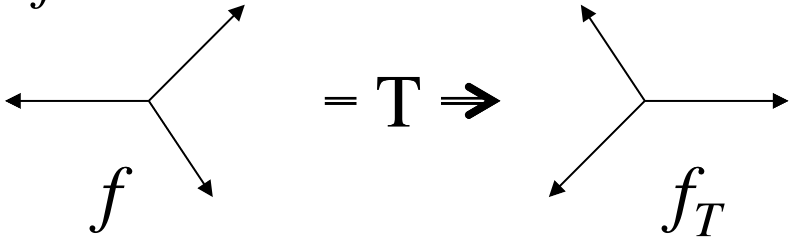
$$= \sum_f \langle P|\vec{T}^\dagger H_w\vec{T}|f\rangle\langle f|\vec{T}^\dagger H_w\vec{T}|\overline{P}\rangle$$

$$= \sum_f \left\{\langle P|\overleftarrow{T}^\dagger H_w\vec{T}|f\rangle\right\}^* \left\{\langle f|\overleftarrow{T}^\dagger H_w\vec{T}|\overline{P}\rangle\right\}^*$$

T transformed state

$$= \sum_f \langle \overline{P}|H_w\boxed{|f_T\rangle\langle_T f|}H_w|P\rangle e^{i(\theta_T-\overline{\theta}_T)}$$

f is summed over all final states of all possible kinematics:



$$\sum_f |f\rangle\langle f| = \sum_f |f_T\rangle\langle_T f|$$

$$\sum_f \langle P|H_w|f\rangle\langle f|H_w|\overline{P}\rangle = \sum_f \langle\overline{P}|H_w|f\rangle\langle f|H_w|P\rangle e^{i(\theta_T-\overline{\theta}_T)}$$

$$\boxed{\text{Therefore,}\quad \Lambda_{12} = \Lambda_{21}e^{i(\theta_T-\overline{\theta}_T)}}$$

If H_W is invariant under CP transformation,

$$\begin{aligned}\left\langle P\middle|H_w\middle|\overline{P}\right\rangle &= \left\langle P\middle|(CP)^{\dagger}CPH_w(CP)^{\dagger}CP\middle|\overline{P}\right\rangle \\ &= \left\langle P\middle|(CP)^{\dagger}H_wCP\middle|\overline{P}\right\rangle \\ &= \left\langle \overline{P}\middle|H_w\middle|P\right\rangle e^{-i2\theta_{CP}}\end{aligned}$$

$$\begin{aligned}\left\langle P\middle|H_w\middle|P\right\rangle &= \left\langle P\middle|(CP)^{\dagger}CPH_w(CP)^{\dagger}CP\middle|P\right\rangle \\ &= \left\langle P\middle|(CP)^{\dagger}H_wCP\middle|P\right\rangle \\ &= \left\langle \overline{P}\middle|H_w\middle|\overline{P}\right\rangle\end{aligned}$$

$$\sum_f \langle P|H_w|f\rangle\langle f|H_w|\overline{P}\rangle$$

$$= \sum_f \left\langle P\middle|(CP)^\dagger CPH_w(CP)^\dagger CP\middle|f\right\rangle\left\langle f\middle|(CP)^\dagger CPH_w(CP)^\dagger CP\middle|\overline{P}\right\rangle$$

$$= \sum_f \left\langle P\middle|(CP)^\dagger H_wCP\middle|f\right\rangle\left\langle f\middle|(CP)^\dagger H_wCP\middle|\overline{P}\right\rangle$$

$$= \sum_{\bar{f}} \langle \overline{P}|H_w|\bar{f}\rangle\langle \bar{f}|H_w|P\rangle e^{-2i\theta_{CP}}$$

f is summed over all final states of all possible kinematics:

$$\sum_f |f\rangle\langle f| = \sum_{\bar{f}} |\bar{f}\rangle\langle\bar{f}|$$

$$\sum_f \langle P|H_w|f\rangle\langle f|H_w|\overline{P}\rangle = \sum_f \langle \overline{P}|H_w|f\rangle\langle f|H_w|P\rangle e^{-2i\theta_{CP}}$$

Therefore, $\Lambda_{12} = \Lambda_{21}e^{-2i\theta_{CP}}$ $\Lambda_{11} = \Lambda_{22}$

Similar for CP and CPT transformations and

If CPT is conserved, $\Lambda_{11} = \Lambda_{22}$

If T is conserved, $|\Lambda_{12}| = |\Lambda_{21}|$

If CP is conserved, $\Lambda_{11} = \Lambda_{22}$ and $|\Lambda_{12}| = |\Lambda_{21}|$

$$\Downarrow$$

If $\Lambda_{11} \neq \Lambda_{22}$, CPT and CP are not conserved

If $|\Lambda_{12}| \neq |\Lambda_{21}|$, T and CP are not conserved

i.e. CP is not conserved for the both cases.

We assume CPT.

$$M_{11} = M_{22} = M,\ \Gamma_{11} = \Gamma_{22} = \Gamma \text{ i.e. } \Lambda_{11} = \Lambda_{22} = \Lambda.$$

NB: for charged particles, i.e. P = P$^+$ and $\overline{\text{P}}$ = P$^-$

no common decay final states between P$^+$ and P$^-$

$$M_{12} = \langle P|H_W|\overline{P}\rangle + \sum_f \mathbf{P}\left(\frac{\langle P|H_W|f\rangle\langle f|H_W|\overline{P}\rangle}{m_0 - E_f}\right) = 0$$

$$\Gamma_{12} = 2\pi\sum_f \langle P|H_W|f\rangle\langle f|H_W|\overline{P}\rangle\delta(m_0 - E_f) = 0$$

$$i\frac{\partial}{\partial t}\begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = \begin{pmatrix} M - i\,\Gamma/2 & 0 \\ 0 & M - i\,\Gamma/2 \end{pmatrix}\begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$$

$$\left.\begin{array}{l} |\,a(t)\,|^2 \propto \exp\,(-\Gamma\,t) \\ |\,b(t)\,|^2 \propto \exp\,(-\Gamma\,t) \end{array}\right\} \textbf{simple exponential decays}$$

Master equation becomes

$$i\frac{\partial}{\partial t}\begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = \begin{pmatrix} \Lambda & \Lambda_{12} \\ \Lambda_{21} & \Lambda \end{pmatrix}\begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$$

$$= \begin{pmatrix} M - \frac{i}{2}\Gamma & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^{*} - \frac{i}{2}\Gamma_{12}^{*} & M - \frac{i}{2}\Gamma \end{pmatrix}\begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$$

$$i\frac{\partial a(t)}{\partial t} = \Lambda a(t) + \Lambda_{12} b(t) \longrightarrow b(t) = \frac{1}{\Lambda_{12}}\left[i\frac{\partial a(t)}{\partial t} - \Lambda a(t)\right]$$

$$i\frac{\partial b(t)}{\partial t} = \Lambda_{21} a(t) + \Lambda b(t)$$

inserting $b(t)$

$$\downarrow$$

$$\frac{\partial^2 a(t)}{\partial t^2} + 2i\Lambda\frac{\partial a(t)}{\partial t} + \left(\Lambda_{12}\Lambda_{21} - \Lambda^2\right)a(t) = 0$$

General solution is given by

$$a(t) = C_+ e^{-i\lambda_+ t} + C_- e^{-i\lambda_- t}$$

$C_\pm$ defined by the initial conditions.

where

$$\lambda_\pm^2 - 2\Lambda\lambda_\pm - \left(\Lambda_{12}\Lambda_{21} - \Lambda^2\right) = 0$$

i.e.

$$\lambda_\pm = \Lambda \pm \sqrt{\Lambda_{12}\Lambda_{21}}$$

are the eigenvalues of $\mathbf{\Lambda}$.

Then,

$$b(t) = \zeta\left(C_+ e^{-i\lambda_+ t} - C_- e^{-i\lambda_- t}\right)$$

where

$$\zeta = \sqrt{\frac{\Lambda_{21}}{\Lambda_{12}}}$$

An initial condition: P is produced at $t = 0$

$$\left|\psi(0)\right\rangle = \left|P\right\rangle, \text{ i.e.} \qquad a(0) = C_+ + C_- = 1,\ b(0) = C_+ - C_- = 0$$

$$C_+ = C_- = \frac{1}{2}$$

$$\begin{aligned}\left|P(t)\right\rangle &= \frac{1}{2}\left(e^{-i\lambda_+ t} + e^{-i\lambda_- t}\right)\left|P\right\rangle + \zeta\frac{1}{2}\left(e^{-i\lambda_+ t} - e^{-i\lambda_- t}\right)\left|\overline{P}\right\rangle \\ &\equiv f_+(t)\left|P\right\rangle + \zeta f_-(t)\left|\overline{P}\right\rangle \\ &= \frac{e^{-i\lambda_+ t}}{2}\left(\left|P\right\rangle + \zeta\left|\overline{P}\right\rangle\right) + \frac{e^{-i\lambda_- t}}{2}\left(\left|P\right\rangle - \zeta\left|\overline{P}\right\rangle\right) \\ &\equiv \frac{\sqrt{1+\left|\zeta\right|^2}}{2}\left(e^{-i\lambda_+ t}\left|P_+\right\rangle + e^{-i\lambda_- t}\left|P_-\right\rangle\right)\end{aligned}$$

$$f_\pm(t) = \frac{1}{2}\left(e^{-i\lambda_+ t} \pm e^{-i\lambda_- t}\right) \qquad\qquad \left|P_\pm\right\rangle = \frac{1}{\sqrt{1+\left|\zeta\right|^2}}\left(\left|P\right\rangle \pm \zeta\left|\overline{P}\right\rangle\right)$$

It follows that

$$\begin{aligned}\mathbf{\Lambda}|P_\pm\rangle &= \begin{pmatrix} \Lambda & \Lambda_{12} \\ \Lambda_{21} & \Lambda \end{pmatrix}\begin{pmatrix} 1 \\ \pm\zeta \end{pmatrix} \\ &= \begin{pmatrix} \Lambda \pm \sqrt{\Lambda_{12}\Lambda_{21}} \\ \Lambda_{21} \pm \Lambda\sqrt{\dfrac{\Lambda_{21}}{\Lambda_{12}}} \end{pmatrix} \\ &= \begin{pmatrix} \Lambda \pm \sqrt{\Lambda_{12}\Lambda_{21}} \\ \left(\sqrt{\Lambda_{12}\Lambda_{21}} \pm \Lambda\right)\sqrt{\dfrac{\Lambda_{21}}{\Lambda_{12}}} \end{pmatrix} \\ &= \left(\Lambda \pm \sqrt{\Lambda_{12}\Lambda_{21}}\right)\begin{pmatrix} 1 \\ \pm\zeta \end{pmatrix} \\ &= \lambda_\pm|P_\pm\rangle\end{aligned}$$

$P_\pm$ are eigenstates of $\lambda_\pm$

$$\boxed{\lambda_\pm \equiv m_\pm - \frac{i}{2}\Gamma_\pm;\ m_\pm = \mathrm{Re}\,\lambda_\pm,\ \Gamma_\pm = -2\,\mathrm{Im}\,\lambda_\pm}$$

mass and decay width

Oscillations between P and $\overline{\mathrm{P}}$

Probability for the initial P remains as P at a given time *t*:

$$\left|\langle P|P(t)\rangle\right|^2 = \left|f_+(t)\right|^2 = \frac{1}{4}\left(e^{-\Gamma_+ t} + e^{-\Gamma_- t} + 2e^{-\overline{\Gamma} t}\cos\Delta m t\right)$$

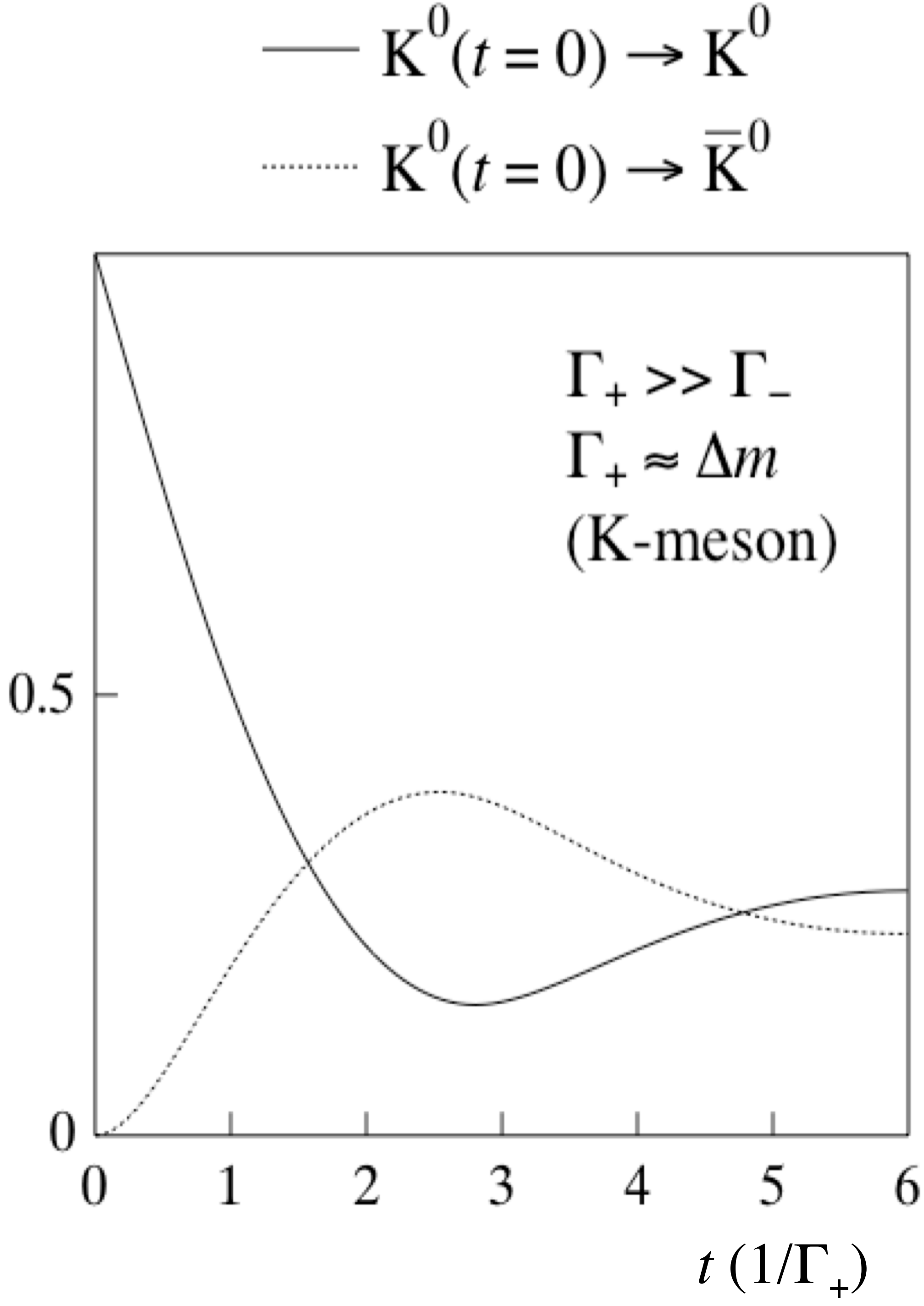
Probability for the initial P changes to $\overline{\mathrm{P}}$ at a given time *t*:

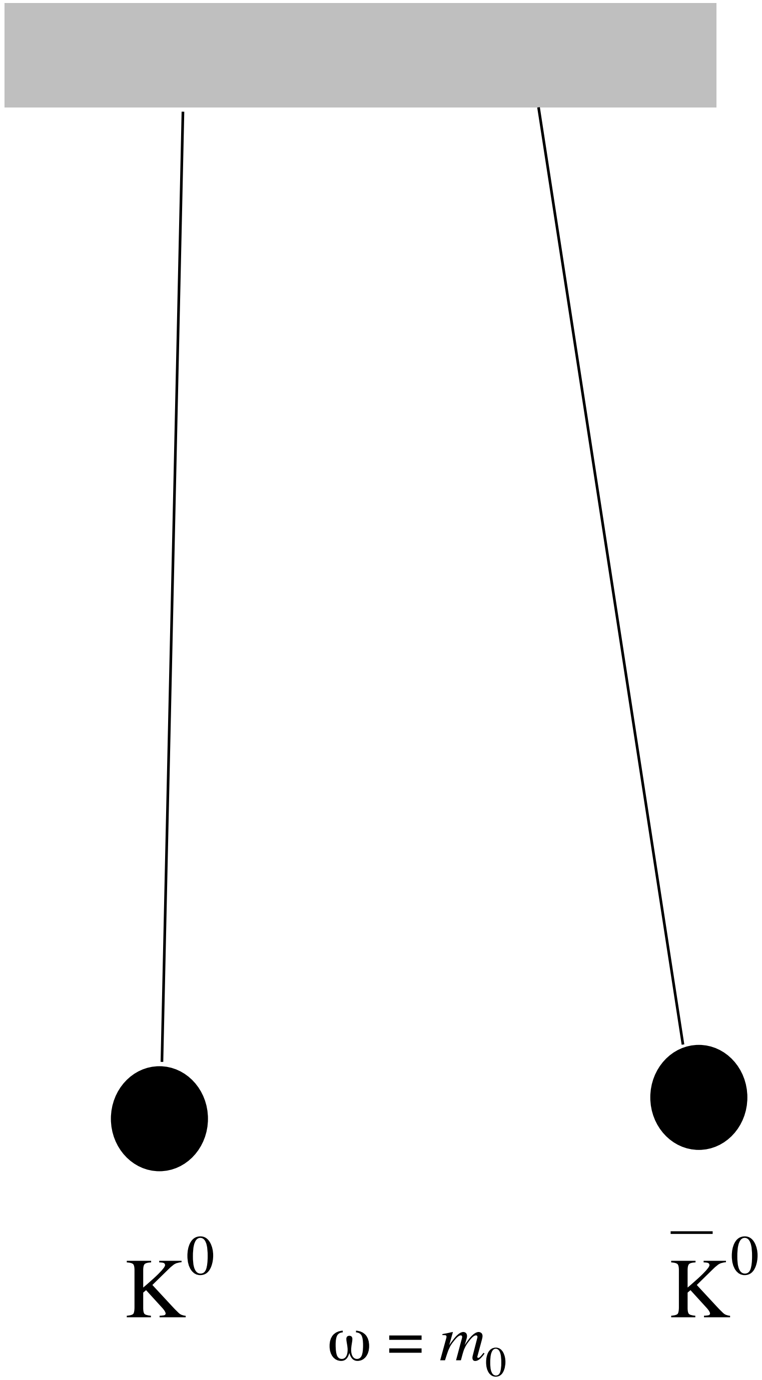
$$\left|\langle \overline{P}|P(t)\rangle\right|^2 = \left|\zeta f_-(t)\right|^2 = \frac{|\zeta|^2}{4}\left(e^{-\Gamma_+ t} + e^{-\Gamma_- t} - 2e^{-\overline{\Gamma} t}\cos\Delta m t\right)$$

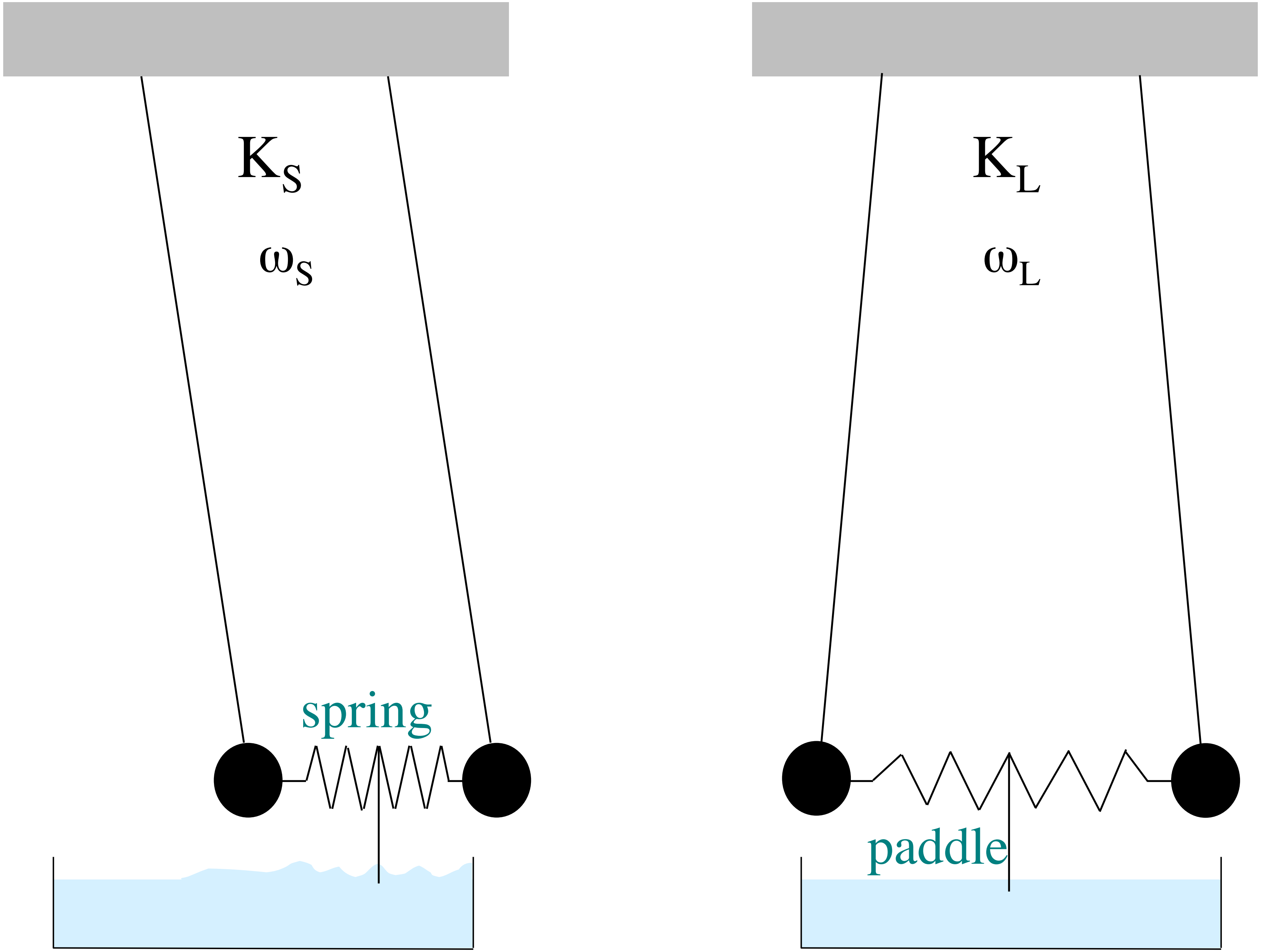
$$\overline{\Gamma} = \frac{\Gamma_+ + \Gamma_-}{2}, \ \Delta m = m_- - m_+$$

Flavour eigenstates P and $\overline{\mathrm{P}}$ are not “physical” states, i.e. have neither definite masses nor decay widths.

Time dependent probabilities
for the neutral kaon case.




K^0
$\bar{K}^0$
$\omega = m_0$


K_S
ω_S
spring
K_L
ω_L
paddle

Probability to find $P_\pm$ in the initial P at a given time *t*:

$$\left|\left\langle P_\pm \middle| P(t)\right\rangle\right|^2 = \frac{1+|\zeta|^2}{4} e^{-\Gamma_\pm t}$$

$P_\pm$ are physical states.

P and $\overline{\mathrm{P}}$ are orthogonal:

$$\langle P|P\rangle = \langle \overline{P}|\overline{P}\rangle = 1,\ \langle P|\overline{P}\rangle = 0$$

P_+ and P_- may not be orthogonal:

$$\langle P_+|P_+\rangle = \langle P_-|P_-\rangle = 1,\ \langle P_+|P_-\rangle = 1-|\zeta|^2$$

Another initial condition: $\overline{\mathrm{P}}$ is produced at $t = 0$

$$\left|\psi(0)\right\rangle = \left|\overline{P}\right\rangle, \text{ i.e.} \qquad a(0) = C_+ + C_- = 0,\ b(0) = C_+ - C_- = 1$$

$$C_+ = -C_- = \frac{1}{2}$$

$$\left|\overline{P}(t)\right\rangle = \frac{1}{\zeta} f_-(t)\left|P\right\rangle + f_+(t)\left|\overline{P}\right\rangle$$

$$= \frac{\sqrt{1+\left|\zeta\right|^2}}{2\zeta}\left(e^{-i\lambda_+ t}\left|P_+\right\rangle - e^{-i\lambda_- t}\left|P_-\right\rangle\right)$$

$$\boxed{\left|P(t)\right\rangle \longleftarrow \text{CP conjugated states} \longrightarrow \left|\overline{P}(t)\right\rangle}$$

CP and T violating case

Probability for the initial P changes to $\overline{\mathrm{P}}$ at a given time *t*:

$$\left|\langle \overline{P} \,|\, P(t) \rangle\right|^2 = \left|\zeta f_-(t)\right|^2 = \frac{|\zeta|^2}{4}\left(e^{-\Gamma_+ t} + e^{-\Gamma_- t} - 2e^{-\overline{\Gamma} t}\cos\Delta m t\right)$$

CP conjugated processes to each other.

Not identical if $\boxed{|\zeta| \neq 1}$ CP violation in the oscillation

NB: If H_w is invariant under CP: $\Lambda_{12} = \Lambda_{21} e^{-2i\theta_{CP}}$

$$\zeta = \sqrt{\frac{\Lambda_{21}}{\Lambda_{12}}} = e^{i(\theta_{CP} + n''\pi)}$$

i.e. $|\zeta| = 1$

Probability for the initial $\overline{\mathrm{P}}$ changes to P at a given time *t*:

$$\left|\langle P \,|\, \overline{P}(t) \rangle\right|^2 = \left|\frac{1}{\zeta} f_-(t)\right|^2 = \frac{1}{4|\zeta|^2}\left(e^{-\Gamma_+ t} + e^{-\Gamma_- t} - 2e^{-\overline{\Gamma} t}\cos\Delta m t\right)$$

Probability for the initial P changes to $\overline{\text{P}}$ at a given time *t*:

$$\left|\langle \overline{P}|P(t)\rangle\right|^2 = \left|\zeta f_-(t)\right|^2 = \frac{|\zeta|^2}{4}\left(e^{-\Gamma_+ t}+e^{-\Gamma_- t}-2e^{-\overline{\Gamma}t}\cos\Delta mt\right)$$

T conjugated processes to each other.
Not identical if $\boxed{|\zeta|\neq 1}$ T violation in the oscillation

NB: If H_w is invariant under T:

$$\Lambda_{12} = \Lambda_{21}e^{i(\theta_T-\overline{\theta}_T)}$$

$$\zeta = \sqrt{\frac{\Lambda_{21}}{\Lambda_{12}}} = e^{i\left(\frac{\overline{\theta}_T-\theta_T}{2}+n''\pi\right)}$$

i.e. $|\zeta| = 1$

Probability for the initial $\overline{\text{P}}$ changes to P at a given time *t*:

$$\left|\langle P|\overline{P}(t)\rangle\right|^2 = \left|\frac{1}{\zeta}f_-(t)\right|^2 = \frac{1}{4|\zeta|^2}\left(e^{-\Gamma_+ t}+e^{-\Gamma_- t}-2e^{-\overline{\Gamma}t}\cos\Delta mt\right)$$

CP and T violation in oscillations for the neutral kaon decays

Flavour eigenstates $\left\{ \begin{array}{l} K^0 = (d\bar{s}) \\ \bar{K}^0 = (\bar{d}s) \end{array} \right.$ CP conjugated

Identification of the initial state:

Initial state at $t = 0$: $p\bar{p}$ annihilation at rest

$$\bar{p}p \rightarrow \begin{cases} K^0 K^- \pi^+ \\ \bar{K}^0 K^+ \pi^- \end{cases}$$

$S = 0$ $S = 0$

$K^- = (\bar{u}s)$

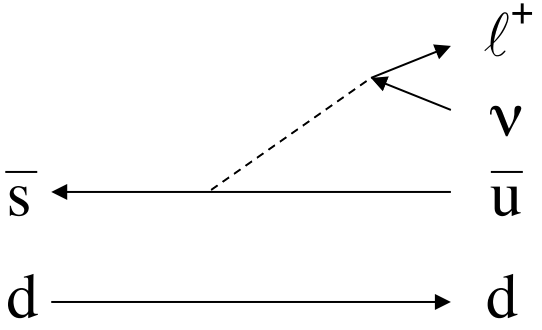
$K^+ = (u\bar{s})$

$$\boxed{K^+\pi^- \rightarrow \bar{K}^0,\ K^-\pi^+ \rightarrow K^0}$$

Identification of the final states:

Semileptonic decays are used to directly measure this...
i.e. flavour specific decay modes

$\Delta q = \Delta s$ rule



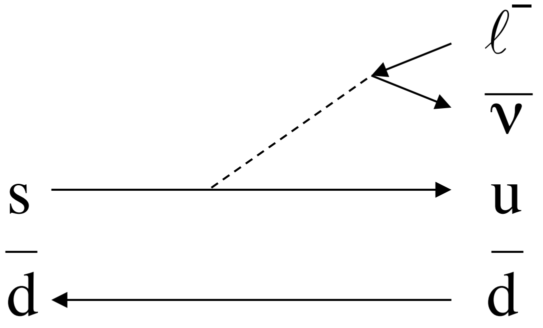
K^0 $\quad$ π^-

$q = 0$ $\quad$ $q = -1$

$s = +1$ $\quad$ $s = 0$

$\Delta q = -1$

$\Delta s = -1$



$\overline{K}^0$ $\quad$ π^+

$q = 0$ $\quad$ $q = +1$

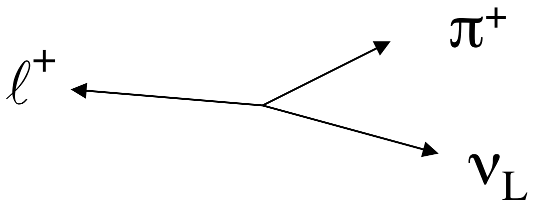
$s = -1$ $\quad$ $s = 0$

$\Delta q = +1$

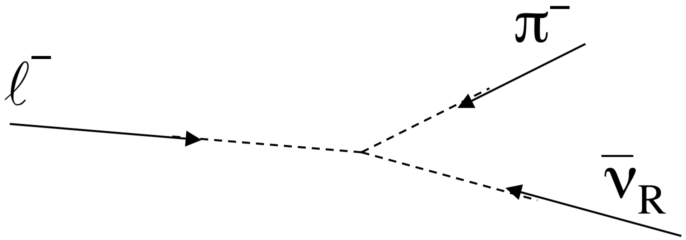
$\Delta s = +1$

final states are specific to the flavour,
i.e. the particle and the anti-particle

(no hadronic decay mode available for the neutral kaons)



$$\Gamma_{\ell^+\upsilon_\mathrm{L}\pi^-} = \int\limits_{\text{phase space}} d\Omega \Big|_{\text{out}} \Big\langle \ell^+(p_1)\upsilon_\mathrm{L}(p_2)\pi^-(p_3)\Big|H_W\Big|\mathrm{K}^0\Big\rangle\Big|^2$$



$$\Big\Downarrow \qquad \text{CPT}$$

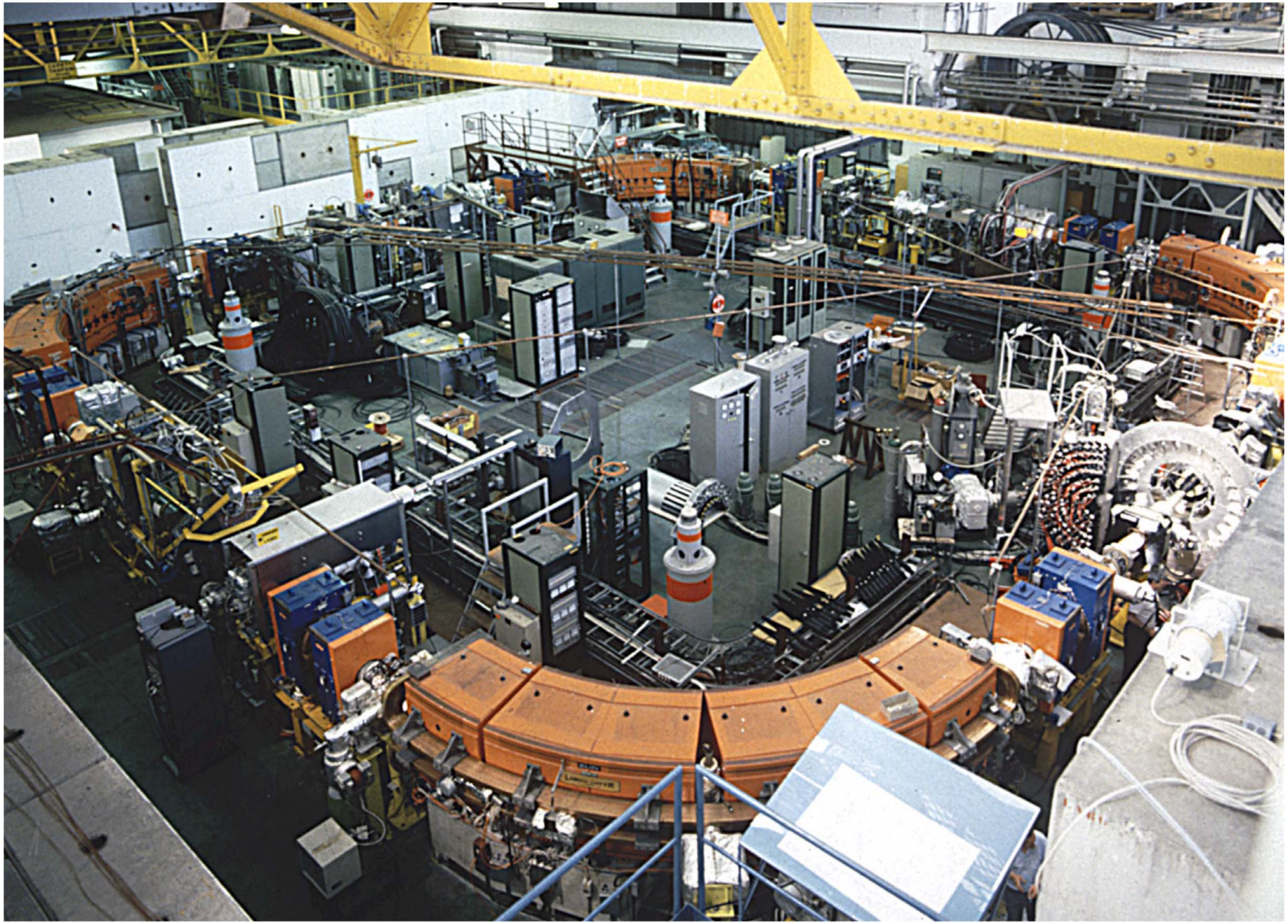
$$\int\limits_{\text{phase space}} d\Omega \Big|_{\text{in}} \Big\langle \ell^-(-p_1)\overline{\upsilon}_\mathrm{R}(-p_2)\pi^+(-p_3)\Big|H_W\Big|\overline{\mathrm{K}}^0\Big\rangle\Big|^2$$

$$= \int\limits_{\text{phase space}} d\Omega \Big|_{\text{out}} \Big\langle \ell^-(p_1)\overline{\upsilon}_\mathrm{R}(p_2)\pi^+(p_3)\Big|H_W\Big|\overline{\mathrm{K}}^0\Big\rangle\Big|^2$$

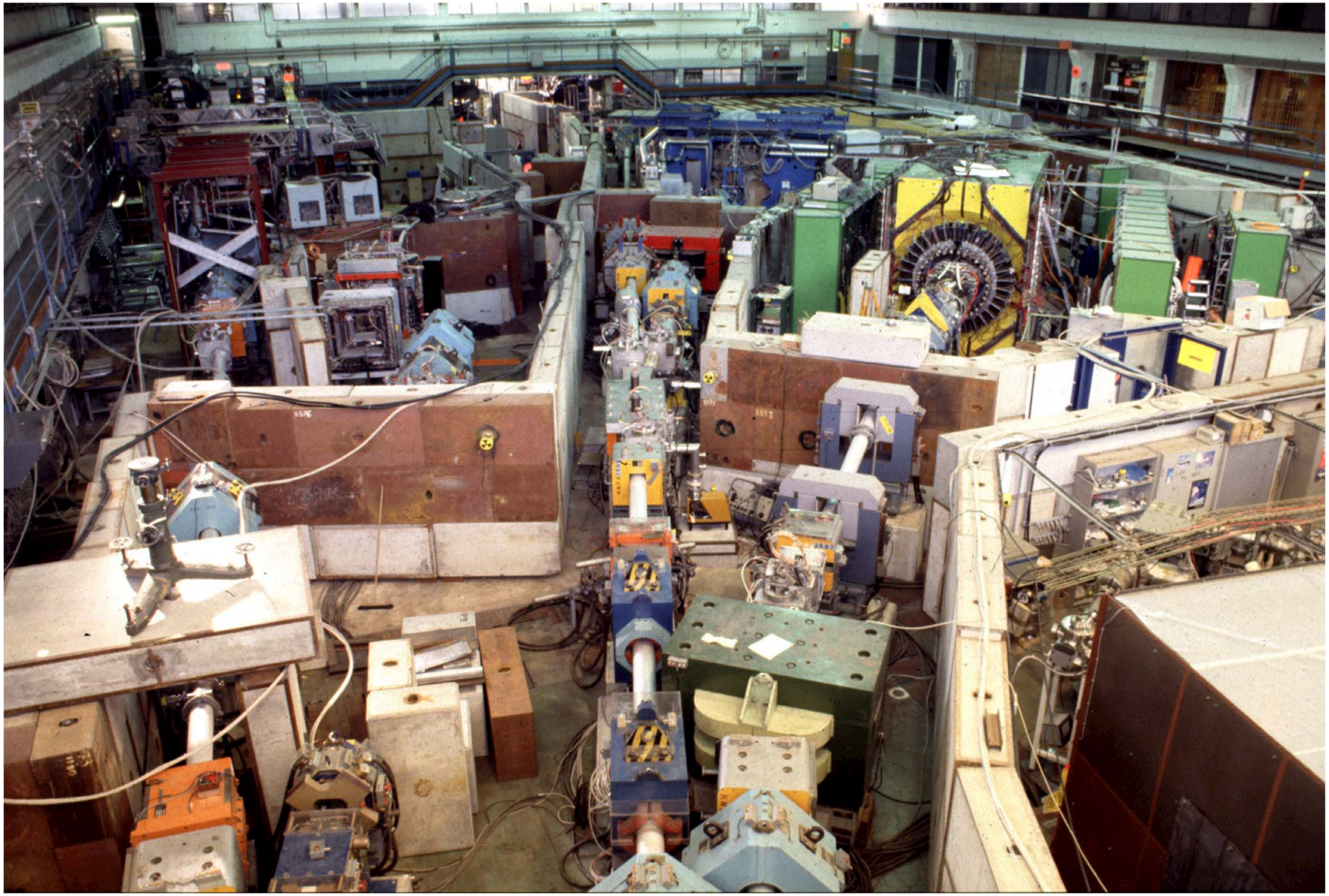
$$= \overline{\Gamma}_{\ell^-\overline{\nu}_R\pi^+}$$

Since the interactions between the final states are weak,

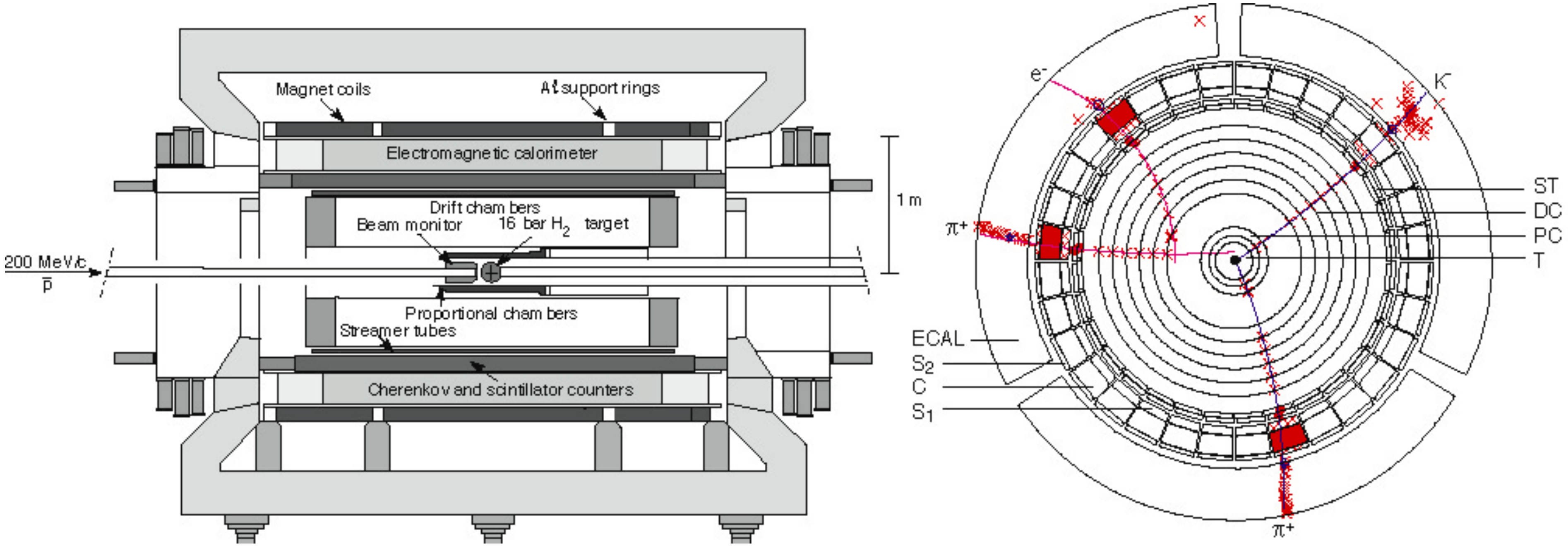
LEAR complex



CPLEAR experiment



CPLEAR experiment



$$p\bar{p} \Rightarrow K^-\pi^+K^0$$

$$K^0 \to \overline{K}^0$$

$$\overline{K}^0 \Rightarrow \pi^+e^-\bar{\nu}$$

$|\zeta|^2 |f_-(t)|^2$ probability for the initial K^0 oscillates to $\overline{\mathrm{K}}^0$

$|f_-(t)|^2 / |\zeta|^2$ probability for the initial $\overline{\mathrm{K}}^0$ oscillates to K^0

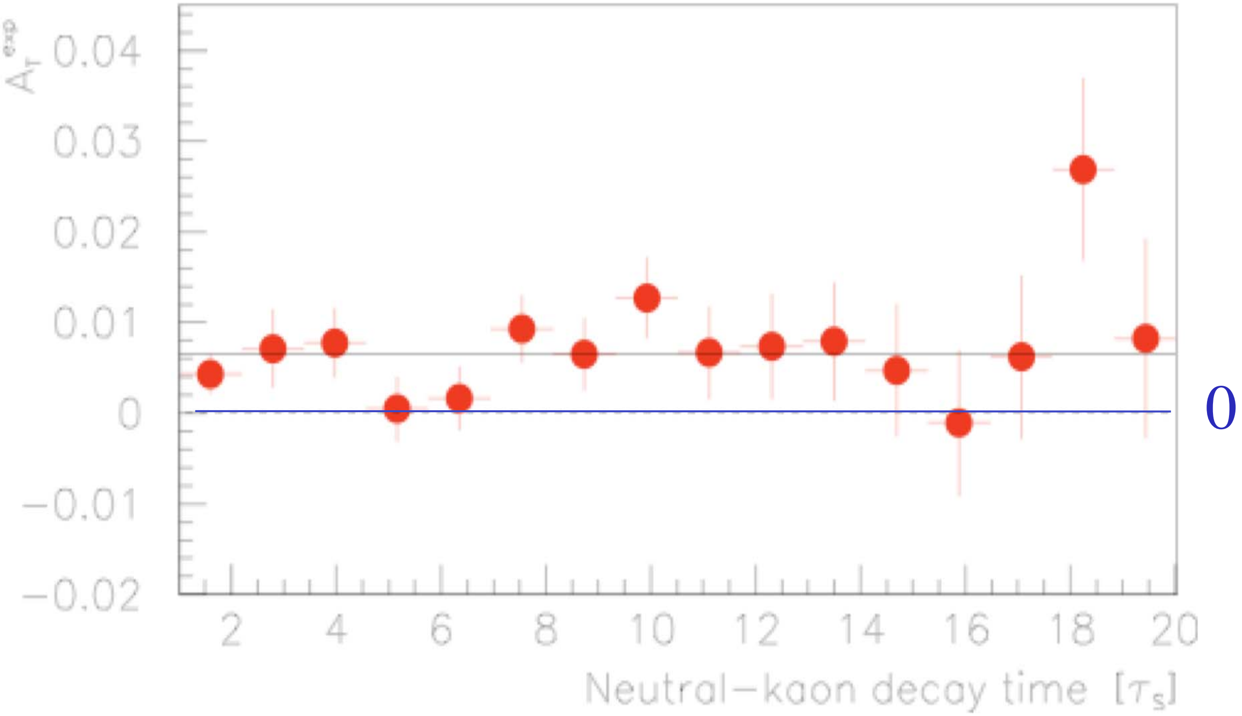
CP and T asymmetry

$$A_T(t) = \frac{\int d\Omega \left| \left\langle \ell^+ \nu_L \pi^- \middle| H_W \middle| \overline{\mathrm{K}}^0(t) \right\rangle \right|^2 - \int d\Omega \left| \left\langle \ell^- \overline{\upsilon}_R \pi^+ \middle| H_W \middle| \mathrm{K}^0(t) \right\rangle \right|^2}{\int d\Omega \left| \left\langle \ell^+ \nu_L \pi^- \middle| H_W \middle| \overline{\mathrm{K}}^0(t) \right\rangle \right|^2 + \int d\Omega \left| \left\langle \ell^- \overline{\upsilon}_R \pi^+ \middle| H_W \middle| \mathrm{K}^0(t) \right\rangle \right|^2}$$

$$= \frac{\frac{1}{|\zeta|^2} - |\zeta|^2}{\frac{1}{|\zeta|^2} + |\zeta|^2}$$

$$= \frac{1 - |\zeta|^4}{1 + |\zeta|^4}$$

CPLEAR



$$A_T = (6.6 \pm 1.6) \times 10^{-3}$$

$$A_T(t) = \frac{1-|\zeta|^4}{1+|\zeta|^4} \xrightarrow{\qquad\qquad} |\zeta| = 0.9967 \pm 0.0008 \neq 1$$

Small $\cancel{\mathrm{CP}}$ and $\cancel{\mathrm{T}}$ in K-$\overline{\mathrm{K}}$ oscillations