Double Beta Decay

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References

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Outline

- Double-beta decay
  - Nuclear beta decay
  - Double-beta decay with neutrinos
  - Neutrinoless double-beta decay
    - What can we learn ?
- Experimental challenges
- Experimental status
- Prospects

Nuclear Beta Decay

In general:

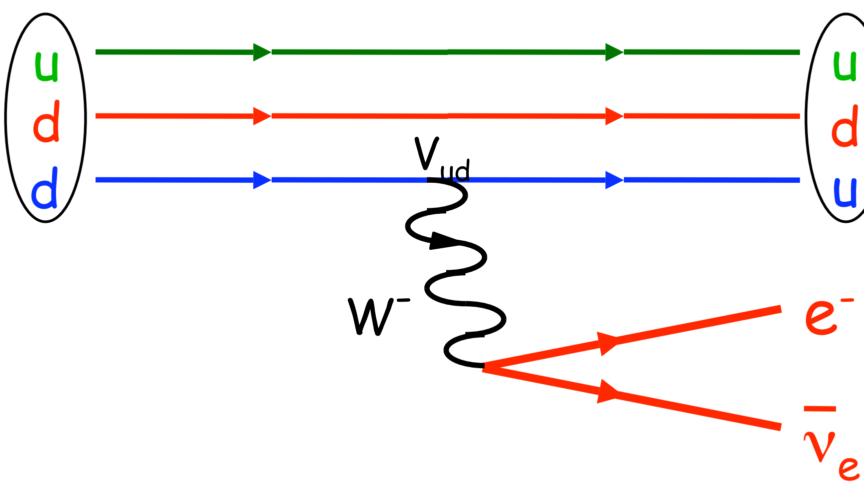
$$\boxed{{}^{A}_{Z}N \rightarrow {}^{A}_{Z+1}N + \beta^{-} + \bar{\nu}_e}$$

$$\boxed{{}^{A}_{Z}N \rightarrow {}^{A}_{Z-1}N + \beta^{+} + \nu_e}$$

Beta decay is actually due to the decay of a nucleon inside the nucleus with emission of a β particle and a (anti)neutrino:

$$n \rightarrow p + e^{-} + \bar{\nu}_e$$

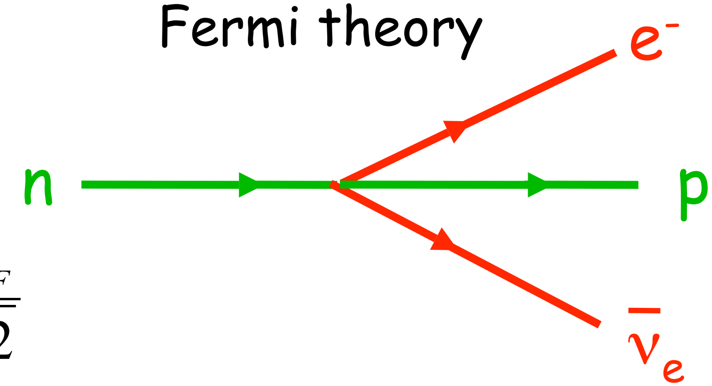
$$p \rightarrow n + e^{+} + \nu_e$$



$k^2 \ll M_W^2$

$$\frac{g^2}{8M_W^2} = \frac{G_F}{\sqrt{2}}$$

Fermi theory



where the Fermi constant,

$G_F = 1.12 \times 10^{-5}\ \mathrm{GeV}^{-2}$.

Classification of Nuclear Beta Decays

$$N_i \rightarrow N_f + \beta^- + \bar{\nu}_e$$

- l = angular momentum between the β^--$\bar{\nu}_e$ pair and N_f
- Allowed transition: l = 0;
- Forbidden transition: l > 0 or $M_f > M_i$
- For allowed transitions:

$$J_{tot} = J_{N_i} = J_{N_f} + S_{leptons} \Rightarrow \Delta J = \left| J_{N_i} - J_{N_f} \right| = S_{leptons}$$

Transition	β^--$\bar{\nu}_e$	Nuclear spin
Fermi	Singlet state	$\Delta J = 0$
Gamow-Teller	Triplet state	$\Delta J = 0, 1$ ($0 \not\rightarrow 0$)

- Examples:

Fermi transition: $^{14}_{8}O(J=0) \rightarrow {}^{14}_{7}N(J=0) + e^+ + \nu_e$

Gamow-Teller transition: $^{6}_{2}He(J=0) \rightarrow {}^{6}_{3}Li(J=1) + e^- + \bar{\nu}_e$

Mixed transition: $^{3}_{1}H(J \neq 0) \rightarrow {}^{3}_{2}He + e^- + \bar{\nu}_e$ $\Delta J = 0$

Nuclear Beta Decay (cont.)

- Apply Fermi's second golden rule, the transition rate is given by

$$W = \frac{2\pi}{\hbar}\left|M_{fi}\right|^2 \rho_f\left(E'\right) = \frac{2\pi}{\hbar}\left|\left\langle \psi_f \left| H_{\text{int}} \right| \psi_i \right\rangle\right|^2 \rho_f\left(E'\right)$$

where

M_{fi} = the matrix element of the underlying physical process,

$\rho_f(E')$ = the phase space of the final state with a total energy E' in the center-of-mass frame.

- For a nuclear beta decay, ignore the low-energy recoiling nucleus, N_f,

$$\frac{1}{T_{1/2}} \propto W = \frac{2\pi}{\hbar}\left|M_{fi}\right|^2 \rho_2\left(Q_\beta, Z\right)$$

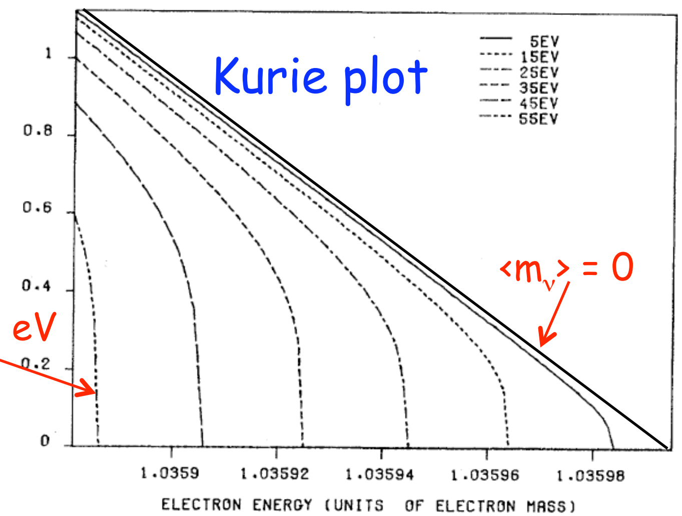
where $\rho_2(Q_\beta,Z)$ = phase space of β^- and $\bar{\nu}_e$ with total kinetic energy Q_β

$$\rho_2(Q_\beta, Z) \propto p_\beta^2 (E' - E_\beta)^2 \sqrt{1 - \left(\frac{\langle m_\nu \rangle c^2}{E' - E_\beta}\right)^2}\, dp_\beta \propto Q_\beta^5$$

Sargent rule

- In general,

$$W \propto G_F^2 \cos^2\theta_c Q_\beta^5$$



Double Beta Decay

- Double Beta Decay ($2\nu\beta\beta$):

$$(1) \qquad {}^{A}_{Z}N \rightarrow {}^{A}_{Z+2}N + 2e^{-} + 2\bar{\nu}_{e}$$

$$(2) \qquad {}^{A}_{Z}N \rightarrow {}^{A}_{Z-2}N + 2e^{+} + 2\nu_{e}$$

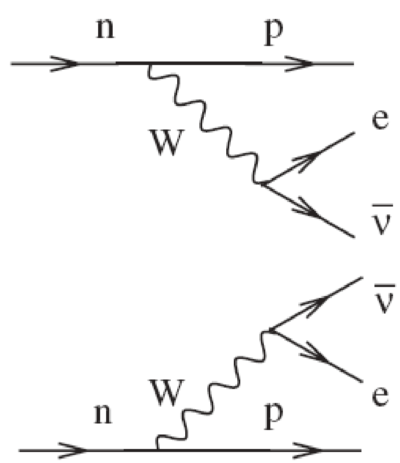
$$(3) \qquad e^{-} + {}^{A}_{Z}N \rightarrow {}^{A}_{Z-2}N + e^{+} + 2\nu_{e} \qquad \text{(electron capture)}$$

$$(4) \qquad 2e^{-} + {}^{A}_{Z}N \rightarrow {}^{A}_{Z-2}N + 2\nu_{e}$$

- The decay rate of process (1) was first estimated by Maria Goeppert-Mayer using the Fermi's theory in 1935:

$$W = \frac{2\pi}{\hbar}\left|M_{fi}(2\nu)\right|^{2}\rho_{4}\left(Q_{2\beta},Z\right) \propto G_{F}^{4}Q_{2\beta}^{11}$$

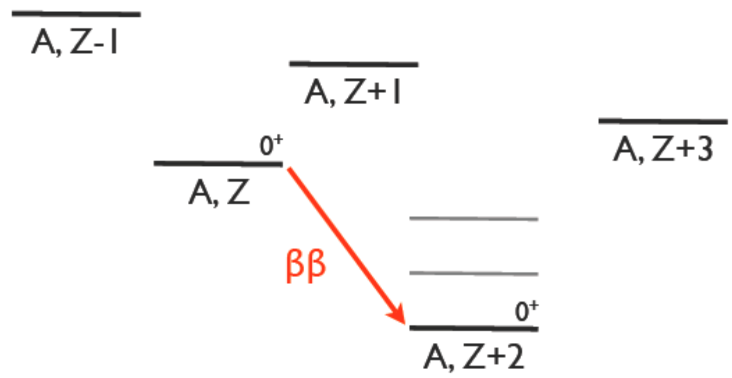
$$\Rightarrow \qquad \left[T_{1/2}^{2\nu\,2\beta}\right]^{-1} \approx 10^{19-25}\,\text{years}$$



where $\rho_4(Q_{2\beta},Z)$ is the four-particle phase space factor, $M_{fi}(2\nu)$ is the nuclear matrix element for the decay.

Some Specifics of Double-beta Decay

- It can only occur when beta decay is energetic forbidden.
- Only even-even nuclei can have 2ν-double-beta decay:

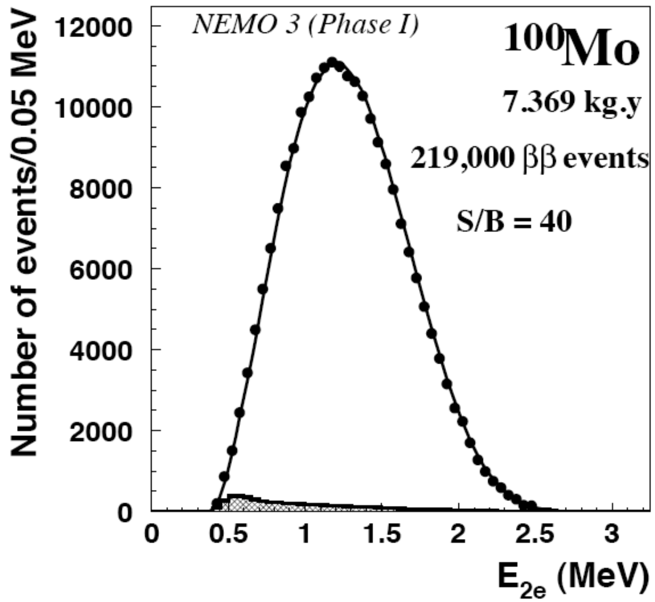


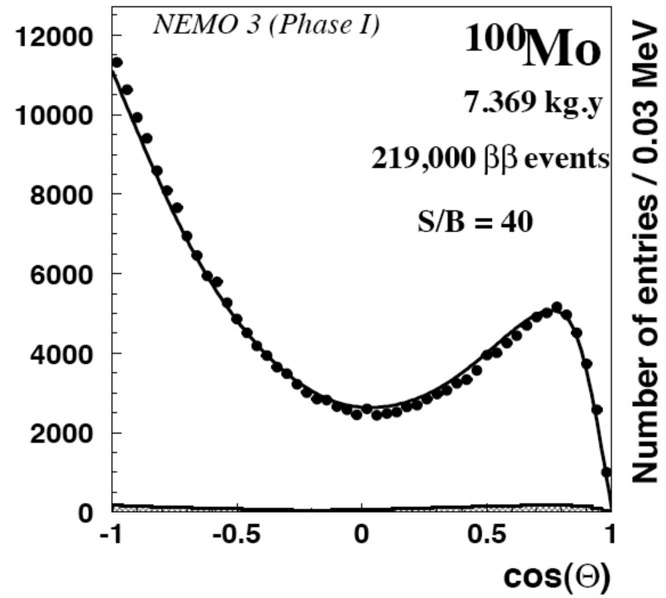
- Only 35 isotopes with 2ν-double-beta decay known in nature:

Transition	Q-value (keV)	nat. ab. (%)
${}^{48}_{20}Ca \rightarrow {}^{48}_{22}Ti$	4271	0.187
${}^{76}_{32}Ge \rightarrow {}^{76}_{32}Se$	2039	7.8
${}^{82}_{34}Se \rightarrow {}^{82}_{36}Kr$	2995	9.2
${}^{96}_{40}Zr \rightarrow {}^{96}_{42}Mo$	3350	2.8
${}^{100}_{42}Mo \rightarrow {}^{100}_{44}Ru$	3034	9.6
${}^{110}_{46}Pd \rightarrow {}^{110}_{48}Cd$	2013	11.8
${}^{116}_{48}Cd \rightarrow {}^{116}_{50}Sn$	2802	7.5
${}^{124}_{50}Sn \rightarrow {}^{124}_{52}Te$	2288	5.64
${}^{130}_{52}Te \rightarrow {}^{130}_{54}Xe$	2533	34.5
${}^{136}_{54}Xe \rightarrow {}^{136}_{56}Ba$	2479	8.9
${}^{150}_{60}Nd \rightarrow {}^{150}_{62}Sm$	3367	5.6

What Have We Learned?

- 2ν-double-beta decays have been observed:





Isotope	$T_{1/2}$ (10^{20} yr)
^{48}Ca	0.43±0.17±0.14
^{76}Ge	11±1.5
^{82}Se	1.03±0.03±0.07
^{100}Mo	0.0768±0.0002±0.0054
^{116}Cd	0.375±0.035±0.021
^{128}T[illegible]	[illegible]

Neutrinoless Double-Beta Decay

- Neutrinoless Double-beta Decay ($0\nu\beta\beta$):

$$(1) \qquad {}^{A}_{Z}N \rightarrow {}^{A}_{Z+2}N + 2e^{-}$$

$$(2) \qquad {}^{A}_{Z}N \rightarrow {}^{A}_{Z+2}N + 2e^{-} + \mathrm{n}\chi \text{ (Majoron)}$$

$$(3) \qquad {}^{A}_{Z}N \rightarrow {}^{A}_{Z-2}N + 2e^{+}$$

$$(4) \qquad e^{-} + {}^{A}_{Z}N \rightarrow {}^{A}_{Z-2}N + e^{+} \qquad \text{(electron capture)}$$

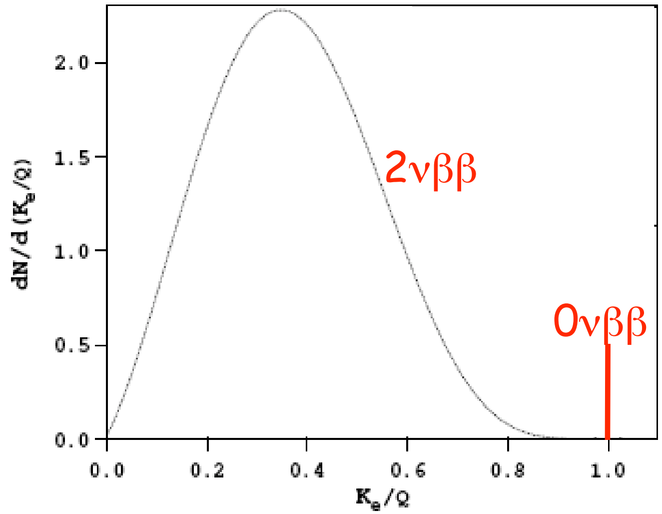
$$(5) \qquad 2e^{-} + {}^{A}_{Z}N \rightarrow {}^{A}_{Z-2}N$$

- Can only happen if
  - neutrinos have mass
  - neutrinos are Majorana particles
- It is a $\Delta L = 2$ process.
- If $0\nu\beta\beta$ decay is observed:
  - it will determine the absolute neutrino mass scale.
  - measurements in a number of different isotopes can reveal the underlying interaction dynamics.
  - may solve the neutrino mass hierarchy problem

Some Features of 0νββ Decay

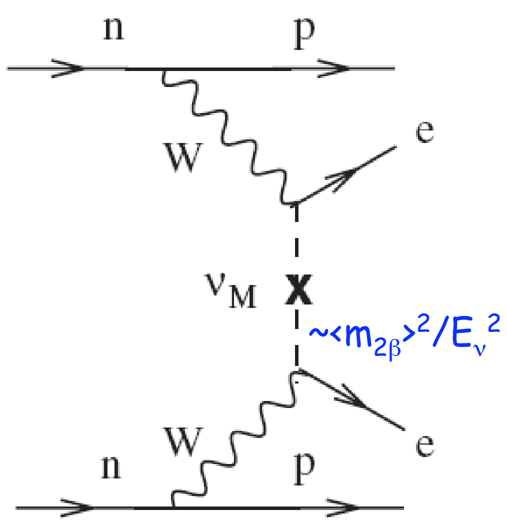
- Neutrinoless Double Beta Decay (0νββ):

$$^{A}_{Z}N \rightarrow {}^{A}_{Z+2}N + 2\beta^-$$



- Kinematics of the 0νββ decay:

$$Q_{2\beta} = K_{Z+2} + K_{e1} + K_{e2} = \frac{p_{Z+2}}{2M_{Z+2}} + K_{e1} + K_{e2} \approx K_{e1} + K_{e2}$$



- The decay rate is given by:

$$W = \frac{2\pi}{\hbar}\left|M_{fi}(0\nu)\right|^2 \rho_2\left(Q_{2\beta}, Z\right)\left\langle m_{2\beta}\right\rangle^2 \propto G_F^4 g_{\nu\bar{\nu}}^2 Q_{2\beta}^5 \left\langle m_{2\beta}\right\rangle^2$$

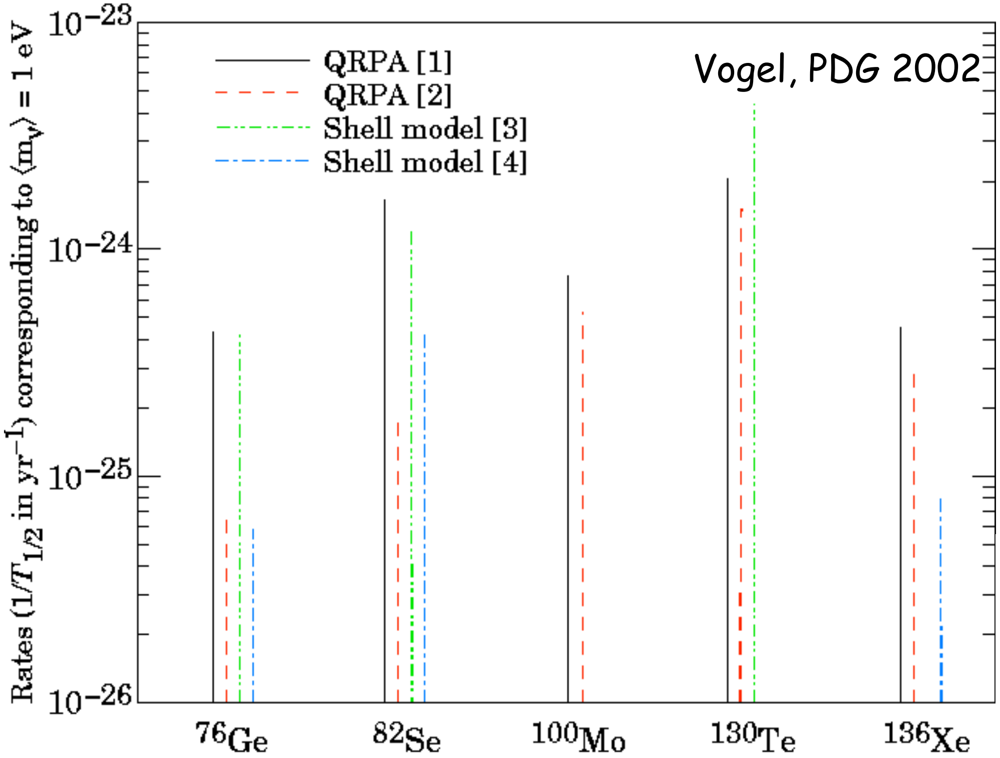
where

$\rho_2(Q_{2\beta},Z)$ is the two-body phase space factor.

$M_{fi}(0\nu)$ is the nuclear matrix element for the decay.

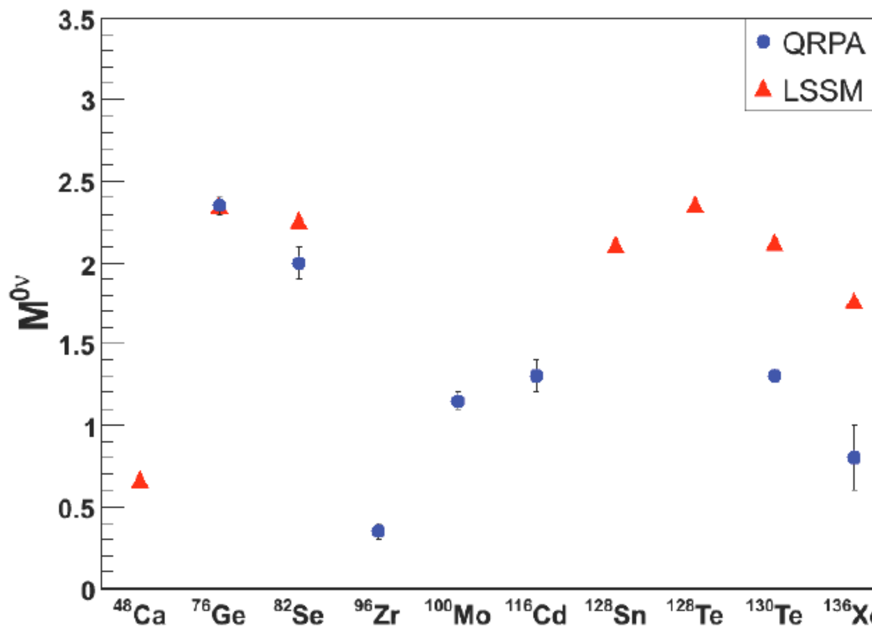
$\langle m_{2\beta}\rangle$ is the effective Majorana mass of ν_e.

Nuclear Matrix Elements



QRPA = Quasi-random Phase Approximation

LSSM = Large Scale Shell Model



Rodin et al., NP**A766**(06)07

Poves, NDM06 talk

- A factor of 3 uncertainty in the nuclear matrix element implies about a factor of 10 uncertainty in $T_{1/2}$.
- Use 2 $\nu\beta\beta$ decay to reduce the uncertainty in the nuclear matrix element

Majorana Neutrino Mixing

- The Majorana neutrino mixing matrix is:

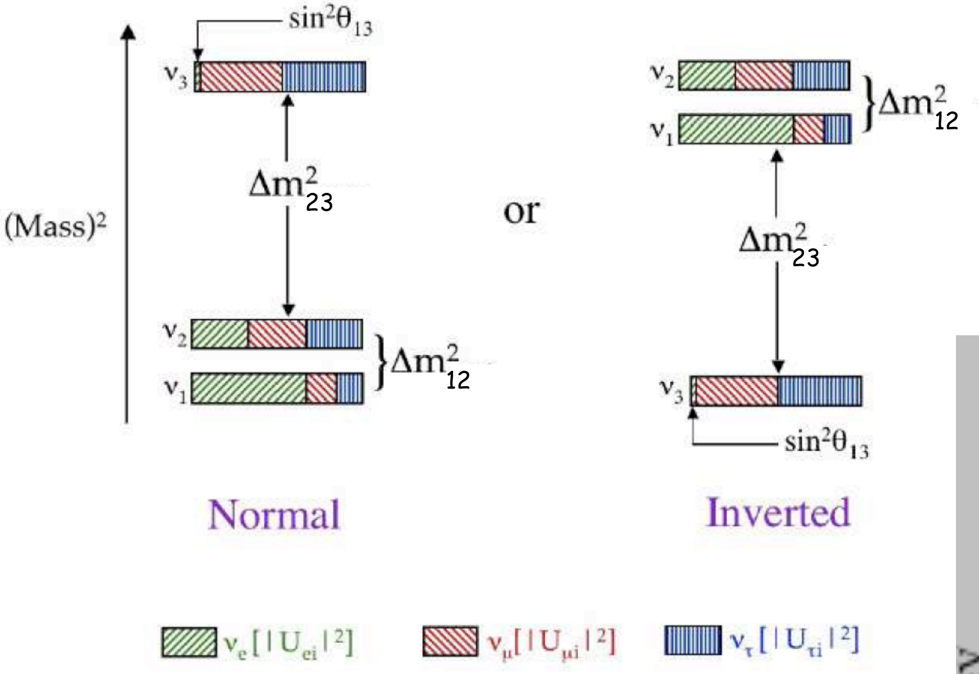
$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{i\frac{\alpha_1}{2}} & 0 & 0 \\ 0 & e^{i\frac{\alpha_2}{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- The diagonal matrix does not play a role in neutrino oscillation.
- The effective neutrino mass:

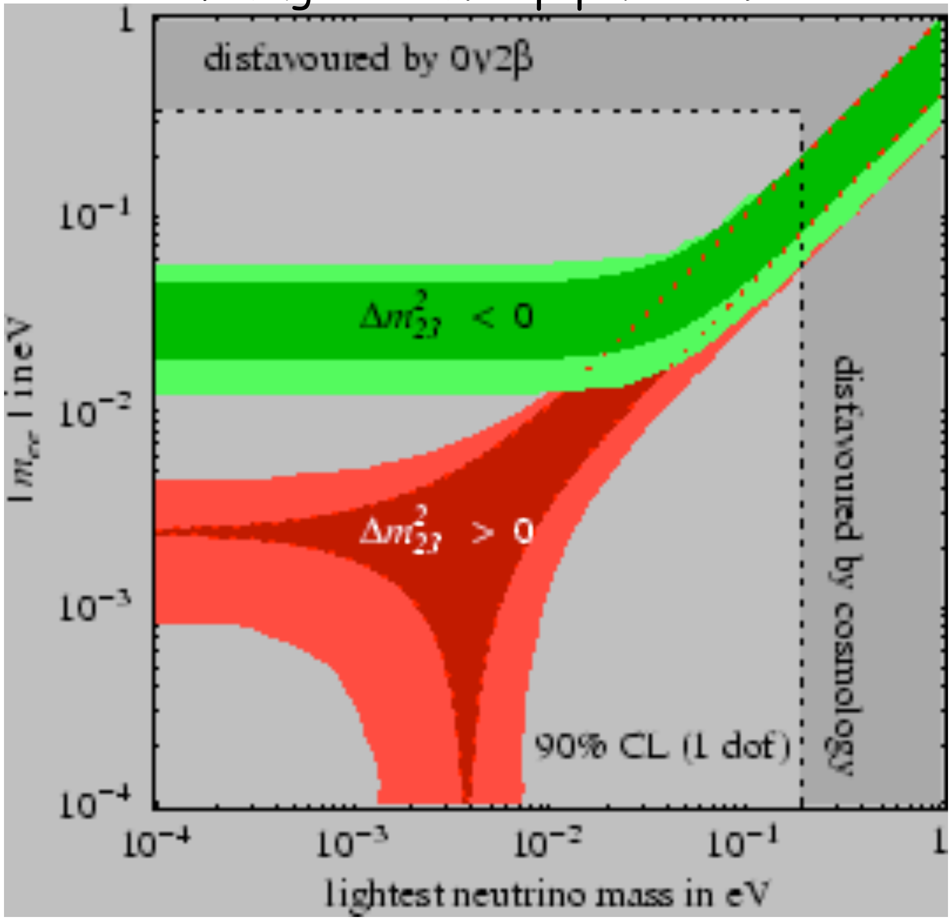
$$\langle m_{2\beta} \rangle = \left| \langle \nu_e | m | \nu_e \rangle \right| = \left| \sum_{i=1}^{3} U_{ei}^2 m_i \right|$$

$$= \left| c_{12}^2 c_{13}^2 e^{i\alpha_1} m_1 + s_{12}^2 c_{13}^2 e^{i\alpha_2} m_2 + s_{13}^2 e^{-i2\delta} m_3 \right|$$

Resolving Mass Hierarchy

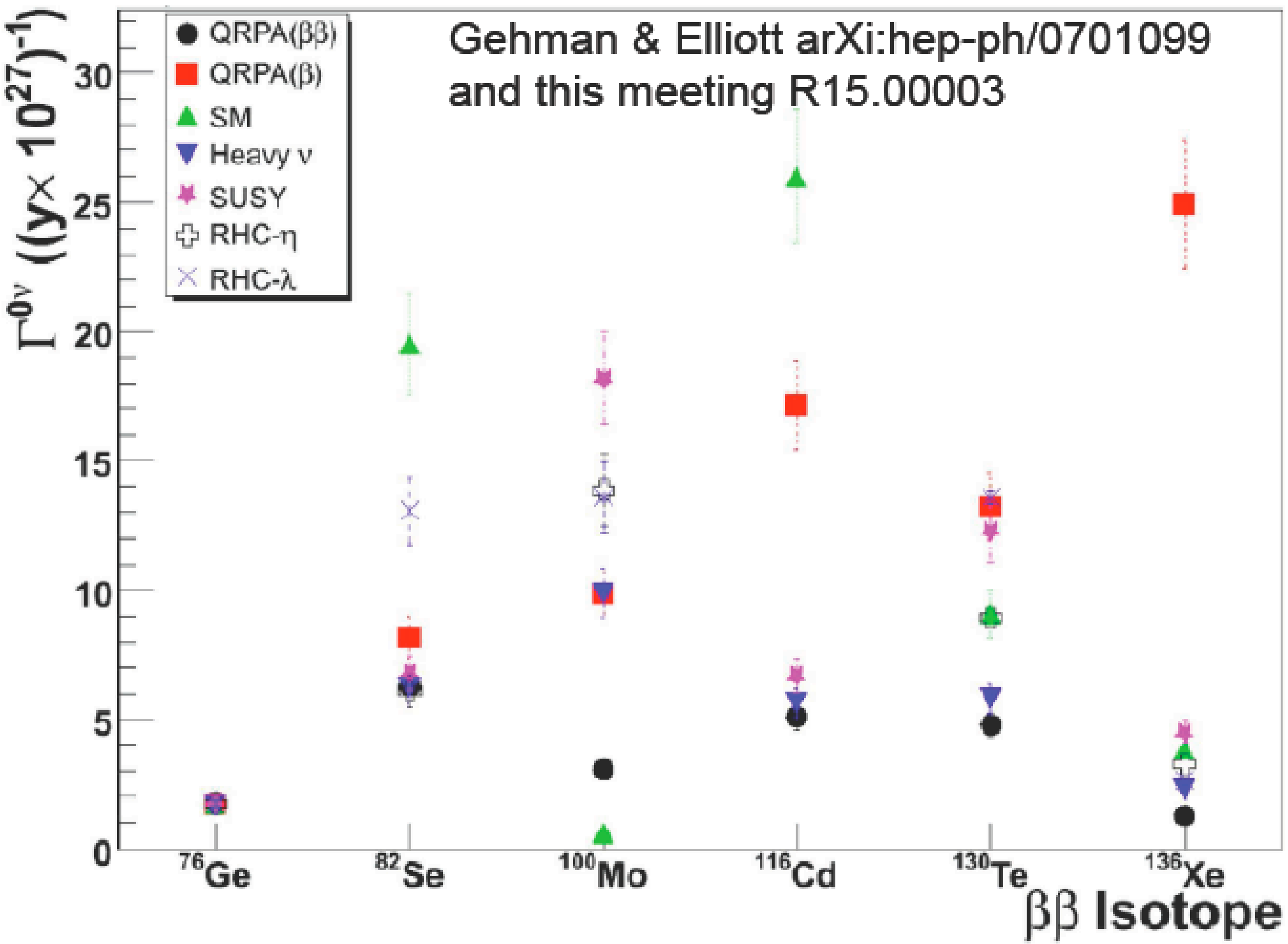


Feruglio et al. hep-ph/020191



If the lightest ν mass is small enough, observation of 0νββ decay can resolve the ordering of m_2 and m_3.

Probing New Physics With 0νββ Decay



- Assume a single dominant mechanism contributing to the 0νββ decay.
- Need to obtain results from several different isotopes and calculation of NME to ~20% to distinguish the potential new physics.

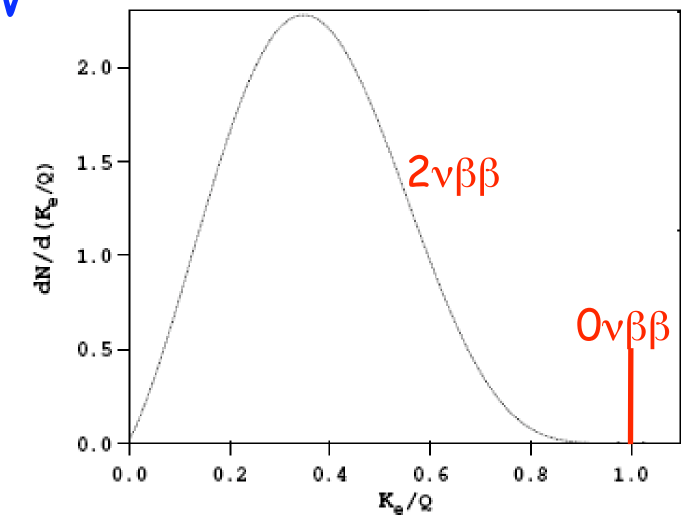
Experimental Consideration

(1) Need massive samples of isotopes.

$$T_{1/2} = \ln 2 \cdot a \cdot N_A \cdot M \cdot t \,/\, N_{\beta\beta} \quad (t \ll T_{1/2})$$

(No background)

For $T_{1/2} = 10^{27} - 10^{28}$ years with $N_{\beta\beta} = 1$, $M \sim 10^5$ (moles)

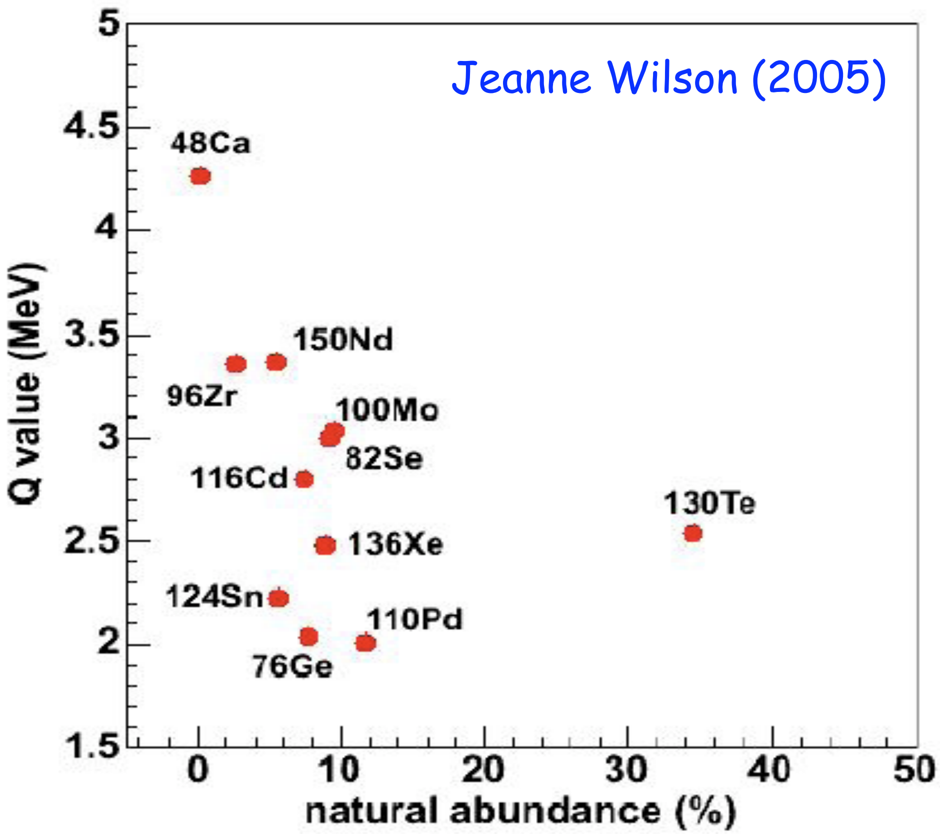


(2) Isolate signal from the irreducible $2\nu\beta\beta$ background, typically at least 8 orders of magnitude larger.

Requires excellent energy resolution at $Q_{2\beta}$.

- Separate signal from external background.
  Require:
  - shielding (go deep underground, self shielding)

Isotopes For Double-beta Decay: Revisit

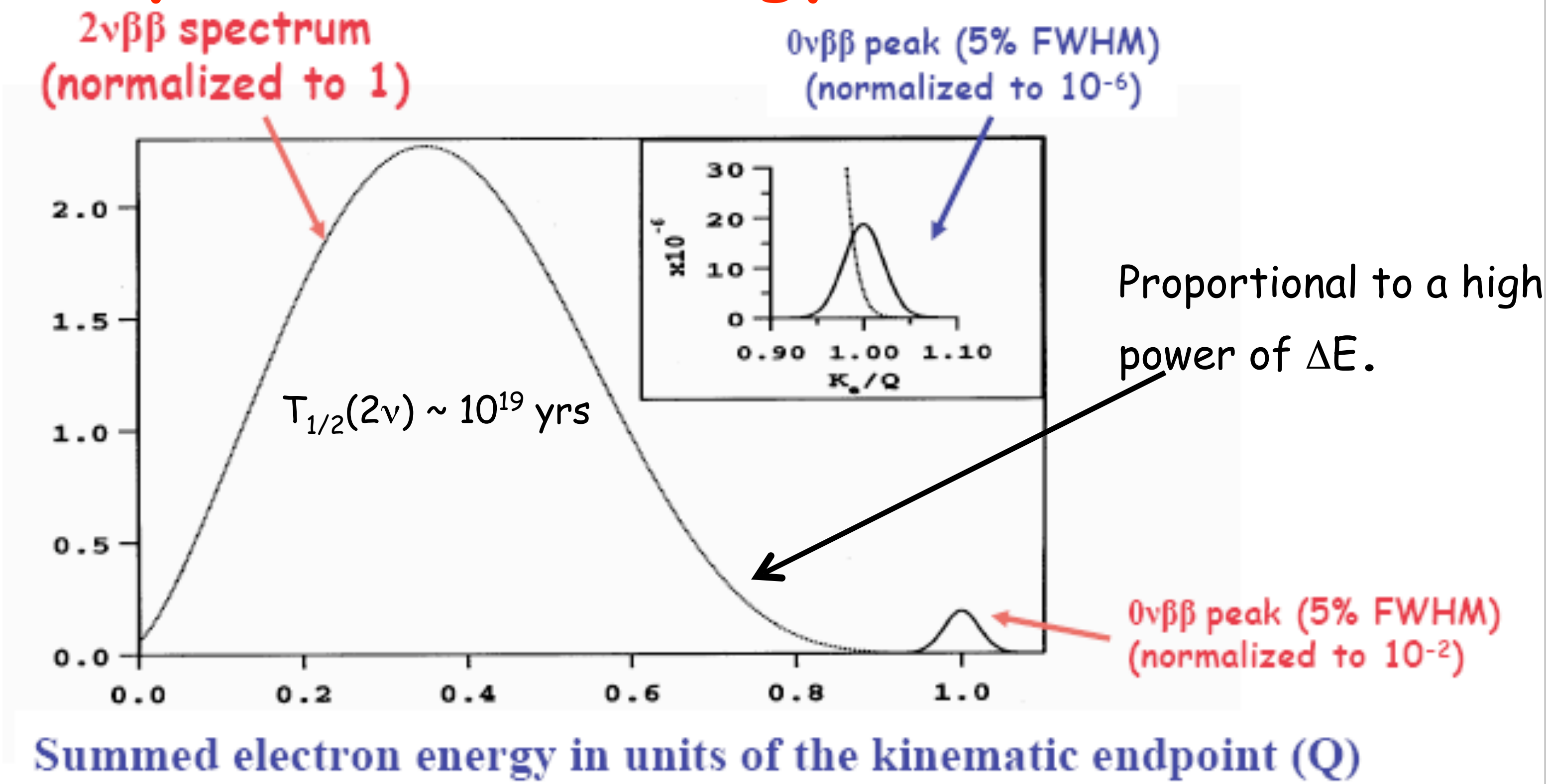


- Can consider enriching the isotopes to reduce the mass of the sample
- However, enrichment is a highly non-trivial endeavour.

Table 4. Compilation of $\beta^+\beta^+$-emitters requiring a Q-value of at least 2048 MeV. The full transition energies Q-$4m_ec^2$ and the natural abundances are shown.

Transition	Q-$4m_ec^2$ (keV)	nat. ab. (%)
${}^{78}_{36}Kr \rightarrow {}^{78}_{34}Se$	838	0.35
${}^{96}_{44}Ru \rightarrow {}^{96}_{42}Mo$	676	5.5
${}^{106}_{48}Cd \rightarrow {}^{106}_{46}Pd$	738	1.25
${}^{124}_{52}Xe \rightarrow {}^{124}_{52}Te$	822	0.10
${}^{130}_{56}Ba \rightarrow {}^{130}_{54}Xe$	534	0.11
${}^{136}_{58}Ce \rightarrow {}^{136}_{56}Ba$	362	0.19

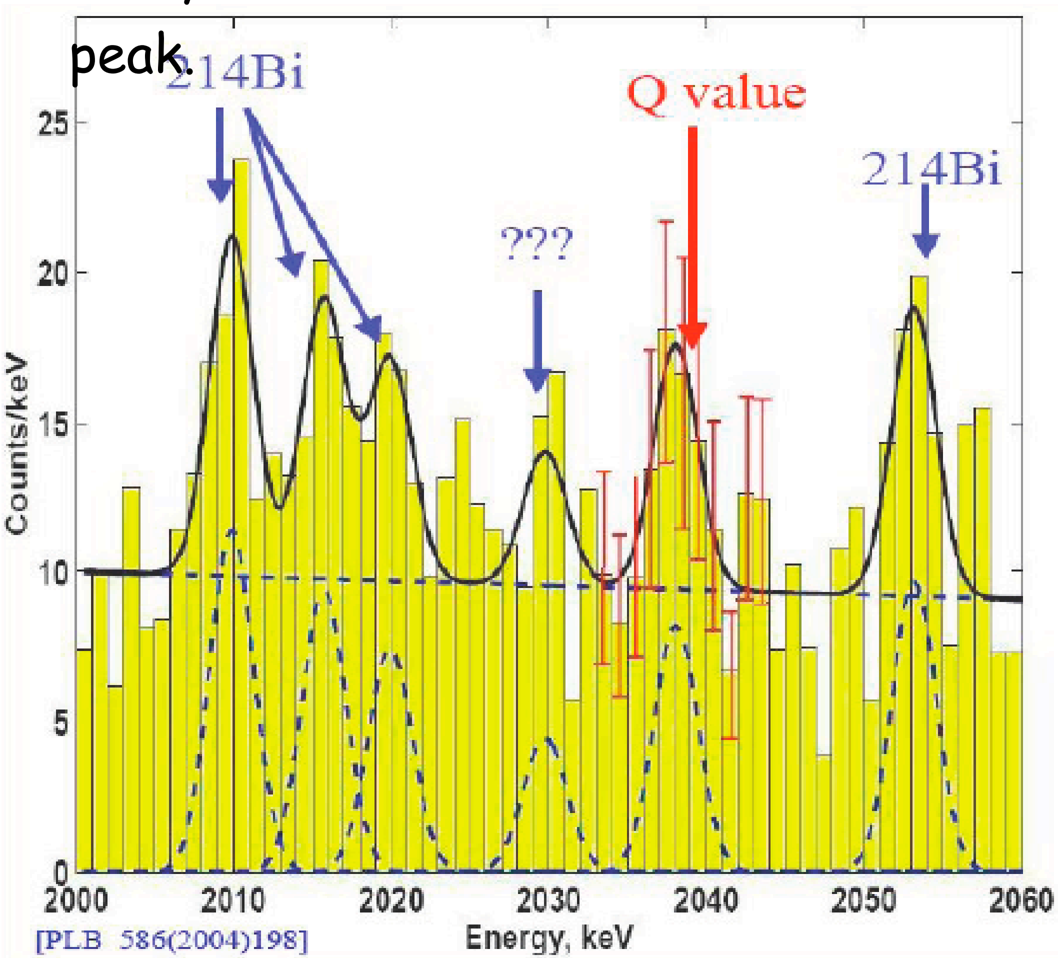
Importance of Energy Resolution



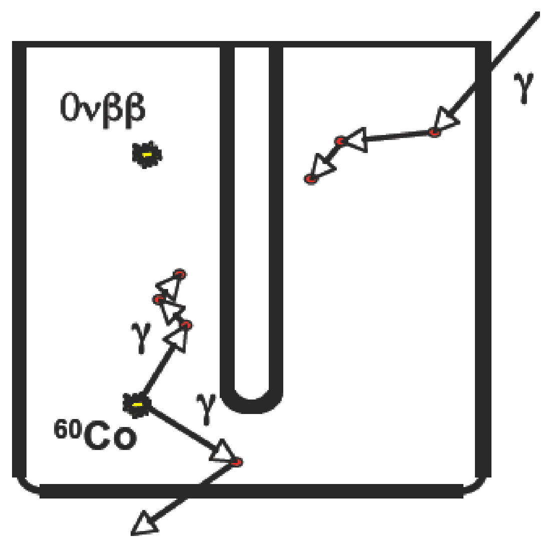
Detector	ΔE at $Q_{2\beta}$ (%)
^{76}Ge (Diode)	~0.2
^{130}Te (Bolometer)	~0.4
^{136}Xe (TPC)	~3
CdZnTe (Semiconductor)	3-4

Energy Resolution: Necessary But Not Sufficient

With energy only, must identify every peak.

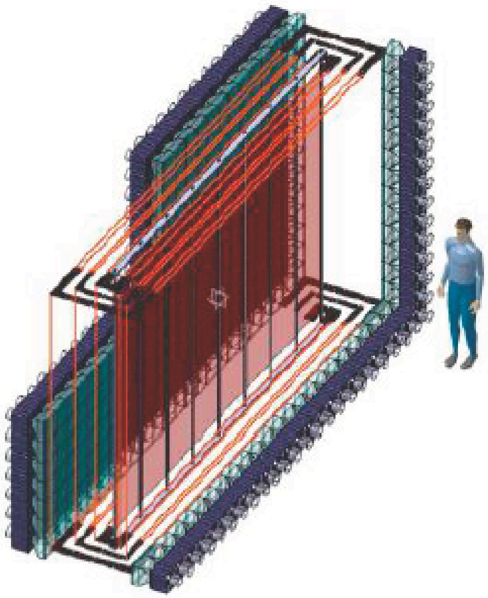


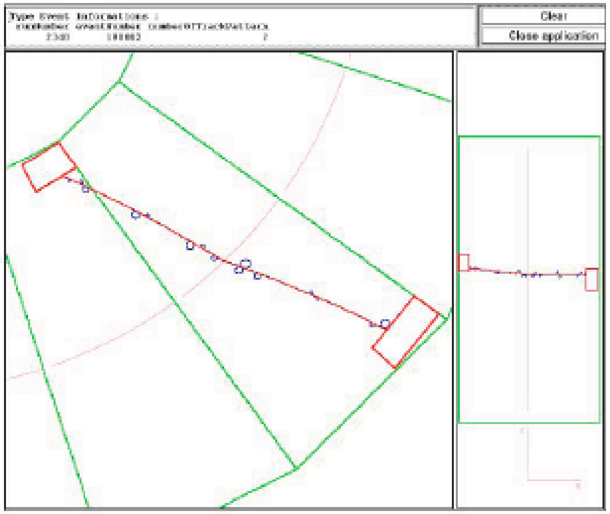
Any event with energy deposited in the signal r[illegible]al.



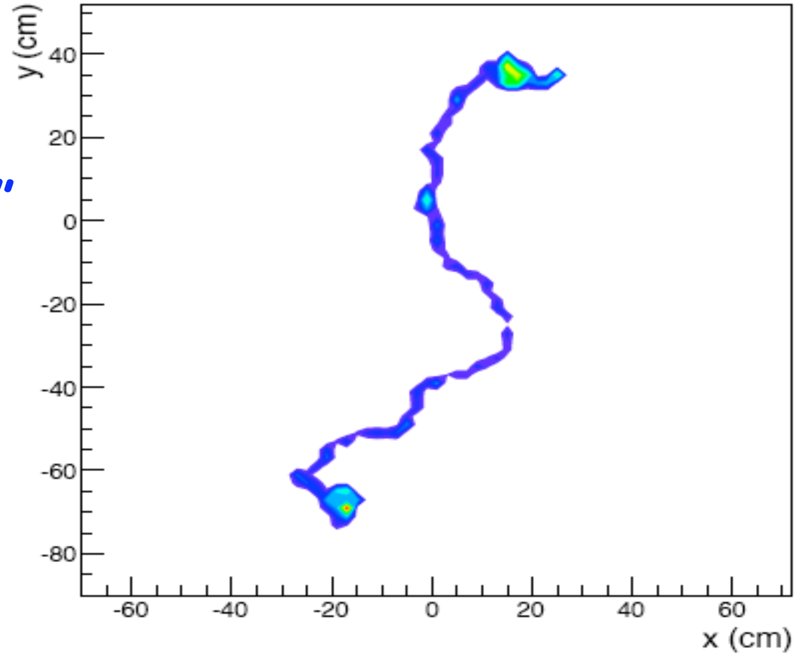
Need additional handles and suppress background.

Examples of Extra Handles





(Super)NEMO tracking the electrons



Topology: "spaghetti, with meatballs"

$\beta\beta$ events: **2**

γ events: **1**

Gotthard's Xe TPC can provide about a factor of 30 rejection

Background And Sensitivity

- Without background, the half-life of a decay is given by

$$T_{1/2} = \ln 2 \cdot \varepsilon \cdot a \cdot N_A \cdot M \cdot t / N_{obs} \qquad (t << T_{1/2})$$

ε = overall efficiency

N_A = Avogadro number

a = natural abundance of the isotope

M = number of moles of the isotope

t = live time of measurement, in years

- With background, the sensitivity of a measurement is related to the statistical error of the background:

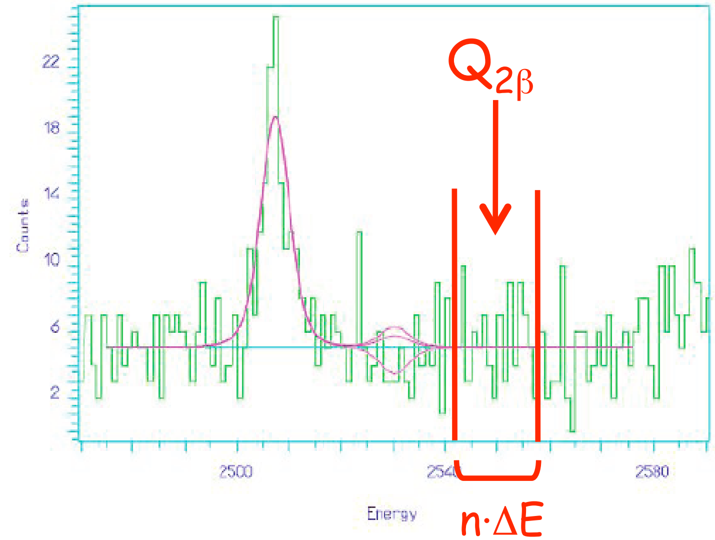
$$N_{obs} = \sqrt{N_{bg}}$$

and

$$N_{bg} = n \cdot B \cdot M \cdot t \cdot \Delta E$$

where n = energy range in units of ΔE

[B] = counts/keV/mole/yr



In this case,

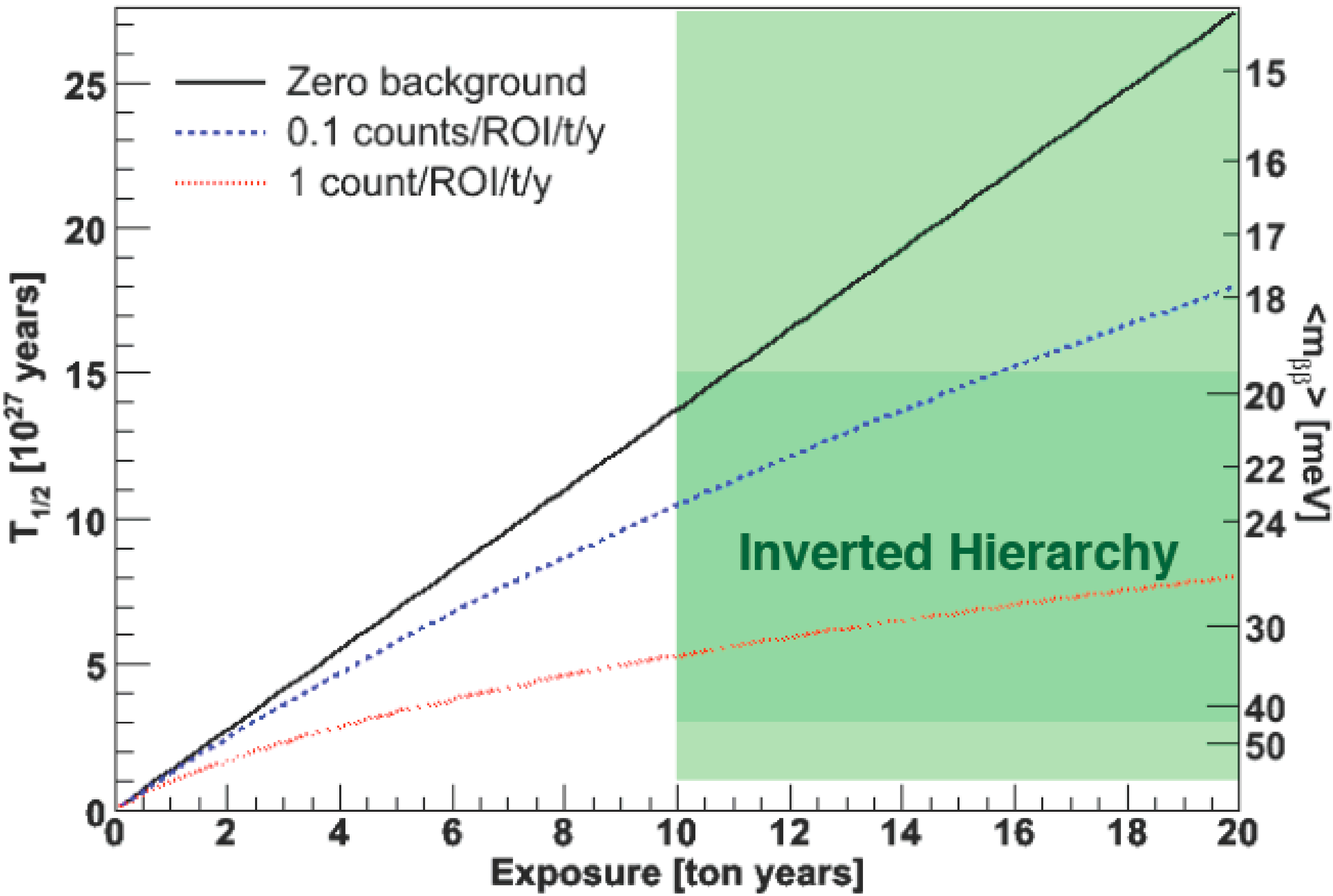
$$T_{1/2} = \ln 2 \cdot \varepsilon \cdot a \cdot N_A \sqrt{\frac{M \cdot t}{n \cdot B \cdot \Delta E}}$$

- Relate the sensitivity of $T_{1/2}$ to $\langle m_{2\beta} \rangle$: $\quad T_{1/2} \propto \dfrac{1}{\langle m_{2\beta} \rangle^2}$

An Example On Sensitivity

^{76}Ge Example

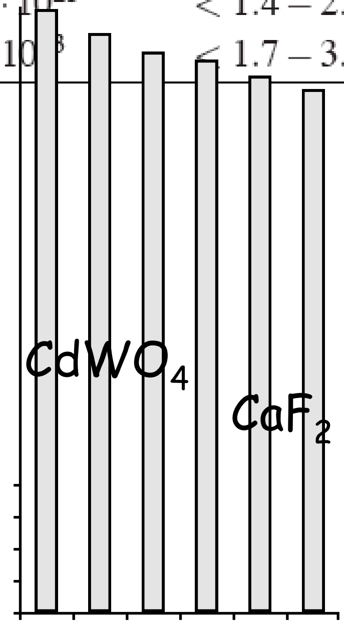
$$T_{1/2}^{0\nu} = \ln(2) N\epsilon t / UL(B)$$



Some Double-beta-decay Experiments

Isotope	$T_{1/2}$, y	$\langle m_\nu \rangle$, eV [44 – 46]	$\langle m_\nu \rangle$, eV [47]	Experiment
^{76}Ge	$> 1.9 \cdot 10^{25}$	$< 0.33 - 0.84$	$< 0.53 - 0.59$	HM [50]
	$\simeq 1.2 \cdot 10^{25} (?)^{*)}$	$\simeq 0.5 - 1.3 (?)^{*)}$	$\simeq 0.7 (?)^{*)}$	Part of HM [41]
	$> 1.6 \cdot 10^{25}$	$< 0.36 - 0.92$	$< 0.58 - 0.64$	IGEX [51]
^{130}Te	$> 1.8 \cdot 10^{24}$	$< 0.4 - 0.9$	$< 1 - 1.6$	CUORICINO$^{**)}$ [52]
^{100}Mo	$> 4.6 \cdot 10^{23}$	$< 0.65 - 1.0$	$< 2.4 - 3.0$	NEMO- 3$^{**)}$ [53]
^{136}Xe	$> 4.5 \cdot 10^{23***)}$	$< 0.8 - 4.7$	$< 2.9 - 5.6$	DAMA [54]
^{116}Cd	$> 1.7 \cdot 10^{23}$	$< 1.4 - 2.5$	$< 3.7 - 4.3$	SOLOTVINO [55]
^{82}Se	$> 1 \cdot 10^{23}$	$< 1.7 - 3.7$	$< 3.8 - 4.7$	NEMO-3$^{**)}$ [53]

Barabash, JINST **1** (2006) P07002



Jeanne Wilson (2005)

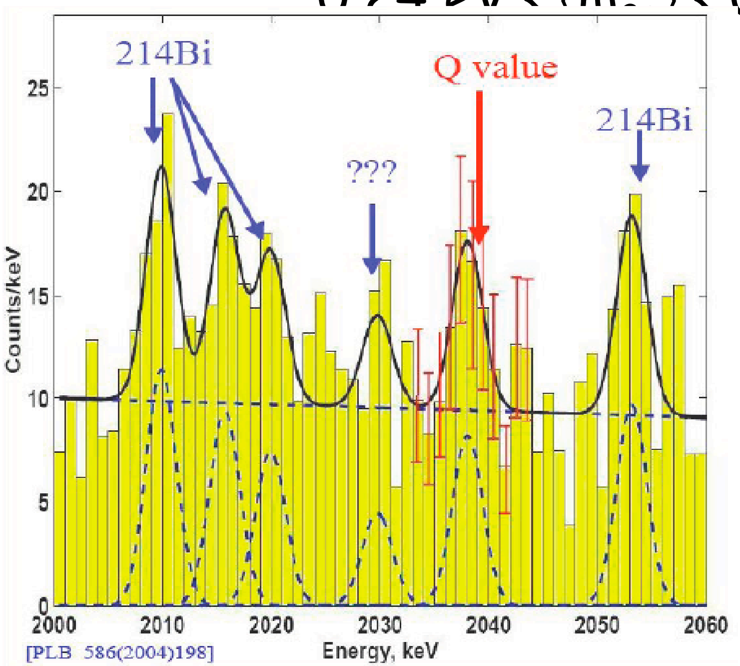
A Claimed Evidence of 0νββ Decay

- Result was obtained with 71 kg-yr of data using five ^{76}Ge crystals with a total mass of 11 kg:

$$T_{1/2} = (1.19^{+2.99}_{-0.5}) \times 10^{25}\ \text{yr}$$

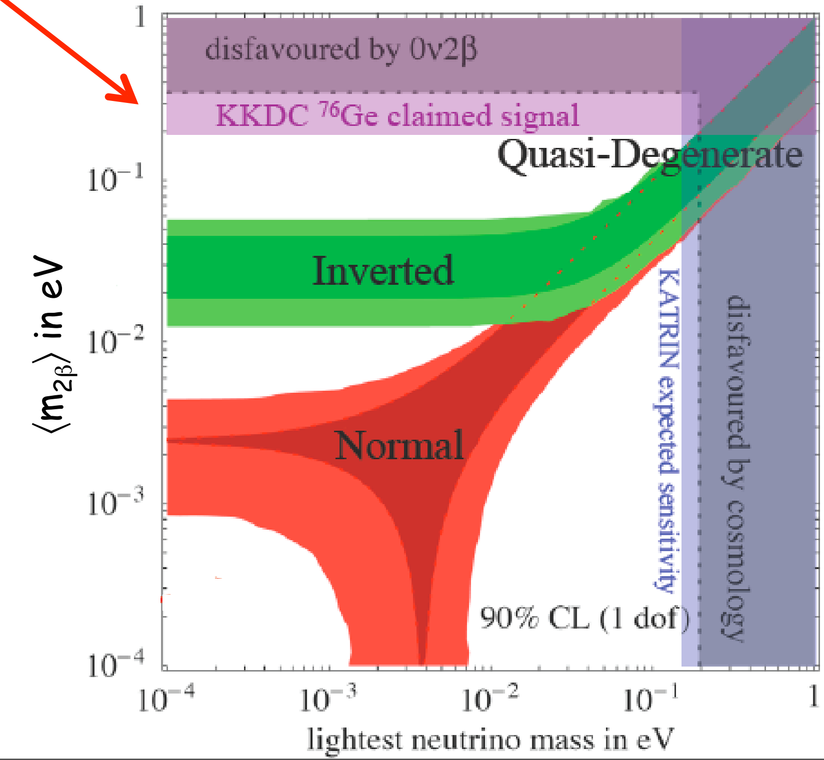
implying

$$0.24\ \text{eV} < \langle m_\nu \rangle < 0.58\ \text{eV}\ \text{(3 st. dev.)}$$



Obtained with 51.4 kg-yr of data using four crystals

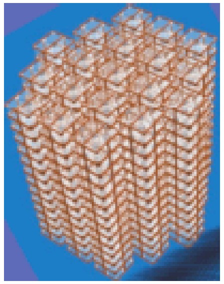
Not consistent with other results?



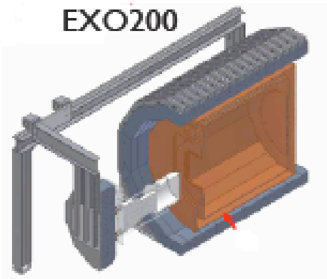
Current And Future Initiatives

0νββ decay Experiments - Efforts Underway

CUORE



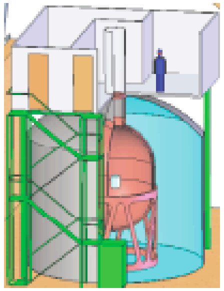
EXO200



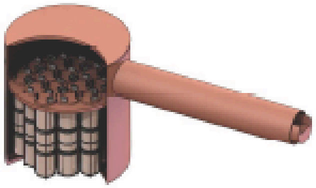
NEMO

Collaboration	Isotope	Technique	Mass	Status
CAMEO	Cd-116	$CdWO_4$ crystals	1 t	
CANDLES	Ca-48	60 CaF_2 crystals in liq. scint	6 kg	Construction
CARVEL	Ca-48	$^{48}CaWO_4$ crystal scint.	100 kg	
COBRA	Cd-116, Te-130	CdZnTe detectors	10 kg	R&D
CUROICINO	Te-130	TeO_2 Bolometer	11 kg	Operating
CUORE	Te-130	TeO_2 Bolometer	206 kg	Construction
DCBA	Nd-150	Nd foils & tracking chambers	20 kg	R&D
EXO200	Xe-136	Xe TPC	200 kg	Construction
EXO	Xe-136	Xe TPC	1-10t	R&D
GEM	Ge-76	Ge diodes in LN	1 t	
GERDA	Ge-76	Seg. and UnSeg. Ge in LAr	35-40 kg 1 t	Construction Future
GSO	Gd-160	Gd_2SiO_5:Ce crystal scint. in liquid scint	2t	
HPXeTPC	Xe-136	High Pressure TPC	1t	R&D
Majorana	Ge-76	Segmented Ge	60 kg 1 t	Proposed Future
NEMO3	Mo-100 Se-82	Foils with tracking	6.9 kg 0.9 kg	Operating
SuperNEMO	Se-82	Foils with tracking	100 kg	Proposed
MOON	Mo-100	Mo sheets	200 kg 1 t	R&D
SNO+ ββ	Nd-150	0.1% suspended in Scint.	56 kg	R&D
Xe	Xe-136	Xe in liq. Scint.	1.56 t	
XMASS ββ	Xe-136	Liquid Xe	10 kg	Feasibility

GERDA



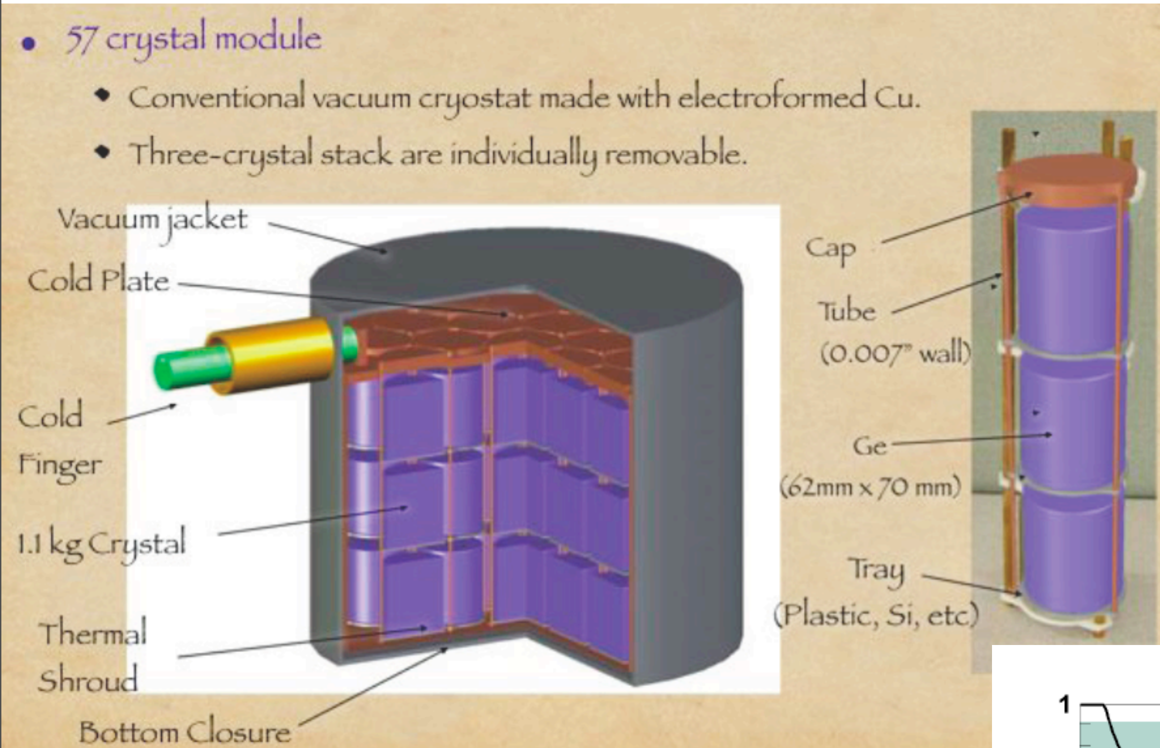
Majorana (R15.00002)

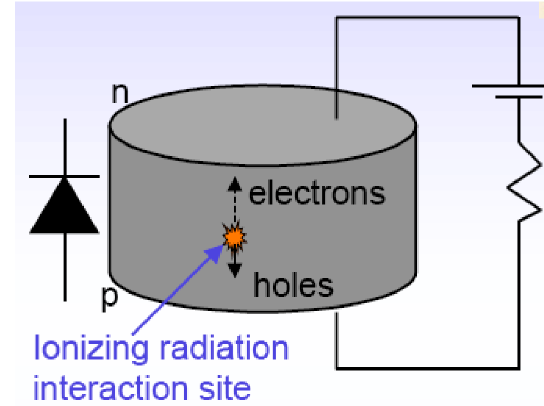


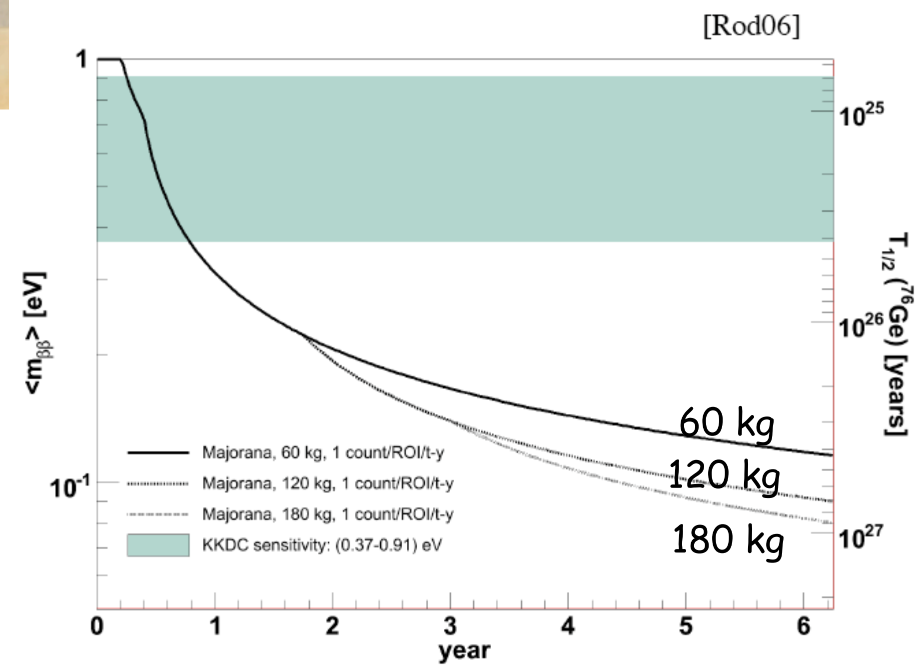
HPXeTPC

Operating | Construction | Proposed/R&D

MAJORANA

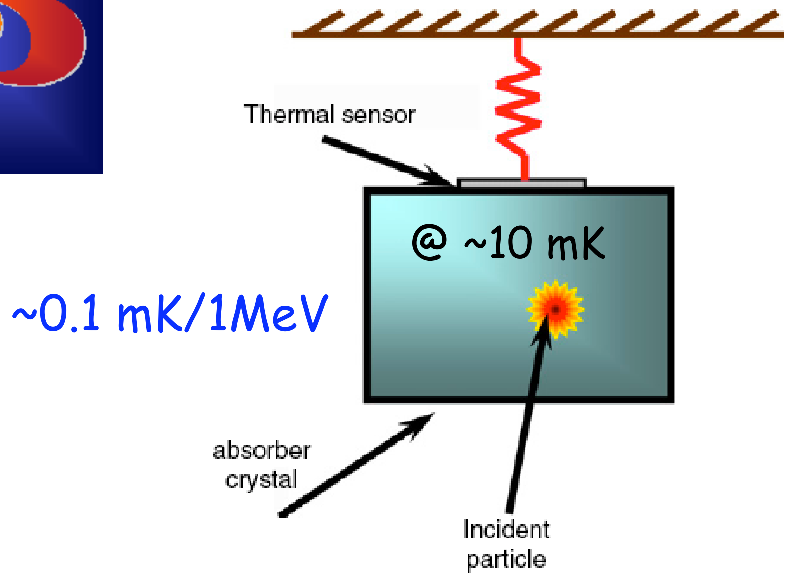


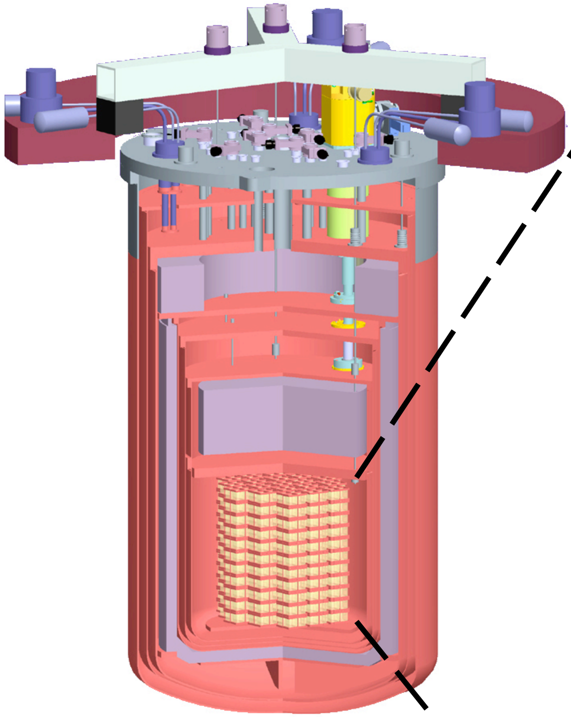




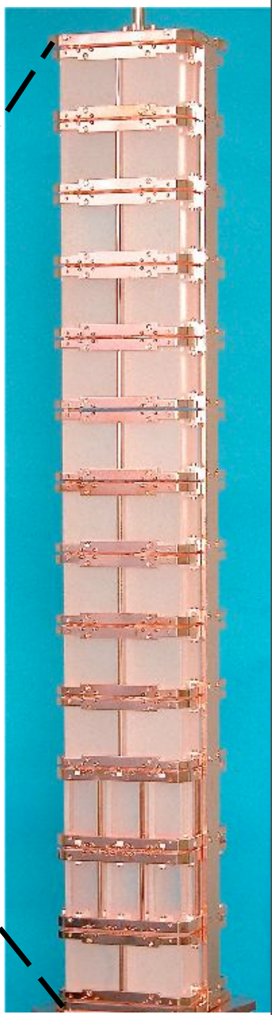
- Goal: 1 event/t-yr in 4 keV ROI.
- Need a factor ~100 reduction in background over what has been achieved.
  - most background events are multi-site; signal is single-site.

CUORE





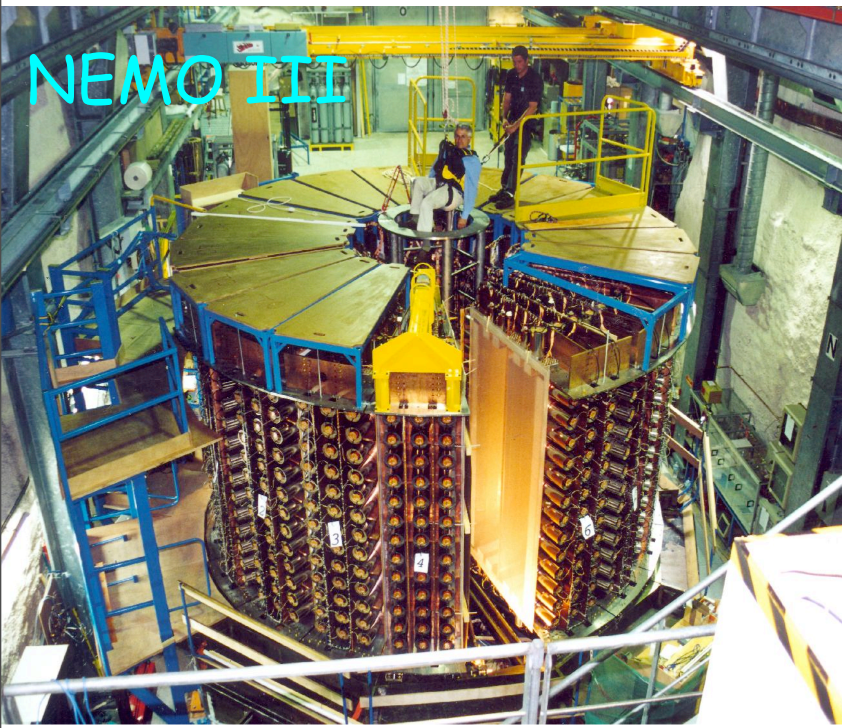
19 towers



- Build on the experience gained from CUORECINO
- Use a cryogenic bolometric detector made of 988 cubic crystals of TeO_2, a total mass of ~750 kg or ~203 kg of ^{130}Te.
- Goal: reach 0.001-0.01 background events/keV/kg/yr.

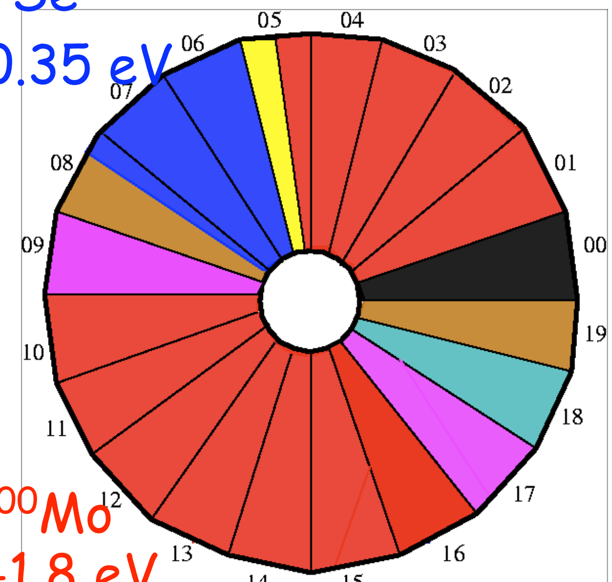
NEMO III

- Running in Frejus since Feb 2003.
- 10 kg double-beta isotopes in a 20 m^2 cylinder.
- Use a magnetic spectrometer:
  - drift chamber
  - plastic-scintillator calorimeter
  - 25 G field



932 g of ^{82}Se

$\langle m_{2\beta} \rangle \sim 0.2-0.35$ eV



- Can perform measurement with multiple sources of isotopes at the same time.

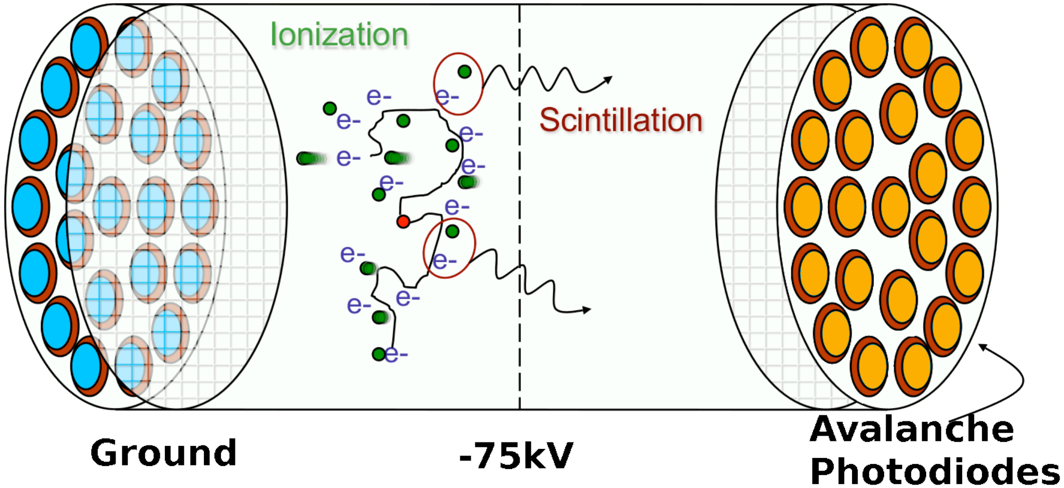
6914 g of ^{100}Mo

$\langle m_{2\beta} \rangle \sim 0.65-1.8$ eV

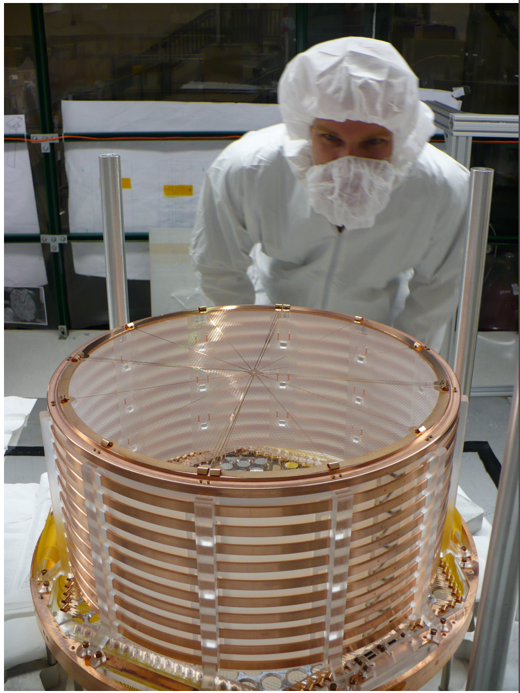
Super-NEMO

- 10 times of NEMO III
- With better energy resolution $\Delta E/E$:
  - from $(14\text{-}16)\%/\sqrt{E}$ to $(7\text{-}9)\%/\sqrt{E}$
- Sources: ^{100}Mo, ^{82}Se, ^{116}Cd and ^{130}Te.
- Only background is events from the tail of $2\nu\beta\beta$ decay.
- With five years of running, plan to reach $\langle m_{2\beta}\rangle \sim 0.03\text{-}0.06$ eV.

EXO

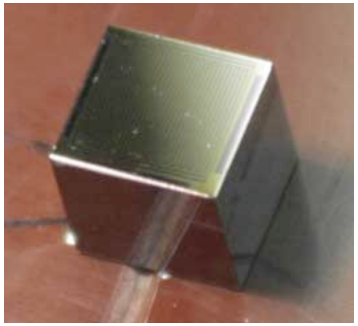


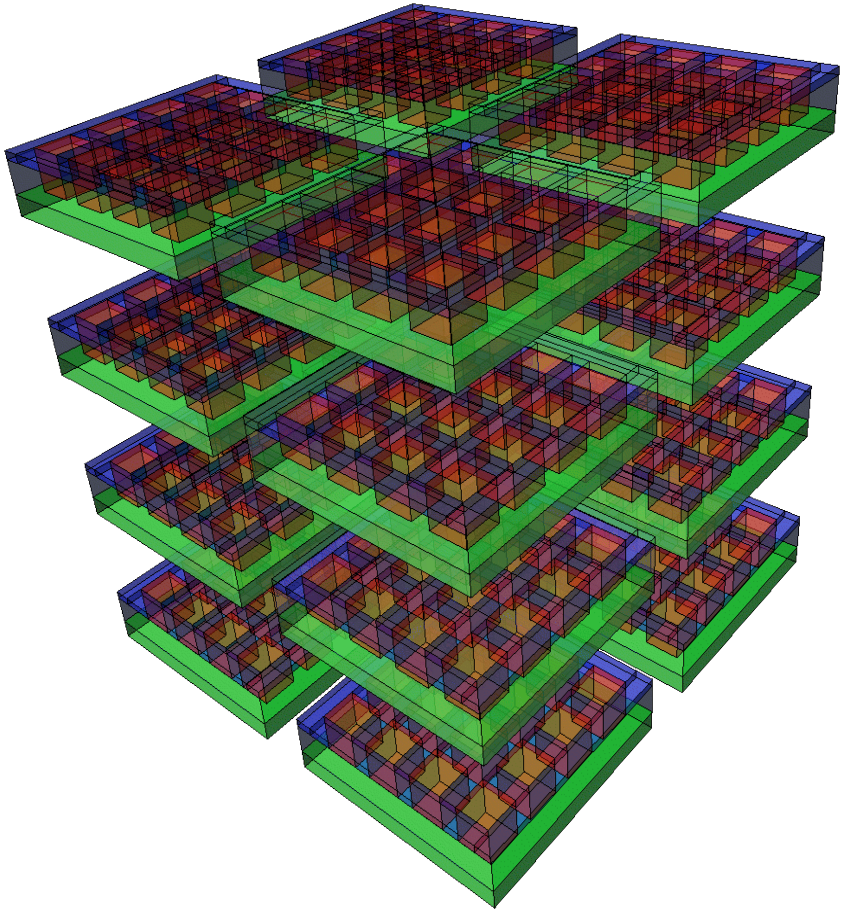
- ~1 t of 90% enriched liquid ^{136}Xe TPC.
- Utilize ionisation and scintillation signals
  - good energy resolution
- Identify the ^{136}Ba daughter
  - electrostatic probe
  - laser fluorescence
- A 200 kg prototype without identification of ^{136}Ba is running in WIPP.



COBRA

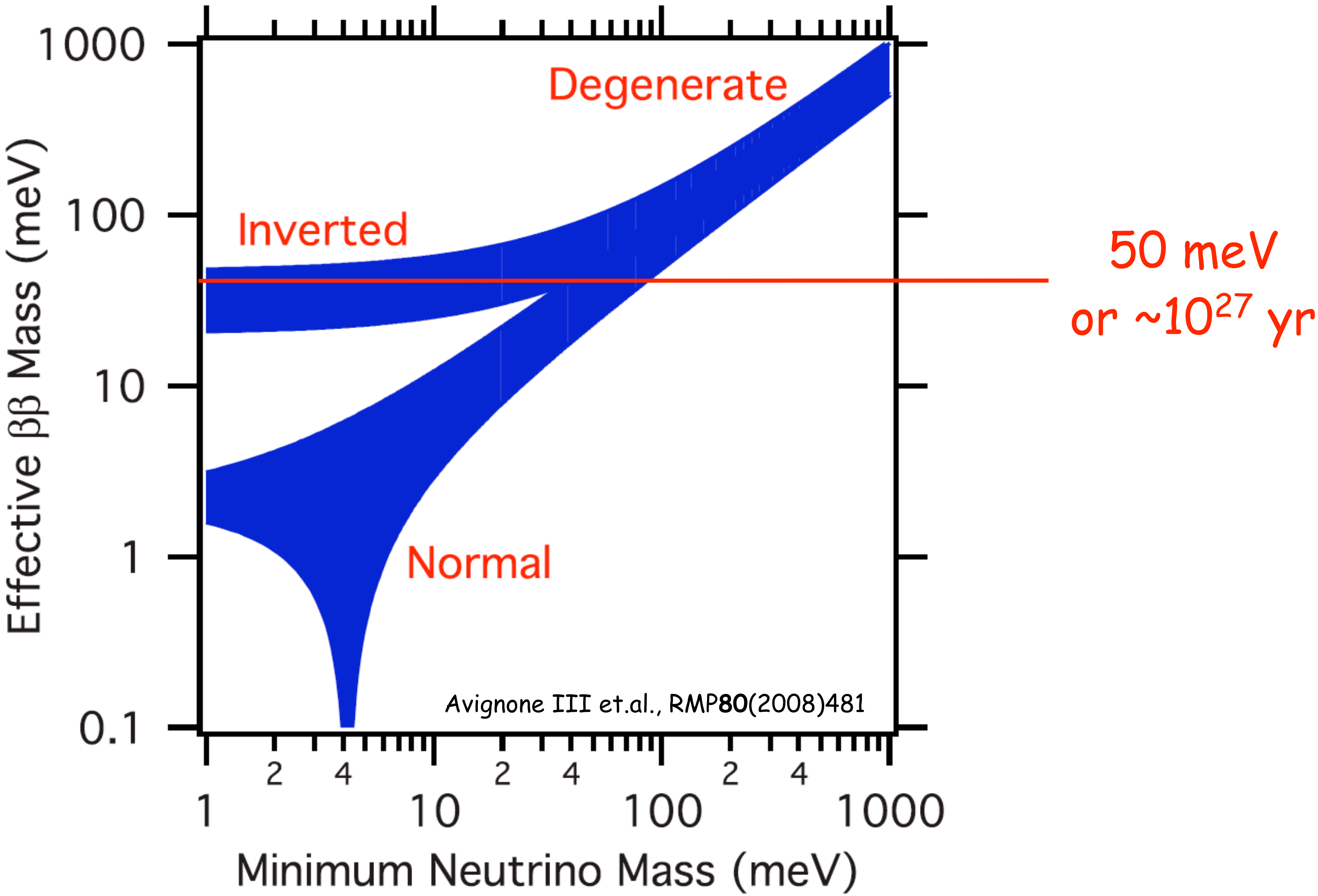
- Large array of 1cm^3 CdZnTe semiconductor crystals operating at room temperature





- Suppress background using
  - pixellisation
  - Coincidence
  - pulse shape analysis
- Good energy resolution (~4%)

Future Prospects



Summary

- Double-beta decay is a very rare process.
- Observation of $0\nu\beta\beta$ decay will lead to the conclusion that
  - Total lepton number, L, is not conserved;
  - Neutrinos are Majorana type;

  and

  - provide a mean to determine the absolute mass scale of the neutrinos.
  - may shed light on physics beyond the Standard Model.
- $0\nu\beta\beta$ experiments are extremely challenging.

$$\mu^- + (A, Z) \to e^+ + (A, Z-2).$$

The current best 90% CL bound comes from SINDRUMII and is given by [49]

$$\frac{\Gamma(\mathrm{Ti} + \mu^- \to \mathrm{Ca}^{GS} + e^+)}{\Gamma(\mathrm{Ti} + \mu^- \to Sc + \nu_\mu)} < 1.7 \times 10^{-12}. \qquad (26)$$

Zuber, K.(2004) 'Double beta decay', Contemporary Physics, 45: 6, 491

$$\nu_\mu N \rightarrow \mu^- \mu^+ \mu^+ X,$$

$$K^+ \rightarrow \pi^- \mu^+ \mu^+.$$

A new upper limit on the branching ratio of

$$\frac{\Gamma(K^+ \rightarrow \pi^- \mu^+ \mu^+)}{\Gamma(K^+ \rightarrow \text{all})} < 3 \times 10^{-9} \quad (90\%\text{CL}),$$

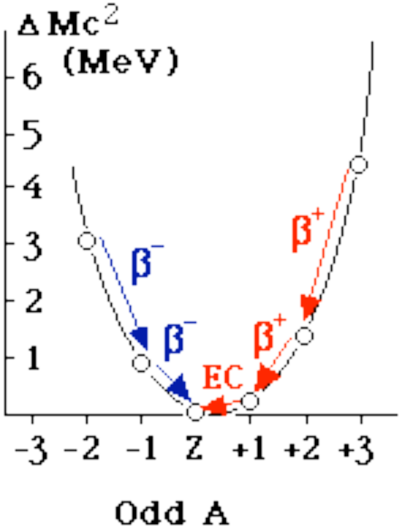
ep-collider HERA [55]. The process studied is

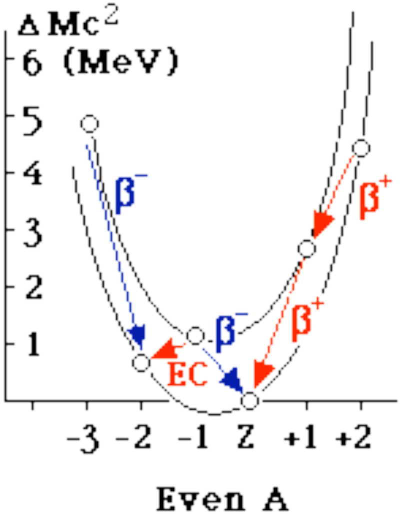
$$e^\pm p \rightarrow \overset{(-)}{\nu_e} \, l^\pm l'^\pm X,$$

$$E_B = a_V A - a_S A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_A \frac{(A-2Z)^2}{A} + \underline{\delta(A,Z)}$$

$$\delta(A,Z) = \begin{cases} +\delta_0 & Z, N \text{ even } (A \text{ even}) \\ 0 & A \text{ odd} \\ -\delta_0 & Z, N \text{ odd } (A \text{ even}) \end{cases}$$

1935 by Carl von Weizsäcker





$$\boxed{(A,Z) \rightarrow (Z+2,A) + 2e^- + 2\nu_e}$$

$(A,Z) \rightarrow (Z-2,A) + 2e^+ + 2\nu_e$

$(A,Z) \rightarrow (Z+2,A) + \gamma + e^+ + 2\nu_e$ (EC)

$(A,Z) \rightarrow (Z+2,A) + 2\gamma + 2\nu_e$ (ECEC)

~ 69 stable and 28 α-unstable ββ isotopes