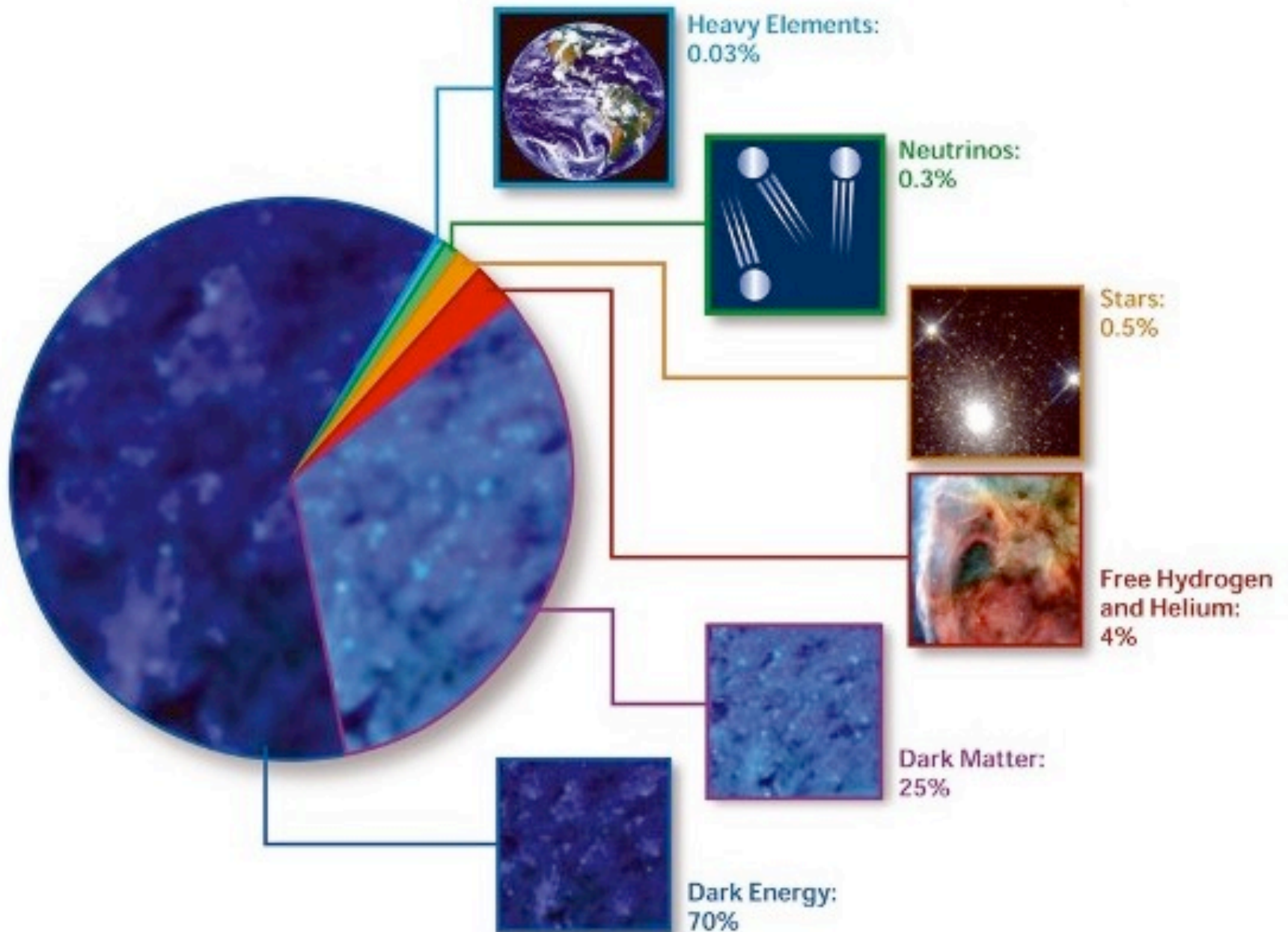


Composition of Present Universe



Some Neutrino Physics

陆锦标 Kam-Biu Luk

Tsinghua University

and

University of California, Berkeley

and

Lawrence Berkeley National Laboratory

Outline

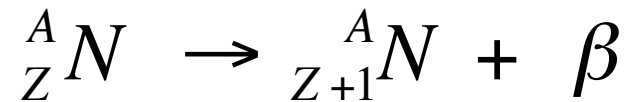
- Some historic findings about neutrinos
- Standard Model of Particle Physics
- Neutrino Oscillation
- Possible modifications of Standard Model
- Open questions in neutrino physics

References

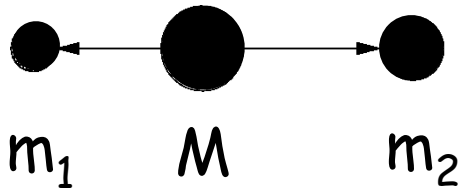
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- 'Weak Interaction of Leptons and Quarks', E. D. Commins and P.H. Bucksbaum, Cambridge University Press (1983).
- M.C. Gonzalex-Garcia and M. Maltoni, Phys. Rep. **460**(2008)1.
- S.F. Novaes, arXiv:hep-ph/0001283 (2000).
- R. Barbieri, arXiv:0706.0684 [hep-ph] (2007).

Beta Decay

Prior to 1930, beta decay was assumed to be a 2-body decay:

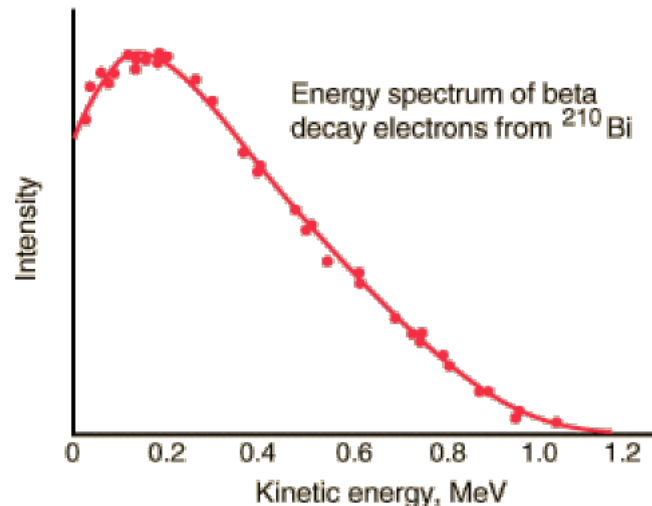


For two-body decay:



$$E_2 = \sqrt{m_2^2 + p^2} = \frac{M^2 + m_2^2 - m_1^2}{2M}$$

But observed continuous energy spectra for β particles :

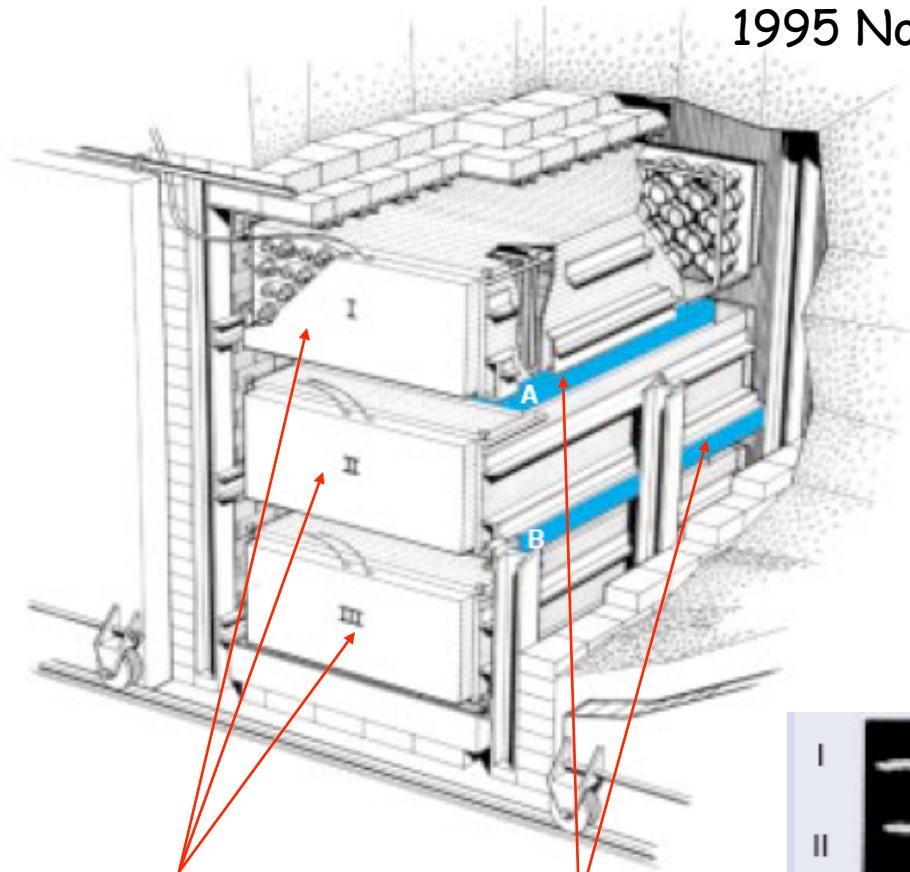


Pauli's solution : there exists a neutral particle - neutrino.

Discovery of Antineutrino

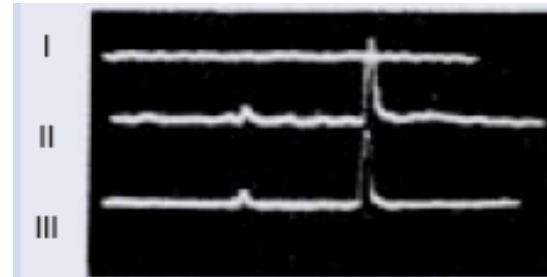
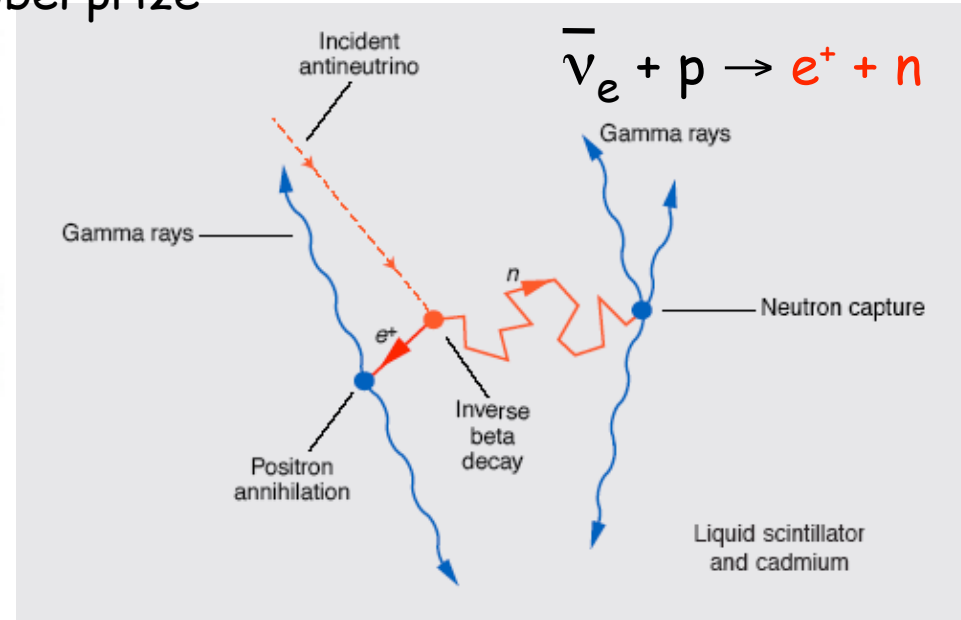
Reines-Cowan Experiment with reactor $\bar{\nu}_e$ at Savannah River (1953-1956)

1995 Nobel prize

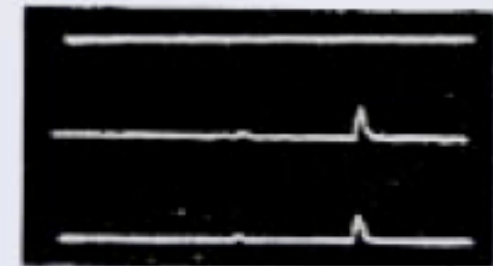


Each with 1000 l of liquid scintillator viewed with PMTs

Each with 200 l of water with CdCl_2



(b) Positron scope

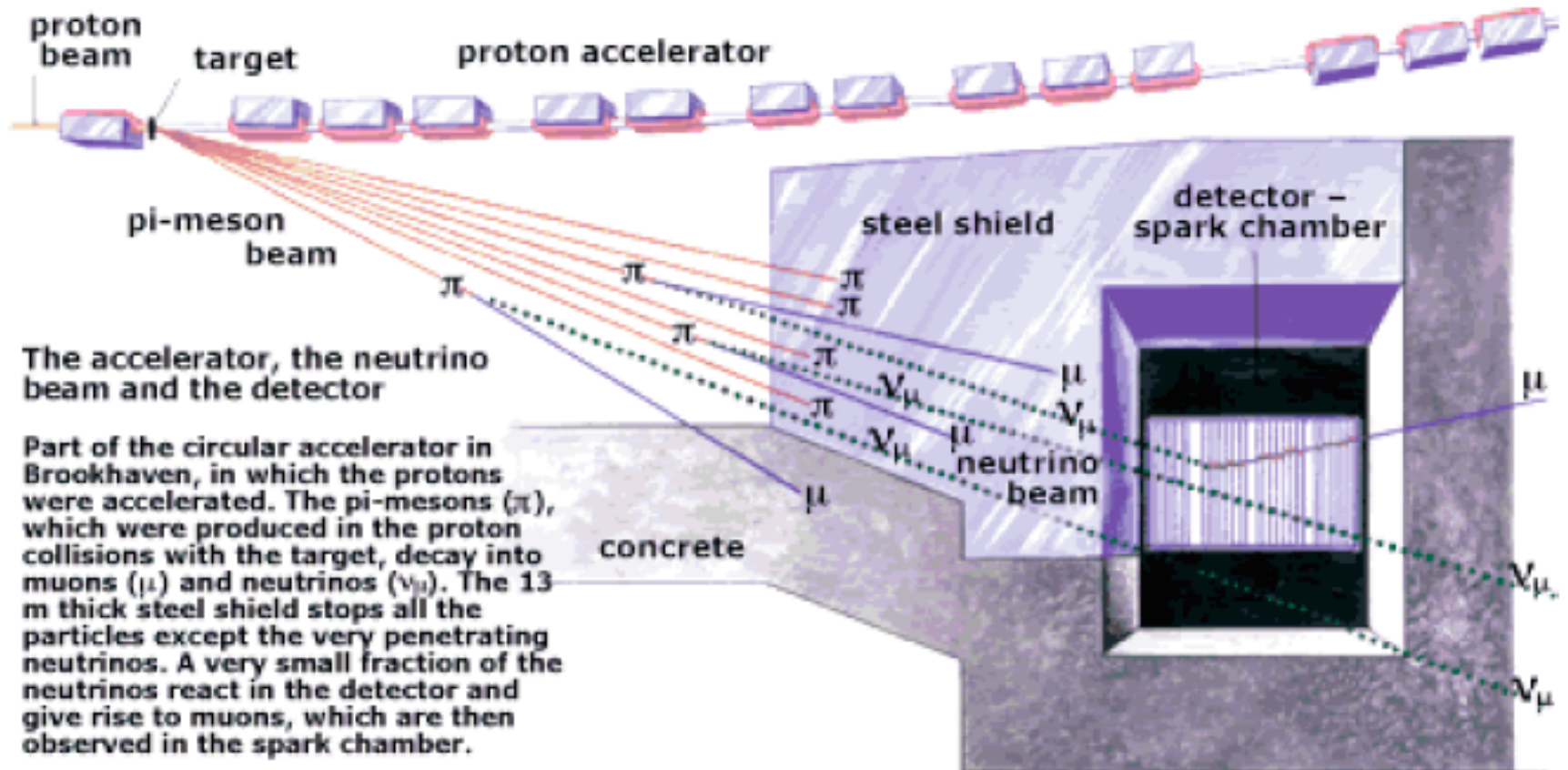


Neutron scope

$\Delta t = 13.5 \mu\text{s}$

$\bar{\nu}_e$ is seen !

Not Just One, But Two types !

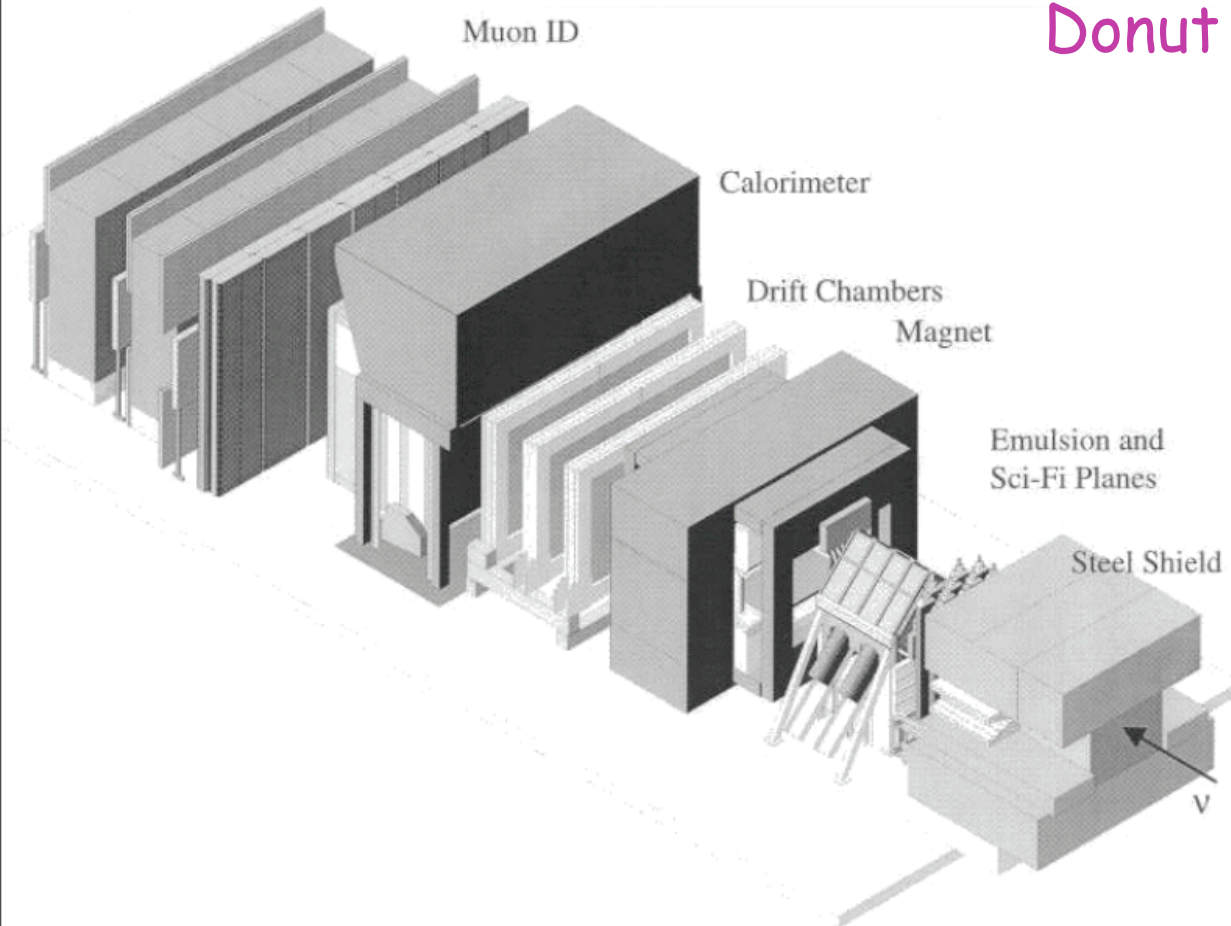


Based on a drawing in Scientific American, March 1963.

1988 Nobel prize

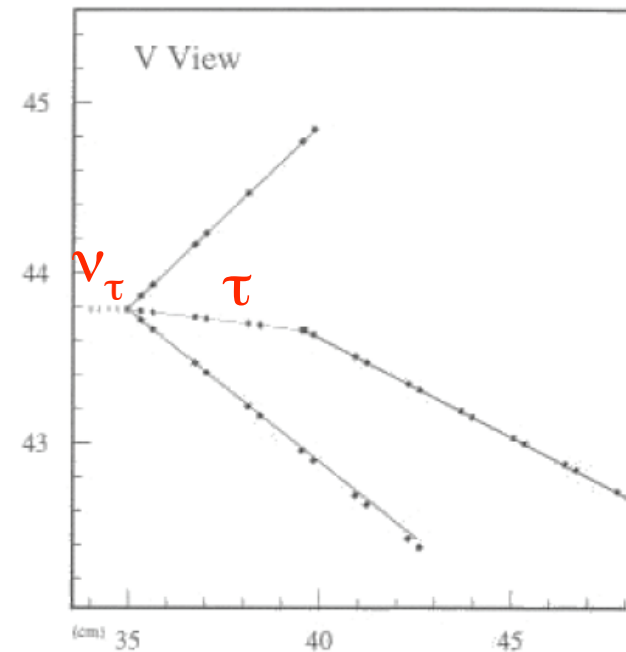
Wait, History Repeats Itself !

Donut at Fermilab (2000):

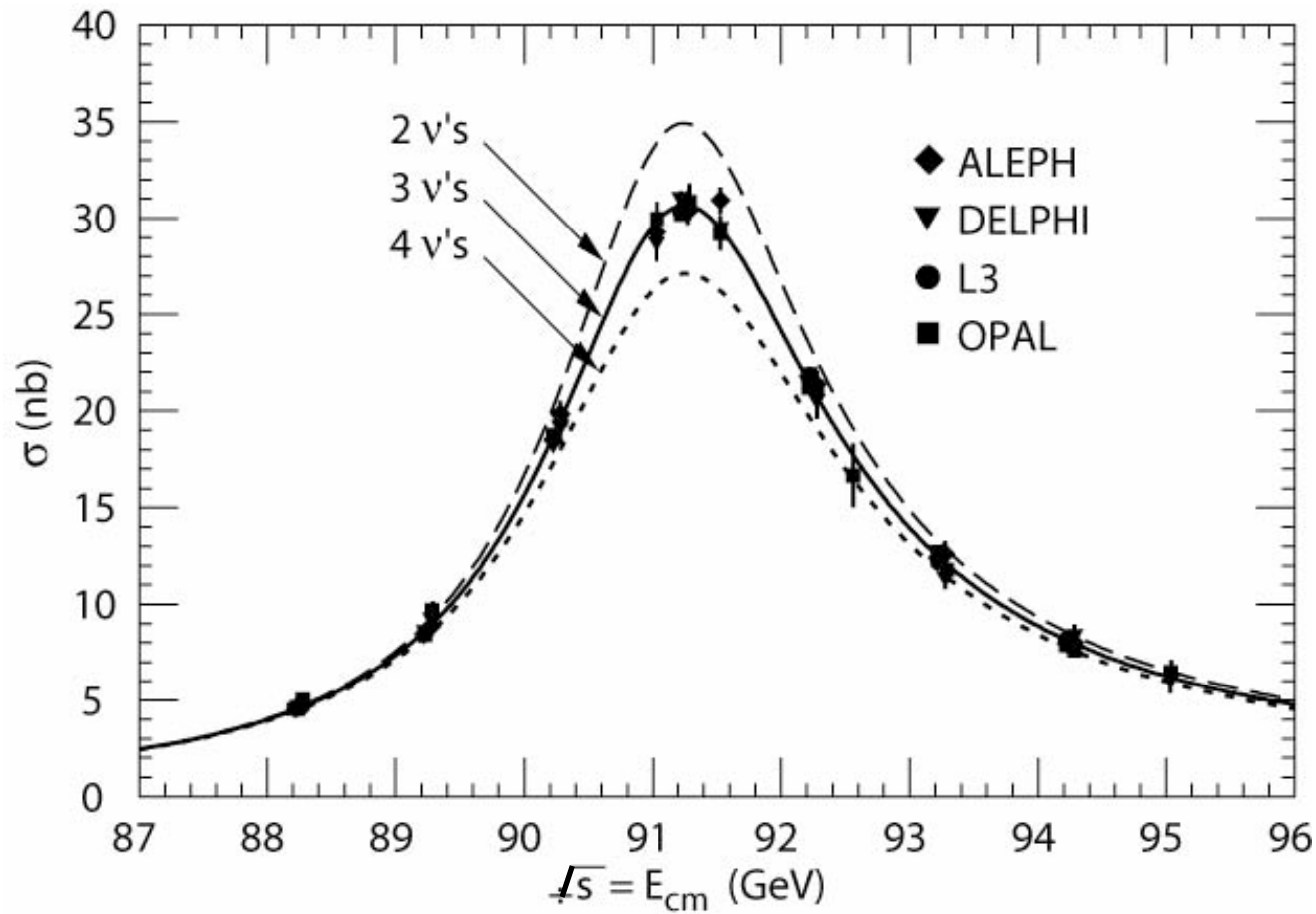


$$\nu_S \rightarrow \ell + \nu_\tau$$

$$\nu_\tau + N \rightarrow \tau + \text{hadrons}$$



Three and No More



Lepton Numbers

| | e^- | ν_e | μ^- | ν_μ | τ^- | ν_τ | e^+ | $\bar{\nu}_e$ | μ^+ | $\bar{\nu}_\mu$ | τ^+ | $\bar{\nu}_\tau$ |
|----------|-------|---------|---------|-----------|----------|------------|-------|---------------|---------|-----------------|----------|------------------|
| L_e | 1 | 1 | 0 | 0 | 0 | 0 | -1 | -1 | 0 | 0 | 0 | 0 |
| L_μ | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | -1 | -1 | 0 | 0 |
| L_τ | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | -1 | -1 |

Total lepton number: $L = L_e + L_\mu + L_\tau$

Conservation of lepton numbers can explain why

- (a) $n \rightarrow p + e^- + \bar{\nu}_e$
- (b) $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$
- (c) $\tau^- \not\rightarrow e^- + \gamma$ ($< 2.6 \times 10^{-6}$, 90% CL)
- (d) $\mu^- \rightarrow e^- + e^- + e^+$ ($< 1 \times 10^{-12}$, 90% CL)

Neutrinos Are Left-handed

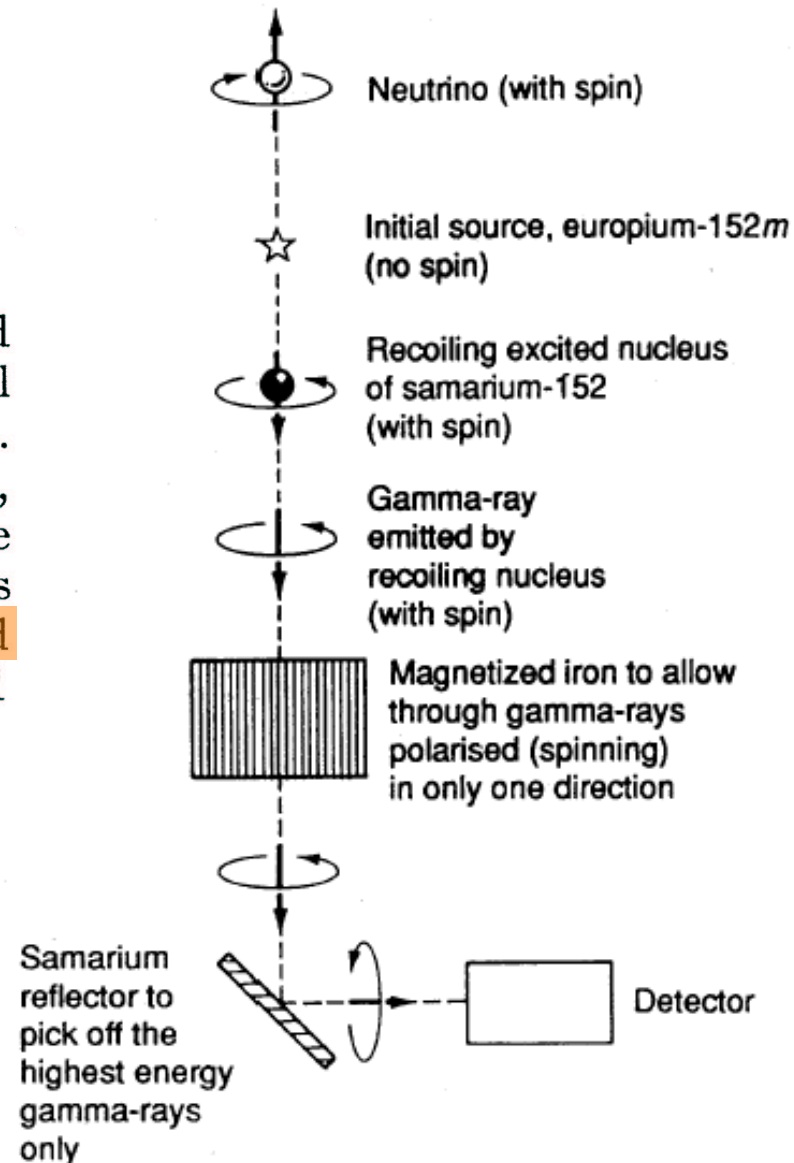
Helicity of Neutrinos*

M. GOLDBABER, L. GRODZINS, AND A. W. SUNYAR

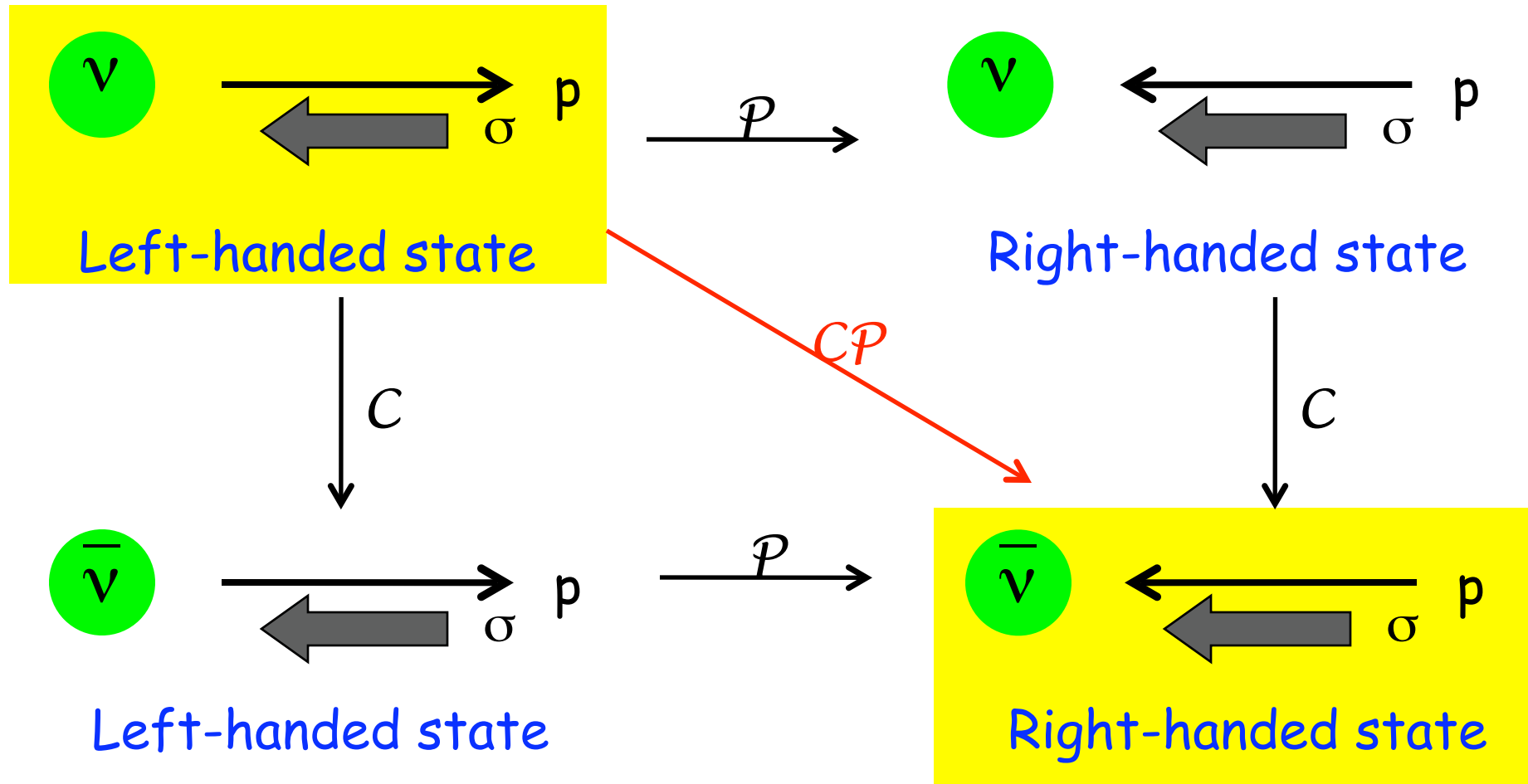
Brookhaven National Laboratory, Upton, New York

(Received December 11, 1957)

A COMBINED analysis of circular polarization and resonant scattering of γ rays following orbital electron capture measures the helicity of the neutrino. We have carried out such a measurement with Eu^{152m} , which decays by orbital electron capture. If we assume the most plausible spin-parity assignment for this isomer compatible with its decay scheme,¹ $0-$, we find that the neutrino is "left-handed," i.e., $\sigma_\nu \cdot \hat{p}_\nu = -1$ (negative helicity).

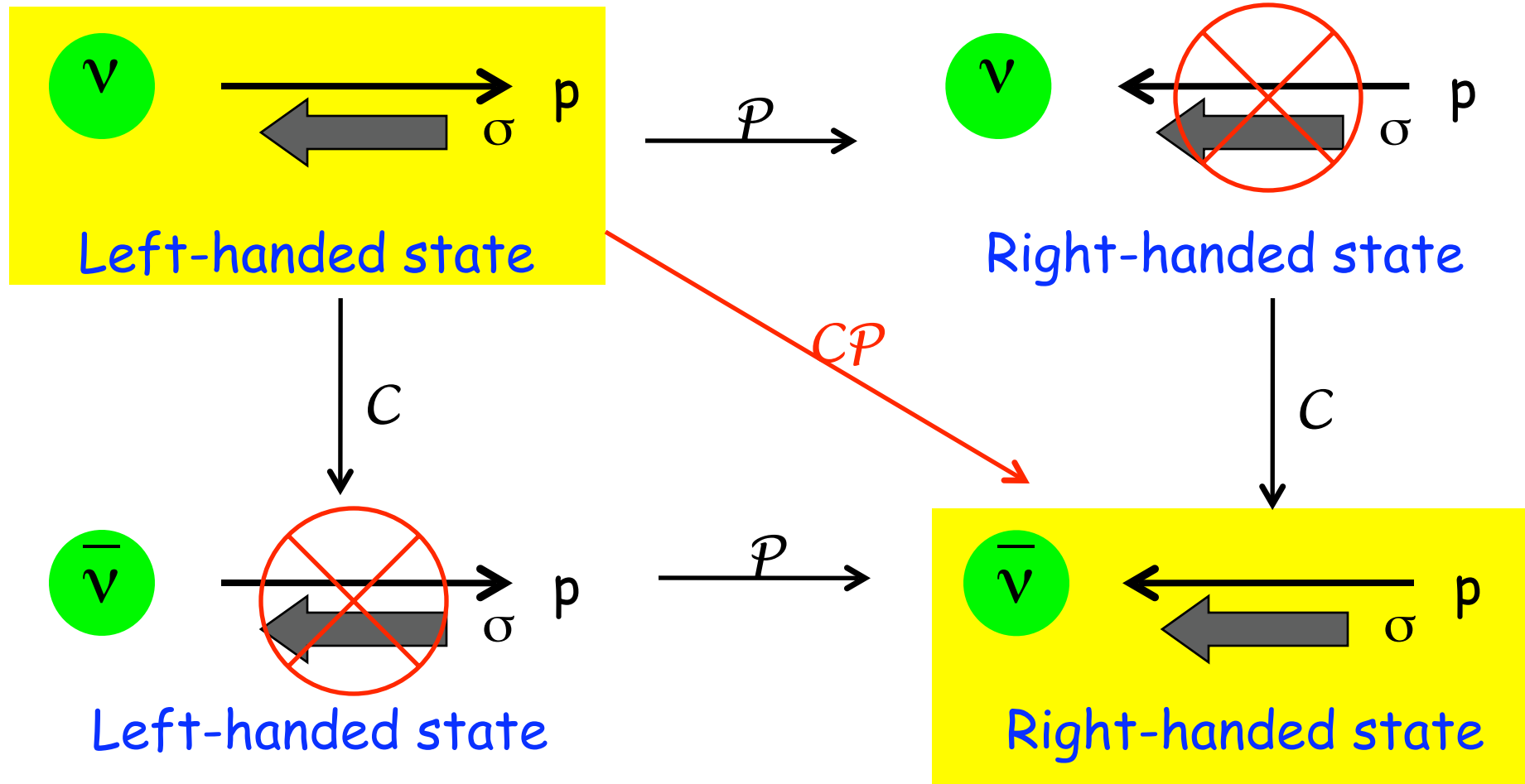


CP Transformation



Left-handed neutrino implies right-handed antineutrino

CP Transformation



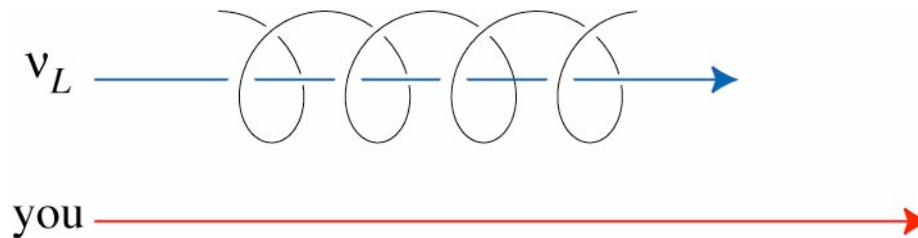
Right-handed neutrino and left-handed antineutrino do not exist.

Neutrinos Believed To Be Massless

- All neutrinos are left-handed \Rightarrow massless
- If they have mass, can't go at speed of light:

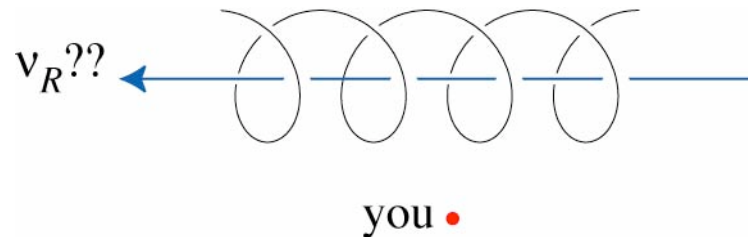
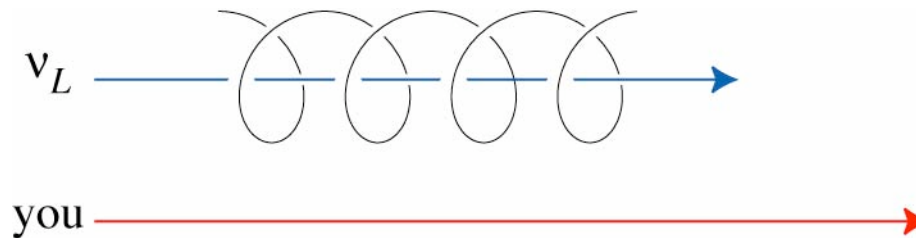
Neutrinos Believed To Be Massless

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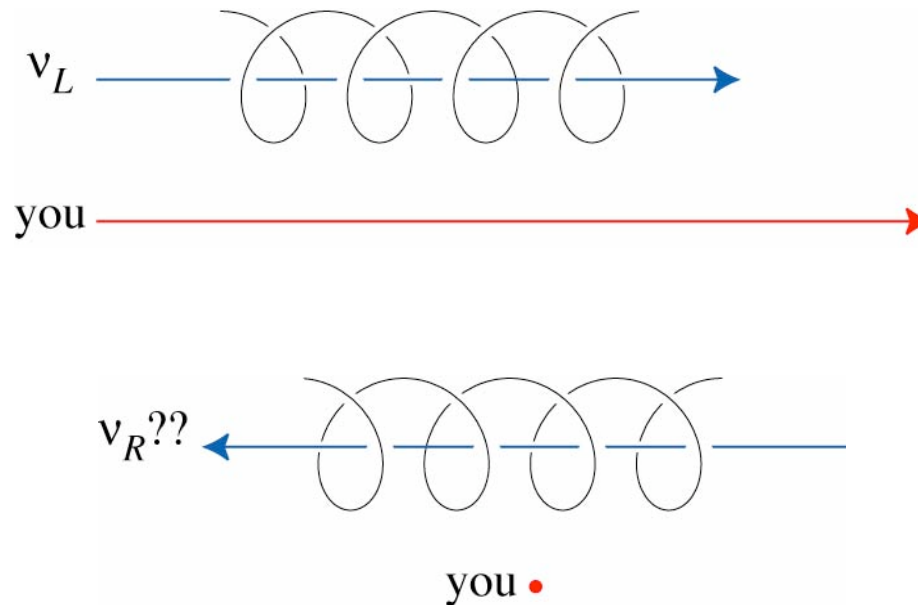
Neutrinos Believed To Be Massless

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Neutrinos Believed To Be Massless

- All neutrinos are left-handed \Rightarrow massless
- If they have mass, can't go at speed of light:



- Now the neutrino becomes right-handed??
 \Rightarrow contradicts experimental findings
 \Rightarrow neutrino can't be massive

Standard Model of Particle Physics

Fermions are grouped into weak-isospin doublets or singlets:

$$\text{Quarks : } \begin{pmatrix} u \\ d' \end{pmatrix}_L \quad u_R \quad d_R \quad \begin{pmatrix} c \\ s' \end{pmatrix}_L \quad c_R \quad s_R \quad \begin{pmatrix} t \\ b' \end{pmatrix}_L \quad t_R \quad b_R$$

$$\text{Leptons : } \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad e_R^- \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \quad \mu_R^- \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L \quad \tau_R^-$$

$$\text{Gauge bosons : } W^\pm, Z, \gamma, g_i$$

$$\text{Coupling constants : } \alpha_s, g, g' \text{ such that } g = |q_e|/\sin\theta_W, \quad g' = g \tan\theta_W$$

$$\text{where } \sin^2\theta_W \approx 0.231$$

$$\text{Higgs : } H = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix} \text{ with vacuum expectation value } \langle H \rangle_0 = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \quad v \approx 246 \text{ GeV}$$

Some Remarks About Standard Model

- All fermions are Dirac particles
 - ν and $\bar{\nu}$ are not identical
- Mass of a Dirac particle arises from its interaction with the Higgs field in the vacuum:
 - mass operator, $m = \lambda_f \langle H \rangle_0 \overset{\text{Yukawa coupling}}{\overbrace{f^c f}^{\substack{\text{annihilates } f_L / \\ \text{creates } f_R}}} \Rightarrow m_e = \lambda_e \langle H \rangle_0$
 - annihilates $\bar{f}_L /$
creates \bar{f}_R
- No ν_R (no ν^c).
 - neutrinos are massless
- W and Z only couple to the left-handed states.

Quark Mixing

- The weak eigenstates of quarks are related to the mass eigenstates via the Cabibbo-Kobayashi-Maskawa (CKM) matrix:

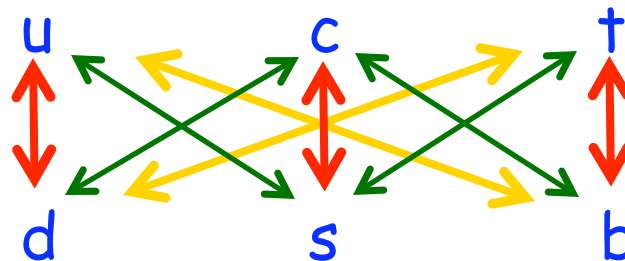
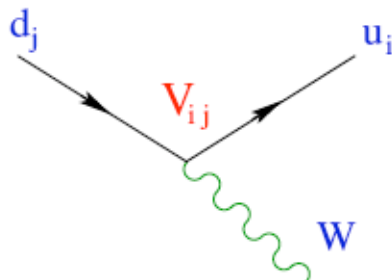
$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

where $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$ with $i, j = 1, 2, 3$

- Measured: $\theta_{12} = 13^\circ$, $\theta_{23} = 2.4^\circ$, $\theta_{13} = 0.21^\circ$, $\delta = 60^\circ \pm 14^\circ$. This yields

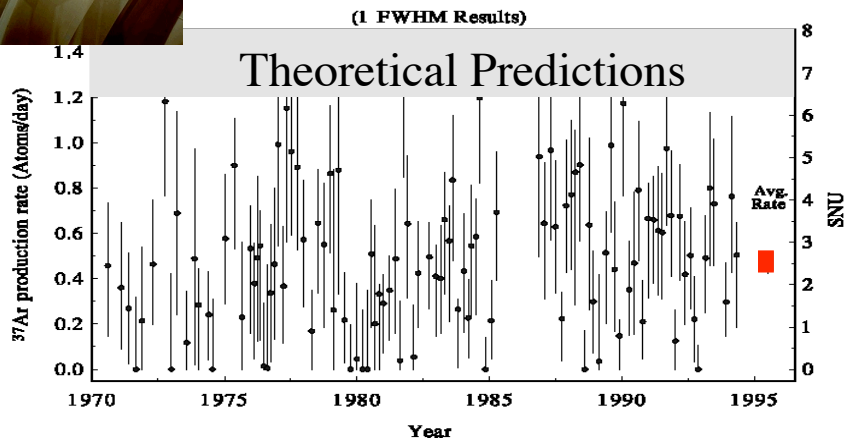
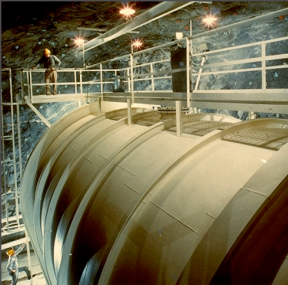
$$V = \begin{pmatrix} 0.9742 & 0.2257 & 0.00359 \\ 0.2256 & 0.9733 & 0.0415 \\ 0.00874 & 0.0407 & 0.9991 \end{pmatrix}$$

- Transition between quarks:

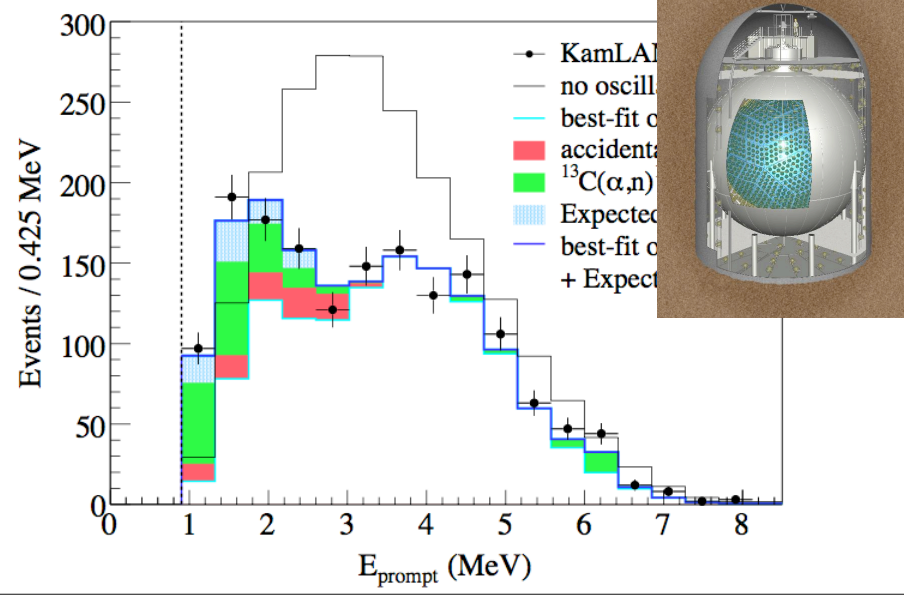
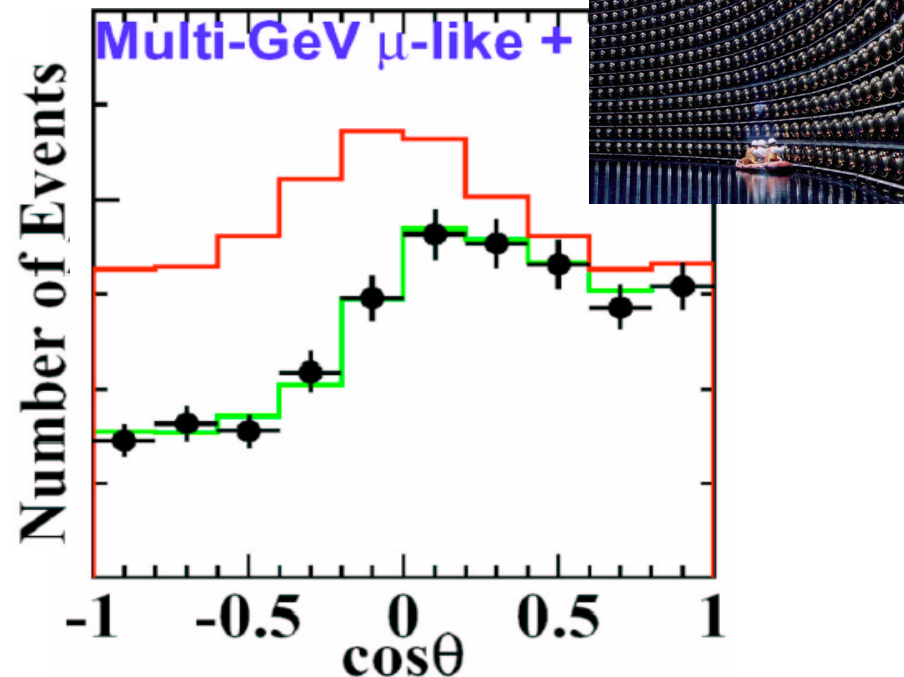
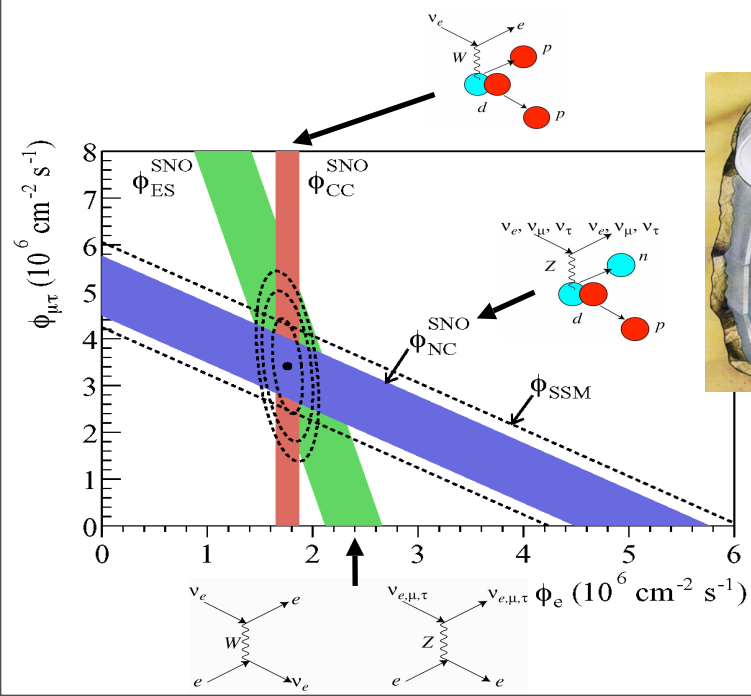


— Most likely
— Likely
— Least likely

Discoveries of Neutrino Oscillation



1 SNU = 10^{-36} interaction/atom/s



Neutrino Mixing

- If neutrinos are massive, it is possible that the **weak eigenstates** are not the same as the **mass eigenstates**:

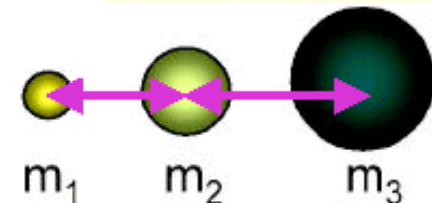
PMNS (Pontecorvo-Maki-Nakagawa-Sakata) matrix

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Weak eigenstates
„flavor eigenstates“



Mass eigenstates



$$\Delta m_{ij}^2 = |m_i^2 - m_j^2| \rightarrow \Delta m_{12}^2, \Delta m_{23}^2$$

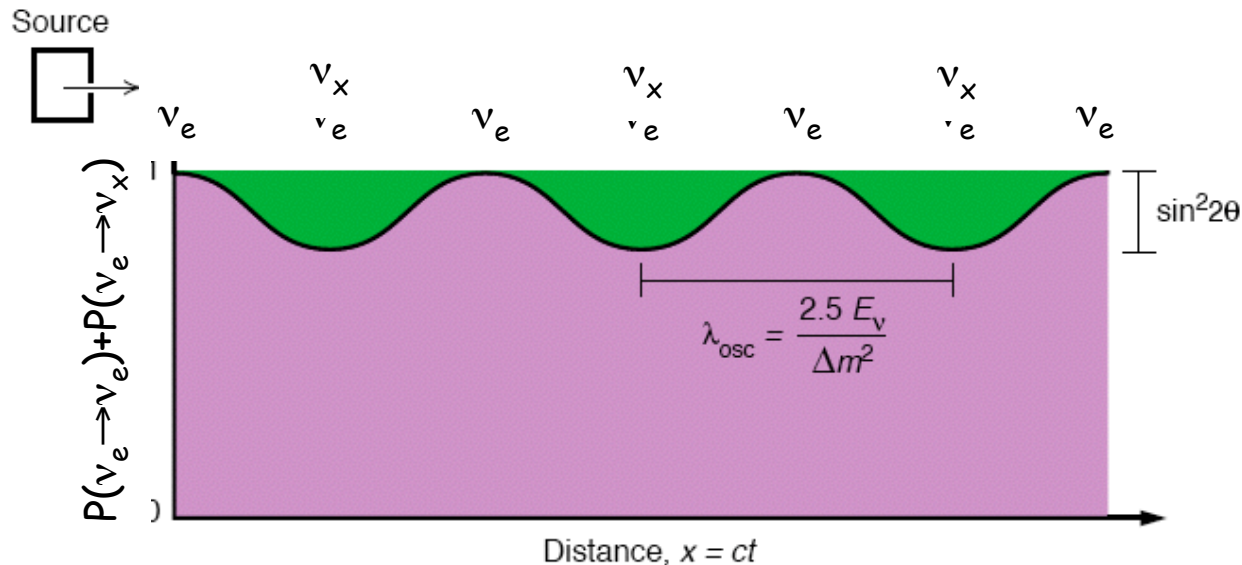
More On Neutrino Mixing

- Parametrize the mixing matrix as:

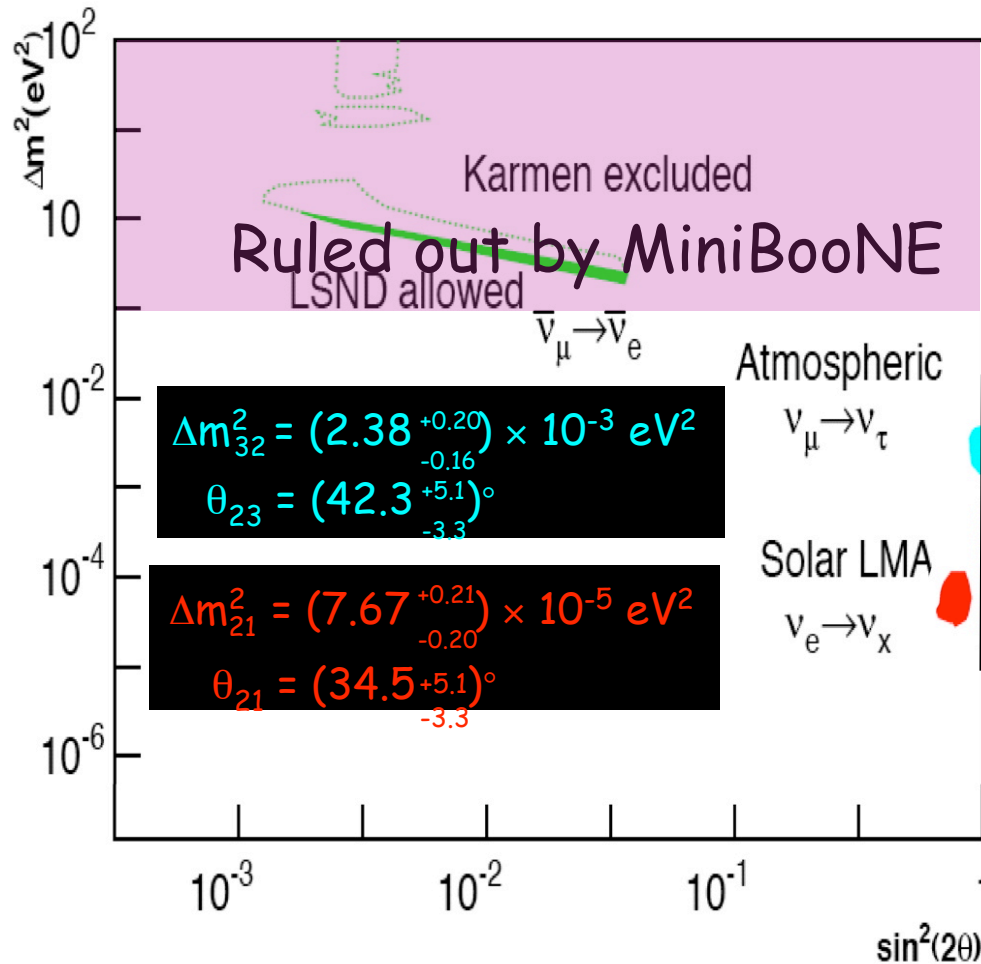
$$\begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta_{13} & 0 & \sin\theta_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -\sin\theta_{13}e^{i\delta} & 0 & \cos\theta_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix}$$

- The survival probability of $\nu_e \rightarrow \nu_e$ is:

$$P(\nu_e \rightarrow \nu_e) = \langle \nu_e | \nu_e(t) \rangle^2 \approx 1 - \sin^2 2\theta_{13} \sin^2\left(\frac{\Delta m_{31}^2 L}{4E}\right) - \cos^4 \theta_{13} \sin^2 2\theta_{12} \sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right)$$

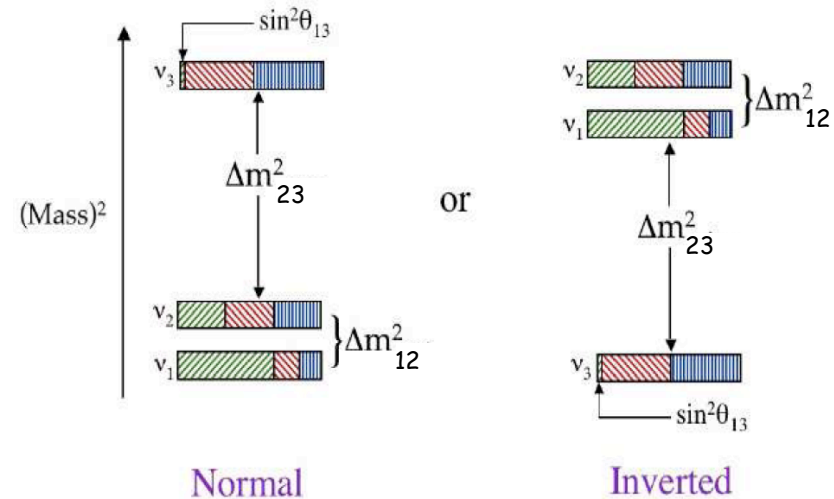


Current Status of Mixing Parameters



Unknowns:

- θ_{13} , δ ,
- sign of Δm_{32}^2 (mass hierarchy)



$$U_{bf} \approx \begin{pmatrix} 0.82 & 0.56 & 0.09 \\ 0.37 - 0.47 & 0.58 - 0.65 & 0.67 \\ 0.32 - 0.43 & 0.52 - 0.59 & 0.74 \end{pmatrix}$$

Implications of Neutrino Oscillation

- Lepton number is not conserved:

$$\Rightarrow \Delta L_i \neq 0, \quad i = e, \mu, \tau$$

$$\Rightarrow \Delta L = 0$$

$$\Rightarrow \mu \rightarrow e + \gamma \text{ could exist}$$

- Neutrinos possess small but finite mass:

\Rightarrow From the measured Δm^2 values and assume normal mass hierarchy:

taking $m_1 \approx 0$, then

$$m_2 \approx 9 \text{ meV}$$

$$m_3 \approx 50 \text{ meV}$$

Fixing Standard Model: Majorana Mass

- Introduce a new effective interaction:

$$\frac{1}{M_{eff}} (h^0 \nu_e - h^+ e)^2$$

yields a **Majorana mass operator**

$$\frac{1}{M_{eff}} \langle H \rangle_0^2 \nu_e \nu_e \quad \text{with } \bar{\nu}_e = \nu_e$$

that, in general, generates **a neutrino mass**

$$m_\nu = \frac{2 \langle H \rangle_0^2}{M_{eff}} = \frac{v^2}{M_{eff}}$$

Remarks:

- (1) the Majorana operator violates lepton-number conservation.
- (2) M_{eff} must be large to make m_ν small
 - Seesaw mechanism

Fixing Standard Model: Sterile Neutrinos

- Introduce right-handed neutrino fields ν^c such that

$$\lambda_\nu \nu_e^c (\nu_e h^0 - e h^+)$$

which yields, in general, a **Dirac mass** for each neutrino:

$$m_\nu = \frac{\lambda_\nu v}{\sqrt{2}}$$

Remarks:

(1) why $m_\nu \ll m_e$?

(2) the field ν^c and its states ν_R and $\bar{\nu}_L$ are weak isospin singlets with

- zero weak hypercharge
- no coupling to W , Z , and γ .

They are called **sterile neutrinos**.

(3) If the ν_L has mass, it may oscillate into a

What Lie Ahead in Neutrino Physics?

- What is the nature of a neutrino?

- a Dirac particle?

$$\nu \neq \bar{\nu}$$

- a Majorana particle?

$$\nu = \bar{\nu}$$

- What is the absolute mass of the neutrinos?
- How do the neutrinos get their mass?
- What is the magnetic moment a neutrino?
- What is the size of the neutrino mixing angle θ_{13} ?
- What is the order of the neutrino masses?
- Is there CP violation in the neutrino sector?
- Are there heavy active neutrinos?

Charge Conjugation and Bar

Bar operator :

$$\bar{\psi} = (\psi^{*\text{T}})\gamma_0$$

Charge conjugation :

$$\psi^C = C \bar{\psi}^{\text{T}} = i\gamma_0\gamma_2 \bar{\psi}^{\text{T}}$$

$$(\psi_R)^C = (\psi^C)_L;$$

Dirac Neutrino



Dirac equation for electrons :

$$(i\gamma^\mu \frac{\partial}{\partial x_\mu} - m)\psi = 0;$$

$$L = \frac{1}{2}(\bar{\psi}(i\gamma^\mu \partial_\mu)\psi - m_D^2 \bar{\psi}\psi) + \hbar c.$$

it is convenient to work with R and L projections :

$$\psi_{R,L} = \frac{1}{2}(1 \pm \gamma_5)\psi = P_{R,L}\psi; P_{R,L} = \frac{1}{2}(1 \pm \gamma_5)$$

$$\psi = \psi_R + \psi_L;$$

$$\bar{\psi}_R \psi_R = \bar{\psi}_L \psi_L = 0$$

$$L_D^M = -\frac{1}{2}m_D^2(\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L)$$

electron is charged, thus no other mass terms allowed.

Applying the same for neutrino, one comes to Dirac neutrino description. In SM ν_R doesn't exist, hence

$$m_\nu = 0!$$

Majorana neutrino



ν is neutral $\Rightarrow \bar{\psi}\psi^c$ and $\bar{\psi}^c\psi$ terms are allowed ($\psi^c = C\psi C^{-1}$)

$$L_M^M = -\frac{1}{2} \left[m_{ML}^2 (\bar{\psi}_L (\psi^c)_R + \overline{(\psi^c)_R} \psi_L) + m_{MR}^2 (\bar{\psi}_R (\psi^c)_L + \overline{(\psi^c)_L} \psi_R) \right]$$

introduce : $\nu_1 = \nu_L + (\nu^c)_R$; $\nu_2 = \nu_R + (\nu^c)_L$

$$L_M^M = -\frac{1}{2} \left[m_{ML}^2 \bar{\nu}_1 \nu_1 + m_{MR}^2 \bar{\nu}_2 \nu_2 \right]$$

$\nu_{1,2}^c = \nu_{1,2} \Rightarrow \text{particle} \equiv \text{anti-particle}$

Majorana description assumes 2 - component neutrino.

In case of $m_\nu = 0$, second one is sterile in SM, thus

Dirac and Majorana descriptions are identical.

See-Saw Mechanism

In the general case :

$$L_M = L_M^D + L_M^M$$

$$L_M = -\frac{1}{2} \begin{pmatrix} \overline{v}_L & \overline{v}_L^C \end{pmatrix} \begin{pmatrix} m_{ML} & m_D \\ m_D & m_{MR} \end{pmatrix} \begin{pmatrix} v_R^C \\ v_R \end{pmatrix}$$

$$L_M = -\frac{1}{2} (m_1 \overline{v}_1 v_1 + m_2 \overline{v}_2 v_2)$$

$$m_{1,2} = \frac{m_{MR} + m_{ML}}{2} \pm \sqrt{\frac{(m_{MR} - m_{ML})^2}{4} + m_D^2}$$

if $m_{MR} \gg m_D, m_{ML}$:

$$m_1 \cong m_{MR}; m_2 \cong \frac{m_D^2}{m_{MR}}$$

This can explain smallness of m_ν .

